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Structural Identification Algorithm Based on Results of Multidimensional Nonlinear Stabilization Plant Test

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***Abstract*—the article proposes a structural identification algorithm based on test results of the multidimensional nonlinear object dynamics models and stochastic disturbance affecting it in service.**

Keywords—structural identification; nonlinear object; stochastic disturbance; stabilization object; transfer function; control system; error vector; frequency response

I. INTRODUCTION

Knowledge of real dynamic behavior of the control objects, their links, impacts and interference in real operating conditions is first of all necessary for successful dynamic design of the optimum control and stabilization systems of aircraft and spacecraft objects, control systems and their flight simulators and trainers. Lack of comprehensive dynamics models of the links, signals and interference that occur in the control loop of these objects during their real long-term flight operation results in shortage of a priori information when creating new and upgrading existing flight control systems, and this can consequently affect the efficiency of control in disturbed critical flight modes. Consideration of such models already at the stage of pilot project (technical proposal) will entail keen improvement or optimization of the disturbed flight control quality [1].

Let us examine a structural identification algorithm of the multidimensional nonlinear stabilization object, its dynamics and disturbance models. Flow chart of the stabilization object under testing is shown in Fig 1.

The following designations are introduced to the flow chart: P_i and M_i are the matrices with dimensions $n \times n$ and $n \times m$ respectively, x_i is frequency response of the output signal vectors in basic operating mode of the object (mode i); u_i is frequency response of the control signal vectors in basic operating mode of the object (mode i); y_i is frequency response of the observation signal vectors in basic operating mode; φ_i is frequency response of the interference signal vectors for the object output measurements in basic operating mode of the object; K is the matrix with dimension $n \times n$ for frequency response of the meters in the system.

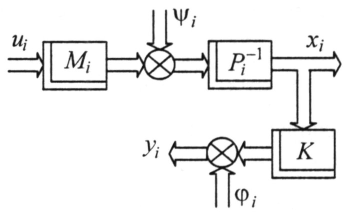

Fig. 1. Flow chart of the stabilization object under identification.

The system of equations for the object motion in the examined mode is as follows:

$$P_i x_i = M_i u_i + \psi_i. \quad (1)$$

It is expedient to present the disturbance behavior vector as $\psi_i = \Psi_0 g$, where Ψ_i is frequency response of the vector $\overline{\psi}_i$, and $g = 1$ for determinated disturbance and equals $g = \sigma_\Delta / \sqrt{\pi}$ for random disturbance.

If disturbance is of random nature, then the transposed matrix of its spectral densities $S'_{\mathring{\psi}\mathring{\psi}}$ can be presented as

$S'_{\overset{\circ}{\psi}\overset{\circ}{\psi}} = \Psi_i^0 S'_{\Delta\Delta}\Psi_{i*}^0$, where Ψ_i^0 is frequency response of the filter that formulates the matrix $S'_{\overset{\circ}{\psi}\overset{\circ}{\psi}}$, and $S'_{\Delta\Delta}$ is spectral density of the single "white" noise that will be:

$$S'_{\Delta\Delta} = \frac{\sigma_\Delta^2}{\pi}. \tag{2}$$

In general case, behavior of the object output signal vector in "i"-th operating mode of the system with known behavior of the vectors y_i and y_i (Fig. 1) should be written as:

$$x_i = K^{-1}(y_i - \varphi_i), \tag{3}$$

and the object dynamics model equation as:

$$x_i = P_i^{-1}M_i u_i + P_i^{-1}\Psi_i g. \tag{4}$$

It is expedient to introduce the following designations:

$$\Phi_i = (\Phi_{1i}, \Phi_{2i}) = (P_i^{-1}M_i, P_i^{-1}\Psi_i); \quad z = \begin{pmatrix} u_i \\ g \end{pmatrix}, \tag{5}$$

$$\tilde{x}_i = \Phi_i z_i. \tag{6}$$

Considering the expressions (3), (5) and (6), behavior of the identification error signal vector ε_i for dynamics models of the object in "i"-th operating mode shall be as follows:

$$\begin{aligned} \varepsilon_i &= \tilde{x}_i - x_i = \Phi_i z_i - K^{-1}(y_i - \varphi_i); \\ \varepsilon_{i*} &= \tilde{z}_{i*} - \Phi_{i*}(y_{i*} - \varphi_{i*})K_*^{-1}, \end{aligned} \tag{7}$$

where "*" is complex conjugation sign.

Identification quality factor, under determinated effects for example, can be the functional:

$$I = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \mathrm{tr}(\varepsilon_i \varepsilon_{i*} R)\, ds. \tag{8}$$

II. Structural Identification of the Object Dynamics Models and Determinated Disturbance (Basic and Convenient Identification Method)

Flow chart of the problem is presented in Fig. 2

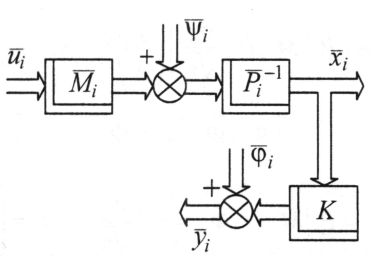

Fig. 2. Flow chart of the control object under study.

In this version of the problem, mode-wise signal vectors and object dynamics model are marked with upper index "–".

For each identification mode "i", signal vector $\overline{u}_i$ is assigned to the object input and data K, $\overline{y}_i$ and $\overline{\varphi}_i$ is measured, based on which the expressions of the identification error vectors (7) can be rewritten in the following form:

$$\begin{aligned} \varepsilon_i &= \overline{\tilde{x}}_i - \overline{x}_i = \overline{\Phi}_i \overline{z}_i - K^{-1}(\overline{y}_i - \overline{\varphi}_i); \\ \varepsilon_{i*} &= \overline{z}_{i*}\overline{\Phi}_{i*} - (\overline{y}_{i*} - \overline{\varphi}_{i*})K_*^{-1}. \end{aligned} \tag{9}$$

After substituting the vectors (9) in the functional (8), the latter will have the following form:

$$\begin{aligned} \overline{I}_i &= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\,\{[\overline{\Phi}_i \overline{z}_i - K^{-1}(\overline{y}_i - \overline{\varphi}_i)][\overline{z}_{i*}\overline{\Phi}_{i*} - \\ &- (\overline{y}_{i*} - \overline{\varphi}_{i*})K_*^{-1}]\,\overline{R}_i\}\, ds = \\ &= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\,\{[\overline{\Phi}_i \overline{z}_i \overline{z}_{i*}\overline{\Phi}_{i*} - \overline{\Phi}_i \overline{z}_i(\overline{y}_{i*} - \overline{\varphi}_{i*})K_*^{-1} - \\ &- K^{-1}(\overline{y}_i - \overline{\varphi}_i)\overline{z}_{i*}\overline{\Phi}_{i*} + \\ &+ K^{-1}(\overline{y}_i\overline{y}_{i*} - \overline{y}_i\overline{\varphi}_{i*} - \overline{\varphi}_i\overline{y}_{i*} + \overline{\varphi}_i\overline{\varphi}_{i*})K_*^{-1}]\,\overline{R}_i\}\, ds. \end{aligned} \tag{10}$$

The problem of functional minimization (10) is solved using Viner-Kolmogorov method [2]. The first variation of the functional (10) has the following form:

$$\begin{aligned} \delta\overline{I}_i &= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\{\overline{R}_i[\overline{\Phi}_i\overline{z}_i\overline{z}_{i*} - K^{-1}(\overline{y}_i - \overline{\varphi}_i)\overline{z}_{i*}]\delta\overline{\Phi}_{i*} + \\ &+ \delta\overline{\Phi}_i[\overline{z}_i\overline{z}_{i*}\overline{\Phi}_{i*} - \overline{z}_i(\overline{y}_{i*} - \overline{\varphi}_{i*})K_*^{-1}]\,\overline{R}_i\}\, ds. \end{aligned} \tag{11}$$

The following symbols are necessary in the variation:

$$\begin{aligned} &\overline{\Gamma}_{i*}\overline{\Gamma}_i = \overline{R}; \qquad \overline{z}_i\overline{z}_{i*} \approx \overline{D}_i\overline{D}_{i*}; \\ &\overline{T}_i = \overline{T}_{i0} + \overline{T}_{i+} + \overline{T}_{i-} = (\overline{\Gamma}_{i*}^{-1})K^{-1}(\overline{y}_i - \overline{\varphi}_i)\,\overline{z}_{i*}(\overline{D}_{i*})^{-1}. \end{aligned} \tag{12}$$

which are performed as factoring and separation operations (by Davis [3]).

As the matrix $\overline{z}_i\overline{z}_{i*}$ is, as a rule, degenerated (special), so the matrices $\overline{D}_i$ and $\overline{D}_{i*}$ are determined approximately.

Considering the designations (12), the variation (11) can be rewritten as follows:

$$\begin{aligned} \delta\overline{I}_i &\approx \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\,[\overline{\Gamma}_{i*}(\overline{\Gamma}_i\overline{\Phi}_i\overline{D}_i - \overline{T}_i)\,\overline{D}_{i*}\delta\overline{\Phi}_{i*} + \\ &+ \delta\overline{\Phi}_i\overline{D}_i(\overline{D}_{i*}\overline{\Phi}_{i*}\overline{\Gamma}_{i*} - \overline{T}_{i*})\,\overline{\Gamma}_i]\, ds. \end{aligned} \tag{13}$$

The condition of approximate equality of the variation (13) to zero will be as follows:

$$\overline{\Gamma}_i \overline{\Phi}_i \overline{D}_i \approx (\overline{T}_{i0} + \overline{T}_{i+}),$$

and the optimized structure identification algorithm $\overline{\Phi}_i$ should be written as:

$$\hat{\overline{\Phi}}_i \approx (\overline{\Gamma}_i)^{-1} (\overline{T}_{i0} + \overline{T}_{i+})\, (\overline{D}_i)^{-1}. \tag{14}$$

This is a basic alternate solution. With potential changes in control signals $\overline{u}$, it is possible to identify the set of dynamics models $\left(\overline{\Phi}_1, \overline{\Phi}_2, \ldots, \overline{\Phi}_i, \ldots, \overline{\Phi}_n\right)$ of the examined system.

But there is also another possibility to obtain more rigorous solutions of identification problem, if all features of the frequency response of vector $\overline{z}_i$ are in only left half plane of complex variable $s = j\omega$.

Practically convenient alternate solution of this problem is considered below.

Thus, if all features of the frequency response of signal vector $\overline{z}_i$ are in only left half plane of complex variable $s = j\omega$, it is possible to change the way of solution of above examined problem and obtain rigorous (optimum) object dynamics model.

Considering the aforesaid, the variation (11) can be rewritten in a new form:

$$\delta \overline{I}_i = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} tr\{\overline{R}_i [\overline{\Phi}_i \overline{z}_i - K^{-1}(\overline{y}_i - \overline{\varphi}_i)]\, \overline{z}_{i*} \delta \overline{\Phi}_{i*} + \\ + \delta \overline{\Phi}_i \overline{z}_i\, [\overline{z}_{i*} \overline{\Phi}_{i*} - (\overline{y}_{i*} - \overline{\varphi}_{i*})\, K_*^{-1}]\, \overline{R}_i \Big\} ds. \tag{15}$$

The following designations are expedient in the variation (15):

$$\overline{\Gamma}_{i*} \overline{\Gamma}_i = \overline{R}_i; \; \overline{T}_i = \overline{T}_{i0} + \overline{T}_{i+} + \overline{T}_{i-} = (\overline{\Gamma}_i) K^{-1} (\overline{y}_i - \overline{\varphi}_i), \tag{16}$$

which are performed as factoring and separation operations by Davis [3].

The condition of equality of the variation (15) to zero has the following form:

$$\overline{\Gamma}_i \overline{\Phi}_i z_i = (\overline{T}_{i0} + \overline{T}_{i+}),$$

and the structure identification algorithm $\hat{\overline{\Phi}}_i$ will be:

$$\hat{\overline{\Phi}}_i = \left(\overline{\Gamma}_i\right)^{-1} \left(\overline{T}_{i0} + \overline{T}_{i+}\right) \left(\overline{z}_i\right)^{\#}, \tag{17}$$

where "#" is a symbol of pseudo inversion of the column matrix $\overline{z}_i$.

Pseudo inversion can be exercised by Gantmaher [2], assuming that:

$$(\overline{z}_i)^{\#} = A^{+} = C^{*} (C \cdot C^{*})^{-1} (B^{*} B)^{-1} B^{*}, \tag{18}$$

where upper index "$*$" is transposition sign.

In the examined version $\overline{z}_i = \begin{pmatrix} \overline{u}_i \\ 1 \end{pmatrix} = B, \; C = (1,0).$

Let the products of matrices equal

$$BC = \begin{pmatrix} \overline{u}_i \\ 1 \end{pmatrix} (1,0), \quad C \cdot C^{*} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot (1,0); \quad B^{*} = (u_i^{*}, 1);$$

$$B^{*} B = \left(\overline{u}_i^{*}, 1\right) \begin{pmatrix} \overline{u}_i \\ 1 \end{pmatrix} = \left(\overline{u}_i^{*} \cdot \overline{u}_i + 1\right) = \sum_{\nu=1}^{m} \overline{u}_{\nu i}^{2} + 1; \quad C \cdot C^{*} = 1,0;$$

By substituting the expressions $C^{*}, \left(CC^{*}\right)^{-1}, \left(B^{*}B\right)^{-1}, B^{*}$ in the formula (18), the matrix A^{+} can be presented in the form:

$$A^{+} = \left(\overline{z}_i\right)^{\#} = \frac{1}{\left(\sum_{\nu=1}^{m} \overline{u}_{\nu i}^{2} + 1\right)} \left(\overline{u}_i^{*}, 1\right) \tag{19}$$

III. Structural Identification of the Object Dynamics Models and Its Random Stationary Disturbance

Flow chart of the identification system shown in Fig. 3 corresponds to the considered version of the problem.

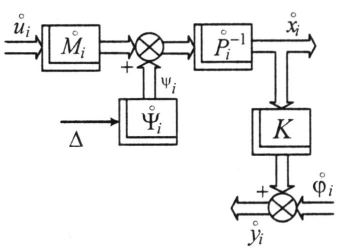

Fig. 3. Flow chart of the mode-wise identification system of the object dynamics models under random effects.

Fourier transformed system of differential equations of the mode-wise linearized identification object has the form:

$$\mathring{x}_i = \mathring{P}_i^{-1} \mathring{M}_i \mathring{u}_i + \mathring{P}^{-1} \frac{\sigma_\Delta}{\sqrt{\pi}} \Delta \\ = \left(\mathring{P}_i^{-1} \mathring{M}_i, \mathring{P}_i^{-1} \frac{\sigma_\Delta}{\sqrt{\pi}} \right) \begin{pmatrix} \mathring{u}_i \\ \Delta \end{pmatrix} \\ = \left(\mathring{\Phi}_{1i}, \mathring{\Phi}_{2i} \right) \mathring{z}_i = \mathring{\Phi}_i \mathring{z}_i; \tag{20}$$

$$\Phi_{1i} = P_i^{-1} M_i, \qquad \Phi_{1i} = P_i^{-1} \frac{\sigma_\Delta}{\sqrt{\pi}}, \qquad \mathring{z}_i = \begin{pmatrix} \mathring{u}_i \\ \Delta \end{pmatrix}.$$

Behavior of the object output signal vector according to the output change data will be:

$$\overset{\circ}{x}_i = K^{-1}\left(\overset{\circ}{y}_i - \overset{\circ}{\varphi}_i\right). \quad (21)$$

Behavior of the dynamics model identification error vector in the examined version of the problem has the form:

$$\overset{\circ}{\varepsilon}_i = \tilde{x}_i - \overset{\circ}{x}_i = \overset{\circ}{\Phi}_i \overset{\circ}{z}_i - \overset{\circ}{x}_i, \qquad \overset{\circ}{\varepsilon}_{i*} = \overset{\circ}{z}_{i*} \overset{\circ}{\Phi}_{i*} - \overset{\circ}{x}_{i*}, \quad (22)$$

where vector $\overset{\circ}{x}_i$ is determined by the expression (21).

By Viner–Hinchin theorem, the matrices of spectral and reciprocal spectral densities of needed signal vectors have the form:

$$\begin{aligned} S'_{\overset{\circ}{\varepsilon}_i \overset{\circ}{\varepsilon}_i} &= <\overset{\circ}{\varepsilon}_i \overset{\circ}{\varepsilon}_{i*}> = \left\langle \left[\overset{\circ}{\Phi}_i \overset{\circ}{z}_i - K^{-1}\left(\overset{\circ}{y}_i - \overset{\circ}{\varphi}_i\right)\right] \times \right. \\ &\times \left.\left[\overset{\circ}{z}_{i*} \overset{\circ}{\Phi}_{i*} - \left(\overset{\circ}{y}_{i*} - \overset{\circ}{\varphi}_{i*}\right) K_*^{-1}\right]\right\rangle = \\ &= \overset{\circ}{\Phi}_i S'_{\overset{\circ}{z}_i \overset{\circ}{z}_i} - \overset{\circ}{\Phi}_i \left(S'_{\overset{\circ}{y}_i \overset{\circ}{z}_i} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{z}_i}\right) K_*^{-1} - K^{-1}\left(S'_{\overset{\circ}{z}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{z}_i \overset{\circ}{\varphi}_i}\right) + \\ &+ K^{-1}\left(S'_{\overset{\circ}{y}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{y}_i \overset{\circ}{\varphi}_i} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{y}_i}\right) K_*^{-1}; \end{aligned} \quad (23)$$

$$S'_{\overset{\circ}{z}_i \overset{\circ}{z}_i} = <\overset{\circ}{z}_i \overset{\circ}{z}_{i*}> = \begin{pmatrix} S'_{\overset{\circ}{u}_i \overset{\circ}{u}_i} & O_{m\times 1} \\ O_{1\times m} & 1 \end{pmatrix};$$

$$\begin{aligned} S'_{\overset{\circ}{z}_i \overset{\circ}{y}_i} &= <\overset{\circ}{y}_i \overset{\circ}{z}_{i*}> = <\overset{\circ}{y}_i (\overset{\circ}{u}_{i*}, \Delta_*)> = (S'_{\overset{\circ}{u}_i \overset{\circ}{y}_i}, S'_{\Delta y_i}); \\ S'_{\overset{\circ}{y}_i \overset{\circ}{z}_i} &= \begin{pmatrix} S'_{\overset{\circ}{y}_i \overset{\circ}{u}_i} \\ S'_{\overset{\circ}{y}_i \Delta} \end{pmatrix}; \end{aligned} \quad (24)$$

$$S'_{\overset{\circ}{z}_i \overset{\circ}{\varphi}_i} = <\overset{\circ}{\varphi}_i \overset{\circ}{z}_{i*}> = <\overset{\circ}{\varphi}_i (\overset{\circ}{u}_{i*}, \Delta_*)> = (S'_{\overset{\circ}{u}_i \overset{\circ}{\varphi}_i}, S'_{\Delta \overset{\circ}{\varphi}_i});$$

$$S'_{\varphi_i z_i} = \begin{pmatrix} S'_{\overset{\circ}{\varphi}_i \overset{\circ}{u}_i} \\ S'_{\overset{\circ}{\varphi}_i \Delta} \end{pmatrix}.$$

The functional of the object dynamics model identification quality:

$$\begin{aligned} e_i &= \frac{1}{j}\int_{-j\infty}^{j\infty} tr(S'_{\varepsilon_i \varepsilon_i} R_i)\, ds \\ &= \frac{1}{j}\int_{-j\infty}^{j\infty} tr\{[\overset{\circ}{\Phi} S'_{\overset{\circ}{z}_i \overset{\circ}{z}_o} \overset{\circ}{\Phi}_{i*} - \overset{\circ}{\Phi}_i (S'_{\overset{\circ}{y}_i \overset{\circ}{z}_o} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{z}_i}) K^{-1} \\ &\quad - K^{-1}(S' - S') \overset{\circ}{\Phi}_{i*} \\ &\quad + K^{-1}(S'_{\overset{\circ}{y}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{y}_i \overset{\circ}{\varphi}_i} + S'_{\overset{\circ}{\varphi}_i \overset{\circ}{\varphi}_i}) \overset{\circ}{K}_*^{-1}] \overset{\circ}{R}\}\, ds. \end{aligned} \quad (25)$$

The first variation of the functional (25) has the form:

$$\begin{aligned} \delta e_i &= \frac{1}{j}\int_{-j\infty}^{j\infty} \mathrm{tr}\, \overset{\circ}{R}_i [\overset{\circ}{\Phi}_i S'_{\overset{\circ}{z}_i \overset{\circ}{z}_i} - K^{-1}(S'_{\overset{\circ}{z}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{z}_i \overset{\circ}{\varphi}_i})]\, \delta \overset{\circ}{\Phi}_{i*} \\ &\quad + \delta \overset{\circ}{\Phi}_i [S'_{\overset{\circ}{z}_i \overset{\circ}{z}_i} \overset{\circ}{\Phi}_{i*} - (S'_{\overset{\circ}{y}_i \overset{\circ}{z}_i} - S'_{\overset{\circ}{\varphi}_i \overset{\circ}{z}_i}) K^{-1}] \overset{\circ}{R}_i\}\, ds. \end{aligned} \quad (26)$$

The following designations are necessary:

$\overset{\circ}{\Gamma}_{i*} \overset{\circ}{\Gamma}_i = \overset{\circ}{R}_i$; $\overset{\circ}{D}_i \overset{\circ}{D}_{i*} = S'_{\overset{\circ}{z}_i \overset{\circ}{z}_i}$ both matrices are factored;

$$\overset{\circ}{T}_i = \overset{\circ}{T}_{i+} + \overset{\circ}{T}_{i+} + \overset{\circ}{T}_{i-} = \overset{\circ}{\Gamma}_i K^{-1}(S'_{\overset{\circ}{z}_i \overset{\circ}{y}_i} - S'_{\overset{\circ}{z}_i \overset{\circ}{\varphi}_i})(\overset{\circ}{D}_{i*})^{-1} \quad (27)$$

the matrix is separated.

Considering the designations (27), the variation (26) can be rewritten as:

$$\begin{aligned} \delta e_i &= \frac{1}{j}\int_{-j\infty}^{j\infty} tr[\overset{\circ}{\Gamma}_{i*}(\overset{\circ}{\Gamma}_i \overset{\circ}{\Phi}_i \overset{\circ}{D}_i - \overset{\circ}{T}_i) \overset{\circ}{D}_{i*}\, \delta \overset{\circ}{\Phi}_{i*} \\ &\quad + \delta \overset{\circ}{\Phi}_i \overset{\circ}{D}_i (\overset{\circ}{D}_{i*} \overset{\circ}{\Phi}_{i*} \overset{\circ}{\Gamma}_{i*} - \overset{\circ}{T}_{i*}) \overset{\circ}{\Gamma}_i]\, ds. \end{aligned}$$

The condition of equality of the variation (26) to zero will be:

$$\overset{\circ}{\Gamma}_i \overset{\circ}{\Phi}_i \overset{\circ}{D}_i = (\overset{\circ}{T}_{i0} + \overset{\circ}{T}_{i+}),$$

and the identification algorithm of the dynamics models will be as follows:

$$\overset{\circ}{\Phi}_i = \overset{\circ}{\Gamma}_i^{-1} (\overset{\circ}{T}_{i0} + \overset{\circ}{T}_{i+})(\overset{\circ}{D}_i)^{-1}. \quad (28)$$

Hereby, the object dynamics models can be obtained for all needed operating modes $i = (1,2,\ldots,k)$ of the nonlinear system.

IV. Conclusions

Proposed identification algorithm may be successfully applied to development of systems for control by unmanned and manned aircrafts of wide class.

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