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ACCURACY ESTIMATION OF INFORMATION PROCESSING RESULTS TAKING INTO ACCOUNT THE MEASUREMENT CORRELATION

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Abstract *– the way of processing the measuring information, taking into account the correlation between the measurements which can be used to estimate the accuracy characteristics of the onboard automatic landing systems at the stages of operational control is considered.*

Index Terms – measuring information, least squared method, correlation function, random value, measurement errors, spectral density, realization of random process.

I. Introduction

To determine the accuracy characteristics of onboard automatic control systems by means of measurement data obtained in flight tests the calculations based on classical least squares method are used. This method is developed in relation to the processing of independent measurements. In this case it is assumed that measurement errors are the system of mutually independent random variables.

The measurement errors actually have in their composition errors, which are characterized by a high degree of dependence. Processing these measurements using the methods developed for independent measurements may lead to considerable errors.

Of particular importance it can have in solving problems associated with the evaluation of the accuracy of positioning the aircraft on the measurement data, or with the choice of an optimal amount of measurement information, providing location determination with a given accuracy.

Methodology of accuracy estimation corresponding to the least squared method, can give top-heavy results. To obtain the actual accuracy it is necessary to apply a special way to solve the problem.

II. Problem definition

The practical interest is connected with the study of influence of the correlation between the measurements on the accuracy of their processing to solve a number of important practical problems, among which are:

- problem identifying the degree of trajectory measurement correlation for which their processing with required accuracy by classical least squared method is possible,
- problem of choosing the composition of the trajectory measurements and the optimal amount of measurement information to ensure positioning of the aircraft with the required accuracy,
- problem of estimating the accuracy of positioning of the aircraft according to the trajectory measurements.

The article analyzes the influence of correlation between measurements oh the accuracy of processing the results with respect to the class of stationary random processes with the correlation function, decreasing exponentially. The corresponding mathematical apparatus and principles of selection of measurement information with known probability characteristics are applied.

III. Methods of processing dependent measurements

To carry out processing the dependent measurements we may use the method of maximum likehood, which gives efficient unbiased estimations (independently of the distribution law of measurements). In this case a minimum variance of parameters obtained in processing is reached.

Let’s consider the basic relationships of this method and their receipt. We introduce the system of random variables:

$$\delta_{ri} \, (i = 1,2,..., n) \, , \qquad (1)$$

expectations of which are equal to Δ_{ri} respectively, and correlation matrix is the correlation matrix of the measurement errors. The dimension of the vector $\boldsymbol{\delta}_r$ is n x 1. As a result of measurement of parameters Δ_{ri} we have the values $\overline{\Delta_{ri}}$ distorted by errors. We may consider the vales $\overline{\Delta_{ri}}$ as a particular realization of the previously introduced system of random variables.

Let’s introduce the system of random variables according to the number of unknown parameters

$$\delta_{qj} \, (j = 1,2,..., N) \, , \qquad (2)$$

the expectations of which are equal to the exact values of the parameters Δ_{qj}. The dimension of the vector $\boldsymbol{\delta}_q$ is N x 1.

Let $\boldsymbol{\delta}_r = \boldsymbol{\delta}_{r_{n1}}$ is n – dimensional random vector with components (2). It is obvious that the system of conditional equations relating the unknown parameters with the measured parameters can be written in the form:

$$\mathbf{A}\boldsymbol{\delta}_q = \boldsymbol{\delta}_r\,, \qquad (3)$$

where the matrix **A** (dimension of the matrix **A** is n x N) is of the form:

$$\mathbf{A} = \mathrm{A}_{nN} = \begin{pmatrix} a_{11}\ a_{12}.....a_{1N} \\ a_{21}\ a_{22}.....a_{2N} \\ ....\,.......... \\ a_{n1}\ a_{n2}.....a_{nN} \end{pmatrix}.$$

Relation (3) is insoluble (with respect to $\boldsymbol{\delta}_q$) connection between the two considered systems of random variables, since $N < n$.

According to the relationship (2) for each particular realization of the system (1) we may determine the system of particular values of unknown parameters (2), which would be optimal in mentioned sense.

We will seek the solution in the form:

$$\boldsymbol{\delta}_q = \mathbf{D}\boldsymbol{\delta}_r\,, \qquad (4)$$

where the matrix **D** (the dimension of the matrix **D** is N x n) has the form:

$$\mathbf{D} = \mathrm{D}_{N\,n} = \begin{pmatrix} d_{11}\ d_{12}.........d_{1n} \\ d_{21}\ d_{22}.........d_{2n} \\ ....\,.......... \\ d_{N1}\ d_{N2}.....d_{N\,n} \end{pmatrix}.$$

Imposing on the system $\boldsymbol{\delta}_q$ continuous of absence of systematic errors and ensuring a minimum variances, we obtain the expression for matrix **D** at which the desired optimum is reached:

$$\mathbf{D} = \mathbf{B}^{-1}\mathbf{A}^T(\tilde{\mathbf{K}}^{-1})^T\,, \qquad (5)$$

where $\tilde{\mathbf{K}}^{-1}$ is squared matrix of dimension n x n inverse to matrix **K**:

$$\tilde{\mathbf{K}} = \tilde{\mathrm{K}}_{nn} = \begin{pmatrix} k_{11}\ \mathrm{k}_{12}.........k_{1n} \\ k_{21}\ k_{22}.........k_{2n} \\ ....\,.......... \\ k_{n1}\ k_{n2}.........k_{nn} \end{pmatrix},$$

where the quantity k_{ij} represents the ratio of the correlation coefficient of *i*–th and *j*–th random variables of the system (1) to the product of the weights of these variables. The weight p_i is assumed as the ratio of the standard deviation of the weight unit σ_0 to the standard deviation σ_i characterizing the respective random variables of the system (1).

The matrix **B** (the dimension N x N) in the equation (5) is expressed in terms of known matrixes as following:

$$\mathbf{B} = \mathbf{A}^T(\tilde{\mathbf{K}}^{-1})^T\mathbf{A}\,. \qquad (6)$$

Introducing the matrix **S** of dimension n x N

$$\mathbf{S} = \mathrm{S}_{nN} = \tilde{\mathbf{K}}^{-1}\mathbf{A} = \begin{pmatrix} s_{11}\ \mathrm{s}_{12}.........s_{1N} \\ s_{21s}\ s_{22}.........s_{2N} \\ ....\,.......... \\ s_{n1}\ \mathrm{s}_{n2}.........s_{nN} \end{pmatrix},$$

the required solution may be written in the form:

$$\boldsymbol{\delta}_q = \mathbf{B}^{-1}\mathbf{S}^T\boldsymbol{\delta}_r\,. \qquad (7)$$

If we substitute the matrix $\overline{\boldsymbol{\Delta}_r}$ in the equation (7) instead of the matrix $\boldsymbol{\delta}_r$, that is the system of random variables (1) is replaced by their particular realization, then we obtain the matrix of needed parameters:

$$\boldsymbol{\Delta}_q^{(0)} = \mathbf{B}^{-1}\mathbf{S}^T\overline{\boldsymbol{\delta}_r}\,, \qquad (8)$$

which at the given composition of measurements and for given form of correlation is optimal in the mentioned sense.

The obtained expression is the solution of the following linear system of equations:

$$\mathbf{A}^T\mathbf{S}\boldsymbol{\Delta}_q^{(0)} = \mathbf{S}^T\overline{\boldsymbol{\Delta}_r}\,, \qquad (9)$$

which we will call the basic system of the considered method/ Note that when the system (1) is the system of independent random variables, the basic system (9) is transformed into known system of normal equations.

IV. The method of evaluating the accuracy of the information

It is very often necessary to evaluate the accuracy of the various linear combinations of the unknown parameters in solving problems related to the processing of the measurements, along with estimate of the accuracy of each parameter. Then the solution of the given problem it is advisable to carry out for a linear function of the desired parameters, representing the most general case of linear combinations of the above.

Let a linear function of the required parameters is given by the expression:

$$\mathbf{F} = \varphi\boldsymbol{\delta}_q\,, \qquad (10)$$

where the matrix φ is defined as

$$\varphi = \varphi_{1N} = (\varphi_1, \varphi_2, ..., \varphi_N)\,.$$

As a characteristic of accuracy of the definition of the linear function (10) we will take its variance σ_F^2. Then the following relation holds

$$\sigma_F^2 = \sigma_0^2\varphi\mathrm{Q}\varphi^T\,, \qquad (11)$$

where the matrix **Q** has the dimension N x N and is equal to

$$\mathbf{Q} = \mathrm{Q}_{NN} = \mathbf{B}^{-1}.$$

Thus, in relation (11) weight unit standard deviation σ_0 remains uncertain. In order to determine the value of σ_0, we introduce two systems of random variables u_i^* and u_i, which are defined by the following matrixes:

$$\mathbf{u}^* = \mathrm{u}_{n1}^* = \mathbf{A}\boldsymbol{\delta}_q - \boldsymbol{\delta}_r , \qquad (12)$$

$$\mathbf{u} = \mathrm{u}_{n1} = \mathbf{J}\boldsymbol{\delta}_q - \boldsymbol{\delta}_r . \qquad (13)$$

The matrix **J** is represented by the product:

$$\mathbf{J} = \mathbf{P}^{-1}\mathbf{S} ,$$

where the diagonal matrix **P** is determined as:

$$\mathbf{P} = P_{nn} = \begin{pmatrix} p_1^2 0 \ldots\ldots\ldots\ldots 0 \\ 0 \quad p_2^2 0 \ldots\ldots\ldots . 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots . \\ 0 \quad 0 \ldots\ldots\ldots\ldots p_n^2 \end{pmatrix} .$$

The random variables u_i^* and u_i represent the errors, characterizing the deviations of the measured parameters from their values obtained by processing. The second group of random variables contains a constant term, which is expectation of these variables. The values of random variables u_i^* and u_i can be calculated for each particular realization. In this case in the expressions (12) and (13) the values δ_r are replaced by their values obtained in the measurements, and the values δ_q are replaced by the values, calculated by the considered above method. Then the standard deviation of weight unit is calculated as:

$$\sigma_0^2 = \frac{\sum_{i=1}^{n} p_i^2 \overline{u_i u_i^*}}{n - N^*} = \frac{\sum_{i=1}^{n} p_i^2 \overline{u_i^* u_i^*}}{n - N^*} . \qquad (14)$$

Here the values $\overline{u_i u_i^*}$ and $\overline{u_i^* u_i^*}$ represent the expectations of the respective random variables and the value N is defined by the expression:

$$N^* = \sum_{i=1}^{n} p_i^2 \sum_{j=1}^{N} a_{ij} \sum_{m=1}^{n} a_{jm} \tilde{k}_{mi} ,$$

where p_i, a_{ij}, d_{jm} and $\tilde{k}_{mi}$ are the elements of the respective matrixes **P**, **A**, **D** and $\tilde{\mathbf{K}}$.

To calculate the numerator in the expression (14) we can only use the approximate values of the sums included in the numerator because the exact value of the expectation of the respective products u_i^* and u_i is unknown for the given composition of the measurements. Then:

$$\sum_{i=1}^{n} p_i^2 \overline{u_i u_i^*} \approx \sum_{i=1}^{n} p_i^2 u_i u_i^* = \sum_{i=1}^{n} p_i^2 \left(\sum_{j=1}^{n} a_{ij} \Delta_{qj}^{(0)} - \tilde{\Delta}_{ri} \right) \left(\sum_{j=1}^{n} d_{ij} \Delta_{qj}^{(0)} - \tilde{\Delta}_{ri} \right)$$

$$\sum_{i=1}^{n} p_i^2 \overline{u_i^* u_i^*} \approx \sum_{i=1}^{n} p_i^2 u_i^* u_i^* = \sum_{i=1}^{n} p_i^2 \left(\sum_{j=1}^{n} a_{ij} \Delta_{qj}^{(0)} - \tilde{\Delta}_{ri} \right)^2$$

Note, that if the system (1) represents the system of the independent random variables, then the relationship (14) is transformed into well-known Gauss formula:

$$\sigma_0^2 = \frac{\sum_{i=1}^{n} p_i^2 \overline{u_i u_i^*}}{n - N} . \qquad (15)$$

V. Choosing a measurement information

In order to analyze the influence of correlation between measurements on the processing accuracy it is necessary to choose a measurement which would provide the ability to compare the results of processing on equal conditions. In particular, it is necessary that the groups of measurements which differ in the degree of correlation would be equaled in size and are based on the same particular realization. In addition, it should be known probability characteristics of measurement errors and the exact values of the measured parameters.

To obtain such measurement information, consider the random function $\boldsymbol{\delta}_r(t)$ with the expectation $\Delta_r(t)$ and the correlation function

$$K_\delta(\tau) = De^{-\varepsilon|\tau|} , \qquad (16)$$

where $D = D[\delta_r] = \mathrm{const}$ is the variance of the random function $\delta_r(t)$, $\tau = t_2 - t_1$ is the difference between two given points in time within the measurement of the considered random function.

It is evident, that the random function $\delta_r(t)$ is a stationary one.

Choosing a stationary function provides the ability to simplify the solution of the given problem with no damage its community.

From random stationary function theory it follows that the spectral density of the random function having a correlation function (16) can be written in the form:

$$S_\delta(\omega) = \frac{D_\varepsilon}{\pi\left(\varepsilon^2 + \omega^2\right)} , \qquad (17)$$

where ω represents the frequency spectrum of the random function $\delta_r(t)$, which may be represented as the following expansion:

$$\delta_r(t) = \sum_{k=0}^{\infty}\left(u_k \cos\omega_k t + v_k \sin\omega_k t\right) + \Delta_r(t) . \qquad (18)$$

In this expression the variables u_k and v_k ($k = 0, 1, 2, \ldots$) are two systems of mutually independent random variables, variance values of which are determined by the respective values of the variance spectrum, and expectations are zero. The spectrum of variances may be defined because the spectral density is known.

To obtain particular realizations of the considered random function we transform the relationship (18). Let's assume that the frequency spectrum is confined by the value ω_m. Next we divide the considered spectrum into m intervals, so that:

$$D_k = \int_{\omega_{k-1}}^{\omega_k} S_\delta(\omega) d\omega = \frac{D}{\pi}\left(\operatorname{arctg}\frac{\omega_k}{\varepsilon} - \operatorname{arctg}\frac{\omega_{k-1}}{\varepsilon}\right), \quad k = 1, 2, ..., m. \quad (19)$$

Let's represent the random values of expansion (18) in the form of products:

$$u_k = \sqrt{D_k \varphi_k}, \qquad v_k \sqrt{D_k \lambda_k}, \qquad (20)$$

where φ_k and λ_k are normalized on (0, 1) mutually independent random values. Substituting the obtained values u_k and v_k into expansion (18) and taking into account the relationship (19) we get:

$$\delta_r(t) = \sqrt{\frac{D}{\pi}} \sum_{k=1}^{m} \sqrt{\operatorname{arctg}\frac{\omega_k}{\varepsilon} - \operatorname{arctg}\frac{\omega_{k-1}}{\varepsilon}} \left[\varphi_k \cos\omega_k t + \lambda_k \sin\omega_k t\right] + \Delta_r(t). \quad (21)$$

Choosing $2p$ values normalized to the (0, 1) random numbers that will be represent particular realization of the system of random values, and substituting these values into the equation (21), we obtain a particular realization of the random function in the form:

$$\tilde{\Delta}_r(t) = \sqrt{\frac{D}{\pi}} \sum_{k=1}^{m} \sqrt{\operatorname{arctg}\frac{\omega_k}{\varepsilon} - \operatorname{arctg}\frac{\omega_{k-1}}{\varepsilon}} \left[\tilde{\varphi}_k \cos\omega_k t + \tilde{\lambda}_k \sin\omega_k t\right] + \Delta_r(t). \quad (22)$$

Choosing from this function the values corresponding specific points in time, we obtain the required particular realization of the system $\delta_{ri} (i = 1, 2, ..., n)$ with known correlation function and expectation.

Note, that for the case where m is sufficiently large, and the intervals are the same, the following relation can be obtained for a particular realization of the random function $\delta_r(t)$:

$$\tilde{\Delta}_r(t) \approx \sqrt{\frac{D}{\pi}\frac{\omega_m}{m}} \sum_{k=1}^{m} \sqrt{\frac{1}{1+\left(\frac{\omega_k}{\varepsilon}\right)^2}} \left[\tilde{\varphi}_k \cos\omega_k t + \tilde{\lambda}_k \sin\omega_k t\right] + \Delta_r(t). \quad (23)$$

VI. Conclusions

The above method of mathematical processing of the dependent measurements and the way of obtaining measuring information with known probability characteristics allow reliably to estimate the influence of correlation between measurements. To estimate this influence it is necessary to have several variants of measuring information, characterized various types of correlation.

In the considered method the main criterion, characterizing the degree of correlation is the parameter ε. When ε - values close to zero, there is a sufficiently high degree of dependence of the individual values of the random function. It is obvious that for large values of the parameter ε we have a weak dependency that is the individual values are practically independent.

The considered method may be used in software for data base to carry out the processing of measuring information, obtained in flight tests of on-board automatic landing systems.

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О.А.Зеленков, А.П.Голік, О.В.Молчанов Оцінка точності результатів обробки інформації з врахуванням впливу кореляції вимірювань.

Розглянуто спосіб обробки вимірювальної інформації, який враховує кореляційний зв'язок між вимірюваннями і може бути застосований для оцінки точнісних характеристик бортових систем автоматичного приземлення на етапах експлуатаційного контролю.

Ключові слова: вимірювальна інформація, метод найменших квадратів, кореляційна функція, випадкова величина, погрішності вимірювання, спектральна густина, реалізація випадкового процесу.

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Оценка точности результатов обработки информации с учетом влияния коррелированности измерений.

Рассмотрен способ обработки измерительной информации, учитывающий корреляционную связь между измерениями, который может быть использован для оценки точностных характеристик бортовых систем автоматического приземления на этапах эксплуатационного контроля.

Ключевые слова: измерительная информация, метод наименьших квадратов, корреляционная функция, случайная величина, погрешности измерений, спектральная плотность, реализация случайного процесса.

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