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### OBSERVER-BASED FLIGHT CONTROL SYSTEM DESIGN UNDER LINEAR MATRIX INEQUALITIES APPROACH

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**Abstract**—The paper examines a problem of observer-based flight control system design. The design procedure allows deriving both the observer and the controller simultaneously. The peculiarity of the proposed approach is that there is no need not need to choose the observer poles. The proposed design procedure is based on linear matrix inequalities technique. To demonstrate the efficiency of the proposed approach a longitudinal motion of unmanned aerial vehicle is used as a case study.

Index Terms—Flight control system; full-order observer; linear matrix inequalities; state feedback; separation principle.

#### I. INTRODUCTION

The wide application of small Unmanned Aerial Vehicles (UAVs) are encouraged to create flight control systems (FCS) that satisfy manifold requirement imposed on the aircraft during flight envelope. Moreover, the small UAVs should easily move in a wide range of velocity and altitude changes. Furthermore, one should care about various problems connected with law cost design and power consumption in order to be implemented onboard computer with restricted abilities. Therefore, the manipulation of such UAVs requires a necessary stability, performance and robustness [1], [2], [4]. A great number of control approaches have been proposed to solve the problem autopilot design [1]-[9], [15]. Among them, it is possible to enumerate some works related to the combination of observer and linear quadratic regulator [5], where the problem of limited number of state vector measurements is considered. To preserve the required level of performance without losing the robustness of the flight control system, the mixed  $\mathbf{H}_2/\mathbf{H}_{\infty}$  – robust optimization procedure is used. The main idea behind this technique is to seek a trade-off between the performance and the robustness of the overall closed loop system [5].

It is necessary admit, that the observer design was originally proposed in works [10], [11], [12]. Lately, the numbers of observer-based control system design approaches were proposed [5], [6] [13].

The autopilot design is also may be performed basing on the available information about the output variables. This circumstance leads to the problem of static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. The SOF problem concerns finding a static or feedback gain to achieve certain desired closedloop characteristics. It is necessary to admit that the output feedback problem is much more difficult to solve in comparison to state feedback control problem [14].

This paper deals with observer and controller development in terms of linear matrix inequalities (LMIs) [16] for aircraft control during flight envelope. It is known that the design procedure of observer deals with selecting desired region poles location. Moreover, the observer eigenvalues should be faster up to ten times in comparison to plant eigenvalues. This condition results in the observer sensitivity to noisy measurement, which is not desirable. To overcome this difficulty procedure of observer design is proposed basing on Lyapunov approach.

The main feature of this paper is that the FCS is designed by applying LMI technique where the observer gains and controller structure are defined by solving the set of LMIs simultaneously.

To prove the efficiency of the proposed approach, the longitudinal motion of the aircraft is considered as a case study.

#### II. PROBLEM STATEMENT

Let us consider a problem of flight control system (FCS) design with incomplete state vector measurement. The aircraft dynamics is represented by the following set of equations

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases}, \ \mathbf{x}(0) = \mathbf{x}_0, \qquad (1)$$

where  $\mathbf{x} \in \mathbf{R}^n$  is the state space vector;  $\mathbf{u} \in \mathbf{R}^m$  is the control vector;  $\mathbf{y} \in \mathbf{R}^p$  is the observation vector;

Besides that, the state space matrices of the controlled plant have the following dimensions  $\mathbf{A} \in \mathbf{R}^{n \times n}$ ,  $\mathbf{B} \in \mathbf{R}^{m \times n}$ ,  $\mathbf{C} \in \mathbf{R}^{p \times n}$ . It could be seen that number of measuring variables  $\mathbf{p}$  is less than number of all phase coordinates  $\mathbf{n}$ . Thus, to design the FCS the full state space vector is necessary to be restored.

In this paper we develop the design procedure of full-order state observer design with further state

feedback construction such that the performance of closed-loop system satisfies selected performance criterion. Thus, the FCS construction is performed under the well known separation principle.

#### III. OBSERVER-BASED FLIGHT CONTROL SYSTEM DESIGN

It is known that the observer estimates the state variables based on measurement of the output y and control u variables [10] – [12]. Let us consider the procedure of observer-based flight control system design under LMI approach.

Consider linear time-invariant system given by (1). Assume that the states  $\mathbf{x}$  are approximated by the states  $\tilde{\mathbf{x}}$ . The observer model takes into account feedback information about observation error and can be represented as

$$\tilde{\mathbf{x}}(t) = \mathbf{A}\,\tilde{\mathbf{x}}(t) + \mathbf{B}\,\mathbf{u}(t) + \mathbf{L}\left(\tilde{\mathbf{y}}(t) - \mathbf{y}(t)\right) = \mathbf{A}\,\tilde{\mathbf{x}}(t) + \mathbf{B}\,\mathbf{u}(t) + \mathbf{L}\,\mathbf{C}\left(\tilde{\mathbf{x}}(t) - \mathbf{x}(t)\right)$$
(2)

where  $(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)) = \mathbf{e}(t)$  is a difference between the real and estimated states (observation error); L is the observer gain matrix that has to be chosen such that the observation error approaches zero as time increases. From (1) and (2) the observation error equation dynamics takes the following form

$$\dot{\mathbf{e}}(t) = (\dot{\mathbf{x}}(t) - \dot{\tilde{\mathbf{x}}}(t)),$$
  
$$\dot{\mathbf{x}}(t) - \tilde{\tilde{\mathbf{x}}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t)$$
  
$$- (\mathbf{A} \tilde{\mathbf{x}}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{L} \mathbf{C} (\tilde{\mathbf{x}}(t) - \mathbf{x}(t))) \qquad (3)$$
  
$$= (\mathbf{A} + \mathbf{L} \mathbf{C}) \mathbf{e}(t)$$

The error decays to zero if it is possible to find observer gain matrix L such that (A+LC) is asymptotically stable. Moreover, the eigenvalues of (A+LC) are the same as those of  $(A+LC)^{T} =$  $= A^{T} + C^{T}L^{T}$ .

The final goal is to control the motion of the plant basing on the estimated states. Thus, for the state feedback control based on observed states  $\tilde{x}$ , namely

$$\mathbf{u} = \mathbf{K} \, \tilde{\mathbf{x}} \,, \tag{4}$$

where  $\mathbf{K}$  is a constant state feedback gain matrix that assures that the system is asymptotically stable, the state equation becomes

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\tilde{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}(\mathbf{x}(t) - \mathbf{e}(t))$$
  
=  $(\mathbf{A} + \mathbf{B}\mathbf{K})\mathbf{x}(t) - \mathbf{B}\mathbf{K}\mathbf{e}(t).$  (5)

Combining together (3) and (5), we obtain

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{B}\mathbf{K} & -\mathbf{B}\mathbf{K} \\ 0 & \mathbf{A} + \mathbf{L}\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix}.$$
 (6)

Equation (6) describes the dynamics of the observed state feedback control system. The characteristic equation for the system is

$$|s\mathbf{I} - \mathbf{A} - \mathbf{B}\mathbf{K}||s\mathbf{I} - \mathbf{A} - \mathbf{L}\mathbf{C}| = 0$$

It is possible to rewrite the system dynamics in terms of plant and observer states, respectively.

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \tilde{\mathbf{x}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{K} \\ -\mathbf{L}\mathbf{C} & \mathbf{A} + \mathbf{B}\mathbf{K} + \mathbf{L}\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \tilde{\mathbf{x}}(t) \end{bmatrix}$$

It is supposed also that the obtained solution given by (4) minimizes performance index as

$$J = \int_{0}^{\infty} \left( \tilde{\mathbf{x}}(t)^{\mathrm{T}} \mathbf{Q} \, \tilde{\mathbf{x}}(t) + \mathbf{u}(t)^{\mathrm{T}} \, \mathbf{R} \, \mathbf{u}(t) \right) dt = , \quad (7)$$
$$\int_{0}^{\infty} \tilde{\mathbf{x}}(t)^{\mathrm{T}} \left( \mathbf{Q} + \mathbf{K}^{\mathrm{T}} \, \mathbf{R} \, \mathbf{K} \right) \tilde{\mathbf{x}}(t) \, dt$$

where Q and R are diagonal matrices, weighting each state and control variables, respectively.

This cost depends on the trajectory, of  $\tilde{\mathbf{x}}(t)$ , taken, such that the worst trajectory will correspond to the worst cost [3].

It is known that the observed-state feedback control system design consists of two stages: (1) design of state feedback control law assuming that all states are available; (2) design a state estimator to estimate states of the system. Replace the states in state feedback control law from stage (1) by the state estimates. Further, they can be combined to form the observed-state feedback control system. This principle of independent state feedback and observer design is referred to as separation principle. Moreover, the observer design deals with choice of poles location. They are usually chosen such that the observer response is much faster that the system response, but very fast observers possess with noise. The proposed approach solves the problem of observed-state feedback design under LMI technique. The main advantage of the proposed design procedure is that there is no need to define the observer poles location. The solution of this problem via LMIs gives the constant state feedback gain matrix K and observer gain L by solving the set of LMIs simultaneously. The proposed design procedure is very simple and utilizes Lyapunov approach.

The simultaneous observer and controller design can be formulated with following theorem.

**Theorem**. The observer-based system (6) is said to be statically stable by means of state feedback (4)

if there exist matrices  $\mathbf{X}_1 = \mathbf{X}_1^T > 0$ , **M** and  $\mathbf{X}_2 = \mathbf{X}_2^T > 0$ , **Z** and satisfy the following conditions:

$$\begin{bmatrix} \mathbf{X}_{1} \, \mathbf{A}^{\mathrm{T}} + \mathbf{A} \mathbf{X}_{1} + \mathbf{M}^{\mathrm{T}} \, \mathbf{B}^{\mathrm{T}} + \mathbf{B} \mathbf{M} & \mathbf{X}_{1} \mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}} \, \mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2} \, \mathbf{X}_{1} & -\mathbf{I} & \mathbf{0} \\ \mathbf{R}^{1/2} \, \mathbf{M} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < \mathbf{0} \,,$$

$$\mathbf{X}_1 = \mathbf{X}_1^* > 0 , \qquad (8)$$

$$\mathbf{A}^{\mathrm{T}} \mathbf{X}_{2} + \mathbf{X}_{2} \mathbf{A} + \mathbf{C}^{\mathrm{T}} \mathbf{Z}^{\mathrm{T}} + \mathbf{Z} \mathbf{C} < 0 ,$$
$$\mathbf{X}_{2} = \mathbf{X}_{2}^{\mathrm{T}} > 0 .$$
(9)

**Proof.** Let  $\mathbf{V}_1(\mathbf{x},t) = \mathbf{x}(t)\mathbf{P}_1\mathbf{x}^T(t)$  with  $\mathbf{P}_1 = \mathbf{P}_1^T > 0$  be a candidate Lyapunov function. The closed loop system (6) preserves stability and minimizes performance index (7) if:

$$\dot{\mathbf{V}}_{1}(\mathbf{x},t) + \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)R\mathbf{u}(t) < 0.$$
(10)

The condition (10) leads to the following inequality:

$$\mathbf{x}^{\mathrm{T}}(t) \Big\{ \mathbf{A}^{\mathrm{T}} \mathbf{P}_{1} + \mathbf{P}_{1} \mathbf{A} + \mathbf{K}^{\mathrm{T}} \mathbf{B}^{\mathrm{T}} \mathbf{P}_{1} + \mathbf{P}_{1} \mathbf{B} \mathbf{K} + \mathbf{Q} + \mathbf{K}^{\mathrm{T}} \mathbf{R} \mathbf{K} \Big\} \\ \times \mathbf{x}(t) < 0$$

Pre-multiplying and post-multiplying right and left sides above written inequality by  $\mathbf{P}^{-1}$ :

$$\mathbf{P}_{l}^{-1}\mathbf{A}^{T} + \mathbf{A}\mathbf{P}_{l}^{-1} + \mathbf{P}_{l}^{-1}\mathbf{K}^{T}\mathbf{B}^{T} + \mathbf{B}\mathbf{K}\mathbf{P}_{l}^{-1} + \mathbf{P}_{l}^{-1}\mathbf{Q}\mathbf{P}_{l}^{-1} + \mathbf{P}_{l}^{-1}\mathbf{K}^{T}\mathbf{R}\mathbf{K}\mathbf{P}_{l}^{-1} < 0.$$
(11)

Let us define the following change of variables  $\mathbf{X}_1 = \mathbf{P}_1^{-1}$ ,  $\mathbf{M} = \mathbf{K} \mathbf{P}_1^{-1}$ ,  $\mathbf{K} = \mathbf{M} \mathbf{P}_1$  and rewrite inequality (11) as

$$\mathbf{X}\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{X} + \mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbf{M} + \mathbf{X}\mathbf{Q}\mathbf{X} + \mathbf{X}\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\mathbf{X} < 0 \quad (12)$$

By applying Shur's Lemma to inequality (12) it is possible to rewrite as matrix inequality:

$$\begin{bmatrix} \mathbf{X}_{1} \, \mathbf{A}^{\mathrm{T}} + \mathbf{A} \mathbf{X}_{1} + \mathbf{M}^{\mathrm{T}} \, \mathbf{B}^{\mathrm{T}} + \mathbf{B} \mathbf{M} & \mathbf{X}_{1} \mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}} \mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2} \, \mathbf{X}_{1} & -\mathbf{I} & \mathbf{0} \\ \mathbf{R}^{1/2} \, \mathbf{M} & \mathbf{0} & -\mathbf{I} \end{bmatrix} < \mathbf{0} \, .$$

This part of the proof considers the design stage (1) according to the separation principle. The second part of the proof considers stage (2) of the design procedure connected with observer construction.

Let  $\mathbf{V}_2(\mathbf{e}(t),t) = \mathbf{e}(t)\mathbf{P}_2 \mathbf{e}^T(t)$  with  $\mathbf{P}_2 = \mathbf{P}_2^T > 0$  be a candidate Lyapunov function. The observer gains can be found if the following inequality is hold:

$$\mathbf{e}^{\mathrm{T}}(t)\left\{\left(\mathbf{A}+\mathbf{LC}\right)^{\mathrm{T}}\mathbf{P}_{2}+\mathbf{P}_{2}\left(\mathbf{A}+\mathbf{LC}\right)\right\}\mathbf{e}(t)<0,$$
$$\mathbf{A}^{\mathrm{T}}\mathbf{P}_{2}+\mathbf{P}_{2}\mathbf{A}+\mathbf{C}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{P}_{2}+\mathbf{P}_{2}\mathbf{LC}<0.$$

The use of the following change of variables 
$$\mathbf{X}_2 = \mathbf{P}_2$$
,  $\mathbf{P}_2 \mathbf{L} = \mathbf{Z}$  reduces to the next LMI:

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_{2} + \mathbf{X}_{2}\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}} + \mathbf{Z}\mathbf{C} < 0 ,$$
$$\mathbf{X}_{2} = \mathbf{X}_{2}^{\mathrm{T}} > 0 .$$

Thus, the observer gains can be evaluated as

$$\mathbf{L} = \mathbf{X}_2^{-1} \mathbf{Z} \ .$$

#### IV. CASE STUDY

To demonstrate the efficiency of the proposed approach a longitudinal channel of the UAV is used as a case study. The state space vector of the longitudinal channel involves the following components:  $\mathbf{x} = [V_t, \alpha, \theta, q, h]^T$ , where  $V_t$  is the true airspeed of UAV,  $\alpha$  is the angle of attack,  $\theta$  is the pitch angle, q is the pitch rate and h is the altitude. The control input vector  $\mathbf{u} = [\delta_e]^T$  is represented only by elevator deflection.

It is considered operating mode with true airspeed at  $V_t = 14.0$  m/s. The linear model in the state space is represented by the matrices [**A**, **B**]:

$$\mathbf{A} = \begin{bmatrix} -0.1816 & 43.9153 & -9.81 & 0 & 0 \\ -0.4292 & -12.7475 & -0.6711 & 0.6898 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0.2988 & -130.2477 & 4.7433 & -21.9445 & 0 \\ 0 & -14.0 & 14.0 & 0 & 0 \end{bmatrix};$$
$$\mathbf{B} = \begin{bmatrix} -0.0408 \\ -0.0553 \\ 0 \\ -14.8151 \\ 0 \end{bmatrix};$$

The output vector of measured variables is given as follows  $\mathbf{y}_{est} = [\theta, q, h]^{\mathrm{T}}$ .

Disturbance, v affecting the longitudinal motion of the aircraft involves the following components: the true airspeed,  $V_t$ , angle of attack,  $\alpha$  and pitch rate, q, so that  $\mathbf{v} = \begin{bmatrix} V_{t_g}, \alpha_g, q_g \end{bmatrix}^T$ . In order to simulate the atmospheric turbulence the Dryden filter is used [3]. It is considered that aircraft flies at moderate turbulence. Parameters appearing in the state space of Dryden filter are given as follows [3]:  $K_V = \sigma_V \sqrt{(2L_V/\pi V)}, \quad \lambda_V = L_V/V, \quad \lambda_\alpha = L_\alpha/V,$  $K_\alpha = 7.2011, \lambda_\alpha = 0.1981, K_q = 1/V, \lambda_q = 4b/\pi V.$ 

The variable *b* represents the aircraft wingspan, b = 2.31 m. The variables  $L_V$ ,  $L_\alpha$  represent the turbulence scale lengths. The variables  $\sigma_v$ ,  $\sigma_\alpha$  represent the

turbulence intensities. The computation of these values depends on the altitude at which the aircraft is flying, wing span and type of turbulence according to standard MIL-F-8785C.

The weighting matrices  $\mathbf{Q}$ ,  $\mathbf{R}$  in (7) have the following form:

$$\mathbf{Q} = \text{diag}([0.0057 \quad 0.0023 \quad 8.9443 \quad 0.0001 \quad 0.7071]);$$
  
 $\mathbf{R} = [0.8367].$ 

By applying proposed approach of observer based controller design via LMI approach, the state feedback gain matrix K and observer gains L are found. Their numerical values are given below: - state feedback gains:

 $\mathbf{K} = \begin{bmatrix} -0.1265 & 1.6762 & -7.7596 & -0.2311 & -0.5043 \end{bmatrix}$ 

- observer gain matrix:

	-2.7634	-5.6055	-4.8286	
	-0.1584	-0.3941	-4.8286 -0.4772	
L =	0.9676	0.9592	1.8864	
	1.3993	8.4330	2.4343	
	2.8823	3.5656	6.1283	

Table 1 reflects standard deviations of the UAV outputs.

TABLE	1
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STANDARD DEVIATIONS OF THE UAV OUTPUTS IN A STOCHASTIC CASE

	Standard deviation					
Plant	$\sigma_v$ , m/sec	$\sigma_{\alpha}$ , <sup>0</sup>	$\sigma_{\vartheta}$ , $^{0}$	$\sigma_q$ , $^0$ /sec	$\sigma_h$ , m	$\sigma_{elev}$ , $^{0}$
V = 14.0  m/s	0.1199	0.6157	0.9882	0.2146	0.1943	1.5160

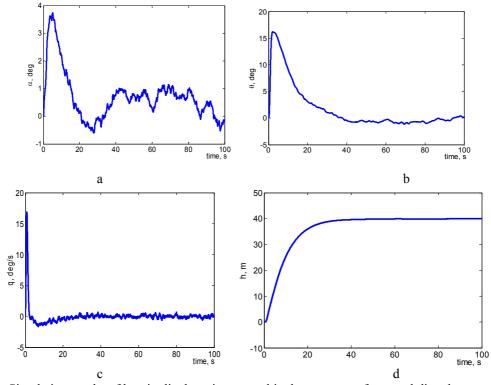
Performance indices of closed loop system with observed state feedback in a loop are given in Table 2.

The simulation results of the closed loop system taking into account the influence of the random wind, simulated according to the standard Dryden model of turbulence confirm the efficiency of proposed approach. Results of the simulation are shown in Figure.

TABLE	2
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PERFORMANCE INDICES OF CLOSED-LOOP SYSTEM

Performance Index	Plant	
Feriormance index	V = 14.0  m/s	
H <sub>2</sub> -norm	0.2545	
$\mathbf{H}_{\infty}$ -norm	0.1981	



Simulation results of longitudinal motion control in the presence of external disturbances: a is the angle of attack, deg; b is the pitch angle, deg; c is the pitch rate, deg/s; d is the altitude, m

#### CONCLUSION

As far as the incomplete state space vector is available for measuring, the flight control system for aircraft can be easily designed by applying observer. Thus, the unavailable states can be suitable approximated by restored states. In turn, the obtained control law is called observed state feedback. The proposed solution is very simple and uses Lyapunov approach. The proposed design procedure can be solved efficiently by applying LMI optimization technique. The main advantage of the proposed approach is that there is no need to define the region of observer poles placement. The proposed approach permits to define the observer gains and state feedback gain matrix directly from set of LMIs, simultaneously.

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## М. М. Комнацька. Синтез системи керування польотом зі спостерігачем на основі лінійних матричних нерівностей

Розглянуто задачу синтезу системи керування польотом зі спостерігачем. Процедура синтезу дозволяє визначити спостерігач та регулятор одночасно. Особливістю запропонованого підходу полягає у тому, що у процесі синтезу спостерігача не виникає потреби вибирати полюса спостерігача. Запропонований підхід до синтезу системи керування польотом зі спостерігачем базується на застосуванні апарату лінійних матричних нерівностей. Ефективність запропонованого підходу демонструється на прикладі керування поздовжнім рухом безпілотного літального апарату.

**Ключові слова:** система керування польотом; спостерігач повного порядку; лінійні матричні нерівності; зворотний зв'язок за станом; теорема розділення. Комнацька Марта Миколаївна. Кандидат технічних наук. Доцент. Кафедра систем управління літальних апаратів, Національний авіаційний університет, Київ, Україна. Освіта: Національний авіаційний університет, Київ, Україна (2007). Напрямок наукової діяльності: системи управління та обробка інформації. Кількість публікацій: 35. E-mail: martakomnatska@gmail.com

# М. Н. Комнацкая. Синтез системы управления полетом с наблюдателем на основе линейных матричных неравенств

Рассмотрено задачу синтеза системы управления полетом с наблюдателем. Процедура синтеза позволяет определить наблюдатель и регулятор одновременно. Особенность предложенного подхода является отсутствие необходимости выбора полюсов наблюдателя. Предложенный подход синтеза системы управления полетом основывается на использовании аппарата линейных матричных неравенств. Эффективность предложенного подхода демонстрируется на примере управления продольным движением беспилотного летательного аппарата.

Ключевые слова: система управления полетом; наблюдатель полного порядка; линейные матричные неравенства; обратная связь по состоянию; теорема разделения.

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Направление научной деятельности: системы управления и обработка информации.

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