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OPTIMAL GUARANTEED COST CONTROL OF AIRCRAFT MOTION

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Abstract**—***The problem of optimal state feedback controller design in terms of quadratic performance index is considered. The problem is formulated in the form of linear matrix inequalities (LMIs). The obtained solution guarantees stabilization of the aircraft during flight mission. During flight envelope the aircraft is subjected to the external stochastic disturbances. The efficiency of the proposed approach is illustrated by a case study of airplane longitudinal motion.*

Index Terms—Aircraft motion control, external disturbances, linear matrix inequality, performance index, robustness, state feedback.

I. INTRODUCTION

The problem of optimal control design has been considered in a number of publications [1]–[3]. Especially, it is very crucial question in the area of aircraft control, where it is necessary to satisfy the manifold requirements imposed on the aircraft during flight envelope. A great number of control approaches have been proposed to solve the problem autopilot design. Among them, it is possible to enumerate some works related to the combination of observer and linear quadratic regulator [3], [4]. Furthermore, to preserve the required level of performance without losing the robustness of the flight control system, the mixed H_2/H_∞ – robust optimization procedure is used. The main idea behind this technique is to seek a trade-off between the performance and the robustness of the overall closed loop system [3], [4]. The autopilot design is also may be performed basing on the available information about the output variables. This circumstance leads to the problem of static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. The SOF problem concerns finding a static or feedback gain to achieve certain desired closed-loop characteristics. It is necessary to admit that the output feedback problem is much more difficult to solve in comparison to state feedback control problem. A survey devoted to this problem is presented in [5].

This paper deals with static state feedback controller design in terms of LMIs [6], [7] for aircraft control during flight envelope. The main feature of this paper is that the obtained state feedback controller stabilizes the set of autonomous systems, simultaneously. Moreover, the designed controller possesses with robustness properties. To prove the efficiency of the proposed technique, the longitudinal motion of the aircraft is considered as a case study.

II. PROBLEM STATEMENT

Let us consider procedure of state feedback design for an aircraft control whose dynamics is described by the following differential equation

$$\dot{\mathbf{x}}(t)=\mathbf{A}_\mathrm{i}\mathbf{x}(t)+\mathbf{B}_\mathrm{i}\mathbf{u}(t),\ \mathbf{x}(0)=\mathbf{x}_0, \qquad (1)$$

where $\mathbf{x}\in\mathbf{R}^\mathbf{n}$ is the state space vector, $\mathbf{u}\in\mathbf{R}^\mathbf{m}$ is the control vector. The uncertainties of the model are represented by the set of matrices $\langle\mathbf{A}_\mathrm{i},\mathbf{B}_\mathrm{i}\rangle$ that satisfy the following requirement:

$$[\mathbf{A}\quad \mathbf{B}]\in Co\{[\mathbf{A}_1\quad \mathbf{B}_1],...,[\mathbf{A}_\mathrm{N}\quad \mathbf{B}_\mathrm{N}]\},\ \mathrm{i}=1,...,\mathrm{N},$$

where Co is a convex set; N is the set of models associated with certain operating conditions within the flight envelope. The main problem is to find the state feedback of the following form

$$\mathbf{u}(t)=\mathbf{K}\,\mathbf{x}(t), \qquad (2)$$

where $\mathbf{K}$ is a constant state feedback gain matrix that assures that the system is asymptotically stable. Thus, the closed-loop system taking into account (1) and (2) takes the well known description in form of differential equation as:

$$\dot{\mathbf{x}}(t)=(\mathbf{A}_\mathrm{i}+\mathbf{B}_\mathrm{i}\mathbf{K})\mathbf{x}(t),\ \mathbf{x}(0)=\mathbf{x}_0.$$

The obtained solution minimizes performances index given by

$$J=\int_0^\infty\left(\mathbf{x}(t)^\mathrm{T}\mathbf{Q}\mathbf{x}(t)+\mathbf{u}(t)^\mathrm{T}\mathbf{R}\mathbf{u}(t)\right)dt =\int_0^\infty\mathbf{x}(t)^\mathrm{T}\left(\mathbf{Q}+\mathbf{K}^\mathrm{T}\mathbf{R}\mathbf{K}\right)\mathbf{x}(t)\,dt, \qquad (3)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are diagonal matrices, weighting each state and control variables, respectively.

This cost depends on the trajectory, of $\mathbf{x}(t)$, taken, such that the worst trajectory will correspond to the worst cost. The control problem is to find the state feedback gain $\mathbf{K}$ and a quadratic Lyapunov function $\mathbf{P}$ that minimizes the bound $\mathbf{x}_0^{\mathrm{T}}\mathbf{P}\mathbf{x}_0$ on the worst cost of J. The problem can be translated into an optimization problem as follows:

$$\text{minimize } \mathbf{x}_0^{\mathrm{T}}\mathbf{P}\mathbf{x}_0;$$

$$\text{subject to } \mathbf{P} > 0,$$

$$\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}\right)^{\mathrm{T}}\mathbf{P} + \mathbf{P}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{K}\right) + \mathbf{Q} + \mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K} \le 0. \quad (4)$$

The goal is to find state feedback controller (2) that simultaneously stabilizes the set of autonomous systems. The obtained solution is optimal and brings to the minimum performance index given by (3). The linear matrix inequality (LMI) technique permits to solve this problem [6], [7]. Thus, it is possible to transform the non-linear inequality (4) into the LMI form. This procedure reduces to the defining new matrices $\mathbf{X}$ and $\mathbf{M}$ such that

$$\mathbf{X} = \mathbf{P}^{-1},\ \mathbf{M} = \mathbf{K}\mathbf{P}^{-1} \text{ and } \mathbf{P} = \mathbf{X}^{-1},$$

$$\mathbf{K} = \mathbf{M}\mathbf{P} = \mathbf{M}\mathbf{X}^{-1}.$$

By substituting new variables $\mathbf{X}$ and $\mathbf{M}$ instead of $\mathbf{P}$ and $\mathbf{K}$ into Lyapunov inequality (4), and then pre-multiplying and post-multiplying the left and the right hand sides by $\mathbf{X}$, the inequality (4) becomes

$$\mathbf{X}\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}_i\mathbf{X} + \mathbf{M}_i^{\mathrm{T}}\mathrm{B}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{M}_i + \mathbf{X}\mathbf{Q}\mathbf{X} + \mathbf{M}_i^{\mathrm{T}}\mathbf{R}\mathbf{M}_i \le 0. \quad (5)$$

Basing on Schur's complement the matrix inequality (5) can be expressed as the LMI

$$\begin{bmatrix} \mathbf{X}\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}\mathbf{X} + \mathbf{B}_i\mathbf{M}_i + \mathbf{M}_i^{\mathrm{T}}\mathrm{B}_i^{\mathrm{T}} & \mathbf{X}\mathbf{Q}^{1/2} & \mathbf{M}_i^{\mathrm{T}}\mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2}\mathbf{X} & -\mathbf{I} & 0 \\ \mathbf{R}^{1/2}\mathbf{M}_i & 0 & -\mathbf{I} \end{bmatrix} \le 0. \quad (6)$$

By applying Schur's complement, the cost

$$\mathbf{x}_0^{\mathrm{T}}\mathbf{P}\mathbf{x}_0 = \mathbf{x}_0^{\mathrm{T}}\mathbf{X}^{-1}\mathbf{x}_0 \le \gamma,$$

is expressed as the LMI

$$\begin{bmatrix} \gamma & \mathbf{x}_0^{\mathrm{T}} \\ \mathbf{x}_0 & \mathbf{X} \end{bmatrix} \ge 0. \quad (7)$$

The following optimization problem can be represented in terms of LMI (6) and (7) minimizes γ subject to

$$\begin{bmatrix} \mathbf{X}\mathbf{A}_{\mathrm{i}}^{\mathrm{T}} + \mathbf{A}\mathbf{X} + \mathbf{B}_{\mathrm{i}}\mathbf{M}_{\mathrm{i}} + \mathbf{M}_{\mathrm{i}}^{\mathrm{T}}\mathrm{B}_{\mathrm{i}}^{\mathrm{T}} & \mathbf{X}\mathbf{Q}^{1/2} & \mathbf{M}_{\mathrm{i}}^{\mathrm{T}}\mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2}\mathbf{X} & -\mathbf{I} & 0 \\ \mathbf{R}^{1/2}\mathbf{M}_{\mathrm{i}} & 0 & -\mathbf{I} \end{bmatrix} \le 0,$$

$$\begin{bmatrix} \gamma & \mathbf{x}_0^{\mathrm{T}} \\ \mathbf{x}_0 & \mathbf{X} \end{bmatrix} \ge 0.$$

III. CASE STUDY

To demonstrate the efficiency of the proposed approach the longitudinal channel of regional jet is used as a case study. The longitudinal dynamics of regional jet in the state space is represented by the phase and control vectors, respectively. The phase and control vector have the following form: $\mathbf{x} = \left[u,\ w,\ q,\ \vartheta,\ h\right]^{\mathrm{T}}$ and $\mathbf{u} = \left[\delta_e\right]^{\mathrm{T}}$, where u is a longitudinal component of true airspeed; w is a vertical component of true airspeed; q is the aircraft pitch rate; ϑ is a pitch angle; h is the aircraft altitude. The control vector $\mathbf{u} = \left[\delta_e\right]^{\mathrm{T}}$ is represented by the elevator defection [8]. It is considered two operating modes of the Boeing 737-100 aircraft with true airspeed at V_1 = 240.88 m/sec and V_2 = 237.26 m/sec. Thus, we have two mathematical models that correspond to these airspeeds. These linear models in the state space are represented by the matrices $\left[\mathbf{A},\ \mathbf{B}\right]$. The set of matrices that correspond to nominal and parametrically perturbed models are given below:

– nominal model

$$\mathbf{A}_n = \begin{bmatrix} -0.674e\text{-}2 & 0.498e\text{-}1 & -0.250e+2 & -0.322e+2 & 0 \\ -0.122e\text{-}1 & -0.738 & 0.788e+3 & -0.102e+1 & 0 \\ 0.245e\text{-}2 & -0.625e\text{-}2 & -0.846 & 0.104e\text{-}13 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.317e\text{-}1 & -0.1e+1 & 0 & 0.790e+3 & 0 \end{bmatrix}; \quad \mathbf{B}_n = \begin{bmatrix} 0.191e-1 \\ -0.601 \\ -0.802e-1 \\ 0 \\ 0 \end{bmatrix};$$

– perturbed model

$$\mathbf{A}_p=\begin{bmatrix}-0.595e\text{-}2 & 0.525e\text{-}1 & -0.290e{+}2 & -0.321e{+}2 & 0\\ -0.156e\text{-}2 & -0.688 & 0.776e{+}3 & -0.120e{+}1 & 0\\ 0.231e\text{-}2 & -0.602e\text{-}2 & -0.771 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0.373e\text{-}1 & -0.999 & 0 & 0.799e{+}3 & 0\end{bmatrix};\quad \mathbf{B}_p=\begin{bmatrix}0.206e\text{-}1\\ -0.553\\ -0.736e\text{-}1\\ 0\\ 0\end{bmatrix}.$$

where the subscript “*n*” corresponds to the nominal model and perturbed model is designated by the subscript “*p*”.

Disturbance, υ affecting the longitudinal motion of the aircraft involves the following components: horizontal and vertical components of true airspeed, u_g and w_g, and pitch rate, q_g such that that $\upsilon=\left[u_g,\ \ w_g,\ \ q_g\right]^{\mathrm{T}}$.

In order to simulate the atmospheric turbulence a Dryden filter is used [8]. The aircraft is considered to fly in a moderate turbulence. The transfer functions of forming filter according to standard MIL–F–8785C [13], [14] used in simulation to account external disturbances have the following structure:

$$H_u(\mathrm{s})=\sigma_u\sqrt{\frac{2L_u}{\pi V}}\frac{1}{1+\frac{L_u}{V}s};\qquad H_w(\mathrm{s})=\sigma_w\sqrt{\frac{L_w}{\pi V}}\frac{1+\frac{\sqrt{3}L_w}{V}s}{(1+\frac{\mathrm{L}_w}{V}\mathrm{s})^2};\qquad H_q(\mathrm{s})=\frac{\pm\frac{\mathrm{s}}{V}}{(1+\left(\frac{4b}{\pi V}\right)\mathrm{s})}H_w(\mathrm{s})..$$

To solve the problem of state feedback design with the control law (2) via LMIs as given by (6) it is necessary to define weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. In our case these matrices are defined as follows

$$\mathbf{Q}=14\cdot10^{-8}\,\mathrm{diag}\left(\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}\right);\ \mathbf{R}=\left[0.011\right].$$

By solving the LMIs given by (6) and (7) the gain matrix $\mathbf{K}$ for the state feedback has the following structure

$$\mathbf{K}=\begin{bmatrix}-0.0090 & -0.1683 & 27.7311 & 195.6772 & 0.0374\end{bmatrix}.$$

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The static state feedback gain matrix obtained with the help of the proposed approach assures simultaneous stabilization the set of autonomous systems that testifies the robust properties of the controller.

Performance indices of closed loop nominal and parametrically perturbed systems with the state feedback are given in Table 1.

Standard deviations of the Boeing 737-100 regional jet with state feedback control for nominal and parametrically perturbed models are given in Table 2.

TABLE 1

STANDARD DEVIATIONS OF THE BOEING 737-100 OUTPUTS IN A STOCHASTIC CASE

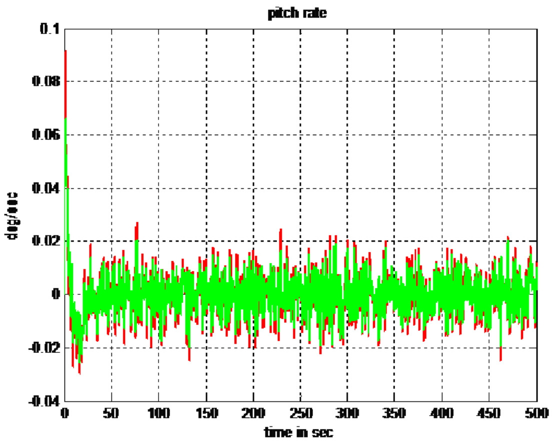
Plant	Standard deviation				
	σ_V, m/sec	σ_q, ^{0}/sec	σ_ϑ, ^{0}	σ_h, m	σ_{el}, ^{0}
V_1= 240.88 m/s	0.3439	0.0094	0.0341	4.5747	0.3439
V_2= 237.26 m/s	0.3087	0.0076	0.0315	4.3585	0.3087

TABLE2

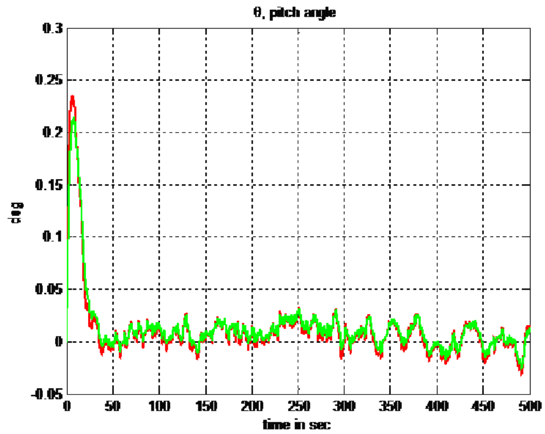
PERFORMANCE INDICES OF CLOSED-LOOP SYSTEMS

Plant	Performance Index	
	$\mathbf{H}_2$-norm	$\mathbf{H}_\infty$-norm
V_1= 240.88 m/s	0.1090	0.3037
V_2= 237.26 m/s	0.0739	0.2096

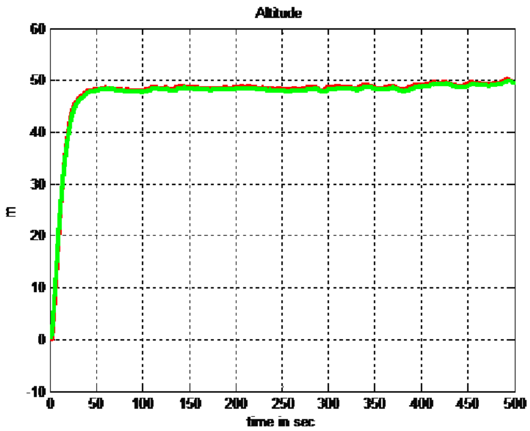
The simulation results of the closed loop systems operation taking into account the influence of the random wind, simulated according to the standard Dryden model of turbulence confirm the efficiency of proposed approach. Results of the simulation are shown in Figure.



a



b



c

Simulation results of Boeing 737-100 aircraft longitudinal motion in the presence of external disturbances: *a* is the pitch rate of nominal and perturbed models, deg/s; *b* is the pitch angle of nominal and perturbed models, deg; *c* is the altitude of nominal and perturbed models, m

CONCLUSIONS

Simulation results prove the efficiency of the proposed approach. It can be seen that the handling quality of the nominal and the perturbed models are satisfied.

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М. М. Комнацька. Оптимальне керування рухом літака з гарантованими мінімальними затратами

Представлено процедуру синтезу оптимального регулятора, що мінімізує обраний квадратичний критерій. Задача розв’язується в термінах лінійних матричних нерівностей. Ефективність запропонованого підходу ілюструється на прикладі керування повздовжнім рухом літака.

Ключові слова: керування рухом літака; зовнішні збурення; лінійна матрична нерівність; показник якості; робастність; зворотній зв’язок за станом.

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М. Н. Комнацкая. Оптимальное управление движением самолета с гарантированными минимальными затратами

Представлена процедура синтеза оптимального регулятора, который минимизирует выбранный показатель качества. Задача решается с использованием аппарата линейных матричных неравенств. Эффективность предложенного подхода иллюстрируется на примере управления продольным движением самолета.
Ключевые слова: управление движением самолета; внешние возмущения; линейное матричное неравенство; показатель качества; робастность; обратная связь по состоянию.

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