National Aviation University

MACHINE ELEMENTS

TERM PAPER DESIGNING

Manual

KYIV 2011

National Aviation University

MACHINE ELEMENTS

TERM PAPER DESIGNING

Manual

KYIV 2011
National aviation university publishing house «NAU-druk»

УДК 621.8 (075.8)
ББК К 420 я 7
М 13

Reviewed by: senior researcher, prof. L.S.Novogrudskyi, prof. O.O.Boronko, associated prof. V.O.Koltsov

The English language adviser: L.V.Budko

Approved by the Methodical and Editorial Board of the National Aviation University ().

M 13 Machine elements. Term paper designing: manual / Authors: A.S.Kryzhanovskyi, A.O.Kornienko, O.V.Bashta, V.M.Pavlov, E.M.Babenko,. – K.: National aviation university publishing house «NAU-druk», 2011. – 128 p.

The manual includes recommendations for carrying out the term paper on the subject “Machine Elements” and examples of analysis and design of double stage speed reducers.

The manual is intended for students of direction 6.070103 “Aircraft Maintenance”.

INTRODUCTION

The term paper on the subject "*Machines Elements"* is one of the basic kinds of the students individual work. The purpose of the term paper is to enhance the knowledge acquired by the student at the lectures, practical classes and laboratory sessions as well as to develop the skills of making research and design of present-day speed reducers.

The term paper is to include the following parts:

1. Determination of speed reducer elements geometrical parameters and carrying out their strength analysis.
2. Designing a speed reducer.
3. Designing a mechanical drive

All calculations are to be presented at the explanatory note that should be carried out according to requirements of State Standard «ДСТУ 3008-95. Державний стандарт України. Документація. Звіти в сфері науки і техніки. Структура і правила оформлення». The explanatory note should be either typed or hand written in blue or black ink on one side of size A4 paper. Every sheet is to be paginated and have the following margins: top – 5 mm, bottom – 5 mm, right – 5 mm, and left – 20 mm.

Besides calculations the explanatory note should have the contents table, the assignment, the reference literature used for making the term paper, specifications. Each new part should be begun with a new page.

Each part must be subdivided into items marked with numerals separated by a point. The first numeral represents the number of the part, the second one shows the number of the item.

All magnitudes that are part of formulas should be explained. Besides, it is necessary to denote measurement units of parameters being calculated.

The graphical part consists of 4 drawings: 1st drawing is a speed reducer (two projections); 2nd drawing is a mechanical drive (two projections); 3rd drawing is shop drawing of a speed reducer shaft; 4th drawing is shop drawing of a speed reducer gear. The 1st and the 2nd drawings are made on A1 whatman paper; the 3rd and the 4th drawings are made on A3 whatman paper. The title block should be drawn in the bottom right hand corner.

1. KINEMATIC AND FORCE ANALYSIS OF A MECHANICAL DRIVE

Let us determine the basic parameters of the mechanical drive (fig. 1.1) if pull of the belt F_t=8 kN; belt speed V=0.7 m/sec; drum diameter D=400 mm.

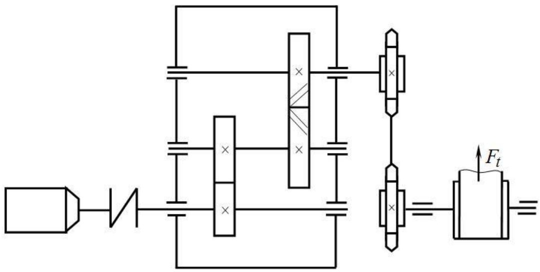


Fig. 1.1. Diagram of double stage speed reducer with chain drive

1.1. Determine the output power of the drive

$$P_{out} = F_t \cdot V = 8 \cdot 0.7 = 5.6 \text{ kW}$$

1.2. Determine the total efficiency of the drive

In general

$$\eta=\eta_1\cdot\eta_2\cdot\ldots\cdot\eta_n ,$$

where $\eta_1, \eta_2, \ldots \eta_n$ are efficiencies of all kinematic pairs and links where the input power is lost.

For our case

$$\eta = \eta_c \cdot \eta_{ssg} \cdot \eta_{hsg} \cdot \eta_{cd} \cdot \eta_b^4,$$

where η_c is the efficiency of the coupling;

η_{ssg} is the efficiency of straight spur gears;

η_{hsg} is the efficiency of helical spur gears;

η_{cd} is the efficiency of the chain drive;

η_b takes into account losses in one pair of bearings.

The magnitudes of all efficiencies are given in table 1.1.

The following components are recommended to use for mechanical drives being analyzed:

- belt drive with flat belt;
- chain drive with roller chain;
- rolling bearings;
- coupling with rubber bushed studs if it is mounted at the input;

- rigid coupling (flange coupling) or flexible coupling (chain coupling) if it is mounted at the output.

Table 1.1

The magnitudes of efficiencies of the drives

Name	Efficiency	
	Closed drive	Opened drive
Gearings:		
- straight spur gears	0.98 - 0.99	0.94 - 0.96
- helical spur gears	0.97 - 0.98	0.94 - 0.95
- bevel gears	0.96 - 0.98	0.92 - 0.94
Worm gearing:		
-one thread worm	0.7 - 0.75	
-two thread worm	0.75 - 0.82	
-four thread worm	0.82 - 0.92	
Belt drives:		
- flat belt drive		0.96 - 0.98
- V-belt drive		0.95 - 0.97
- toothed belt drive		0.94 - 0.97
Chain drives:		
- roller chain		0.94 - 0.96
- toothed chain		0.96 - 0.97
Couplings:		
- with rubber bushed studs	0.996	
- flexible coupling	0.985 - 0.995	
- rigid coupling	1	
Bearings:		
- rolling bearings	0.99 - 0.995	
- sliding bearings	0.98 - 0.985	

Let us assume that η_{c} = 0.996, η_{ssg} = 0.98, η_{hsg} = 0.97, η_{cd} = 0.94, η_b= 0.99. Then

$$\eta = 0.996 \cdot 0.98 \cdot 0.97 \cdot 0.94 \cdot 0.99^4 = 0.8549.$$

Pay attention that the magnitude of the total efficiency must be rounded off to thousandth or ten thousandth.

1.3. Determine the input power

Taking into account that the efficiency is determined as ratio of the output power to the input one

$$\eta = \frac{P_{out}}{P_{inp}}$$

we can find needed power of the electrical motor

$$P_{inp} = \frac{P_{out}}{\eta} = \frac{5.6}{0.8549} = 6.55\,kW\,.$$

1.4. Select the electrical motor

For given mechanical drives we will use asynchronous electrical motor. It is explained by the fact that in comparison with the other types of motors asynchronous electrical motors are simpler in design and maintenance, more reliable and less expensive. The most widely spread asynchronous motors are series 4A squirrel cage induction motors.

Asynchronous motors are chosen by means of table 1.2 depending on the input power P_{inp} of a mechanical drive and the synchronous rotational speed n_s (rotational speed of a magnetic field that characterizes operation of the motor without load). For given mechanical drives asynchronous motors are used either synchronous rotational speed n_s= 1500 rpm or n_s= 1000 rpm.

Table 1.2.

Parameters of asynchronous motors

Rated power P_r, kW	Synchronous rotational speed n_s, rpm					
	3000		1500		1000	
	Type designation	S,%	Type designation	S,%	Type designation	S,%
0.55	63B2	8.5	71A4	7.3	71B6	10
0.75	71A2	5.9	71B4	7.5	80A6	8.4
1.1	71B2	6.3	80A4	5.4	80B6	8.0
1.5	80A2	4.2	80B4	5.8	90L6	6.4
2.2	80B2	4.3	90L4	5.1	100L6	5.1
3.0	90L2	4.3	100S4	4.4	112MA6	4.7
4.0	100S2	3.3	100L4	4.7	112MB6	5.1
5.5	100L2	3.4	112M4	3.7	132S2	3.3
7.5	112M2	2.5	132S4	3.0	132M6	3.2
11.0	132M2	2.3	132M4	2.8	160S6	2.7
15	160S2	2.1	160S4	2.3	160M6	2.6
18.5	160M2	2.1	160M4	2.2	180M6	2.7
22	180S2	2.0	180S4	2.0	200M6	2.8
30	180M2	1.9	180M4	1.9	200L6	2.1

For our mechanical drive we select 4A132S4 Induction Motor (P_r = 7.5 kW, n_s = 1500 rpm).

1.5. Determine the motor rated rotational speed n_r

$$n_r = n_s (1 - \frac{S}{100}),$$

where S is relative speed loss that is determined according to table 1.2. In our case S = 3 %. After substituting corresponding magnitudes we obtain

$$n_r = 1500 \cdot (1 - \frac{3}{100}) = 1455 \text{ rpm}.$$

1.6. Determine the output rotational speed

$$n_{out} = \frac{60 \cdot V}{\pi \cdot D} = \frac{60 \cdot 0.7}{3.14 \cdot 0.4} = 33.44 \text{ rpm}.$$

1.7. Determine the total velocity ratio of the mechanical drive

$$u = \frac{n_{inp}}{n_{out}} = \frac{1455}{33.44} = 43.51 \cdot$$

1.8. Distribute the total velocity ratio between mechanical drive steps

The total velocity ratio can be found by the formula

$$u = u_{red} \cdot u_{cd},$$

where u_{red} is the speed reducer velocity ratio; u_{cd} is the chain drive velocity ratio.

First, determine the velocity ratio of open transmissions, in particular of the belt drive or the chain drive. In general for the belt drive u_{bd} is ranged from 2 to 4, for the chain drive u_{cd} is ranged from 1.5 to 4.

For our mechanical drive let u_{cd} = 2.8.

Determine the speed reducer velocity ratio u_{red}

$$u_{red} = \frac{u}{u_{cd}} = \frac{43.51}{2.8} = 15.54.$$

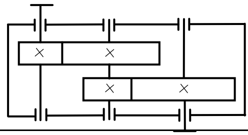
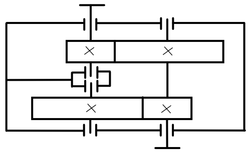
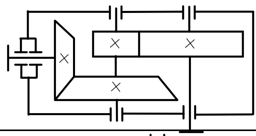
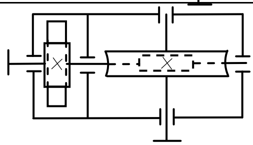
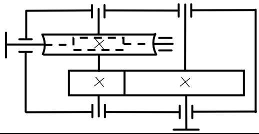
On the other hand

$$u_{red} = u_{ssg} \cdot u_{hsg},$$

where u_{ssg} is the straight spur gears velocity ratio; u_{hsg} is the helical spur gears velocity ratio. These velocity ratios are found by means of table 1.3 (here u_h – velocity ratio of high-speed transmission, u_l – velocity ratio of low- speed transmission) depending upon the speed reducer type.

Table 1.3

Velocity ratios of speed reducers

Type of speed reducer	Diagram	Formula	Recommended magnitudes
Double-stage spur gear speed reducer		$u_h = \frac{u_{red}}{u_\ell}$	From 2 to 5
		$u_l = 0.88\sqrt{u_{red}}$	
Double-stage coaxial spur gear speed reducer		$u_h = \frac{u_{red}}{u_\ell}$	From 2 to 5
		$u_l = 0.95\sqrt{u_{red}}$	
Double-stage bevel-spur gear speed reducer		$u_h = 0.25u_{red}$	From 2 to 4
		$u_l = \frac{u_{red}}{u_h}$	From 3 to 5
Double-stage spur-worm gear speed reducer		u_h from 2 to 2.5	
		u_l from 14 to 30	
Double-stage worm-spur gear speed reducer		$u_h = \frac{u_{red}}{u_\ell}$	From 8 to 30
		(0.03 to 0.06) $\cdot u_{red}$	From 3 to 5

For mentioned above mechanical drive as high-speed transmission the straight spur gears are considered and as low-speed transmission the helical spur gears are used. According to the table 1.3

$$u_{hsg} = 0.88\sqrt{u_{red}} = 0.88\sqrt{15.54} = 3.47;$$

$$u_{ssg} = \frac{u_{red}}{u_{hsg}} = \frac{15.54}{3.47} = 4.48.$$

If obtained magnitudes of velocity ratios are greater than recommended values it is necessary to decrease the motor synchronous rotational speed n_s.

1.9. Determine the rotational speed of all shafts

$n_1 = n_r = 1455$ rpm;

$$n_2 = \frac{n_1}{u_{ssg}} = \frac{1455}{4.48} = 324.77 \ \text{rpm};$$

$$n_3 = \frac{n_2}{u_{hsg}} = \frac{324.77}{3.47} = 93.6 \ \text{rpm};$$

$$n_4 = \frac{n_3}{u_{cd}} = \frac{93.6}{2.8} = 33.43 \ \text{rpm}.$$

Obtained magnitude of n_4 must be equal to n_{out} calculated according to p. 1.6. Error ε must be not more than 1%. In our case ε=0.03%.

1.10. Determine the angular velocity of all mechanical drive shafts

$$\omega_1 = \frac{\pi n_1}{30} = \frac{3.14 \cdot 1455}{30} = 152.29 \sec^{-1};$$

$$\omega_2 = \frac{\omega_1}{u_{ssg}} = \frac{152.29}{4.48} = 34 \ \sec^{-1};$$

$$\omega_3 = \frac{\omega_2}{u_{hsg}} = \frac{34}{3.47} = 9.798 \ \sec^{-1};$$

$$\omega_4 = \frac{\omega_3}{u_{cd}} = \frac{9.798}{2.8} = 3.499 \ \sec^{-1}.$$

1.11. Determine the power at mechanical drive shafts.
Calculation is carried out for P_{inp}, determined in p.1.3.

$P_1 = P_{inp} \cdot \eta_c \cdot \eta_b = 6.55 \cdot 0.996 \cdot 0.99 = 6.458$ kW;
$P_2 = P_1 \cdot \eta_{ssg} \cdot \eta_b = 6.458 \cdot 0.98 \cdot 0.99 = 6.265$ kW;
$P_3 = P_2 \cdot \eta_{hsg} \cdot \eta_b = 6.265 \cdot 0.97 \cdot 0.99 = 6.017$ kW;
$P_4 = P_3 \cdot \eta_{cd} \cdot \eta_b = 6.017 \cdot 0.94 \cdot 0.99 = 5.6$ kW.

Obtained magnitude of P_4 must be equal to P_{out} calculated according to the p.1.1. Error should not be greater than 1%. In our case ε=0.

1.12. Determine the torques at all shafts.

$$T_1 = \frac{P_1}{\omega_1} = \frac{6.458 \cdot 10^3}{152.29} = 42.406 \ \text{N} \cdot \text{m};$$

$$T_2 = \frac{P_2}{\omega_2} = \frac{6.265 \cdot 10^3}{34} = 184.265 \ \text{N} \cdot \text{m};$$

$$T_3 = \frac{P_3}{\omega_3} = \frac{6.017 \cdot 10^3}{9.8} = 613.98 \ \text{N} \cdot \text{m};$$

$$T_4 = \frac{P_4}{\omega_4} = \frac{5.6 \cdot 10^3}{3.499} = 1600.45 \ \text{N} \cdot \text{m}.$$

Checking.

The output torque T_{out} can also be found as

$$T_{out} = F_t \frac{D}{2} = \frac{8 \cdot 10^3 \cdot 0.4}{2} = 1600 \ \text{N} \cdot \text{m}.$$

Determine the error. It should be less than 1%.

$$\varepsilon = \frac{1600.45 - 1600}{1600.45} \cdot 100\% = 0.03 \ \%.$$

2. Analysis of allowable stresses

Let us analyze the speed reducer of the mechanical drive when: n^p = 550.19 rpm; n^g = 183.4 rpm.

We will begin from the selection of the material of gears and determination of their allowable contact and bending stresses.

2.1. Select the material of toothed wheels.

The main material of toothed wheels is carbon or alloy steels. Depending on material's hardness toothed wheels are subdivided into two groups:

- toothed wheels with surface hardness $H \le 350\,\text{BHN}$;
- toothed wheels with surface hardness $H > 350\,\text{BHN}$.

For general purpose speed reducers the following alternatives are possible:

1. A pinion and a gear are produced from identical carbon or alloy steel, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni). Heat treatment of both the gear and the pinion is martempering. The pinion hardness is ranged from 269 to 302 BHN and the gear hardness is ranged from 235 to 262 BHN.
2. A pinion and a gear are produced from identical alloy steel, such as 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 35XM (0.35C-Cr-Mo). Heat treatment of the gear is martempering to hardness ranged from 269 to 302 BHN. Heat treatment of the pinion is martempering and surface (induction) hardening to hardness ranged from 45 to 50 HRC.

Toothed wheels of the straight spur gears are recommended to produce according to the 1st alternative. If we deal with either helical spur gears or bevel gears we can use either the 1st or the 2nd alternative.

2.2. Determine the mean magnitude of the hardness of the gear and the pinion:

- for the pinion $$H^p_m = \frac{H^p_{min} + H^p_{max}}{2};$$
- for the gear $$H^g_m = \frac{H^g_{min} + H^g_{max}}{2}.$$

In our case a pinion and a gear are produced from identical alloy steel 40XH and we use the 2nd alternative (heat treatment of the gear is martempering to hardness ranged from 269 to 302 BHN; heat treatment of the pinion is martempering and surface hardening to hardness ranged from 45 to 50 HRC):

$$H_m^p = \frac{45+50}{2} = 47.5\,\text{HRC}; \qquad H_m^g = \frac{269+302}{2} = 285.5\,\text{BHN}.$$

2.3. Determine the allowable contact stress for the pinion and for the gear.

Determine the limit of contact endurance for the pinion $\sigma_{H\,lim}^p$ and for the gear $\sigma_{H\,lim}^g$ according to the table 2.1.

$$\sigma_{H\lim}^p = 17 \cdot H_m^p + 200 = 17 \cdot 47.5 + 200 = 1007.5\text{MPa}$$

$$\sigma_{H\lim}^g = 2 \cdot H_m^g + 70 = 2 \cdot 285.5 + 70 = 641\text{MPa}$$

Determine the base number of stress cycles for the pinion N_{H0}^p and for the gear N_{H0}^g. For this purpose we use table 2.2.

$N_{H0}^p = 68.9 \cdot 10^6$ stress cycles;

$N_{H0}^g = 22.5 \cdot 10^6$ stress cycles.

2.3.3. Determine the service life in hours for the gearing:

$$t = L \cdot 365 \cdot K_a \cdot 24 \cdot K_d,$$

where L is the service life in years; K_a is the annual utilization factor that takes into account use of the gearing during a year; K_d is the daily utilization factor that takes into account use of the gearing for 24 hours. These parameters should be given in the specification for the term paper.

In our case the service life of the gearing is 8 years, $K_a = 0.7$, $K_d = 0.3$

$$t = 8 \cdot 365 \cdot 0.7 \cdot 24 \cdot 0.3 = 14716.8\,\text{hours}.$$

2.3.4. Determine the factor K_{HE} that reduces variable load conditions to the constant load equivalence.

$$K_{HE} = \sum_{i=1}^{n} \frac{t_i}{t} \cdot \left(\frac{T_i}{T_{max}} \right)^3,$$

where T_{max} and T_i are correspondingly maximum and acting torques; t_i is the time of action of the torque T_i.

Note: If the time of the torque action is less than $0.03 \cdot t$, this torque should not be taken into account.

Table 2.1

Contact and bending limits of endurance

Heat treatment	Tooth hardness		Gear material	$\sigma_{H\,lim}$, MPa	$\sigma_{b\,lim}$, MPa
	case	core and root			
Normalizing, martempering	Brinell 180 to 350		Carbon and alloy steels, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 50XH (0.50C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$2H_m + 70$	$1.8H_m$
Full hardening	Rockwell C, 40 to 55		Carbon and alloy steels, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$18H_m + 150$	500
Surface hardening	Rockwell C, 40 to 58	Rockwell C, 25 to 35	Alloy steels, such as 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 50XH (0.50C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$17H_m + 200$	650
Case hardening	Rockwell C, 54 to 64	Rockwell C, 30 to 45	Alloy steels, such as 20XH2M (0.20C-Cr-2Ni-Mo)	$23H_m$	950
Nitriding	Rockwell C, 50 to 60	Rockwell C, 24 to 40	Alloy steels, such as 40XH2MA (0.40C-Cr-2Ni-Mo, quality)	1050	$300 + 1.2H_m$ (of tooth core)

Table 2.2

Base number of stress cycles

BHN_m	up to 200	250	300	350	400	450	500	550	600
HRC_m	-	25	32	38	43	47	52	56	60
$N_{H0}\cdot 10^6$	10	16.5	25	36.4	50	68	87	114	143

In our case according to the load diagram:

t_i: *0.003t 0.15t; 0.25t; 0.6t;*

T_i: *1.3T T; 0.7T; 0.5T.*

$$K_{HE} = \frac{0.15t}{t}\cdot\left(\frac{T}{T}\right)^3 + \frac{0.25t}{t}\left(\frac{0.7T}{T}\right)^3 + \frac{0.6t}{t}\left(\frac{0.5T}{T}\right)^3 =$$

$$= 0.15 + 0.25\cdot 0.7^3 + 0.6\cdot 0.5^3 = 0.311$$

2.3.5. Determine the equivalent number of cycles for the pinion and the gear.

$$N_{HE}^{p} = 60\cdot n^{p}\cdot c\cdot t\cdot K_{HE};$$

$$N_{HE}^{g} = 60\cdot n^{g}\cdot c\cdot t\cdot K_{HE}$$

where n^p and n^g are correspondingly rotational speeds of the pinion and the gear; c is the number of gears meshing with the gear being analyzed. In our case $c = 1$.

2.3.6. Determine the durability factor for the pinion and for the gear.

if $N_{HE} \geq N_{HO}$ then $K_{HL}=1$,

if $N_{HE} < N_{HO}$ then $K_{HL} = \sqrt[6]{\frac{N_{H0}}{N_{HE}}}$.

$$N_{HE}^{p} = 60\cdot 550.19\cdot 1\cdot 14716.8\cdot 0.311 = 151.091\cdot 10^6;\ N_{H0}^{p} = 68.9\cdot 10^6,$$

$$N_{HE}^{p} > N_{H0}^{p}\text{, consequently } K_{HL}^{p} = 1;$$

$$N_{HE}^{g} = 60\cdot 183.4\cdot 1\cdot 14716.8\cdot 0.311 = 50.364\cdot 10^6;\ N_{H0}^{g} = 22.5\cdot 10^6,$$

$$N_{HE}^{g} > N_{H0}^{g}\text{, consequently } K_{HL}^{g} = 1.$$

2.3.7. Determine the safety factor S_H for the pinion and for the gear.

- for homogeneous structure of the material (heat treatment is normalizing, martempering and full hardening) $S_H = 1.1$;
- for heterogeneous structure of the material (heat treatment is surface hardening, case hardening, nitriding) $S_H = 1.2$.

2.3.8. Determine the contact allowable stresses for the gear and for the pinion

$$\left[\sigma_H^p\right] = \frac{\sigma_{Hlim}^{p}\cdot K_{HL}}{S_H^p}, \qquad \left[\sigma_H^g\right] = \frac{\sigma_{Hlim}^{g}\cdot K_{HL}}{S_H^g}.$$

In our case: $S_H^p = 1.2;\ S_H^g = 1.1;$

$$\left[\sigma_H^p\right] = \frac{1007.5 \cdot 1}{1.2} = 839.58\text{MPa}; \quad \left[\sigma_H^g\right] = \frac{641 \cdot 1}{1.1} = 582.73\text{MPa}.$$

If H^p-$H^g \leq 70$BHN we assume as the design allowable contact stress the less magnitude of above calculated stresses, where H^p and H^g are correspondingly hardness of the pinion and gear materials.

Otherwise, the design allowable contact stress is determined by the following formula

$$\left[\sigma_H\right] = 0.45 \cdot \left(\left[\sigma_H^p\right] + \left[\sigma_H^g\right]\right) \leq 1.23 \cdot \left[\sigma_H^g\right].$$

In our case H^p-$H^g > 70$BHN. That is why

$$\left[\sigma_H\right] = 0.45 \cdot (839.58 + 582.73) = 640.04\text{MPa} \leq 1.23 \cdot \left[\sigma_H^g\right] = 716.76\text{MPa}.$$

Thus, for further calculations we assume as the design allowable contact stress $\left[\sigma_H\right] = 640.04\text{MPa}$.

2.4. Determine the allowable bending stresses of the pinion and for the gear.

2.4.1 . Determine the limits of the bending endurance for the pinion $\sigma_{b\,lim}^p$ and for the gear $\sigma_{b\,lim}^g$. For this purpose we use table 2.1.

$$\sigma_{b\,lim}^p = 650\text{MPa}; \quad \sigma_{b\,lim}^g = 1.8 \cdot 285.5 = 513.9\text{MPa}.$$

2.4.2. Determine the base number of stress cycles N_{b0}.

For steels $N_{b0} = 4 \cdot 10^6$.

2.4.3. Determine the factor K_{bE} that reduces variable load conditions to the constant load equivalence.

$$K_{bE} = \sum_{i=1}^{n} \frac{t_i}{t} \cdot \left(\frac{T_i}{T_{max}}\right)^k,$$

where k=3 for toothed wheels with hardness $H \leq 350$ BHN.

k=9 for toothed wheels with hardness $H > 350$ BHN.

In our case k=3 for the gear material, and k=9 for the pinion material.

That is why

$$K_{bE}^p = \frac{0.15t}{t} \cdot \left(\frac{T}{T}\right)^9 + \frac{0.25t}{t}\left(\frac{0.7T}{T}\right)^9 + \frac{0.6t}{t}\left(\frac{0.5T}{T}\right)^9 =$$

$$= 0.15 + 0.25 \cdot 0.7^9 + 0.6 \cdot 0.5^9 = 0.161$$

$$K_{bE}^{g} = \frac{0.15t}{t} \cdot \left(\frac{T}{T}\right)^3 + \frac{0.25t}{t}\left(\frac{0.7T}{T}\right)^3 + \frac{0.6t}{t}\left(\frac{0.5T}{T}\right)^3 = 0.311$$

2.4.4. Determine the equivalent number of cycles for the pinion and the gear.

$N_{bE}^{p} = 60 \cdot n_p \cdot c \cdot t \cdot K_{bE};$

$N_{bE}^{g} = 60 \cdot n_g \cdot c \cdot t \cdot K_{bE};$

$N_{bE}^{p} = 60 \cdot 550.19 \cdot 1 \cdot 14716.8 \cdot 0.161 = 78.22 \cdot 10^6;$

$N_{bE}^{g} = 60 \cdot 183.4 \cdot 1 \cdot 14716.8 \cdot 0.311 = 50.364 \cdot 10^6$.

2.4.5. Determine the durability factor for the pinion and for the gear.

if $N_{bE} \geq N_{bO}$ then K_{bL}=1,

if $N_{bE} < N_{bO}$ then $K_{bL} = \sqrt[m]{\frac{N_{b0}}{N_{bE}}}$,

where m=3 for toothed wheels with hardness $H \leq 350$ BHN and m=9 if $H > 350$ BHN.

In our case: $N_{bE}^{p} > N_{b0}^{p}$, consequently $K_{bL}^{p} = 1$;

$N_{bE}^{g} > N_{b0}^{g}$, consequently $K_{bL}^{g} = 1$.

2.4.6. Determine the safety factor S_b for the pinion and for the gear.

- for wheels made of forged blanks (our case) $S_b = 1.75$;
- for wheels made of cast blanks $S_b = 2.3$.

2.4.7. Determine the bending allowable stresses for the gear and for the pinion

$\left[\sigma_b^p\right] = \frac{\sigma_{blim}^{p} \cdot K_{bL}}{S_b^p}$, $\left[\sigma_b^g\right] = \frac{\sigma_{blim}^{g} \cdot K_{bL}}{S_b^g}$.

In our case: $S_b^p = S_b^g = 1.75$;

$\left[\sigma_b^p\right] = \frac{650 \cdot 1}{1.75} = 371.43\text{MPa}$; $\left[\sigma_b^g\right] = \frac{513.9 \cdot 1}{1.75} = 293.657\text{MPa}$

For further calculations we assume as the design allowable bending stress the less magnitude of above calculated stresses $\left[\sigma_b\right] = 293.657\text{MPa}$.

3. Analysis of the straight spur gears for strength

Let us carry out the analysis of the straight spur gears for strength if torque at the pinion shaft T^p =74 N·m; torque at the gear shaft T^g =370 N·m; velocity ratio of the gearing u=5; allowable contact stress $[\sigma_H]$=515 MPa; allowable bending stress $[\sigma_b]$=255 MPa; hardness of the gear material H^g=285 BHN, angular velocity of the gear shaft ω^g = 40 rad/sec.

3.1. Determine the centre distance of the straight spur gears

$$a_w = 0.85 \cdot (u \pm 1) \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}},$$

where upper sign ("+") is right for gears with external toothing and down sign ("-") is right for gears with internal toothing; u is the velocity ratio of the gearing; T^g is the torque at the gear shaft in N·mm; $[\sigma_H]$ is the allowable contact stress in MPa; E_{tr} is the transformed modulus of elasticity in MPa; $K_{H\beta}$ is the load concentration factor; $\psi_{ba}= b^g/a_w$ is the gear face width factor.

Transformed modulus of elasticity E_{tr} is determined as

$$E_{tr} = \frac{2 \cdot E^p \cdot E^g}{E^p + E^g},$$

where E^p and E^g are correspondingly moduli of elasticity of pinion and gear materials. Since the pinion and the gear are made of steel we can make the conclusion that $E_{tr} = E^p = E^g = 2.1 \cdot 10^5$ MPa.

Load concentration factor $K_{H\beta}$ is determined by means of table 3.1 depending upon disposition of toothed wheels with respect to bearings and factor $\psi_{bd}= b^g/d^p$. Since b^g and d^p were not determined we find this factor by the following formula

$$\psi_{bd} = \frac{b^g}{d^p} = \frac{0.5 \cdot b^g}{a_w} \cdot (u \pm 1) = 0.5 \cdot \psi_{ba} \cdot (u \pm 1),$$

where the gear face width factor ψ_{ba} is determined from table 3.2 depending upon the disposition of the gear relative to bearings and taking into account that the value of ψ_{ba} should correspond to standard. The greater ψ_{ba} the less overall dimensions of the gearing. That is why we select the greater magnitude of ψ_{ba} = 0,4.

In our case $\psi_{bd} = 0.5 \cdot 0.4 \cdot (5+1) = 1.2$, as the gear is located non-symmetrically with respect to supports we take $K_{H\beta}$=1.19.

$$a_w = 0.85 \cdot (5+1) \cdot \sqrt[3]{\frac{370 \cdot 10^3 \cdot 1.19 \cdot 2.1 \cdot 10^5}{515^2 \cdot 5^2 \cdot 0.4}} = 163\text{mm}$$

Obtained magnitude of a_w we round off to the nearest greater side according to the series given in table 3.3. We assume a_w=180 mm.

Table 3.1

Approximate values of $K_{H\beta}$

Gear arrangement with respect to bearings	Tooth surface hardness, BHN	$\psi_{bd} = \frac{b^g}{d^p}$					
		0.2	0.4	0.6	0.8	1.2	1.6
On cantilevers, ball bearings	up to 350 over 350	1.08 1.22	1.17 1.44	1.28 -	- -	- -	- -
On cantilevers, roller bearings	up to 350 over 350	1.06 1.11	1.12 1.25	1.19 1.45	1.27 -	- -	- -
Symmetrical	up to 350 over 350	1.01 1.01	1.02 1.02	1.03 1.04	1.04 1.07	1.07 1.16	1.11 1.26
Nonsymmetrical	up to 350 over 350	1.03 1.06	1.05 1.12	1.07 1.20	1.12 1.29	1.19 1.48	1.28 -

Table 3.2

Recommended values of the gear face width factor ψ_{ba}

Gear arrangement with respect to bearings	Tooth hardness	ψ_{ba}
Symmetrical	Any	0.315; 0.4; 0.5
Non-symmetrical	Brinell BHN, up to 350 Rockwell C, 40 upwards	0.315; 0.4 0.25; 0.315
On shaft cantilevers	Brinell BHN, up to 350 Rockwell C, 40 upwards	0.25 0.2
For herringbone gears	Any	0.4 ; 0.5; 0.63
For internal gears	Any	0.2

Table 3.3

Standard values of the centre distance a_w

Series 1	63	80	100	125	160	200	250	315	400	500
Series 2	71	90	112	140	180	224	280	355	450	560

Note. Series 1 should be preferred to Series 2

3.2. Determine the nominal pitch circle diameter of the gear

$$d^g = \frac{2 \cdot a_w \cdot u}{u \pm 1} = \frac{2 \cdot 180 \cdot 5}{5+1} = 300\text{mm}.$$

3.3. Determine the face width of the gear

$$b^g = \psi_{ba} \cdot a_w = 0.4 \cdot 180 = 72 \text{ mm}.$$

3.4. Determine the module according to the strength condition for bending

$$m \geq \frac{2 \cdot K_m \cdot T^g}{d^g \cdot b^g \cdot [\sigma_b]} = \frac{2 \cdot 6.8 \cdot 370 \cdot 10^3}{300 \cdot 72 \cdot 255} = 0.91 \text{mm},$$

where K_m is taken as 6.8 for straight spur gears.

Obtained magnitude of the module should be rounded off to the greater side according to the standard series given in table 3.4. It is necessary to note that for general-purpose speed reducers the minimum value of the module is m_{min} = 2 mm. In our case we assume that m = 2 mm.

Table 3.4

Standard values of m_n

Series 1	1.0	1.25	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	12.0
Series 2	1.125	1.375	1.75	2.25	2.75	3.5	4.5	5.5	7.0	9.0	11.0	14.0

Note. Series 1 should be preferred to Series 2

3.5. Determine the total number of teeth

$$z_\Sigma = \frac{2 \cdot a_w}{m} = \frac{2 \cdot 180}{2} = 180.$$

Obtained value of z_Σ should be rounded off to the nearest integer numeral.

3.6. Determine the number of teeth of the pinion

$$z^p = \frac{z_\Sigma}{u \pm 1} \geq z_{min},$$

where z_{min}=17 for straight spur gears.

Obtained value of z^p should be rounded off to the nearest integer numeral. If z^p <17 it is necessary to decrease the module or to use nonstandard toothed wheels. For our case

$$z^p = \frac{z_\Sigma}{u \pm 1} = \frac{180}{6} = 30 \geq z_{min} = 17.$$

3.7. Determine the number of teeth of the gear

$$z^g = z_\Sigma \mp z^p = 180 - 30 = 150.$$

Upper sign is right for gears with external toothing and down sign is used for gears with internal toothing.

3.8. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p} = \frac{150}{30} = 5.$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less or equal to 4%. Otherwise the number of teeth z^p, z^g and z_Σ must be rounded off to the other side.

3.9. Determine the nominal pitch circles diameters for the pinion and the gear

$$d^p = m \cdot z^p = 2 \cdot 30 = 60 \text{ mm},$$

$$d^g = 2 \cdot a_w \mp d^p = 2 \cdot 180 - 60 = 300 \text{ mm}.$$

3.10. Determine the addendum circles diameters for the pinion and the gear

$$d_a^p = d^p + 2 \cdot m = 60 + 2 \cdot 2 = 64 \text{ mm},$$

$$d_a^g = d^g \pm 2 \cdot m = 300 + 2 \cdot 2 = 304 \text{ mm}.$$

3.11. Determine the dedendum circles diameters for the pinion and the gear

$$d_f^p = d^p - 2.5 \cdot m = 60 - 2.5 \cdot 2 = 55 \text{ mm},$$

$$d_f^g = d^g \mp 2.5 \cdot m = 300 - 2.5 \cdot 2 = 295 \text{ mm}.$$

3.12. Determine forces that act in the engagement of straight spur gears:

- turning force $F_t = \frac{2 \cdot T^g}{d^g} = \frac{2 \cdot 370}{0.3} = 2467\text{N}$;
- radial force $F_r = F_t \cdot tg\alpha_w = 2467 \cdot tg 20° = 898\text{N}$,

where $\alpha_w = 20°$ is the pressure angle for the pitch circle.

3.13. Determine the maximum contact stress that develops in the contact zone of teeth

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T^p \cdot K_H \cdot E_{tr}}{\left(d^p\right)^2 \cdot b^g \cdot \sin 2\alpha_w} \cdot \left(\frac{u_{act} \pm 1}{u_{act}}\right)} =$$

$$= 1.18 \cdot \sqrt{\frac{74 \cdot 10^3 \cdot 1.19 \cdot 1.24 \cdot 2.1 \cdot 10^5}{60^2 \cdot 72 \cdot \sin 40°} \cdot \left(\frac{5+1}{5}\right)} = 465\text{MPa},$$

where T^p is the torque at the pinion shaft in N·mm; K_H is the design load factor that is determined as $K_H = K_{H\beta} \cdot K_{HV}$, where $K_{H\beta}$ is the load concentration factor; K_{HV} is the dynamic load factor.

The load concentration factor $K_{H\beta}$ is specified by table 3.2 depending upon $\psi_{bd}=\frac{b^g}{d^p}=\frac{72}{60}=1.2$. That is why $K_{H\beta}$=1.19.

In order to determine K_{HV} it is necessary to find the peripheral speed V^g of the gear

$$V^g=\frac{\omega^g\cdot d^g}{2}=\frac{40\cdot 0.3}{2}=6\,\text{m/sec},$$

where ω^g is the angular velocity of the gear and the gearing accuracy of manufacturing (table 3.5).

The dynamic load factor K_{HV} is determined by table 3.6.

Table 3.5

Gearing accuracy of manufacturing

Types of gear drives	Peripheral speed V, m/sec			
	under 5	5 - 8	8 – 12.5	over 12.5
Straight spur gears	9	8	7	6
Helical spur gears	9	9	8	7
Straight bevel gears	8	7	-	-
Spiral bevel gears	9	9	8	7

Table 3.6

Dynamic load factor K_{HV}

Gearing accuracy of manufacturing	Tooth surface hardness, BHN	Peripheral speed V, m/sec					
		1	2	4	6	8	10
7	up to 350 over 350	1.04/1.02 1.03/1.00	1.07/1.03 1.05/1.01	1.14/1.05 1.09/1.02	1.21/1.06 1.14/1.03	1.29/1.07 1.19/1.03	1.36/1.08 1.24/1.04
8	up to 350 over 350	1.04/1.01 1.03/1.01	1.08/1.02 1.06/1.01	1.16/1.04 1.10/1.02	1.24/1.06 1.16/1.03	1.32/1.07 1.22/1.04	1.40/1.08 1.26/1.05
9	up to 350 over 350	1.05/1.01 1.04/1.01	1.10/1.03 1.07/1.01	1.20/1.05 1.13/1.02	1.30/1.07 1.20/1.03	1.40/1.09 1.26/1.04	1.50/1.12 1.32/1.05

Note: The figures in the numerators refer to straight spur gears and those in the denominators, to helical spur gears.

Obtained value of σ_H should correspond to the following condition:

$$\sigma_H=(0.8\ldots1.1)\cdot[\sigma_H].$$

Otherwise it is necessary to change the center distance a_w and recalculate the gearing.

3.14. Determine the maximum bending stress

$$\sigma_b = \frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Y_b}{m \cdot b^g} = \frac{2467 \cdot 1.42 \cdot 1.58 \cdot 3.6}{2 \cdot 72} = 138.4\text{MPa} \le [\sigma_b] = 255\text{MPa},$$

where $K_{b\beta}$ is the load concentration factor that is determined by table 3.7; K_{bv} is the dynamic load factor determined from table 3.8; Y_b is the tooth form factor that is determined by means of table 3.9 depending upon the number of teeth of the gear for the case when the offset factor x=0

If obtained magnitude of σ_b>[σ_b] it is necessary to increase the module.

Table 3.7

Approximate values of $K_{b\beta}$

Gear arrangement with respect to bearings	Tooth surface hardness, BHN	$\psi_{bd} = \frac{b^g}{d^p}$					
		0.2	0.4	0.6	0.8	1.2	1.6
On cantilevers, ball bearings	up to 350 over 350	1.16 1.33	1.37 1.70	1.64 -	- -	- -	- -
On cantilevers, roller bearings	up to 350 over 350	1.10 1.20	1.22 1.44	1.38 1.71	1.57 -	- -	- -
Symmetrical	up to 350 over 350	1.01 1.02	1.03 1.04	1.05 1.08	1.07 1.14	1.14 1.30	1.26 -
Nonsymmetrical	up to 350 over 350	1.05 1.09	1.10 1.18	1.17 1.30	1.25 1.43	1.42 1.73	1.61 -

Table 3.8

Dynamic load factor K_{bV}

Gearing accuracy of manufacturing	Tooth surface hardness, BHN	Peripheral speed V, m/sec					
		1	2	4	6	8	10
7	up to 350 over 350	1.08/1.03 1.03/1.01	1.16/1.06 1.05/1.02	1.33/1.11 1.09/1.03	1.50/1.16 1.13/1.05	1.62/1.22 1.17/1.07	1.80/1.27 1.22/1.08
8	up to 350 over 350	1.10/1.03 1.04/1.01	1.20/1.06 1.06/1.02	1.38/1.11 1.12/1.03	1.58/1.17 1.16/1.05	1.78/1.23 1.21/1.05	1.96/1.29 1.26/1.08
9	up to 350 over 350	1.13/1.04 1.04/1.01	1.28/1.07 1.07/1.02	1.50/1.14 1.14/1.04	1.72/1.21 1.21/1.06	1.98/1.28 1.27/1.08	1.25/1.35 1.34/1.09

Note: The figures in the numerators refer to straight spur gears and those in the denominators, to helical spur gears.

Table 3.9

Tooth form factor Y_b

z or z_v	17	20	22	24	26	28	30	35	40	45	50	65	80	100
Y_b	4.27	4.07	3.98	3.92	3.88	3.81	3.8	3.75	3.7	3.66	3.65	3.62	3.61	3.6

4. Analysis of the helical spur gears for strength

Let us carry out the analysis of the helical spur gears for strength if torque at the pinion shaft T^p =161.12 N·m; torque at the gear shaft T^g =464.3 N·m; velocity ratio of the gearing u=4; allowable contact stress $[\sigma_H]$= 640 MPa; allowable bending stress $[\sigma_b]$= 293.657 MPa; hardness of the gear material H^g=285,5 BHN, angular velocity of the gear shaft ω^g = 19.19 rad/sec, symmetrical disposition of gears between supports.

4.1. Determine the center distance of the helical spur gears

$$a_w = 0.75 \cdot (u+1) \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}},$$

where u is the velocity ratio of the gearing; T^g is the torque at the gear shaft in N·mm; $[\sigma_H]$ is the allowable contact stress in MPa; E_{tr} is the transformed modulus of elasticity in MPa; $K_{H\beta}$ is the load concentration factor; $\psi_{ba}= b^g/a_w$ is the gear face width factor.

Transformed modulus of elasticity E_{tr} is determined as

$$E_{tr} = \frac{2 \cdot E^p \cdot E^g}{E^p + E^g},$$

where E^p and E^g are correspondingly modules of elasticity of pinion and gear materials. Since the pinion and the gear are made of steel we can make the conclusion that $E_{tr} = E^p = E^g = 2.1 \cdot 10^5$ MPa.

Load concentration factor $K_{H\beta}$ is determined by means of table 3.2 depending on disposition of toothed wheels with respect to bearings and factor $\psi_{bd}= b_g/d_p$. Since b_g and d_p were not determined we find this factor by the following formula

$$\psi_{bd} = \frac{b^g}{d^p} = \frac{0.5 \cdot b^g}{a_w} \cdot (u \pm 1) = 0.5 \cdot \psi_{ba} \cdot (u \pm 1),$$

where gear face width factor ψ_{ba} is determined from table 3.1 depending on the disposition of the gear relative to bearings taking into account that the value of this factor should correspond to standard. The greater ψ_{ba} the less overall dimensions of the gearing. That is why we select the greater magnitude of ψ_{ba}.

Obtained magnitude of a_w should be rounded off to the nearest greater side according to the series given in table 3.3.

From table 3.1 we take ψ_{ba} = 0.5; then $\psi_{bd} = 0.5 \cdot 0.5 \cdot (4+1) = 1.25$,

From table 3.2 we take $K_{H\beta}$ = 1.073 (for symmetrical gear arrangement and tooth surface hardness up to 350).

Than

$$a_w = 0.75 \cdot (4+1) \cdot \sqrt[3]{\frac{464300 \cdot 1.073 \cdot 2.1 \cdot 10^5}{640^2 \cdot 4^2 \cdot 0.5}} = 118.965 \text{mm}$$

Round off obtained magnitude of to the nearest greater side according to table 3.3. That is why we take a_w = 125 mm for further calculations.

4.2. Determine the nominal pitch circle diameter of the gear

$$d^g = \frac{2 \cdot a_w \cdot u}{u+1} = \frac{2 \cdot 125 \cdot 4}{4+1} = 200 \text{mm}.$$

4.3. Determine the face width of the gear

$$b^g = \psi_{ba} \cdot a_w = 0.5 \cdot 125 = 62.5 \text{ mm}.$$

4.4. Determine the normal module according to the strength condition for bending

$$m_n \geq \frac{2 \cdot K_m \cdot T^g}{d^g \cdot b^g \cdot [\sigma_b]},$$

where K_m is taken as 5.8 for helical spur gears.

Obtained magnitude of the module should be rounded off to the greater side according to the standard series given in table 3.4. It is necessary to note that for general-purpose speed reducers the minimum value of the module is m_{min} = 2 mm, in our case

$$m_n = \frac{2 \cdot 5.8 \cdot 464300}{200 \cdot 62.5 \cdot 293.657} = 1.467 \text{mm},$$

we take m_n = 2 mm for further calculations.

4.5. Determine the helix angle

$$\beta = \arcsin\left(\frac{3.5 \cdot m_n}{b^g}\right) = \arcsin\left(\frac{3.5 \cdot 2}{62.5}\right) = 6.43 = 6^\circ 25'.$$

For helical spur gears this angle should be ranged from 8 to 18°. Otherwise, it is necessary to change the normal module m_n. As in our case $\beta = 6^\circ 25'$ that is less than 8°, that's why we take m_n=2.5 mm, than

$$\beta = \arcsin\left(\frac{3.5 \cdot 2.5}{62.5}\right) = 8.048 = 8^\circ 2'.$$

4.6. Determine the total number of teeth

$$z_\Sigma = \frac{2 \cdot a_w \cdot \cos\beta}{m_n} = \frac{2 \cdot 125 \cdot \cos 8^\circ 2'}{2.5} = 99.19 .$$

Obtained value of z_Σ should be rounded off to the nearest integer numeral, assume $z_\Sigma = 99$.

4.7. Specify the helix angle according to the integer number of z_Σ

$$\beta = \arccos\left(\frac{m_n \cdot z_\Sigma}{2 \cdot a_w}\right) = \arccos\left(\frac{2.5 \cdot 99}{2 \cdot 125}\right) = 8.11 = 8^\circ 6'$$

Obtained value of this angle is ranged from 8 to 18°. Condition is satisfied.

4.8. Determine the number of teeth of the pinion

$$z^p = \frac{z_\Sigma}{u \pm 1} \geq z_{min},$$

where for helical spur gears $z_{min}=17 \cdot \cos^3\beta$.

Obtained value of z^p should be rounded off to the nearest integer numeral. If $z^p < 17 \cdot \cos^3\beta$ it is necessary to decrease the module or to use nonstandard toothed wheels.

In our case

$$z^p = \frac{99}{4 \pm 1} = 19.8 \Rightarrow z^p = 20 > z_{min} = 17 \cdot \cos^3 8^\circ 6' = 16.5.$$

4.9. Determine the number of teeth of the gear

$$z^g = z_\Sigma - z^p = 99 - 20 = 79 .$$

4.10. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p} = \frac{79}{20} = 3.95 .$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less or equal to 4%. Otherwise the number of teeth z^p, z^g and z_Σ must be rounded off to the other side.

$$\varepsilon = \left|\frac{3.95 - 4}{4}\right| \cdot 100\% = 1.25 < 4\%$$

Condition is satisfied.

4.11. Determine the nominal pitch circle diameters for the pinion and the gear

$$d^p = \frac{m_n}{\cos\beta} \cdot z^p = \frac{2.5}{\cos 8^\circ 6'} \cdot 20 = 50.5\text{mm},$$

$$d^g = 2 \cdot a_w - d^p = 2 \cdot 125 - 50.5 = 199.5\text{mm}.$$

4.12. Determine the addendum circle diameters for the pinion and the gear

$$d_a^p = d^p + 2m_n = 50.5 + 2 \cdot 2.5 = 55.5\text{mm},$$

$$d_a^g = d^g + 2m_n = 199.5 + 2 \cdot 2.5 = 204.5\text{mm}.$$

4.13. Determine the dedendum circle diameters for the pinion and the gear

$$d_f^p = d^p - 2.5 \cdot m_n = 50.5 - 2.5 \cdot 2.5 = 44.25\text{mm},$$

$$d_f^g = d^g - 2.5 \cdot m_n = 199.5 - 2.5 \cdot 2.5 = 193.25\text{mm}.$$

4.14. Determine forces that act in the engagement of the helical spur gears:

- turning force $F_t = \frac{2 \cdot T^g}{d^g} = \frac{2 \cdot 464300}{199.5} = 4654.64\text{N}$;
- radial force $F_r = \frac{F_t}{\cos\beta} \cdot \text{tg}\alpha_w = \frac{4654.64}{\cos 8^\circ 6'} \cdot \text{tg}20^\circ = 1711.22\text{N}$;
- axial force $F_a = F_t \cdot \text{tg}\beta = 4654.64 \cdot \text{tg}8^\circ 6' = 662.45\text{N},$

where $\alpha_w = 20^\circ$ is the pressure angle for the pitch circle.

4.15. Determine the maximum contact stress that develops in the contact zone of teeth

$$\sigma_H = 1.18 \cdot Z_{H\beta} \cdot \sqrt{\frac{T^p \cdot K_H \cdot E_{tr}}{\left(d^p\right)^2 \cdot b^g \cdot \sin 2\alpha_w} \cdot \left(\frac{u_{act} \pm 1}{u_{act}}\right)},$$

where $Z_{H\beta}$ takes into account rising the contact strength of the helical spur gears in comparison with the straight spur gears; T^p is the torque at the pinion shaft in N·mm; K_H is the design load factor that is determined as

$$K_H = K_{H\beta} \cdot K_{HV},$$

where $K_{H\beta}$ is the load concentration factor; K_{HV} is the dynamic load factor.

The load concentration factor $K_{H\beta}$ is specified by table 3.2 depending upon $\psi_{bd} = \frac{b^g}{d^p} = \frac{62.5}{50.5} = 1.238$, then $K_{H\beta}$ = 1.072.

In order to determine K_{HV} it is necessary to find the peripheral speed V^g of the gear

$$V^g = \frac{\omega^g \cdot d^g}{2} = \frac{19.19 \cdot 0.1995}{2} = 1.914 \text{m/sec} \Rightarrow K_{HV} = 1.01$$

where ω^g is the angular velocity of the gear and the gearing accuracy of manufacturing (table 3.5).

Accuracy of manufacturing gear drive is 9.

The dynamic load factor K_{HV} is determined by table 3.6.

$$K_H = 1.072 \cdot 1.01 = 1.0827;$$

Factor $Z_{H\beta}$ is determined in the following way

$$Z_{H\beta} = \sqrt{\frac{K_{H\alpha} \cdot \cos^2 \beta}{\varepsilon_\alpha}} = \sqrt{\frac{1.13 \cdot \cos^2 8^\circ 6'}{1.663}} = 0.816,$$

where $K_{H\alpha}$ takes into account non-uniform load distribution between several pairs of teeth; ε_α is the contact ratio.

$K_{H\alpha}$ depends upon the accuracy of manufacturing and the peripheral speed and is determined according to table 4.1.

$$K_{H\alpha} = 1.13.$$

Table 4.1

Factors $K_{H\alpha}$, $K_{b\alpha}$ that take into account non-uniform load distribution between some pairs

Peripheral speed V, m/sec	Accuracy of manufacturing	$K_{H\alpha}$	$K_{b\alpha}$
To 5	7	1.03	1.07
	8	1.07	1.22
	9	1.13	1.35
From 5 to 10	7	1.05	1.2
	8	1.10	1.3
From 10 to 15	7	1.08	1.25
	8	1.15	1.40

Contact ratio ε_α is found by the following formula

$$\varepsilon_\alpha = \left[1.88 - 3.2 \cdot \left(\frac{1}{z^p} + \frac{1}{z^g}\right)\right] \cdot \cos\beta =$$
$$= \left[1.88 - 3.2 \cdot \left(\frac{1}{20} + \frac{1}{79}\right)\right] \cdot \cos 8^\circ 6' = 1.663.$$

Obtained value of σ_H should correspond to the following condition:

$$\sigma_H = (0.8\ldots1.1)\cdot[\sigma_H].$$

Otherwise it is necessary to change the center distance a_w and recalculate the gearing.

$$\sigma_H = 1.18 \cdot 0.816 \cdot \sqrt{\frac{161120 \cdot 1.0827 \cdot 210000}{50.5^2 \cdot 62.5 \cdot \sin(2 \cdot 20^\circ)} \cdot \left(\frac{3.95+1}{3.95}\right)} = 644.539 \text{MPa};$$

$\sigma_H < 1.1[\sigma_H]$, strength condition is satisfied.

4.16. Determine the maximum bending stress

$$\sigma_b = \frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Z_{b\beta} \cdot Y_b}{m_n \cdot b^g} \le [\sigma_b],$$

where $K_{b\beta}$ is the load concentration factor that is determined by table 3.7; K_{bv} is the dynamic load factor determined from table 3.8; Y_b is the tooth form factor that is determined by means of table 3.9 depending on the number of teeth of the equivalent straight spur gear $z_v^g = \dfrac{z^g}{\cos^3\beta}$ for the case when the shift factor x=0.

Factor $Z_{b\beta}$ is the analogy of $Z_{H\beta}$ and is determined as

$$Z_{b\beta} = \frac{K_{b\alpha} \cdot Y_\beta}{\varepsilon_\alpha},$$

where $K_{b\alpha}$ is chosen from table 4.1; $Y_\beta = 1 - \frac{\beta^\circ}{140}$ is the correction factor.

If obtained magnitude of $\sigma_b > [\sigma_b]$ it is necessary to increase the module.

In our case: $K_{b\beta}$ = 1.155; K_{bv} = 1.07; $z_v^g = \frac{79}{\cos^3 8^\circ 6'} = 81.42 \Rightarrow 81$;

Y_b = 3.61; $K_{b\alpha}$ = 1.35; $Y_\beta = 1 - \frac{8^\circ 40'}{140} = 0.939$; $Z_{b\beta} = \frac{1.35 \cdot 0.939}{1.663} = 0.762$;

$$\sigma_b = \frac{4654.64 \cdot 1.155 \cdot 1.07 \cdot 0.762 \cdot 3.61}{2.5 \cdot 62.5} = 101.273\text{MPa} < [\sigma_b] = 293.657\text{MPa}.$$

Strength condition is satisfied.

5. Analysis of the bevel gears for strength

Let us analyze the bevel gears for strength if torque at the gear shaft T^g =460 N·m; velocity ratio of the gearing u=3; allowable contact stress $[\sigma_H]$=620 MPa; allowable bending stress $[\sigma_b]$=168 MPa, hardness of the gear material H^g=285 BHN.

5.1. Determine the external pitch diameter of the gear

$$d_e^g = 1.7 \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr} \cdot u}{\nu_H \cdot [\sigma_H]^2 \cdot \psi_{bR} \cdot (1-\psi_{bR})}},$$

where T^g is the torque at the gear shaft in N·mm; E_{tr} is the transformed modulus of elasticity; $K_{H\beta}$ is the load concentration factor; u is the velocity ratio; ν_H = 0.85 is the correction factor that takes into account reducing bevel gears strength in comparison with spur gears; $[\sigma_H]$ is the allowable contact stress; $\psi_{bR}=b^g/R_e$ is the gear face width factor that determines proportions of the face width of the gear with respect to the external cone distance. Factor ψ_{bR} must be less than 0.3. Recommended value of ψ_{bR} = 0.285.

Since both pinion and gear are made of steel, the transformed modulus of elasticity E_{tr} =2.1·10^5 MPa.

Load concentration factor $K_{H\beta}$ depends upon the hardness of the gear material. If $H^g \le 350$ BHN $K_{H\beta}$ is ranged from1.23 to 1.35. Otherwise ($H^g>350$ BHN) $K_{H\beta}$ is ranged from 1.25 to 1.45. It is necessary to note that greater values of $K_{H\beta}$ are assumed for the case when one of toothed wheels is on the cantilever shaft.

$$d_e^g = 1.7 \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr} \cdot u}{\nu_H \cdot [\sigma_H]^2 \cdot \psi_{bR} \cdot (1-\psi_{bR})}} = 1,7 \cdot \sqrt[3]{\frac{460 \cdot 10^3 \cdot 1.3 \cdot 2.1 \cdot 10^5 \cdot 3}{0.85 \cdot 620^2 \cdot 0.285 \cdot (1-0.285)}} = 302.9\text{mm.}$$

After calculation the obtained magnitude of d_e^g should be rounded off to the greater side according to standard series given in table 5.1. In our case we assume d_e^g =315 mm.

Table 5.1

Standard values of the external pitch diameter d_e^g

Series 1	40	50	63	80	100	125	160	200	250	315	400	500
Series 2	-	-	71	90	112	140	180	224	280	355	450	560

<u>Note.</u> Series 1 should be preferred to Series 2.

5.2. Determine pitch angles for the pinion and for the gear.

δ_2 = arctg u= arctg 3=71°36’, δ_1 = 90°- δ_2=90-71.6=18°24’.

5.3. Determine the external cone distance

$$R_e = \frac{d_e^g}{2 \cdot \sin \delta_2} = \frac{315}{2 \cdot \sin 71°36'} = 165.98\text{mm} .$$

5.4. Determine the face width of the gear

$$b^g = \psi_{bR} \cdot R_e = 0.285 \cdot 165.98 = 47.3 \text{ mm}.$$

5.5. Determine the external module

$$m_e = \frac{14 \cdot T^g \cdot K_{b\beta}}{\nu_b \cdot \mathrm{d}_\mathrm{e}^\mathrm{g} \cdot b^g \cdot [\sigma_b]} = \frac{14 \cdot 460 \cdot 10^3 \cdot 1,32}{0.85 \cdot 315 \cdot 47.3 \cdot 168} = 3.99\text{mm},$$

where ν_b = 0.85 is the correction factor; $K_{b\beta}$ is the load concentration factor that is determined according to table 3.7 depending upon ψ_{bd} factor, where the latter is found as

$$\psi_{bd} = \frac{b^g}{d_m^p} = 0.166 \cdot \sqrt{u^2 + 1} = 0.166 \cdot \sqrt{3^{2\cdot} + 1} = 0.53 .$$

5.6. Determine the number of teeth of the gear

$$\mathrm{z}^\mathrm{g} = \frac{d_e^g}{m_e} = \frac{315}{3.99} = 78.9$$

and round off z^g to the integer numeral. Assume $z^g = 79$.

5.7. Determine the number of teeth of the pinion

$$z^p = \frac{z^g}{u} = \frac{79}{3} = 26.3$$

and round off z^p to the integer numeral too. In our case $z^p = 26$.

5.8. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p} = \frac{78}{26} = 3.04 .$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less or equal to 4%. Otherwise, we should round off values of z^p and z^g to the other side.

In our case $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\% = \left|\frac{3.04 - 3}{3}\right| \cdot 100\% = 1.33 < 4\%.$

5.9. Specify pitch angles for the pinion and the gear

δ_2 = arctg u_{act}=arctg 3.04=71°48’, δ_1 = 90°- δ_2=18°12’

5.10. Determine external pitch diameters of the pinion and the gear.

$$d_e^p = m_e \cdot z^p = 3.99 \cdot 26 = 103.74\text{mm},$$

$$\mathrm{d}_e^g = m_e \cdot z^g = 3.99 \cdot 79 = 315.21\text{mm}.$$

5.11. Determine diameters of addendum circles at the outer section for the pinion and the gear

$$d_{ae}^p = d_e^p + 2 \cdot m_e \cdot \cos\delta_1 = 103.74 + 2 \cdot 3.99 \cdot \cos 18°12' = 111.32 \text{ mm},$$

$$d_{ae}^g = d_e^g + 2 \cdot m_e \cdot \cos\delta_2 = 315.21 + 2 \cdot 3.99 \cdot \cos 71°48' = 317.70 \text{ mm}.$$

5.12. Determine diameters of dedendum circles in the outer section for the pinion and the gear.

$$d_{fe}^p = d_e^p - 2.4 \cdot m_e \cdot \cos\delta_1 = 103.74 – 2.4 \cdot 3.99 \cdot \cos 18°12' = 94.64 \text{ mm},$$

$$d_{fe}^g = d_e^g - 2.4 \cdot m_e \cdot \cos\delta_2 = 315.21 – 2.4 \cdot 3.99 \cdot \cos 71°48' = 312.22 \text{ mm}.$$

5.13. Specify the external cone distance

$$R_e = 0.5 \cdot m_e \cdot \sqrt{\left(z^p\right)^2 + \left(z^g\right)^2} = 0.5 \cdot 3.99 \cdot \sqrt{26^2 + 79^2} = 165.92\text{mm}.$$

5.14. Specify the face width of the gear

$$b^g = \psi_{bR} \cdot R_e = 0.285 \cdot 165.92 = 47.23 \text{ mm}.$$

5.15. Determine mean pitch diameters for the pinion and for the gear

$$d_m^p = \frac{d_e^p \cdot (R_e - 0.5 \cdot b^g)}{R_e} = d_e^p \cdot \left(1 - 0.5 \cdot \psi_{bR}\right) = 103.74 \cdot (1 - 0.5 \cdot 0.285) = 88.96\text{mm},$$

$$d_m^g = \frac{d_e^g \cdot (R_e - 0.5 \cdot b^g)}{R_e} = d_e^g \cdot \left(1 - 0.5 \cdot \psi_{bR}\right) = 315.21 \cdot (1 - 0.5 \cdot 0.285) = 270.29\text{mm}.$$

5.16. Determine forces that act in the engagement of the bevel gears

- turning force $F_t = \frac{2 \cdot T^g}{d_m^g} = \frac{2 \cdot 420 \cdot 10^3}{270.29} = 3108\,\text{N}$;

- radial force at the gear

$$F_r^g = F_t \cdot tg\,\alpha_w \cdot \cos\delta_2 = 3108 \cdot tg20° \cdot \cos 71°48' = 353.3\ \text{N};$$

- axial force at the gear

$$F_a^g = F_t \cdot tg\,\alpha_w \cdot \sin\delta_2 = 3108 \cdot tg20° \cdot \sin 71°48' = 1074.4\ \text{N}.$$

5.17. Determine the maximum contact stress that develops in the contact zone of teeth:

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T^p \cdot K_H \cdot E_{tr}}{\nu_H \cdot \left(d_m^p\right)^2 \cdot b^g \cdot \sin 2\alpha_w} \cdot \left(\frac{\sqrt{u_{act}^2 + 1}}{u_{act}}\right)} =$$

$$= 1.18 \cdot \sqrt{\frac{153 \cdot 10^3 \cdot 1.29 \cdot 2.1 \cdot 10^5}{0.85 \cdot 88.96^2 \cdot 47.23 \cdot \sin 40°} \cdot \left(\frac{\sqrt{3.04^2 + 1}}{3.04}\right)} = 545.3\,\text{MPa},$$

where T^p is in N·mm; K_H is the design load factor determine as

$$K_H = K_{H\beta} \cdot K_{HV}.$$

Load concentration factor $K_{H\beta}$ is specified by means of table 3.2 depending upon factor $\psi_{bd} = \frac{b^g}{d_m^p}$.

Dynamic load factor K_{HV} is determined according to table 3.6 depending upon the peripheral speed of the gear ($V^g = \frac{\omega^g \cdot d_m^g}{2}$) and the accuracy of manufacturing (table 3.5). In order to use table 3.6 for bevel gears we should reduce the degree of accuracy by 1.

In our case $\psi_{bd} = \frac{b^g}{d_m^p} = \frac{47.23}{88.96} = 0.53$, $V^g = \frac{\omega^g \cdot d_m^g}{2} = \frac{25 \cdot 0.27}{2} = 3.4\,\text{m/sec}$.

$K_H = K_{H\beta} \cdot K_{HV} = 1.16 \cdot 1.11 = 1.29$.

Obtained value of σ_H should correspond to the following condition

$$\sigma_H = (0.8\ldots1.1) \cdot [\sigma_H] = (0.8\ldots1.1) \cdot 620 = 496\ldots682\ \text{MPa}.$$

Otherwise it is necessary to change the external pitch diameter and recalculate the gearing. In our case strength condition is satisfied.

5.18. Determine the maximum bending stress

$$\sigma_b = \frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Y_b}{\nu_b \cdot m_m \cdot b^g} = \frac{3108 \cdot 1.32 \cdot 1.27 \cdot 3.6}{0.85 \cdot 3.42 \cdot 47.23} = 136.6\,\text{MPa} \le [\sigma_b] = 168\,\text{MPa}\ ,$$

where $K_{b\beta}$ is the load concentration factor that is determined by table 3.7; K_{bV} is the dynamic load factor determined from table 3.8 (for bevel gears we should reduce the accuracy of manufacturing by 1); Y_b is the tooth form factor that is determined by means of table 3.9 depending upon the number of teeth of the equivalent straight spur gear $z_v^g = \frac{z^g}{\cos\delta_2} = \frac{79}{\cos 71°48'} = 253$ for the case when the offset factor x=0; ν_b = 0.85 is the correction factor; $m_m = \frac{d_m^g}{z^g} = \frac{270.29}{79} = 3.42\,\text{mm}$ is the mean module.

6. Analysis of the worm gearing

Let us carry out the analysis of the worm gearing for strength if torque at the worm shaft $T^w = 105$ N·m; torque at the gear shaft $T^g = 1750$ N·m; frequency of the worm shaft rotation n^w = 441.75 rpm, frequency of the gear shaft rotation n^g = 22.1 rpm; velocity ratio of the gearing u=20; angular velocity of the gear shaft ω^g = 19.19 rad/sec.

6.1. Determine approximately the slippage speed in the worm gearing

$$V_{sl} = 4.5 \cdot 10^{-4} \cdot n^w \cdot \sqrt[3]{T^g} \ ,$$

where n^w is the rotational speed of the worm in rpm; T^g is the torque at the worm gear shaft in N·m.

$$V_{sl} = 4.5 \cdot 10^{-4} \cdot 441.75 \cdot \sqrt[3]{1750} = 2.4 \text{m/sec}$$

6.2. Select the material of the worm and the worm gear.

Worm.

The best performance is obtained when worms are made of carbon or alloy steels of grade 45 (0.45 C) and 40XH (0.40-C-Cr-Ni) surface-hardened to hardness ranged from 50 to 55 HRC or of grade 20X (0.20 C-Cr) and 18XГТ (0.18 C-Cr-Mn-Ti) case-hardened to hardness ranged from 58 to 63 HRC. The immunity to seizure improves with increasing hardness of the working surfaces of threads. Also, the surface roughness of the threads should be kept to a minimum (usually Ra 0.2). For this purpose the threads are ground and polished.

Worm gear

Material of the worm gear depends on the slippage speed.

If V_{sl}>5 m/sec, the worm gear is made of tin bronzes such as Bronze(10Sn-1Ni-1P), Bronze(10Sn-1P).

If V_{sl} is ranged from 2 to 5 m/sec, the worm gear is made of tinless (aluminum-iron) bronzes such as Bronze (9Al-4Fe).

If V_{sl}<2 m/sec, the worm gear is produced from cast irons such as Grey cast iron 12 or Grey cast iron 18.

Mechanical characteristics of materials of the worm gear are given in table 6.1. It is recommended to choose the material of the worm gear with higher mechanical characteristics.

It is necessary to note that calculation of the worm gearing is carried out by the material of the worm gear because it has less strength in comparison with the material of the worm.

Thus the worm is made of carbon steel of grade 45 (0.45 C) and surface-hardened to hardness ranged from 50 to 55 HRC; the worm gear is made of tinless (aluminum-iron) bronze such as Bronze (9Al-4Fe) with chill casting to σ_y = 195 MPa, ultimate strength in tension $\sigma_{ul\,t}$ = 490 MPa.

Table 6.1

Mechanical characteristics of materials of the worm gear ring

Material	Slippage speed *Vs*, m/sec	Casting method	Mechanical characteristics, MPa		
			Yield point σ_y	Ultimate strength in tension $\sigma_{ul\,t}$	Ultimate strength in bending $\sigma_{ul\,b}$
Tin bronzes					
Bronze(10Sn-1Ni-1P)	Over 5	Centrifugal casting	165	285	
Bronze(10Sn-1P)	Over 5	Chill casting	195	245	
Bronze(10Sn-1P)	Over 5	Sand casting	132	215	
Tinless bronzes					
Bronze(9Al-4Fe)	2…5	Centrifugal casting	200	500	
Bronze(9Al-4Fe)	2…5	Chill casting	195	490	
Bronze(9Al-4Fe)	2…5	Sand casting	195	392	
Cast-irons					
Grey cast iron 12	Up to 2	Sand casting	-		280
Grey cast iron 18	Up to 2	Sand casting	-		360

6.3. Determine the allowable contact stress.

a) For the worm gear made of tin bronzes the allowable contact stress is determined from the condition to prevent fatigue pitting

$$[\sigma_H] = \sigma_{lim} \cdot C_v \cdot K_{HL},$$

where σ_{lim} is the limit of contact endurance that is determined as

$$\sigma_{lim} = 0.9 \cdot \sigma_{ul\,t};$$

$\sigma_{ul\,t}$ is the ultimate strength in tension (table 6.1); C_v is the factor that takes into account the wear rate of a worm gear tooth depending on the slippage speed (table 6.2); K_{HL} is the durability factor.

Table 6.2

Values of C_v

Slippage speed V_{sl}, m/sec	5	6	7	8
C_v	0.95	0.88	0.83	0.8

Durability factor K_{HL} is found in the following way:

$$K_{HL} = \sqrt[8]{\frac{N_{H0}}{N_{HE}}},$$

where $N_{HO} = 10^7$ is the base number of cycles;

$N_{HE} = 60 \cdot n^g \cdot t \cdot K_{HE}$ is the equivalent number of cycles;
n^g is the rotational speed of the worm gear;

$t = L \cdot 365 \cdot K_a \cdot 24 \cdot K_d$ is the service life in hours;
L is the service life in years; K_a is the annual utilization factor; K_d is the daily utilization factor; K_{HE} is the factor that reduces variable load conditions to the constant equivalent.

$$K_{HE} = \sum_{i=1}^{n} \frac{t_i}{t} \cdot \left(\frac{T_i}{T_{max}} \right)^4,$$

where T_i and T_{max} are correspondingly acting and maximum torques; t_i is time of action of the torque.

Obtained magnitude of K_{HL} should satisfy to the following condition:

$$0.67 \le K_{HL} \le 1.15.$$

Otherwise, for further calculations we take the extreme values of the mentioned above inequality.

b) For the worm gear made of either tinless bronzes or cast irons, the allowable contact stress is determined to avoid seizure:

- for tinless bronzes $[\sigma_H] = 300 - 25 \cdot V_{sl}$;
- for cast iron $[\sigma_H] = 175 - 35 \cdot V_{sl}$.

In our case: $[\sigma_H] = 300 - 25 \cdot 2.4 = 240$ MPa

6.4. Determine the allowable bending stress.

For this purpose we use table 6.3, where $\sigma_{ul\,t}$ is the ultimate strength in tension; $\sigma_{ul\,b}$ is the ultimate strength in bending; σ_y is the yield point; K_{bL} is the durability factor.

Table 6.3

Allowable bending stresses

Material	Non-reversed gearing	Reversed gearing
Bronze	$[\sigma_b] = (0.08 \cdot \sigma_{ul\,t} + 0.25 \cdot \sigma_y) \cdot K_{bL}$	$[\sigma_b] = 0.16 \cdot \sigma_{ul\,t} \cdot K_{bL}$
Cast-iron	$[\sigma_b] = 0.12 \cdot \sigma_{ul\,b} \cdot K_{bL}$	$[\sigma_b] = 0.075 \cdot \sigma_{ul\,b} \cdot K_{bL}$

Mechanical characteristics of the worm gear materials are given in table 6.1.

Durability factor K_{bL} is determined as

$$K_{bL} = \sqrt[9]{\frac{N_{b0}}{N_{bE}}},$$

where $N_{b0} = 1 \cdot 10^6$ is the base number of cycles;

$N_{bE} = 60 \cdot n^g \cdot t \cdot K_{bE}$ is the equivalent number of cycles;

n^g is the rotational speed of the worm gear;

$t = L \cdot 365 \cdot K_a \cdot 24 \cdot K_d$ is the service life in hours;

L is the service life in years; K_a is the annual utilization factor; K_d is the daily utilization factor; K_{bE} is the factor that reduces variable load conditions to the constant equivalent.

$$K_{bE} = \sum_{i=1}^{n} \frac{t_i}{t} \cdot \left(\frac{T_i}{T_{max}} \right)^9,$$

where T_i and T_{max} are correspondingly acting and maximum torques; t_i is time of the torque action.

Note: If the time of the torque action is less than $0.03 \cdot t$, this torque should not be taken into account.

Obtained magnitude of K_{bL} should satisfy to the following condition:

$$0.543 \le K_{bL} \le 1.$$

Otherwise, for further calculations we take the extreme values of the mentioned above inequality.

In our case: t_i: $0.003t$ $0.15t$; $0.25t$; $0.6t$;

T_i: $1.3T$ T; $0.7T$; $0.5T$.

the service life of the gearing is 8 years, $K_a = 0.7$, $K_d = 0.3$

$t = 8 \cdot 365 \cdot 0.7 \cdot 24 \cdot 0.3 = 14716.8\,\text{hours}.$

Then $K_{bE} = \frac{0.15t}{t} \cdot \left(\frac{T}{T}\right)^9 + \frac{0.25t}{t} \cdot \left(\frac{0.7T}{T}\right)^9 + \frac{0.6t}{t} \cdot \left(\frac{0.5T}{T}\right)^9 =$

$= 0.15 + 0.25 \cdot 0.7^9 + 0.6 \cdot 0.5^9 = 0.161$

$N_{bE} = 60 \cdot 22.1 \cdot 14716.8 \cdot 0.161 = 3.142 \cdot 10^6$; $N_{b0} = 1 \cdot 10^6$;

$K_{bL} = \sqrt[9]{\frac{1 \cdot 10^6}{3.142 \cdot 10^6}} = 0.881$ Condition $0.543 \le K_{bL} \le 1$ is satisfied.

For bronze and non-reversed transmission

$[\sigma_b] = (0.08 \cdot 490 + 0.25 \cdot 195) \cdot 0.881 = 77.484$ MPa

6.5.Calculate the worm gear for strength.

6.5.1. Determine the center distance of the worm gearing

$$a_w = 0.625 \cdot \left(\frac{q^w}{z^g} + 1\right) \cdot \sqrt[3]{\frac{T^g \cdot E_{tr}}{[\sigma_H]^2 \cdot \left(\frac{q^w}{z^g}\right)}},$$

where T^g is the torque at the worm gear shaft in N·mm;

$z^g = z^w \cdot u \ge 28$ is the number of teeth of the worm gear (it should be rounded off to the nearest integer numeral);

z^w is the number of threads of the worm that is determined according to table 6.4.

Table 6.4

Number of threads of the worm

Velocity ratio u	8 to 14	over 14 to 30	over 30
Number of threads z^w	4	2	1

If $u = 20$, than $z^w = 2$ and $z^g = 2 \cdot 20 = 40 \ge 28$

q^w is the worm diameter factor whose minimum value is found as $q^w_{min} = 0.212 \cdot z^g$ (obtained magnitude of q^w must be rounded off to the greater side according to the following standard series 8; 10; 12.5; 14; 16; 20);

So $q^w_{min} = 0.212 \cdot 40 = 8.48$ round off to the $q^w_{min} = 10$.

E_{tr} is the transformed modulus of elasticity that is determined by the formula

$$E_{tr} = \frac{2 \cdot E^w \cdot E^g}{E^w + E^g} = \frac{2 \cdot 2.1 \cdot 10^5 \cdot 0.9 \cdot 10^5}{2.1 \cdot 10^5 + 0.9 \cdot 10^5} = 1.26 \cdot 10^5 \text{ MPa}.$$

where E^w is the worm material modulus of elasticity (for steels E = $2.1 \cdot 10^5$ MPa); E^g is the worm gear material modulus of elasticity (for bronzes and cast irons $E \approx 0.9 \cdot 10^5$ MPa).

$$a_{\text{w}} = 0.625 \cdot \left(\frac{10}{40} + 1\right) \cdot \sqrt[3]{\frac{1750 \cdot 10^3 \cdot 1.26 \cdot 10^5}{240^2 \cdot \left(\frac{10}{40}\right)}} = 194\text{mm}$$

Obtained value of a_w should be rounded off to the greater side according to standard series given in table 6.5.

Table 6.5

Standard values of the centre distance a_w of the worm gearing

Series 1	40	50	63	80	100	125	160	200	250	315	400	500
Series 2	-	-	-	-	-	140	180	225	280	355	450	-

Note. Series 1 should be preferred to Series 2.

The obtained value of a_w round off to the 200 mm.

6.5.2. Determine the axial module

$$m = \frac{2 \cdot a_w}{q^w + z^g} = \frac{2 \cdot 200}{10 + 40} = 8\text{mm}$$

and round off obtained value according to standard series given in table 6.6.

Table 6.6

Standard values of module m for worm gearing

m ,mm	2.5; 3.15; 4; 5	6.3; 8; 10; 12.5
q^w	8; 10; 12.5; 16; 20	8; 10; 12.5; 14; 16; 20

6.5.3. In order to ensure standard value of the centre distance we use modified worm gear with offset factor x determined as

$$x = \frac{a_{\text{w}}}{m} - 0.5 \cdot (q^w + z^g) = \frac{200}{8} - 0.5 \cdot (10 + 40) = 0 .$$

To avoid undercutting the following condition should be carried out

$$-1 \leq x \leq 1 .$$

Otherwise it is necessary to change a_w, q^w or z^g.

6.5.4. Specify the velocity ratio of the worm gearing

$$u_{act} = \frac{z^g}{z^w} = \frac{40}{2} = 20$$

and determine the error $\varepsilon = \left| \frac{u_{act} - u}{u} \right| \cdot 100\%$ that must be less or equal to 4%.

$$\varepsilon = 0\%$$

6.5.5. Determine the pitch diameter of the worm

$$d^w = m \cdot q^w = 8 \cdot 10 = 80\text{mm}.$$

6.5.6. Determine the addendum circle diameter of the worm

$$d_a^w = d_w + 2 \cdot m = 80 + 2 \cdot 8 = 96\text{mm}.$$

6.5.7. Determine the dedendum circle diameter of the worm

$$d_f^w = d^w - 2.4 \cdot m = 80 - 2.4 \cdot 8 = 60.8\text{mm}.$$

6.5.8. Determine the threaded length of the worm by means of table 6.7.

Table 6.7

Threaded length of the worm b^w

Shift factor x	Number of threads of the worm z^w	
	1; 2	4
0	$b^w \geq (11+0.06 \cdot z^g) \cdot m$	$b^w \geq (12.5+0.09 \cdot z^g) \cdot m$
-0.5	$b^w \geq (8+0.06 \cdot z^g) \cdot m$	$b^w \geq (9.5+0.09 \cdot z^g) \cdot m$
-1.0	$b^w \geq (10.5+z^w) \cdot m$	$b^w \geq (10.5+z^w) \cdot m$
+0.5	$b^w \geq (11+0.1 \cdot z^g) \cdot m$	$b^w \geq (12.5+0.1 \cdot z^g) \cdot m$
+1.0	$b^w \geq (12+0.1 \cdot z^g) \cdot m$	$b^w \geq (13+0.1 \cdot z^g) \cdot m$

Note. From manufacturing consideration, the threaded length of milled and ground worms is increased by 25 mm at m <10 mm, by 35 to 40 mm at m =10 mm to 16 mm and by 50 mm at m >16 mm.

$$b^w \geq (11+0.06 \cdot 40) \cdot 8 = 107.2 \text{ mm}$$

Assume $b^w = 107.2 + 25 \approx 132\text{mm}$

6.5.9. Determine the lead angle of the worm

$$\gamma = \text{arctg}\left(\frac{z^w}{q^w}\right) = \text{arctg}\left(\frac{2}{10}\right) = 11^\circ 18'$$

6.5.10. Determine the pitch diameter of the worm gear

$$d^g = m \cdot z^g = 8 \cdot 40 = 320\text{mm}.$$

6.5.11. Determine the addendum circle diameter of the worm gear

$$d_a^g = d^g + 2 \cdot m \cdot (1 + x) = 320 + 2 \cdot 8 \cdot (1 + 0) = 336\text{mm}.$$

6.5.12. Determine the dedendum circle diameter of the gear

$$d_f^g = d^g - 2 \cdot m \cdot (1.2 - x) = 320 - 2 \cdot 8 \cdot (1.2 - 0) = 300.8\text{mm}$$

6.5.13. Determine the maximum diameter of the worm gear

$$d_{amax}^g = d_a^g + \frac{6 \cdot m}{z^w + 2} = 336 + \frac{6 \cdot 8}{2 + 2} = 348\text{mm}$$

6.5.14. Determine the face width of the worm gear

$$b^g \le 0.75 \cdot d_a^w \quad \text{for} \quad z^w = 1;\ 2$$

$$b^g \le 0.67 \cdot d_a^w \quad \text{for} \quad z^w = 4.$$

$$b^g \le 0.75 \cdot 96 = 72\text{mm}$$

6.5.15. Determine the peripheral speed at the worm and the worm gear

$$V^w = \frac{\pi \cdot d^w \cdot n^w}{60} = \frac{\pi \cdot 80 \cdot 10^{-3} \cdot 441.75}{60} = 1.85\text{m/sec};$$

and

$$V^g = \frac{\pi \cdot d^g \cdot n^g}{60} = \frac{\pi \cdot 320 \cdot 10^{-3} \cdot 22.1}{60} = 0.37\text{m/sec}$$

6.5.16. Specify the slippage speed

$$V_{sl} = \frac{V^w}{\cos\gamma} = \frac{1.85}{\cos 11^\circ 18'} = 1.89\text{m/sec}$$

6.5.17. Determine the efficiency of the worm gearing

$$\eta = \frac{\text{tg}\,\gamma}{\text{tg}\,(\gamma + \rho')},$$

where ρ' is the friction angle determined by means of table 6.8.

Table 6.8

Angle of friction ρ'

Slippage speed V_{sl}, m/s	Angle of friction, ρ'	Slippage speed V_{sl}, m/s	Angle of friction, ρ'
0.1	4°30′ - 5°10′	2.5	1°40′ - 2°20′
0.5	3°10′ - 3°40′	3	1°30′ - 2°10′
1.0	2°30′ - 3°10′	4	1°20′ - 1°40′
1.5	2°20′ - 2°50′	7	1°00′ - 1°30′
2.0	2°00′ - 2°30′	10	0°55′ - 1°20′

Note. Greater values of ρ' correspond to tinless worm gears.

In our case: ρ' = 2°10'; $\eta = \dfrac{\mathrm{tg}11^\circ18'}{\mathrm{tg}\,(11^\circ18' + 2^\circ10')} = 0.834$

6.5.18. Determine forces in the engagement of the worm gearing

- turning force at the worm F_t^w and axial force at the worm gear F_a^g :

$$F_t^w = F_a^g = \frac{2 \cdot T^w}{d^w} = \frac{2 \cdot 105 \cdot 10^3}{80} = 2625\text{N};$$

- axial force at the worm F_a^w and turning force at the worm gear F_t^g :

$$F_a^w = F_t^g = \frac{2 \cdot T^g}{d^g} = \frac{2 \cdot 1750 \cdot 10^3}{320} = 10937{,}5\text{N};$$

- radial force Fr :

$$F_r = F_t^g \cdot \mathrm{tg}\,\alpha_w = 10937.5 \cdot \mathrm{tg}20^\circ = 3980.9\text{N}.$$

6.5.19. Determine the maximum contact stress

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T^g \cdot K_H \cdot E_{tr} \cdot \cos^2\gamma}{\left(d^g\right)^2 \cdot d^w \cdot \varepsilon_\alpha \cdot \xi \cdot \delta \cdot \sin 2\alpha_w}},$$

where T^g is in N·mm; ξ = 0.75 is the factor which takes into account the fact that contact of the worm and the worm gear occurs not along the whole arc of contact defined by the gear face angle 2δ (2δ ≈ 100º = 1.75 rad); ε_α is the contact ratio determined as

$$\varepsilon_\alpha = \left(\sqrt{0.03 \cdot \left(z^g\right)^2 + z^g + 1} - 0.17 \cdot z^g + 2.9\right) / 2.95 =$$
$$= \left(\sqrt{0.03 \cdot (40)^2 + 40 + 1} - 0.17 \cdot 40 + 2.9\right) / 2.95 = 1{,}876;$$

$K_H = K_{H\beta} \cdot K_{HV}$ is the design load factor.

Table 6.9

Worm deformation factor Θ

Number of threads of the worm z^w	Worm deformation factor at *q* of					
	8	10	12.5	14	16	20
1	72	108	157	176	225	248
2	57	86	125	152	171	197
3	51	76	110	134	148	170
4	47	70	101	123	137	157

$K_{H\beta} = 1 + \left(\frac{z^g}{\Theta} \right)^3 \cdot (1 - x_1)$ is the load concentration factor; Θ is the worm deformation factor (table 6.9);

x_1 takes into account load nature

$$x_1 = \frac{\sum_{i=1}^{n} t_i \cdot T_i}{T_{max} \cdot \sum_{i=1}^{n} t_i} = \frac{(0{,}15t \cdot T) + (0.25t \cdot 0.7T) + (0.6t \cdot 0.5T)}{T \cdot (0.15t + 0.25t + 0.6t)} = 0.625;$$

$$K_{H\beta} = 1 + \left(\frac{40}{86} \right)^3 \cdot (1 - 0{,}625) = 1{,}0377 .$$

K_{HV} is the dynamic load factor determined by means of table 6.10 (for worm gearings we take 7 or 8 accuracy of manufacturing).

Table 6.10

Dynamic load factor K_{HV}

Degree of accuracy	Dynamic load factor K_{HV} at V_{sl} (m/sec) of			
	up to 1.5	1.5 - 3	3 - 7.5	7.5 - 12
7	1.0	1.0	1.1	1.2
8	1.15	1.25	1.4	-
9	1.25	-	-	-

Thus if $V_{sl} = 1.89 \text{m/sec}$, and accuracy is 8 K_{HV} = 1.25;

K_H = 1.0377 · 1.25 = 1.297; ξ = 0.75; $2\delta \approx 100^\circ$;

Obtained value of σ_H should correspond to the following condition

$$\sigma_H = (0.8\ldots1.1) \cdot [\sigma_H].$$

Otherwise it is necessary to change the center distance a_w and recalculate the gearing.

$$\sigma_H = 1.18 \cdot \sqrt{\frac{1750 \cdot 10^3 \cdot 1.297 \cdot 1.26 \cdot 10^5 \cdot \cos^2 11^\circ 18'}{(320)^2 \cdot 80 \cdot 1.876 \cdot 0.75 \cdot 1.75/2 \cdot \sin 2 \cdot 20^\circ}} = 243.04 \text{MPa}$$

$\sigma_H < 1.1 \cdot [\sigma_H] = 264 \text{MPa}$

Strength condition is satisfied.

6.5.20. Determine the maximum bending stress

$$\sigma_b = 0.7 \cdot \frac{F_t^g \cdot K_b \cdot Y_b}{b^g \cdot m_n} \le [\sigma_b],$$

where K_b is the design load factor ($K_b = K_H$); K_b =1.297.
$m_n = m \cdot \cos\gamma$ is the module at the normal section;

$$m_n = 8 \cdot \cos 11^0 18' = 7.846$$

Y_b is the tooth form factor determined by means of table 6.11 depending on the number of teeth of the equivalent straight spur gear

$$z_v^g = \frac{z^g}{\cos^3\gamma} = \frac{40}{\cos^3 11°18'} = 42,397 \Rightarrow 42 .$$

Table 6.11

Tooth form factor Y_b

z_v	28	30	32	35	40	45	50	60	80	100	150	300
Y_b	1.8	1.76	1.71	1.61	1.55	1.48	1.45	1.4	1.34	1.3	1.27	1.24

From table 6.11 tooth form factor Y_b = 1.52;

$$\sigma_b = 0.7 \cdot \frac{10937.5 \cdot 1.297 \cdot 1.52}{72 \cdot 7.846} = 26.72\text{MPa} < [\sigma_b]$$

Strength condition is satisfied.

6.6. Determine the temperature of the oil containing in the casing

$$t_{oil} = t_{air} + \frac{P^w \cdot (1-\eta)}{K_t \cdot A} \le [t_{oil}],$$

where t_{air} = 20°C is the temperature of the air;

P^w is the power at the worm in W; $P^w = \frac{T^g \cdot \omega^g}{\eta} = \frac{1750 \cdot 1.92}{0.834} = 4.03 \cdot 10^3\,\text{W}$

η is the efficiency of the worm gearing;

K_t is the heat transfer factor (for cast iron casings K_t =15…18 W/m^2·C°);

A is the area of the cooling surface determined approximately depending upon the center distance a_w by means of table 6.12;

[t_{oil}] is the allowable temperature of the oil (for industrial oils [t_{oil}] = 80…95°C).

Table 6.12

Cooling surface area *A* of the worm gear speed reducer

a_w, mm	80	100	125	140	160	180	200	225	250	280
A, m^2	0.19	0.24	0.36	0.43	0.54	0.67	0.8	1.0	1.2	1.4

Assume that K_t = 17; than from table 6.12 the area of cooling surface A = 0.78 m^2;

Thus $$t_{oil} = 20 + \frac{4.03 \cdot 10^3 \cdot (1 - 0.834)}{17 \cdot 0.78} = 70.45^\circ \mathrm{C} < [t_{oil}].$$

Condition is satisfied.

7. Analysis of the flat belt drive

Let us carry out the analysis of the flat belt drive if input power P_1=6.6 kW; torque at the driving pulley T_1=42 N·m; velocity ratio of the belt drive u_{bd}=2.15; rotational speed of the driving pulley n_1=1555 rpm.

7.1. Determine the diameter of the smaller(driving) pulley

$$d_1 \approx 6 \cdot \sqrt[3]{T_1} = 6 \cdot \sqrt[3]{42 \cdot 10^3} = 208.6\,\text{mm},$$

where T_1 is in N·mm.

Round off the diameter to the nearest standard value according to the following series: 63, 71, 80 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120.

Assume d_1=224 mm.

7.2. Determine the diameter of the larger pulley taking into account a relative speed loss ε = 0.01%

$$d_2 = d_1 \cdot u_{bd} \cdot (1-\varepsilon) = 224 \cdot 2.15 \cdot (1-0.0001) = 481.55\text{mm}.$$

and round off obtained magnitude according to the series of standard values.

Assume d_2=500 mm.

7.3. Specify the velocity ratio of the belt drive

$$u_{bd} = \frac{d_2}{d_1 \cdot (1-\varepsilon)} = \frac{500}{224 \cdot (1-0.0001)} = 2.232\,.$$

Error should be ε ≤ 4 %.

$$\varepsilon = \left| \frac{2.232-2.15}{2.15} \right| \cdot 100\% = 3.6\% < 4\ \%.$$

7.4. Determine the center distance

$$a = 2 \cdot (d_1 + d_2) = 2 \cdot (224 + 500) = 1448\text{mm}.$$

7.5. Compute the contact angle

$$\alpha_1 = 180 - 60 \cdot \frac{d_2 - d_1}{a} = 180 - 60 \cdot \frac{500-224}{1448} = 168.56°\,.$$

7.6. Determine the belt length

$$L = 2 \cdot a + 0.5 \cdot \pi \cdot (d_1 + d_2) + \frac{(d_2 - d_1)^2}{4 \cdot a} = 2 \cdot 1448 + 0.5 \cdot 3.14 \cdot 724 + \frac{276^2}{4 \cdot 1448} = 4032.7\,\text{mm}\,.$$

7.7. Determine the belt speed

$$V = \frac{\pi \cdot d_1 \cdot n_1}{60} = \frac{3.14 \cdot 0.224 \cdot 1455}{60} = 17.056 \text{ m/sec}.$$

7.8. Determine the turning (tangential) force

$$F_t > \frac{P_1}{V} = \frac{6.6}{17.056} = 388.06 \text{ N}.$$

7.9. Choose the rubberized fabric belt according to table 7.1.

In our case we take the rubberized fabric belt Б 800 with the number of plies z = 3, δ_0 = 1.5 mm thick each (including the rubber interlayers); the maximum allowable load to the ply p_0 = 3 N/mm of width.

Table 7.1

Rubberized fabric belts

	Fabric plies			
	Б 800	БКНЛ	ТА-150	ТК-20
Nominal strength in N per mm of width	55	55	150	200
Maximum allowable load p_0 to a ply in N per mm of width	3	3	10	13
Design thickness δ_0 of fabric plies with rubber interlayers, mm	1.5	1.2	1.2	1.3
Number of plies if belt width b, mm				
20-71	3-5	3-5	--	--
80-112	3-6	3-6	--	--
125-560	3-6	3-6	3-4	3-4

Check the requirement

$$\delta=\delta_0 \cdot z \le 0.025 \cdot d_1;$$

1.5·3≤0.025·224; 4.5≤5.6mm. Condition is satisfied.

7.10. Determine contact angle factor C_α

$$C_\alpha = 1 - 0.003 \cdot (180 - \alpha_1) = 1 - 0.003 \cdot (180 - 168.56) = 0.966.$$

7.11. Determine factor C_v, with taks into account the effect of the belt speed

C_v = 1.04 – 0.0004 $\cdot V^2$=1.04 – 0.0004 $\cdot 17.056^2$=0,9236.

7.12. Determine service factor C_s:

- C_s = 1 for steady operation (belt conveyers, lathes and grinding machines);

- C_s=0.9 in the case of moderate vibration (chain conveyers and milling machines);

- C_s=0.8 in the case of considerable vibration (flight conveyers, planing machines).

The value of C_s is to be reduced by 0.1 in two-shift operation and by 0.2 in three-shift operation.

Let us assume that we have moderate vibration. That is why C_s=0.9.

7.13. Determine factor C_Θ, that takes into account the belt position in the space.

- $C_\Theta = 1$ for horizontal drives and inclined at up to 60°;
- $C_\Theta = 0.9$ for drives inclined at over 60° to 80°;
- $C_\Theta = 0.8$ for drives inclined at over 80° to 90°.

In our case we assume $C_\Theta = 1$.

7.14. Determine the allowable load to 1mm of ply width, N/mm

$$[p]=p_0 \cdot C_\alpha \cdot C_V \cdot C_s \cdot C_\Theta=3\cdot 0.966\cdot 0.9236\cdot 0.9\cdot 1=2.409 \text{ N/mm.}$$

7.15. Find the belt width

$$b \geq \frac{F_t}{z \cdot [p]} = \frac{388.06}{3 \cdot 3} = 43.12 \,\text{mm}$$

and round off obtained magnitude according to series of standard values: 20; 25; 32; 40; 50; 63; 71; 80; 90; 100; 112; 125; 140; 160; 180; 200 and so on. Assume b=50 mm.

7.16. Determine the pulley width B according to table 7.2.

Table 7.2

Determining the pulley width *B*

Belt width b, mm	20	25	32	40	50	63	71	80	90	100	112	125	140	160
Pulley width B, mm	25	32	40	50	63	71	80	90	100	112	125	140	160	180

In our case we assume B = 63 mm.

7.17. Determine the pretension in the belt

$$F_0 = \sigma_0 \cdot b \cdot \delta = 1.8\cdot 50\cdot 4.5=405 \text{ N,}$$

where σ_0 =1.8 MPa- tensile prestress.

7.18. Determine tension of the belt

on the tight side $F_1=F_0 + 0.5\cdot F_t$=405+0.5·388.06=599.03 N,

on the slack side $F_2=F_0 - 0.5\cdot F_t$=405-0.5·388.06=270.97 N

7.19. Calculate the force acting on the shaft and bearings

$$F_b = 3 \cdot F_0 \cdot \sin\frac{\alpha_1}{2} = 3 \cdot 405 \cdot \sin\frac{168.56}{2} = 1208.9\,\text{N}.$$

7.20. Determine tensile stress

$$\sigma_1 = \frac{F_1}{b \cdot \delta} = \frac{599.03}{50 \cdot 4.5} = 2.662\,\text{MPa}.$$

7.21. Determine bending stress

$$\sigma_b = E \cdot \frac{\delta}{d_1} = 100 \cdot \frac{4.5}{224} = 2.009\,\text{MPa},$$

where E is modulus of elasticity of the belt material. For rubberized fabric belts E =100÷200 MPa.

7.22. Determine tensile stress due to action of centrifugal force F_c

$$\sigma_c = \rho \cdot V^2 \cdot 10^{-6} = 1100 \cdot 17.056^2 \cdot 10^{-6} = 0.32\,\text{MPa},$$

where ρ is density of belts. For rubber-impregnated flat belts ρ = 1100 ÷ 1200 kg /m^3.

7.23. Determine the maximum stress

$$\sigma_{max} = \sigma_1 + \sigma_b + \sigma_c \le [\sigma].$$

For rubberized fabric belts [σ] = 7 MPa.

$$\sigma_{max} = \sigma_1 + \sigma_b + \sigma_c = 2.662 + 2.009 + 0.32 = 4.991\,\text{MPa} < [\sigma] = 7\,\text{MPa}.$$

Condition is satisfied.

7.24. Determine the service life of the belt

$$H_0 = \frac{\sigma_{-1}^6 \cdot 10^7 \cdot C_i \cdot C_l}{\sigma_{max}^6 \cdot 2 \cdot 3600 \cdot \lambda} \ge 2000\,\text{hours},$$

where σ_{-1} is limit of endurance (for rubberized belts σ_{-1}=7 MPa); $\lambda = \frac{V}{L} = \frac{17.056}{4032.7} = 4.229$ is the number of belt runs per second; $C_i \approx 1.5 \cdot \sqrt[3]{u_{bd}} - 0.5 = 1.5 \cdot \sqrt[3]{2.232} - 0.5 = 1.44$ takes into account the velocity ratio; C_l takes into account the nature of the load (for constant load C_l =1; for variable load C_l=2). We assume that load is variable.

$$H_0 = \frac{\sigma_{-1}^6 \cdot 10^7 \cdot C_i \cdot C_l}{\sigma_{max}^6 \cdot 2 \cdot 3600 \cdot \lambda} = \frac{7^6 \cdot 10^7 \cdot 1.44 \cdot 2}{4.991^6 \cdot 2 \cdot 3600 \cdot 4.229} = 5568.2 > 2000\,\text{hours}.$$

Condition is satisfied.

8. Analysis of the chain drive

Let us carry out the analysis of the chain drive for strength if torque at the driving sprocket T_1 = 464.5 N·m; torque at the driven sprocket T_2 = 1105 N·m; rotational speed of the driving sprocket n_1 = 122.25 rpm, rotational speed of the driven sprocket n_2 = 48.9 rpm; velocity ratio of the chain drive u_{cd} = 2.5; input power of the chain drive P_1 = 5.947 kW.

8.1. Determine the number of teeth of the driving sprocket

$$z_1 = 31 - 2 \cdot u_{cd} \geq 17$$

$$z_1 = 31 - 2 \cdot 2.5 = 26$$

and round off obtained magnitude to the nearest integer numeral.
Assume $z_1 = 26 > 17$.

8.2. Determine the number of teeth of the driven sprocket

$$z_2 = z_1 \cdot u_{cd} \leq 120$$

and round off to the nearest integer numeral

$$z_2 = 26 \cdot 2.5 = 65$$

Assume $z_2 = 65 < 120$.

8.3. Specify the velocity ratio and determine the error

$$u_{cd} = \frac{z_2}{z_1} = \frac{65}{26} = 2.5 .$$

The error should be $\varepsilon \leq 4$ %.
In our case $\varepsilon = 0$ %.

8.4. Determine the service factor K_s

$$K_s = K \cdot K_a \cdot K_{lub} \cdot K_\gamma \cdot K_d \cdot K_{ten},$$

where K takes into account the load nature taken as 1 in quiet operation and as 1.2 to 1.5 in the case of shocks and impacts; K_a is the center distance factor assumed as K_a=1 for a = 30·t to 50·t and K_a=0.8 for a = 60·t to 80·t; K_{lub} is lubrication factor (K_{lub}=0.8 for immersion lubrication, K_{lub}=1 for drop-feed lubrication and K_{lub}=1.5 for periodic greasing); K_γ accounts for the angle that the shaft centre line makes with the horizontal (K_γ=1 for $\gamma \le 60^\circ$ and K_γ=1.25 for $\gamma > 60^\circ$); K_d is a duty factor (K_d=1 for one-shift operation, K_d=1.25 for two-shift operation and K_d=1.5 for three-shift operation); K_{ten} accounts for the manner of tension control (K_{ten}=1 for drives with chain tighteners, K_{ten}=1.15 for drives with adjustable bases, and K_{ten}=1.25 for fixed-base drives).

In our case we have small shocks and impacts (K=1.2); the center distance is a = 40 t (K_a=1); periodic greasing (K_{lub}=*1.5*); $\gamma \le 60^\circ$ and (K_γ=1); one-shift operation (K_d=1); fixed base drive (K_{ten}=1.25).

$$K_s = 1.2 \cdot 1 \cdot 1.5 \cdot 1 \cdot 1 \cdot 1.25 = 2.25$$

8.5. Approximately determine the allowable mean pressure on the hinges by means of table 8.1 depending on the rotational speed of the smaller sprocket.

In our case for rotational speed n_1=122.25 rpm, $[p] \approx 29$ MPa.

Table 8.1

Allowable mean pressure [p], in MPa

n_1, rpm	Chain pitch, mm							
	12.7	15.875	19.05	25.4	31.75	38.1	44.45	50.8
50	46	43	39	36	34	31	29	27
100	37	34	31	29	27	25	23	22
200	29	27	25	23	22	19	18	17
300	26	24	22	20	19	17	16	15
500	22	20	18	17	16	14	13	12
750	19	17	16	15	14	13	-	-
1000	17	16	14	13	13	-	-	-
1250	16	15	13	12	-	-	-	-

8.6. Determine the chain pitch

$$t \ge 2.8 \cdot \sqrt[3]{\frac{T_1 \cdot K_s}{z_1 \cdot [p]}} = 2.8 \cdot \sqrt[3]{\frac{464.5 \cdot 10^3 \cdot 2.25}{26 \cdot 29}} = 31.22\text{mm},$$

where T_1 is in $N{\cdot}m$.

Round off the pitch to the nearest standard value according to the table 8.1. Assume t = 31.75 mm.

8.7. Specify the allowable mean pressure according to table 8.1 by interpolation [p]. On multiplying it by

$K_p = 1 + 0.01 \cdot (z_1\text{-}17) = 1 + 0.01 \cdot (26\text{-}17) = 1.09$ we get finite magnitude of $[p] = 29 \cdot 1.09 = 31.61$MPa

8.8. Determine the effective mean pressure.

For that we

- find chain speed

$$V = \frac{z_1 \cdot t \cdot n_1}{60 \times 10^3} = \frac{26 \cdot 31.75 \cdot 122.25}{60 \cdot 10^3} = 1.68 \text{m/sec};$$

- find turning (tangential) force

$$F_t = \frac{P_1}{V} = \frac{5.947 \cdot 10^3}{1.68} = 3539.9 \text{N};$$

- look up the projected hinge area S_h using table 8.2. In our case S_h=262 mm^2

Then the effective mean pressure

$$p = \frac{F_t \cdot K_s}{S_h} \le [p].$$

If this inequality is not right it is necessary to increase the pitch t.

$$p = \frac{3539.9 \cdot 2.25}{262} = 30.4 \text{MPa} < [p];$$

inequality is right.

8.9. Determine the number of links in the chain

$$L_t = 2 \cdot a_t + 0.5 \cdot z_\Sigma + \frac{\Delta^2}{a_t},$$

where $L_t = \dfrac{L}{t}$ is the chain length in pitches;

$a_t = \dfrac{a}{t}$; $a \approx (30 \ldots 50) \cdot t$; $a_t = 45$;

$z_\Sigma = z_1 + z_2$; $z_\Sigma = 26 + 65 = 91$;

$\Delta = \dfrac{z_2 - z_1}{2 \cdot \pi}$; $\Delta = \dfrac{62 - 26}{2 \cdot \pi} = 6.21$.

Round off obtained magnitude of L_t to even integer numeral.

$$L_t = 2 \cdot 45 + 0.5 \cdot 91 + \frac{6.21^2}{45} = 136.36 \Rightarrow 136$$

8.10. Specify the centre distance

$$a = 0.25 \cdot t \cdot (L_t - 0.5 \cdot z_\Sigma + \sqrt{(L_t - 0.5 \cdot z_\Sigma)^2 - 8 \cdot \Delta^2}\ ) =$$

$$= 0.25 \cdot 31.75 \cdot (136 - 0.5 \cdot 91 + \sqrt{(136 - 0.5 \cdot 91)^2 - 8 \cdot 6.21^2}\ ) = 1423.03\text{mm}$$

The slack side of the chain should have a slight sag $f \approx 0.01 \cdot a$, for which purpose the design centre distance is reduced by 0.2 to 0.4 %.

Assume that a = 1418.76 mm.

8.11. Determine the pitch diameters

- of the driving sprocket $d_{p1} = \dfrac{t}{\sin\left(\dfrac{180^\circ}{z_1}\right)} = \dfrac{31.75}{\sin\left(\dfrac{180}{26}\right)} = 263.41\text{mm}$;
- of the driven sprocket $d_{p2} = \dfrac{t}{\sin\left(\dfrac{180^\circ}{z_2}\right)} = \dfrac{31.75}{\sin\left(\dfrac{180}{65}\right)} = 657.17\text{mm}$.

8.12. Determine the addendum diameters

$$D_{e_1} = t \cdot (\text{ctg}\left(\frac{180}{z_1}\right) + 0.7) - 0.31 \cdot d_1 =$$

$$= 31.75 \cdot (\text{ctg}\left(\frac{180}{26}\right) + 0.7) - 0.31 \cdot 19.05 = 277.81\text{mm}$$

$$D_{e_2} = \text{t} \cdot (\text{ctg}\left(\frac{180}{z_2}\right) + 0.7) - 0.31 \cdot d_1 =$$

$$= 31.75 \cdot (\text{ctg}\left(\frac{180}{65}\right) + 0.7) - 0.31 \cdot 19.05 = 672.77\text{mm}$$

where d_1 is the roller diameter (table 8.2).

8.13. Determine the dedendum diameter

$$D_{i_1} = d_{p_1} - (d_1 + 0.175 \cdot \sqrt{d_{p_1}}) = 263.41 - (19.05 + 0.175 \cdot \sqrt{263.41}) = 241.52\text{mm},$$

$$D_{i_2} = d_{p_2} - (d_1 + 0.175 \cdot \sqrt{d_{p_2}}) = 657.17 - (19.05 + 0.175 \cdot \sqrt{657.17}) = 633.63\text{mm}.$$

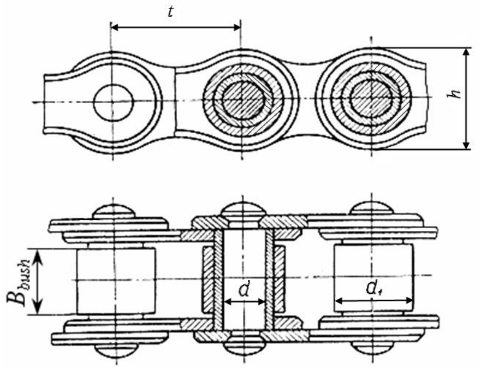
8.14. Determine the web thickness of sprocket

$$C = 0.93 \cdot B_{bush} = 0.93 \cdot 19.05 = 17.72\text{mm}$$

where B_{bush} is determined according to table 8.2.

Table 8.2

Leading particulars of Soviet-made roller chains PR



Pitch t	B_{bush}, mm	Pin diameter d	Roller diameter d_1	h, max	b, max	Breaking load F_{br}, kN	Mass per meter run q, kg	Projected hinge area S_h, mm^2
9.525	5.72	3.28	6.35	8.5	17	9.1	0.45	28.1
12.7	7.75	4.45	8.51	11.8	21	18.2	0.75	39.6
15.875	9.65	5.08	10.16	14.8	24	22.7	1.0	54.8
19.05	12.7	5.96	11.91	18.2	33	31.8	1.9	105.8
25.4	15.88	7.95	15.88	24.2	39	60.0	2.6	179.7
31.75	19.05	9.55	19.05	30.2	46	88.5	3.8	262
38.1	25.4	11.12	22.23	36.2	58	127.0	5.5	394
44.45	25.4	12.72	25.4	42.4	62	172.4	7.5	473
50.8	31.75	14.29	28.58	48.3	72	226.8	9.7	646

8.15. Determine forces acting to the links

- turning force $F_t = \frac{P_1}{V} = \frac{5.947 \cdot 10^3}{1.68} = 3539.88\text{N}$;

- centrifugal force $F_c = q \cdot V^2 = 3.8 \cdot 1.68^2 = 10.73$ N, where q is the mass per meter run of the chain in kg (table 8.2);

- load due to chain deflection $F_f = 9.81 \cdot K_f \cdot q \cdot a$, where $K_f = 1$ for vertical centre line arrangements, $K_f = 6$ for horizontal centre line arrangements and $K_f = 1.5$ for the centre line arrangement on the angle 45^o.

In our case centre line is arranged on the angle 45^o, thus $K_f = 1.5$

$$F_f = 9.81 \cdot 1.5 \cdot 3.8 \cdot 1.423 = 79.57 \text{ N}.$$

8.16. Determine the design load on the shaft

$$F_{shaft} = F_t + 2 \cdot F_f = 3539.88 + 2 \cdot 79.57 = 3699.02 \text{ N}.$$

8.17. Determine the safety factor

$$S = \frac{F_{br}}{F_t \cdot K + F_c + F_f} \geq [S],$$

where F_{br} is the breaking load in N(table 8.2) F_{br} = 88.5 kN;
K is the dynamic factor taking into account the load nature (p.8.4) K = 1.2; [S] is standard safety factor (table 8.3), [S] = 7.9.

Table 8.3

Standard factors of safety [S] for PR Roller chains

n_1, rpm	Chain pitch t, mm							
	12.7	15.875	19.05	25.4	31.75	38.1	44.45	50.8
50	7.1	7.2	7.2	7.3	7.4	7.5	7.6	7.6
100	7.3	7.4	7.5	7.6	7.8	8.0	8.1	8.3
300	7.9	8.2	8.4	8.9	9.4	9.8	10.3	10.8
500	8.5	8.9	9.4	10.2	11.0	11.8	12.5	-
750	9.3	10.0	10.7	12.0	13.0	14.0	-	-
1000	10.0	10.8	11.7	13.3	15.0	-	-	-
1250	10.6	11.6	12.7	14.5	-	-	-	-

$$S = \frac{88.5 \cdot 10^3}{3539.88 \cdot 1.2 + 10.73 + 79.57} = 20.4 > [S].$$

Condition is satisfied.

9. Analysis of shafts

Let us design the shafts construction and carry out strength analysis of the output shaft if torque at the output shaft T=400 N·m.

9.1 Determine the minimum diameter of speed reducer shafts

$$d_{min} = \sqrt[3]{\frac{T}{0.2 \cdot [\tau]}},$$

where T is the torque at the shaft in N·mm; [τ] is the allowable tangential stress due to torsion in MPa.

In order to compensate action of bending stresses the allowable tangential stress due to torsion is assumed as down rated. For steels [τ] = 15…20 MPa.

Obtained magnitude of d_{min} is rounded off to the greater side according to the following standard series: 20, 21, 22, 23, 24, 25, 26, 28, 30, 32, 34, 36, 38, 40, 42, 45, 48, 50, 52, 55, 58, 60, 65, 70, 75, 80, 85, 90, 95, 100,105, 110, 115, 120, 130, 140, 150.

If the speed reducer shaft is joined with the electric motor shaft the following condition should be carried out

$$d_{motor} - d_{min} \leq 10 \text{ mm},$$

where d_{motor} is the diameter of the electric motor shaft (table 9.1).

9.2. Design the construction of speed reducer shafts.

In general purpose speed reducers stepped shafts with solid cross-section are used as a rule.

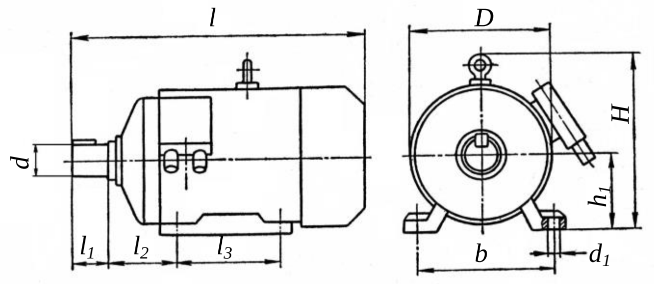
For the input shaft d_{min} is the diameter of the shaft cantilever portion where such elements as a half coupling, a pulley, a sprocket or a pinion may be mounted (Fig. 9.1). In order to fix above mentioned elements in the axial direction we use a shoulder which height t_1 may be ranged from 2 to 5 mm depending on the shaft diameter. Recommended values of t_1 are given in table 9.2.

The next shaft portion of diameter $d_2=d_1+2 \cdot t_1$ (the value of d_2 must correspond to standard series) is for installing a seal. Seals are used to prevent bearing assemblies from finding dust and dirt and to remain lubrication of bearings. For general purpose speed reducer commercial seals are used more frequently.

In order to reduce friction at the point of contact of the seal with the shaft corresponding portion should be polished. For this purpose this portion is additionally surface hardened to hardness 45-50 HRC.

Table 9.1

Overall and mounting dimensions of series 4A three-phase induction motors (GOST 19523-81)



Type designation	Number of poles	Overall dimensions			Mounting dimensions						
		L	H	D	d	h_1	l_1	l_2	l_3	b	d_1
4A90L	2;4;6;8	350	243	208	24	90	50	56	125	140	10
4A100S		365	265	235	28	100	60	63	132	160	12
4A100L		395	280						140		
4A112M		452	310	260	32	112	80	70	140	190	12
4A132S		480	350	302	38	132	80	89	178	216	12
4A132M		530									
4A160S	2	624	430	358	42	160	110	121	178	254	15
	4;6;8				48						
4A160M	2	667			42				210		
	4;6;8				48						

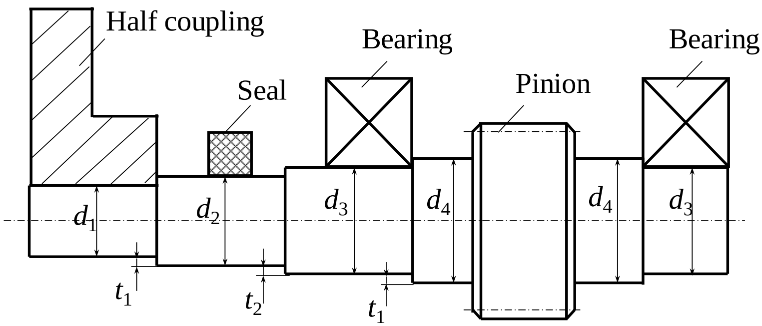


Fig.9.1. Input shaft

Table 9.2

Recommended values of t_1 and t_2

d, mm	20 - 50	55 - 120
t_1, mm	2; 2.5	5
t_2, mm	1; 1.5	2.5

The next portion of the shaft is for mounting a bearing. The diameter of this portion is determined as

$$d_3 = d_2 + 2 \cdot t_2,$$

where t_2 is the height of the shoulder that is used for differentiation of shaft surfaces by hardness and roughness. Recommended values of t_2 are given in table 9.2. It is necessary to note that t_2 should be chosen to obtain shaft diameter d_3 ended by 0 or 5. It is explained by the fact that bearings are standard elements with the inner ring diameter ended by 0 or 5.

Bearings must be fixed in the axial direction. That is why the diameter of the next portion of the shaft, where a pinion or gear is installed, is determined as

$$d_4 = d_3 + 2 \cdot t_1.$$

Obtained value of d_4 should correspond to standard series.

A pinion may be made either as solid with the shaft or as an individual part. In order to increase shaft strength and rigidity it is recommended to use pinion shafts.

The last portion of the shaft is for installing the second bearing. The diameter of this portion should be the same as for the first bearing. In our case it is d_3.

The output shaft has the same design as the input one. But in contrast to the latter a gear is mounted on the shaft portion of diameter d_4 (Fig.9.2). In order to fix the gear in the axial direction we should provide for the shoulder of height t_1. That is why the diameter of the next portion of the shaft is $d_5 = d_4 + 2 \cdot t_1$.

For our case we should design the output shaft where a helical spur gear is mounted. We will have the following diameters:

$d_{min} = \sqrt[3]{\dfrac{400 \cdot 10^3}{0.2 \cdot [20]}} = 46{,}4\ \text{mm}$, that's why d_1 = 48 mm (according to the standard series);

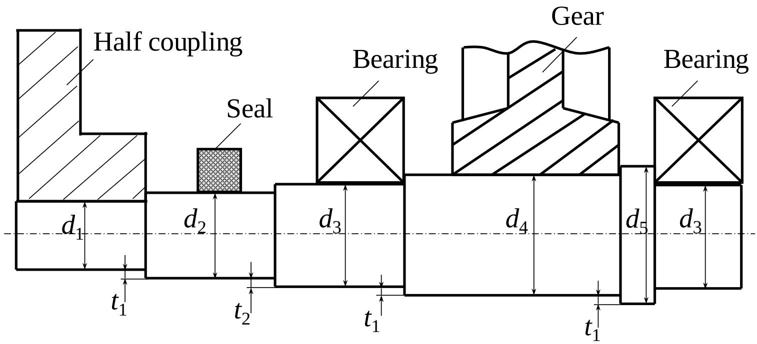


Fig.9.2. Output shaft

$$d_2 = d_1 + 2 \cdot t_1 = 48 + 2 \cdot 2.5 = 53 \text{ mm}, \ d_2 = 55 \text{ mm};$$
$$d_3 = d_2 + 2 \cdot t_2 = 55 + 2 \cdot 2.5 = 60 \text{ mm};$$
$$d_4 = d_3 + 2 \cdot t_1 = 60 + 2 \cdot 5 = 70 \text{ mm};$$
$$d_5 = \mathrm{d}_4 + 2 \cdot t_1 = 70 + 2 \cdot 5 = 80 \text{ mm}.$$

If we design an intermediate shaft d_{min} is the diameter of the portion where bearings are installed (Fig.9.3).

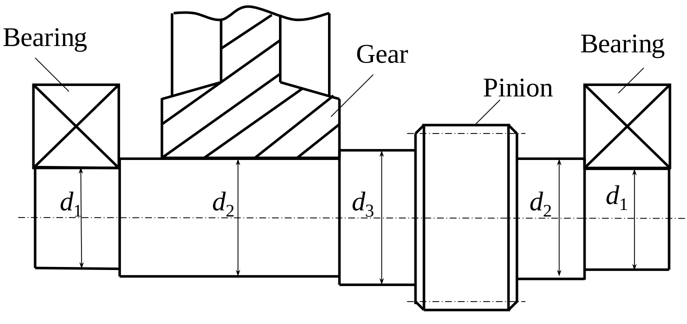


Fig.9.3. Construction of the intermediate shaft

For bevel pinion shafts (Fig.9.4) and worm shafts (Fig.9.5) we should introduce the additional portion of diameter d_2' between portions of diameters d_2 and d_3. This portion is necessary to install a slotted nut for adjusting clearances in the bearing. Diameter d_2' should be chosen according to table 12.1.

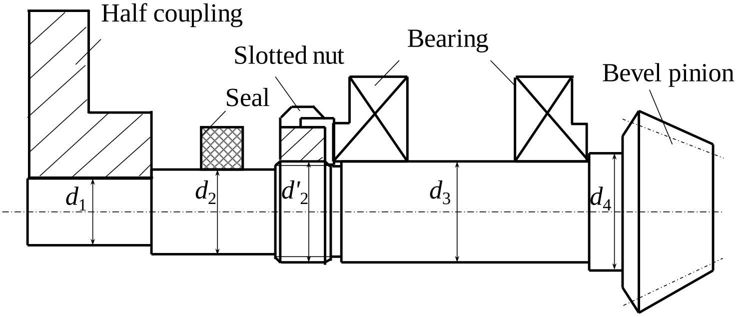


Fig.9.4. Bevel pinion shaft

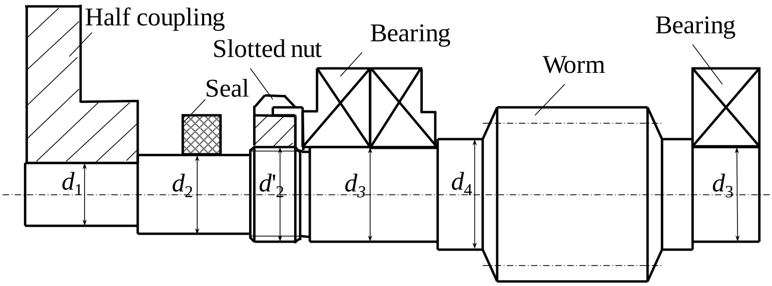


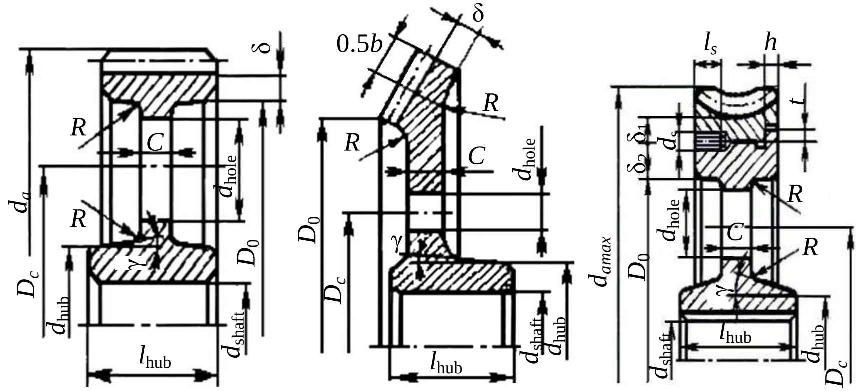
Fig.9.5. Worm shaft

9.3. Determine sizes of elements that are installed on the shaft.

9.3.1. Pinion.

Face width of the pinion $b^p = b^g + 5$.

9.3.2. Spur and bevel gears (Fig.9.6, *a*, *b*):



a *b* *c*

Fig.9.6. Spur gear (*a*), bevel gear (*b*), worm gear (*c*)

- thickness of the rim $\delta = (3\ldots4)\cdot m$;
- thickness of the web $C = (0.2\ldots0.3)\cdot b^g$;
- diameter of the hub $d_{hub}=(1.5\ldots1.7)\cdot d_{shaft}$;
- length of the hub $l_{hub}=(1.2\ldots1.5)\cdot d_{shaft}$;
- diameter of the hole $d_{hole} = \frac{D_0 - d_{hub}}{4}$;
- diameter of the hole centre line $D_c = \frac{D_0 + d_{hub}}{2}$;
- fillet radii $R \geq 6$ mm and angle $\gamma \geq 7^o$.

9.3.3. Worm gear (Fig 9.6, *c*).
- thickness of the bronze ring $\delta_1= 2\cdot m$;
- thickness of the steel rim $\delta_2= 2\cdot m$;
- thickness of the web $C = (0.2\ldots0.3)\cdot b^g$;
- diameter of the hub $d_{hub}=(1.5\ldots1.7)\cdot d_{shaft}$;
- length of the hub $l_{hub}=(1.2\ldots1.5)\cdot d_{shaft}$;
- diameter of the screw $d_s=(1.2\ldots1.4)\cdot m$;
- length of the screw $l_s=(0.3\ldots0.4)\cdot b^g$;
- diameter of the hole $d_{hole} = \frac{D_0 - d_{hub}}{4}$;
- diameter of the hole centre line $D_c = \frac{D_0 + d_{hub}}{2}$;
- width and height of the collar $h =0.15\cdot b^g$; $t= 0.8\cdot h$
- fillet radii $R \geq 6$ mm and angle $\gamma \geq 7^o$.

9.3.4. Bearings.

The type of a bearing depends on the load it can withstand.

Shafts where **straight spur gears** are located should be installed in bearings that withstand radial load only. For this purpose we use radial ball bearings of lightweight series (table 9.3).

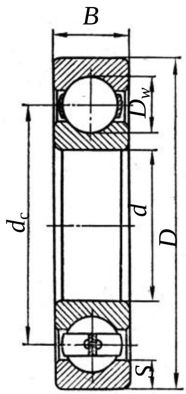
Shafts of **helical spur gears** should be mounted on angular contact ball bearings of lightweight series (table 9.4) because they can perceive both radial and axial loads.

Shafts of **bevel gears** and a **worm gear** are mounted on tapered roller bearings of lightweight series (table 9.5). It is explained by the fact that they can withstand heavy axial loads.

Worm shafts are installed in two tapered roller bearings (table 9.5) and one radial ball bearing of lightweight series (table 9.3).

Table 9.3

Single - Row Radial Ball Bearings. Lightweight series (GOST 8338-75), mm

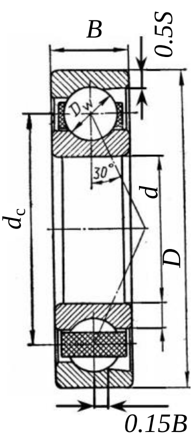


$d_c = 0.5\cdot(D+d)$
$D_w = 0.32\cdot(D\text{-}d)$
$S = 0.15\cdot(D\text{-}d)$

Type designation	d	D	B	r	Basic load rating C_r, kN	Static load rating C_0, kN
204	20	47	14	1.5	12.7	6.2
205	25	52	15	1.5	14.0	6.95
206	30	62	16	1.5	19.5	10.0
207	35	72	17	2	25.5	13.7
208	40	80	18	2	32.0	17.8
209	45	85	19	2	33.2	18.6
210	50	90	20	2	35.1	19.8
211	55	100	21	2.5	43.6	25.0
212	60	110	22	2.5	52.0	31.0
213	65	120	23	2.5	56.0	34.0
214	70	125	24	2.5	61.8	37.5
215	75	130	25	2.5	66.3	41.0
216	80	140	26	3	70.2	45.0
217	85	150	28	3	83.2	53.0
218	90	160	30	3	95.6	62.0
219	95	170	32	3.5	108.0	69.5
220	100	180	34	3.5	124.0	79.0

Table 9.4

Single-row angular-contact ball bearing (GOST 831-75). Lightweight narrow series (α=12° for 36000 and α=26° for 46000), mm

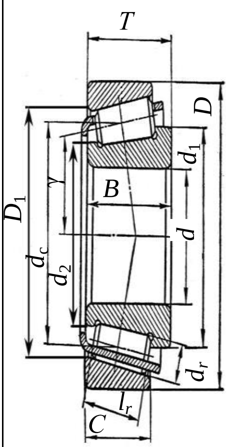


$d_c = 0.5\cdot(D+d)$
$D_w = 0.32\cdot(D\text{-}d)$
$S = 0.15\cdot(D\text{-}d)$

Type designation	d	D	B	T	r	r_1	Load rating, kN	
							C_r	C_0
36204	20	47	14	14	1.5	0.5	15.7	8.31
36205	25	52	15	15	1.5	0.5	16.7	9.10
36206	30	62	16	16	1.5	0.5	22.0	12.0
36207	35	72	17	17	2	1	27.8	17.8
36208	40	80	18	18	2	1	31.1	23.2
36209	45	85	19	19	2	1	36.2	25.1
36210	50	90	20	20	2	1	43.2	27.0
36211	55	100	21	21	2.5	1.2	48.4	34.2
36212	60	110	22	22	2.5	1.2	54.4	39.3
46213	65	120	23	23	2.5	1.2	61.5	46.8
36214	70	125	24	24	2.5	1.2	70.0	54.8
46215	75	130	25	25	2.5	1.2	80.2	59.8
36216	80	140	26	26	3	1.5	93.6	65.0
36217	85	150	28	28	3	1.5	101.0	70.8
36218	90	160	30	30	3	1.5	118.0	83.0
36219	95	170	32	32	3.5	2	134.0	95.0
36220	100	180	34	34	3.5	2	124	118
36222	110	200	38	38	3.5	2	146	137

Table 9.5

Single-row tapered-roller bearings (GOST 333-79). Lightweight series, α=12÷18°, mm



$d_c = 0.5\cdot(D+d)$
$d_r = 0.25\cdot(D\text{-}d)$
$l_r = 0.68\cdot B$

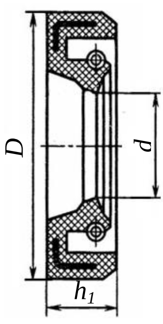
Type designation	d	D	T	B	C	d_1	d_2	D_1	Load rating, kN		e
									C_r	C_0	
7202	15	35	11.75	11	9	25	20	27	10.5	6.1	0.45
7203	17	40	13.25	12	11	27	22	32	14.0	9.0	0.31
7204	20	47	15.25	14	12	32	26	38	21.0	13.0	0.36
7205	25	52	16.25	15	13	37	31	43	24.0	17.5	0.36
7206	30	62	17.25	16	14	45	38	52	31.5	22.0	0.36
7207	35	72	18.25	17	15	52	44	60	38.5	26.0	0.37
7208	40	80	19.25	19	16	58	50	67	46.5	32.5	0.38
7209	45	85	20.75	20	16	63	55	72	50.0	33.0	0.41
7210	50	90	21.75	21	17	67	58	78	56.0	40.0	0.37
7211	55	100	22.75	21	18	75	65	85	65.0	46.0	0.41
7212	60	110	23.75	23	19	82	72	95	78.0	58.0	0.35
7214	70	125	25.25	26	21	95	83	108	96.0	82.0	0.37
7215	75	130	27.25	26	22	100	88	113	107.0	84.0	0.39
7216	80	140	28.25	26	22	110	97	121	112.0	95.2	0.42
7217	85	150	30.50	28	24	113	100	129	130.0	109.0	0.43
7218	90	160	32.50	31	26	121	107	138	158.0	125.0	0.38
7219	95	170	34.50	32	27	129	114	145	168.0	131.0	0.41
7220	100	180	37.00	34	29	137	122	154	185.0	146.0	0.41

9.3.5. Commercial seal.

Dimensions of commercial seals are given in table 9.6.

Table 9.6

Standard commercial seals (*GOST* 8752-79), mm



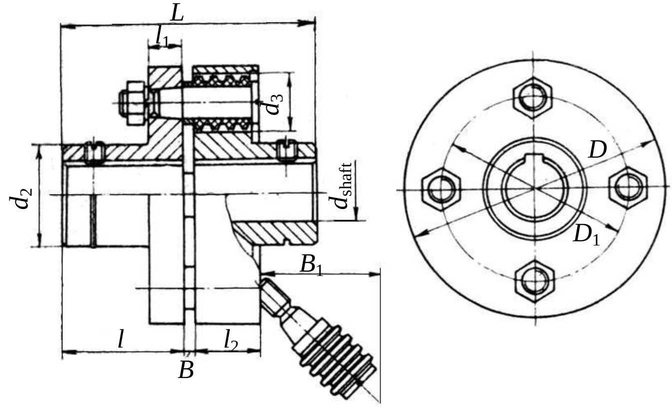
d	D	h_1	d	D	h_1
20;21;22	40		55;56;58	80	
24	41		60	85	
25	42		63;65	90	
26	45		70;71	95	
30;32	52		75	100	
35;36;38	58	10	80	105	12
40	60		85	110	
42	62		90;95	120	
45	65		100	125	
48;50	70		105	130	
52	75		110	135	

9.3.6. Coupling.

Dimensions of the couplings may be found by means of tables 9.7 (coupling with rubber-bushed studs), 9.8 (chain coupling) and 9.9 (flanged coupling).

Table 9.7

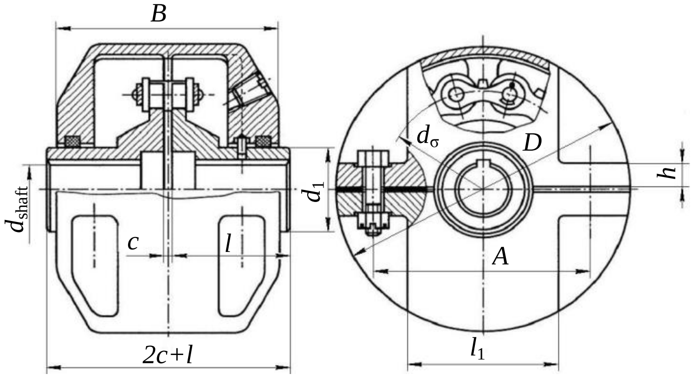
Standard couplings with rubber bushed studs (GOST 21424 – 75), mm



<table>
<tr><th rowspan="3">Torque T, N·m</th><th rowspan="3">d</th><th rowspan="3">D, not more</th><th colspan="2">L</th><th colspan="2">l</th><th rowspan="3">D_1</th><th rowspan="3">l_1</th><th rowspan="3">l_2</th><th rowspan="3">d_2</th><th rowspan="3">d_3</th><th rowspan="3">B</th><th rowspan="3">B_1</th><th rowspan="3">Number of studs</th></tr>
<tr><th colspan="4">Modification</th></tr>
<tr><th>1</th><th>2</th><th>1</th><th>2</th></tr>
<tr><td rowspan="2">31.5</td><td>16</td><td rowspan="2">90</td><td rowspan="2">81</td><td rowspan="2">60</td><td rowspan="2">40</td><td rowspan="2">28</td><td rowspan="2">63</td><td rowspan="7">16</td><td rowspan="7">20</td><td>30</td><td rowspan="7">20</td><td rowspan="7">4</td><td rowspan="7">28</td><td rowspan="4">4</td></tr>
<tr><td>18</td><td>32</td></tr>
<tr><td rowspan="2">63.0</td><td>20</td><td rowspan="2">100</td><td rowspan="2">104</td><td rowspan="2">76</td><td rowspan="2">50</td><td rowspan="2">36</td><td rowspan="2">71</td><td rowspan="2">36</td></tr>
<tr><td>22</td></tr>
<tr><td rowspan="3">125</td><td>25</td><td rowspan="3">120</td><td rowspan="3">125</td><td rowspan="3">89</td><td rowspan="3">60</td><td rowspan="3">42</td><td rowspan="3">90</td><td rowspan="2">40</td><td rowspan="10">6</td></tr>
<tr><td>28</td></tr>
<tr><td>30</td><td>50</td></tr>
<tr><td rowspan="7">250</td><td>32</td><td rowspan="7">140</td><td rowspan="7">165</td><td rowspan="7">121</td><td rowspan="7">80</td><td rowspan="7">58</td><td rowspan="7">105</td><td rowspan="10">18</td><td rowspan="10">32</td><td>56</td><td rowspan="10">28</td><td rowspan="10">5</td><td rowspan="10">42</td></tr>
<tr><td>35</td><td>56</td></tr>
<tr><td>36</td><td>63</td></tr>
<tr><td>38</td><td>71</td></tr>
<tr><td>40</td><td>71</td></tr>
<tr><td>42</td><td>75</td></tr>
<tr><td>45</td><td>80</td></tr>
<tr><td rowspan="3">500</td><td>40</td><td rowspan="3">170</td><td rowspan="3">225</td><td rowspan="3">160</td><td rowspan="3">110</td><td rowspan="3">82</td><td rowspan="3">130</td><td>71</td><td rowspan="3">8</td></tr>
<tr><td>42</td><td>75</td></tr>
<tr><td>45</td><td>80</td></tr>
<tr><td rowspan="4">1000</td><td>48</td><td rowspan="4">220</td><td rowspan="4">226</td><td rowspan="4">170</td><td rowspan="4">140</td><td rowspan="4">105</td><td rowspan="4">160</td><td rowspan="4">24</td><td rowspan="4">40</td><td>80</td><td rowspan="4">36</td><td rowspan="4">6</td><td rowspan="4">56</td><td rowspan="4">10</td></tr>
<tr><td>50</td><td>90</td></tr>
<tr><td>55</td><td>90</td></tr>
<tr><td>60</td><td>100</td></tr>
</table>

Table 9.8

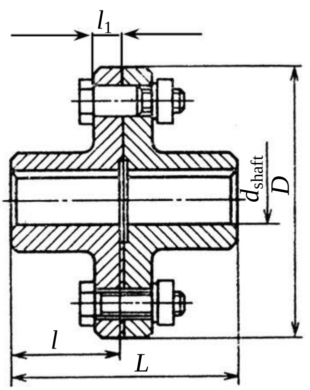
Chain couplings (GOST 20742-81), mm



Torque, N·m	d_{shaft}	D	l	c	d_1	d_σ	l_1	B	A	h
1000	55	210	85	1.8	90	147	120	150	144	15
	60		90							
2000	70	280	105	2.0	100	196	170	200	202	17
	80		120		115					
	90		135		130					
4000	100	350	150	2.0	150	260	208	200	240	18
	110		165		170					
	125		180		180					

Table 9.9

Flanged coupling (GOST 20761-80), mm



Torque T, N·m	d_{shaft}	D	L		l		l_1
			Modification				
			1	2	1	2	
63	20	100	104	76	50	36	15
	22						
125	25	112	124	83	60	42	17
	28						
250	30	140	170	120	80	58	20
	32						
	35						
	36						
400	40	150	230	170	110	82	22
	45						
	50						
630	55	170	290	220	140	105	22
	60						

End of the table 9.9

	1000	60	180	290	220	140	105	25
		63						
		70						
	1600	70	190	290	220	140	105	25
		75						
		80						
	2500	80	224	350	270	170	130	38
		85						
		90						
		95						
	5000	100	224	430	340	210	165	55

9.3.7. Sprocket (Fig 9.7):
- diameter of the hub $d_{hub}=(1.5\ldots1.7)\cdot d_{shaft}$;
- length of the hub $l_{hub}=(1.2\ldots1.5)\cdot d_{shaft}$;
- thickness of the web $C = 0.93\cdot B_{bush}$,

where B_{bush} is the bush width (table 8.2).

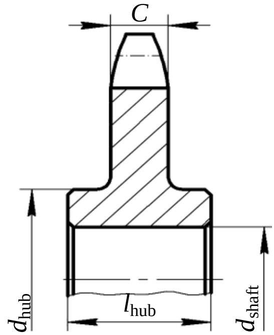


Fig. 9.7. Sprocket

9.3.8. Pulley (Fig. 9.8).
- diameter of the hub $d_{hub}=(1.8\ldots2.2)\cdot d_{shaft}$;
- length of the hub $l_{hub}=(1.5\ldots2.2)\cdot d_{shaft} \le B$, where B is the pulley width;
- thickness of the web $C=(1.2\ldots1.3)\delta$

Thickness of the rim $\delta=0.02(d+2B)$

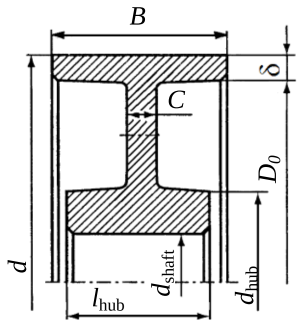


Fig. 9.8. Pulley

SKETCH LAYOUT

After designing a shaft construction and determination of sizes of all elements mounted on a shaft it is necessary to find distances between elements which are located on a shaft. For this purpose a sketch layout of a speed reducer should be made. In this case a speed reducer is drawn in one projection (top or front view) to scale 1:1 on profile paper.

Let us consider as an example plotting a sketch layout of double stage spur gear speed reducer.

1. Plot a straight spur gears taking into account dimensions which were determined during strength analysis and in p.9.3.1 and 9.3.2. We

will begin from plotting the centre distance, pitch circle diameters, addendum and dedendum circle diameters of a pinion and a gear. The engagement of gears has to be represented as in Fig. 9.9.

2. Plot a helical spur gears. It is recommended that a distance between straight and helical spur gears should not be less than 10 mm.

3. Determine disposition of inner walls of a speed reducer. In order to eliminate contact of gears with a wall it is recommended to locate the inner wall by distance 10 mm with respect to a gear hub and by distance 20 mm with respect to gear face end.

4. Determine disposition of bearings. In this case we should take into account that inner surface of a bearing assembly has to be protected from grease washing out. For this purpose grease retaining rings are used. The width of these rings is ranged from 10 to 12 mm. That is why bearings are located by distance 10 mm with respect to speed reducer inner wall.

5. Plot bearings for input, intermediate and output shafts using dimensions from tables 9.3-9.5.

6. Determine disposition of outer walls of a speed reducer. Outer wall is located by distance $0.5 \cdot B_{max,}$ where B_{max} is the width of the largest bearing.

7. In a speed reducer bearing assemblies have to be protected by bearing caps from the side of speed reducer outer walls. The width of these caps is ranged from 8 to 12 mm. Let us draw straight lines by distance 10 mm with respect to speed reducer outer walls to determine approximate disposition of bearing caps.

8. Plot commercial seals for input and output shafts using dimensions from table 9.6. A seal is located in a bearing cap by distance 3-4 mm with respect to bearing cap face end.

9. Make an arc along input and output shaft axes by distance 20 mm with respect to bearing caps to determine disposition of the first shoulder of a shaft.

10. Lay out length of an element mounted on the cantilever portion of the shaft along input and output shaft axes. In this case we obtain extreme points of the input and output shafts.

11.Draw speed reducer shafts taking into account their construction and diameters of corresponding portions

Examples of sketch layout of double stage coaxial spur gear speed reducer and double stage bevel and spur gear speed reducer are correspondingly shown in Fig. 9.9-9.11.

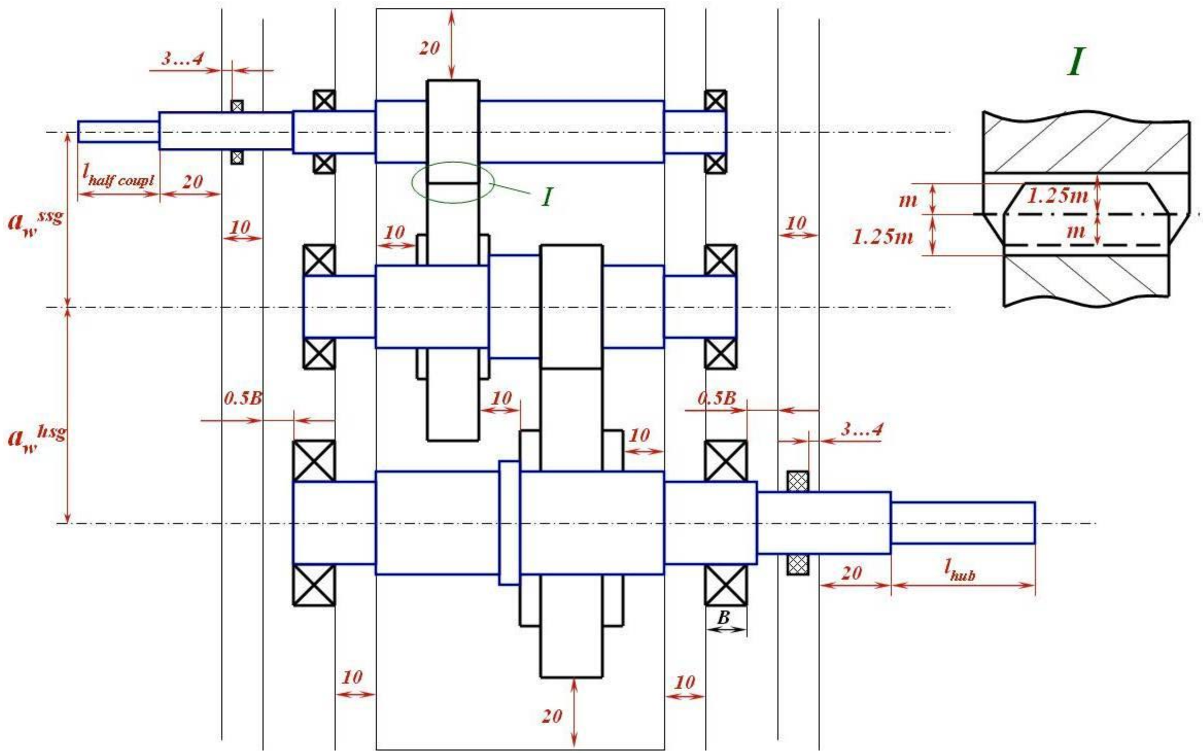


Fig. 9.9. Sketch layout of double stage spur gear speed reducer

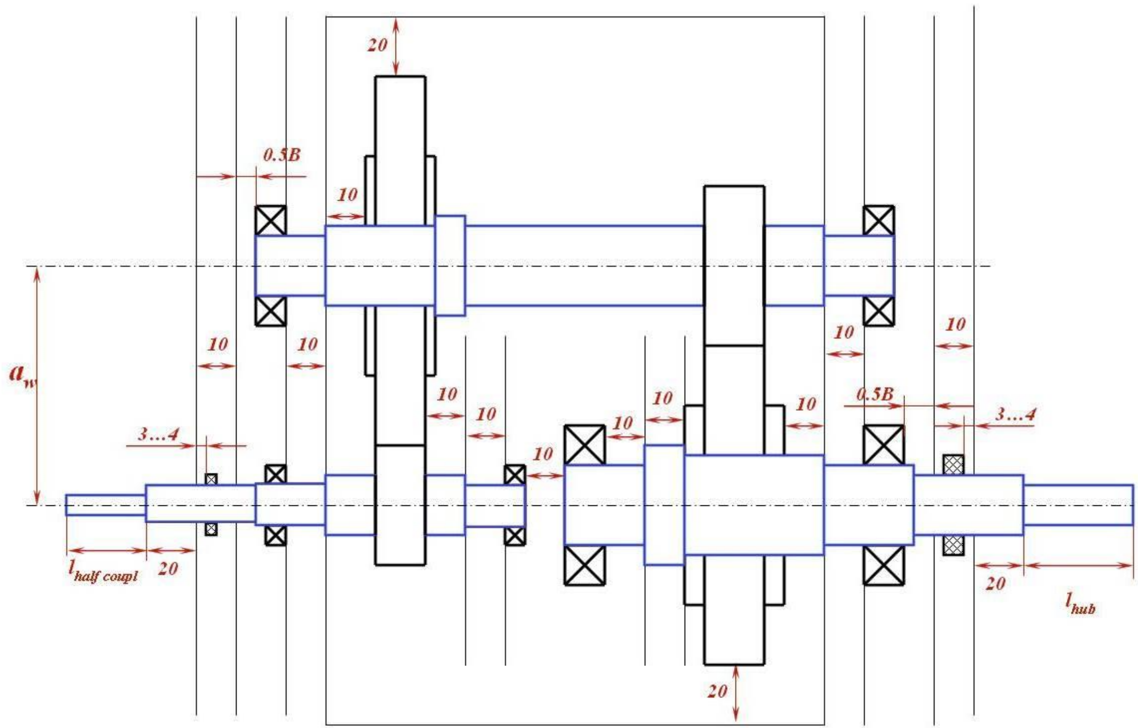


Fig. 9.10. Sketch layout of double stage coaxial spur gear speed reducer

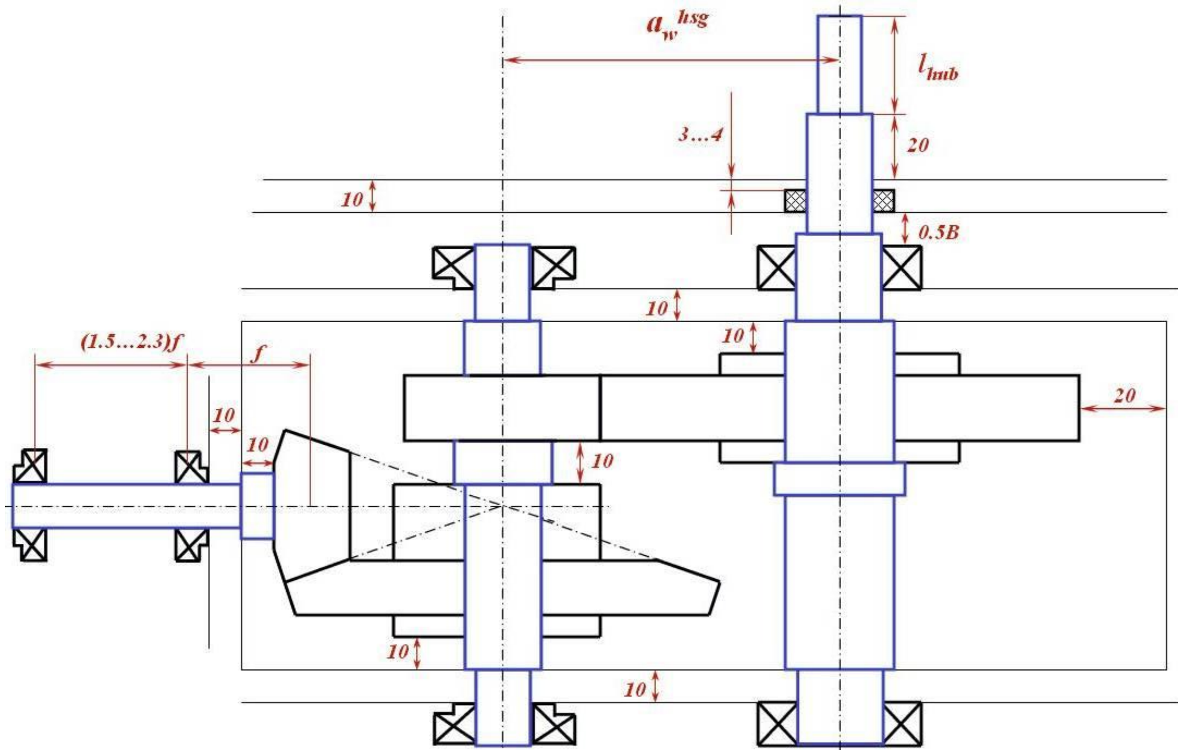


Fig. 9.11. Sketch layout of double stage bevel and spur gear speed reducer

The order of plotting a bevel gears is shown in Fig. 9.12.

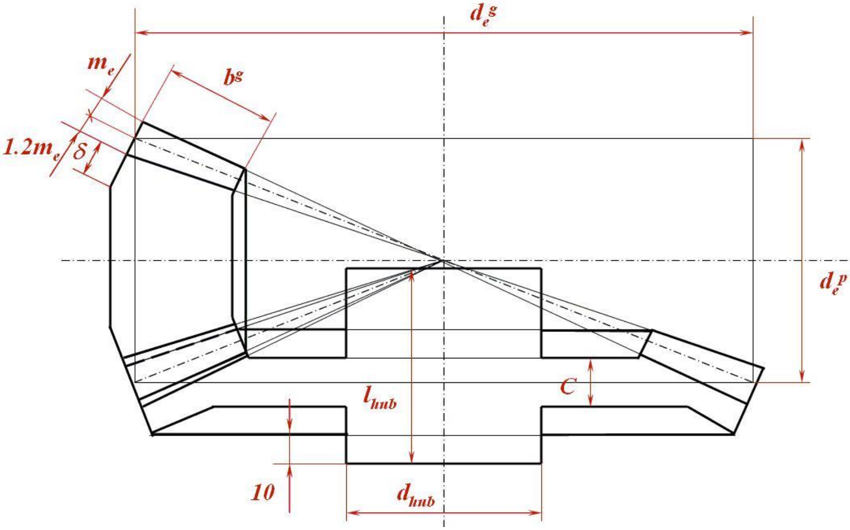


Fig. 9.12. Sketch layout of bevel gears

10. Shaft analysis for strength

Let us carry out the analysis of the shaft for strength if torque at the pinion shaft T_p = 370 N·m and from the previous analysis we have that turning (tangential) force F_t = 2467 N; F_r = 898 N; F_a = 346.7 N; d_g = 300 mm; d = 55 mm (the shaft diameter under the gear), and from the sketch layout a = 63 mm; b = 63 mm; c = 164 mm.

10.1. Select the material of the shaft.

The main material of shafts is medium-carbon or alloy steels of grade Steel 0.4C (40), Steel 0.45C (45), Steel 0.4C-Cr (40X) and others with hardness H ≥ 200 BHN.

Mechanical characteristics of steels are given in table 10.1.

Table 10.1

Mechanical Characteristics of Basic Shaft Materials

Steel grade	Blank diameter, mm(max)	Brinell hardness,	σ_{ul}	σ_y	Heat treatment
			MPa		
0.45C	160	170 to 217 BHN	600	340	Normalizing
0.45C	200	192 to 240 BHN	750	450	Martempering
0.40C-Cr	200	230 to 260 BHN	850	550	Martempering
0.40C-Cr	120	260 to 280 BHN	950	700	Martempering
0.40C-Cr-Ni	200	230 to 300 BHN	850	600	Martempering
0.35C-Cr-Mo	200	>240 BHN	900	800	Martempering
0.40C-Cr-Ni-Mo quality	160	>302 BHN	1100	900	Martempering

For example, let us choose Steel 0.4C-Cr (40X) heat treated by martempering to hardness ranged from 230 to 260 BHN, σ_{ul} = 850 MPa, σ_y = 550 MPa.

10.2. Analyze a shaft for static strength.

10.2.1. Plot the analytical model of a shaft and apply all acting forces (Fig. 10.1). In this case a shaft is considered as a beam mounted on two supports, in particular on one immovable hinge support and one movable hinge support.

Let us analyze forces which may act on a shaft.

It is necessary to remember that in the engagement of straight spur gears turning force F_t and radial force F_r develop (radial force is always directed to the centre of rotation of the gear). In the engagement of helical gears, bevel gears and worm gearing besides turning and radial forces axial force F_a develops. This force is parallel to the shaft axis. Values of mentioned above forces were found during of analysis corresponding gear drives for strength.

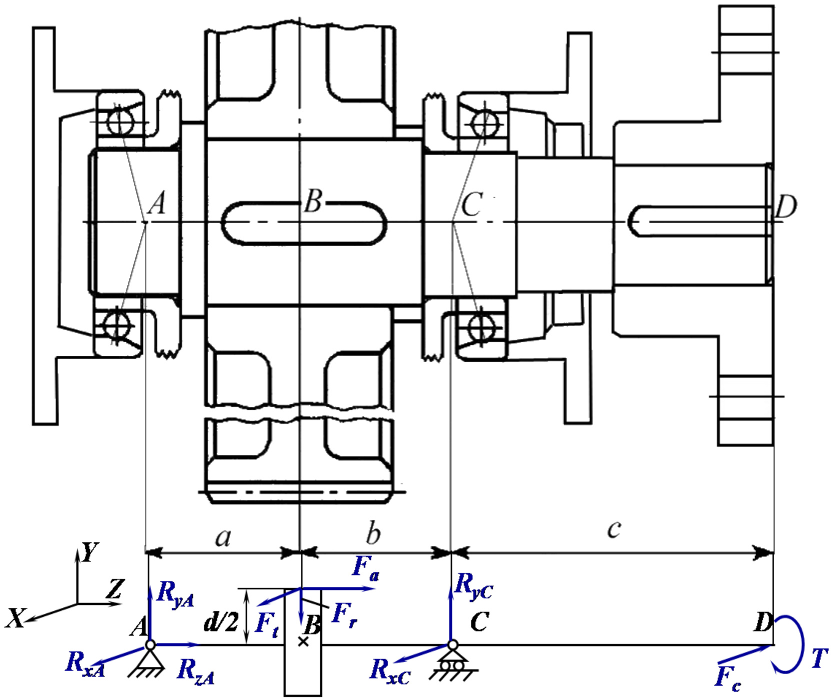


Fig. 10.1. Analytical model of the output shaft

On the cantilever portion of the shaft either a pulley or a sprocket or a half coupling may be mounted.

If a pulley or sprocket is installed on the shaft a force acting to the shaft from the side of the element is directed along the centre line of the mechanical drive. The value of this force was determined during analysis of the corresponding mechanical drive.

If a half coupling is installed on the shaft the first loads the shaft by the torque. Additionally, because of misalignment of shafts joined by the coupling the latter exerts upon the shaft an additional force F_c. This force may be directed to any side (with respect to turning force F_t). But for analytical models we will consider the worst case when this force is directed opposite to F_t. In this case the shaft deformations are maximum.

The value of this force is determined in the following way:

- for single stage speed reducers $F_c = 125 \cdot \sqrt{T}$;
- for double stage speed reducers $F_c = 250 \cdot \sqrt{T}$,

where T is the torque at the shaft in N·m.

10.2.2. Plot the analytical model of the shaft in the vertical plane and transfer all forces to the shaft (Fig. 10.2). It is necessary to note that according to the theoretical mechanics as a result of transferring parallel forces to any point the additional moment develops. In our case it is axial force F_a that is parallel to the shaft axis. That is why the additional moment is

$$M_a = F_a \cdot \frac{d^g}{2},$$

where d^g is the pitch circle diameter of the gear.

So $M_a = 346.7 \cdot \frac{300}{2} = 52.005 \cdot 10^3 \text{ N} \cdot \text{mm}$

$$F_c = 250 \cdot \sqrt{T} = 250 \cdot \sqrt{370} = 4808.8\text{N}$$

10.2.3. Determine vertical support reacting R_{yA} and R_{yC}. For this purpose we should consider the equilibrium of the beam and set up equations of moments with respect to points A and C:

$$\sum M_A^v = 0, \quad \sum M_C^v = 0.$$

$$\sum M_A = 0: \quad -F_r \cdot a - M_a + R_{yC} \cdot (a+b) = 0;$$

$$R_{yC} = \frac{F_r \cdot a + M_a}{a+b} = \frac{898 \cdot 63 + 52.005 \cdot 10^3}{63+63} = 861.74\text{N};$$

$$\sum M_c = 0: \quad -R_{yA} \cdot (a+b) + F_r \cdot b - M_a = 0;$$

$$R_{yA} = \frac{F_r \cdot b - M_a}{a+b} = \frac{898 \cdot 63 - 52.005 \cdot 10^3}{63+63} = 36.26\text{N};$$

For checking we set up equation of forces that act in the vertical plane of the shaft. The sum of these forces should give zero ($\sum F_i^v = 0$).

$$\sum F_{yi} = 0: \qquad R_{yA} - F_r + R_{yC} = 0.$$

$$36.26 - 898 + 861.74 = 0.$$

10.2.4. Plot the diagram of bending moments in the vertical plane (M_b^v) (Fig. 10.2).

$$0 \le x \le a; \qquad M_y = R_{yA} \cdot x;$$

$$M_y(0) = 0; \qquad M_y(a) = R_{yA} \cdot a = 36.26 \cdot 63 = 2.28 \cdot 10^3\,\mathrm{N \cdot mm};$$

$$a \le x \le a + b; \qquad M_y = R_{yA} \cdot x + M_a - F_r \cdot (x - a);$$

$$M_y(a) = R_{yA} \cdot a + M_a = 36.26 \cdot 63 + 52.005 \cdot 10^3 = 54.29 \cdot 10^3\,\mathrm{N \cdot mm};$$

$$M_y(a+b) = R_{yA} \cdot (a+b) + M_a - F_r \cdot b =$$

$$= 36.26 \cdot (63 + 63) + 52.005 \cdot 10^3 - 898 \cdot 63 = 0;$$

10.2.5. Plot the analytical model of the shaft in the horizontal plane and transfer all forces to the shaft (Fig. 10.2). In this case as a result of parallel transferring force F_t the torque T develops

$$\mathrm{T} = \mathrm{F_t} \cdot \frac{\mathrm{d^g}}{2} = 2467 \cdot \frac{300}{2} = 370 \cdot 10^3\,\mathrm{N \cdot mm}.$$

10.2.6. Determine horizontal support reacting forces R_{XA} and R_{XC}. For this purpose we should set up equations of moments with respect to points *A* and *C*:

$$\sum M_A^h = 0, \qquad \sum M_C^h = 0.$$

$$\sum M_A = 0: \qquad F_t \cdot a + R_{xC} \cdot (a+b) - F_c \cdot (a+b+c) = 0;$$

$$R_{xC} = \frac{-F_t \cdot a + F_c \cdot (a+b+c)}{a+b} =$$

$$= \frac{-2467 \cdot 63 + 4808.8 \cdot (63 + 63 + 164)}{63 + 63} = 9834.37\,N;$$

$$\sum M_c = 0: \qquad -R_{xA} \cdot (a+b) - F_t \cdot b - F_c \cdot c = 0;$$

$$R_{xA} = \frac{-F_t \cdot b - F_c \cdot c}{a+b} = \frac{-2467 \cdot 63 - 4808.8 \cdot 164}{63 + 63} = -7492.57\,\mathrm{N};$$

For checking we find the sum of all forces that acts in the horizontal plane of the shaft. This sum should be equal to zero ($\sum F_i^h = 0$).

$$\sum F_{xi} = 0: \quad -R_{xA} + F_t + R_{xC} - F_c =$$
$$-7492.57 + 2467 + 9834.37 - 4808.8 = 0.$$

10.2.7. Plot the diagram of bending moments in the horizontal plane ($\mathrm{M_b^h}$) (Fig. 10.2).

$$0 \le x \le a; \quad M_x = -R_{xA} \cdot x; \quad M_x(0) = 0;$$
$$M_x(a) = -R_{xA} \cdot a = -7492.57 \cdot 63 = -472 \cdot 10^3\,\mathrm{N \cdot mm};$$

$$a \le x \le a + b; \quad M_x = -R_{xA} \cdot x + F_t \cdot (x - a);$$
$$M_x(a) = -R_{xA} \cdot a = -7492.57 \cdot 63 = -472 \cdot 10^3\,\mathrm{N \cdot mm};$$
$$M_x(a+b) = -R_{xA} \cdot (a+b) + F_t \cdot b =$$
$$= -7492.57 \cdot (63 + 63) + 2467 \cdot 63 = -788.64 \cdot 10^3\,\mathrm{N \cdot mm};$$
$$0 \le x \le c; \quad M_x = F_c \cdot x; \quad M_x(0) = 0;$$
$$M_x(c) = -F_c \cdot c = -4808.8 \cdot 164 = -788.64 \cdot 10^3\,\mathrm{N \cdot mm}.$$

10.2.8. Plot the diagram of total bending moments (Fig. 10.2) taking into account that

$$M_\Sigma = \sqrt{(M_x)^2 + (M_y)^2}\,.$$
$$M_\Sigma(A) = 0\,;$$
$$M_\Sigma(B) = \sqrt{\left(2.28 \cdot 10^3\right)^2 + \left(472 \cdot 10^3\right)^2} = 472 \cdot 10^3\,\mathrm{N \cdot mm}\,;$$
$$M_\Sigma(B) = \sqrt{\left(54.29 \cdot 10^3\right)^2 + \left(472 \cdot 10^3\right)^2} = 475.11 \cdot 10^3\,\mathrm{N \cdot mm}\,;$$
$$M_\Sigma(C) = \sqrt{0 + \left(788.64 \cdot 10^3\right)^2} = 788.64 \cdot 10^3\,\mathrm{N \cdot mm}\,;$$
$$M_\Sigma(D) = 0$$

10.2.9. Plot the twisting moment diagram (Fig. 10.2).

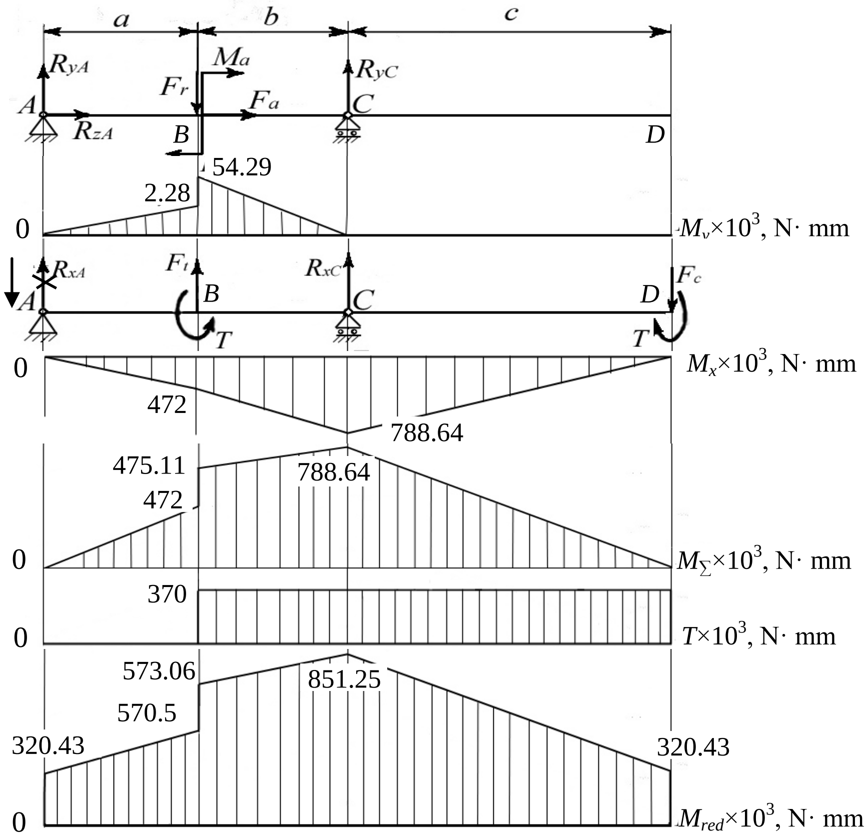


Fig. 10.2. Diagrams of moments for the shaft analytical model

10.2.10. Plot the reduced moments (Fig. 10.2) diagram taking into account that M_{red}

$$M_{red} = \sqrt{M_{\Sigma}^2 + 0.75 \cdot T^2}\ .$$

$$M_{red}(A) = \sqrt{0 + 0.75 \cdot (370 \cdot 10^3)^2} = 320.43 \cdot 10^3\,\text{N}\cdot\text{mm}\,;$$

$$M_{red}(B) = \sqrt{(472 \cdot 10^3)^2 + 0.75 \cdot (370 \cdot 10^3)^2} = 570.5 \cdot 10^3\,\text{N}\cdot\text{mm}\,;$$

$$M_{red}(B) = \sqrt{(475.11 \cdot 10^3)^2 + 0.75 \cdot (370 \cdot 10^3)^2} = 573.06 \cdot 10^3\,\text{N}\cdot\text{mm}\,;$$

$$M_{red}(C) = \sqrt{(788.64 \cdot 10^3)^2 + 0.75 \cdot (370 \cdot 10^3)^2} = 851.25 \cdot 10^3\,\text{N}\cdot\text{mm}$$

$$M_{red}(D) = \sqrt{0 + 0.75 \cdot (370 \cdot 10^3)^2} = 320.43 \cdot 10^3\,\text{N}\cdot\text{mm}$$

10.2.11. For the critical section of the shaft (where the reduced moment is maximum) we check the shaft for static strength

$$\sigma_b = \frac{M_{rmax}}{0.1 \cdot d^3} \le [\sigma_b], \ \sigma_b = \frac{851.25 \cdot 10^3}{0.1 \cdot 55^3} = 51.16\text{MPa},$$

$$51.16 \le 120\text{MPa}$$

where d is the diameter of the shaft at the critical section; $[\sigma_b]$ is the allowable bending stress. For steels $[\sigma_b]$ = 120 MPa.

Condition is satisfied.

If $\sigma_b > [\sigma_b]$ we must increase the diameter of the shaft at the critical section.

10.3. Analyze the shaft for fatigue strength.

10.3.1. Determine the limit of endurance in bending and in torsion for the shaft material:

for carbon steels: $\sigma_{-1} = 0.43 \cdot \sigma_{ul}$, $\sigma_{-1} = 0.43 \cdot 850 = 365.5\text{MPa}$

for alloy steels: $\sigma_{-1} = 0.35 \cdot \sigma_{ul} + 120$,

$\tau_{-1} = (0.2 \ldots 0.3) \cdot \sigma_{ul}$, $\tau_{-1} = 0.25 \cdot 850 = 212.5\text{MPa}$

where σ_{ul} is the ultimate strength of the material (table 10.1).

10.3.2. Determine peak magnitudes of bending and torsion stresses (σ_p, τ_p) at the critical sections of the shaft.

Critical section is a section where the total moment M_Σ is maximum. It may be a section where a gear or a bearing is located.

$$\sigma_p = \frac{M_\Sigma}{0.1 \cdot d^3} = \frac{788.64 \cdot 10^3}{0.1 \cdot 55^3} = 47.4\text{MPa},$$

$$\tau_p = 0.5 \cdot \frac{T}{0.2 \cdot d^3} = 0.5 \cdot \frac{370 \cdot 10^3}{0.2 \cdot 55^3} = 5.56\text{MPa},$$

where d is the diameter of the shaft at the critical section.

10.3.3. Determine mean components of the bending and torsion stresses (σ_m, τ_m).

If axial force F_a<1000 N we assume $\sigma_m = 0$ and $\tau_m = \tau_p$.

Otherwise

$$\sigma_m = \frac{F_a}{\frac{\pi \cdot d^2}{4}}, \qquad \tau_m = \tau_p.$$

In our case F_a = 346.7 N < 1000 N, so $\sigma_m = 0$ and $\tau_m = \tau_p$ = 5.56 MPa.

10.3.4. Determine factors ψ_σ and ψ_τ of mean stress components

- for carbon steels $\psi_\sigma = 0.1$ and $\psi_\tau = 0.05$;
- for alloy steels $\psi_\sigma = 0.15$ and $\psi_\tau = 0.1$.

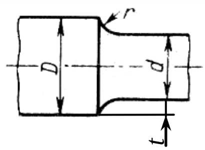
In our case $\psi_\sigma = 0.15$ and $\psi_\tau = 0.1$.

10.3.5. Determine effective stress concentration factors K_σ and K_τ. For this purpose we use table 10.2.

If the critical section of the shaft is a section where a bearing is mounted we will use as stress concentrator interference fit. If a gear is installed at the critical section a keyed portion is considered as the stress concentrator.

Table 10.2

Stress Concentration Factors for Shafts



A. Filleted Transition Regions

Values of fillet radius *r*

d	r_{max}
Over 18 to 30	1.6
Over 30 to 50	2.0
Over 50 to 80	2.5
Over 80 to 120	3.0

end of the table 10.2

Values of K_σ and K_τ

t/r	*r/d*	K_σ at σ_{ul} (MPa) of				K_τ at σ_{ul} (MPa) of			
		500	700	900	1200	500	700	900	1200
2	0.01	1.55	1.6	1.65	1.7	1.4	1.4	1.45	1.45
	0.02	1.8	1.9	2.0	2.15	1.55	1.6	1.65	1.7
	0.03	1.8	1.95	2.05	2.25	1.55	1.6	1.65	1.7
3	0.01	1.9	2.0	2.1	2.2	1.55	1.6	1.65	1.75
	0.02	1.95	2.1	2.2	2.4	1.6	1.7	1.75	1.85
	0.03	1.95	2.1	2.25	2.45	1.65	1.7	1.75	1.9

B. Values of K_σ and K_τ for Keyed portions of Shafts

σ_{end}, MPa	K_σ for keyseats cut with		K_τ
	End mills	Side mills	
500	1.60	1.40	1.40
700	1.90	1.55	1.70
900	2.15	1.70	2.05
1200	2.50	1.90	2.40

C. Values of K_σ and K_τ for Splined and Threaded Portions of Shafts

σ_{end}, MPa	K_σ for		K_τ for		
	Splined portions	Threaded portions	Parallel-sides splines	Involute splines	Threaded portions
500	1.45	1.80	2.25	1.45	1.50
700	1.60	2.20	2.45	1.50	1.65
900	1.70	2.45	2.65	1.55	2.10
1200	1.75	2.90	2.80	1.60	2.39

D. Values of K_σ/K_d and K_τ/K_d at Interference-Fit Joints

Shaft diameter *d*, mm	K_σ/K_d at σ_{ul}, (MPa) of				K_τ/K_d at σ_{ul}, (MPa) of			
	500	700	900	1200	500	700	900	1200
30	2.5	3.0	3.5	4.25	1.9	2.2	2.5	3.0
50	3.05	3.65	4.3	5.2	2.25	2.6	3.1	3.6
100 up	3.3	3.95	4.6	5.6	2.4	2.8	3.2	3.8

In our case the critical section of the shaft is a section where a bearing is mounted so we will use as stress concentrator interference fit. Then $\frac{K_\sigma}{K_d} = 4.35$; $\frac{K_\tau}{K_d} = 2.97$ from the table 10.2 D.

10.3.6. Determine the surface roughness factor K_F. For that we use Fig.10.3.

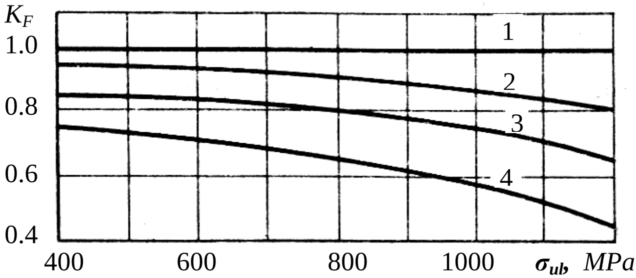


Fig. 10.3. Values of K_F: 1 - polished portions; 2 - ground portions; 3 - portions made with finish turning; 4 - portions made with rough turning

It is necessary to note that the portion of the shaft where a bearing is installed should be ground while the shaft portion for a toothed wheel is made with finish turning.

Thus portion of the shaft where a bearing is installed grounded and $K_F = 0.9$.

10.3.7. Determine factor K_d, that takes into account absolute dimensions of the shaft cross-section. For this purpose we use Fig.10.4.

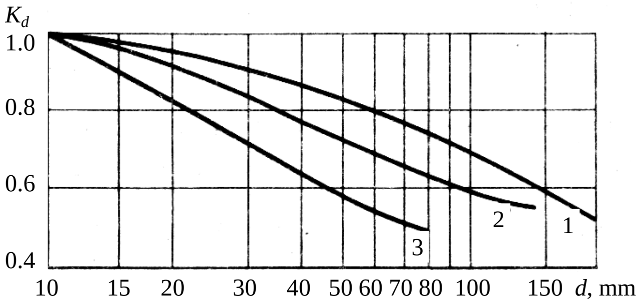


Fig. 10.4. Values of K_d : 1- carbon steels without stress concentrators; 2 – alloy steels without stress concentrators or carbon steels with stress

concentrators (K_σ>2); 3 - alloy steels with stress concentrators (K_σ>2).

Thus we have carbon steel without stress concentrators and K_d = 0.82.

10.3.8. Determine safety factors in terms of bending and torsion

$$S_\sigma = \frac{\sigma_{-1}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_p + \psi_\sigma \cdot \sigma_m} = \frac{365.5}{\frac{4.35}{0.9} \cdot 47.4 + 0} = 1.6;$$

$$S_\tau = \frac{\tau_{-1}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_p + \psi_\tau \cdot \tau_m} = \frac{212.5}{\frac{2.97}{0.9} \cdot 5.56 + 0.1 \cdot 5.56} = 11.40.$$

10.3.9. Determine the safety factor of the shaft at the critical section

$$S = \frac{S_\sigma \cdot S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}} \geq [S];$$

$$S = \frac{S_\sigma \cdot S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}} = \frac{1.6 \cdot 11.40}{\sqrt{1.6^2 + 11.40^2}} = 1.58$$

Allowable values of the safety factor [*S*] are given in table 10.3.

Table 10.3

Values of [*S*]

Degree of accuracy of design loads, analytical models, and mechanical characteristics	[*S*]
High	1.2-1.5
Approximate (shafts of most general-purpose mechanisms)	1.5-1.8
Reduced(also for shafts with *d*>200 mm)	1.8-2.2

Condition is satisfied.

11. Analysis of the rolling contact bearings

Let us analyze the rolling contact bearings for strength. Initial dates are:

– type designation of the bearing and its sizes (# and $d \times D \times B$);

– rotational speed n of the bearing inner ring;

– components of reacting forces in supports R_{xA}, R_{yA}, R_{xC}, R_{yC} and R_{zA};

– basic load rating C_r and static load rating C_0 for radial ball bearings and angular-contact bearings with pressure angle $\alpha \le 18^o$ (tables 9.3 and 9.4);

– basic load rating C_r and axial load parameter e for angular-contact bearings with pressure angle $\alpha > 18^o$ (tables 9.4 and 11.1);

– basic load rating C_r, axial load parameter e and axial load factor Y for tapered roller bearings (table 9.5).

In our case for the output shaft we will use angular-contact boll bearings 36214 (70×125×24); n = 93.6 rpm; R_{xA} = 7491.5 N, R_{yA} = 36.26 N, R_{xC} = 9835.5 N, R_{yC} = 861.74 N and $R_{zA} = F_a$ = 346.7 N; C_r =80.2 kN, C_0 = 54.8 kN.

11.1. Determine the total radial reacting forces which act to the bearings

$$F_{r1} = \sqrt{R_{xA}^2 + R_{yA}^2} = \sqrt{7491.5^2 + 36.26^2} = 7491.6 \text{ N};$$

$$F_{r2} = \sqrt{R_{xC}^2 + R_{yC}^2} = \sqrt{9835.5^2 + 861.74^2} = 9870.2 \text{ N}.$$

11.2. Determine the total axial forces acting to the bearings

11.2.1. Calculate additional axial forces S_1 and S_2 that develop as a result of action of radial forces F_{r1} and F_{r2}

$$S_1 = F_{r1} \cdot e'; \quad S_2 = F_{r2} \cdot e',$$

where – $e'=e$ for radial ball bearings and angular contact ball bearings; $e'=0.83 \cdot e$ for tapered roller bearings.

It is necessary to note that for radial ball bearings and angular contact ball bearings with pressure angle $\alpha \le 18^o$ axial load parameter e is determined by table 11.1 depending upon ratio F_a/C_o.

In our case F_a/C_o=346.7/57800 = 0.006; $e'=e$ = 0.3 (table 11.1); $S_1 = F_{r1} \cdot e'$=7491.6·0.3 = 2247.5 N; $S_2 = F_{r2} \cdot e'$ = 9870.2·0.3= 2961.1 N.

11.2.2. Plot the analytical model of the shaft and show all forces acting on the shaft in the axial direction (Fig. 11.1).

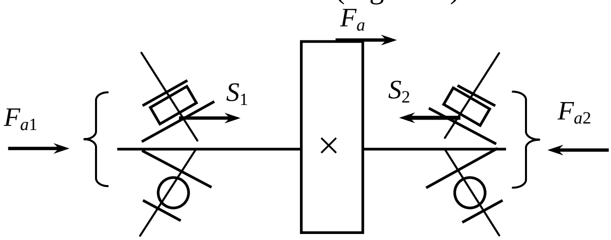


Fig. 11.1. Forces acting to the shaft in the axial direction.

Table 11.1

Values of X, Y and e for some types of bearings

Type of bearing	α°	F_a/C_o	$F_a/(VF_r)\le e$		$F_a/(VF_r)>e$		e
			X	Y	X	Y	
		0.014				2.30	0.19
		0.028				1.99	0.22
		0.056				1.71	0.26
		0.084				1.55	0.28
Single Row radial Ball Bearing	0	0.11	1	0	0.56	1.45	1.30
		0.17				1.31	0.34
		0.28				1.15	0.38
		0.42				1.04	0.42
		0.56				1.00	0.44
		0.014				1.81	0.30
		0.029				1.62	0.34
		0.057				1.46	0.37
		0.086				1.34	0.41
Single Row Angular Contact Ball Bearing	12	0.11	1	0	0.45	1.22	0.45
		0.17				1.13	0.48
		0.29				1.14	0.52
		0.43				1.01	0.54
		0.57				1.00	0.54
	26	–	1	0	0.41	0.87	0.68
	36	–	1	0	0.37	0.66	0.95
Single row Tapered Roller Bearing		–	1	0	0.4	0.4ctgα	1.5ctgα

11.2.3. Determine total axial forces F_{a1} and F_{a2}. For that we should find the sum $F_a + S_1$ and compare with force S_2. There are 3 possible cases:

– if $F_a + S_1 > S_2$ $F_{a1} = S_1$ and $F_{a2} = F_a + S_1$;
– if $F_a + S_1 = S_2$ $F_{a1} = S_1$ and $F_{a2} = S_2$;
– if $F_a + S_1 < S_2$ $F_{a1} = S_2 – F_a$ and $F_{a2} = S_2$.

In our case $F_a + S_1$ = 346.7 + 2247.5 = 2594.2 N < S_2 = 2961.1 N, therefore F_{a1} = 2961.1 – 346.7 = 2614.4 N and F_{a2} = 2961.1 N.

11.3. Determine factor V that takes into account what ring of the bearing is movable.

– for bearings with movable inner ring $V = 1$;
– for bearings with movable outer ring $V = 1.2$.

In general purpose speed reducers bearings with movable inner ring are used only.

11.4. Determine safety factor K_s that takes into account the load nature. It may be ranged from 1 to 2.5 depending upon the type of designing machine. For general purpose speed reducers the safety factor is assumed as 1.3.

11.5. Determine temperature factor K_t according to table 11.2.

Table 11.2

Values of temperature factor K_t

Temperature of the bearing t, ºC	100	125	150	175	200	225	250
K_t	1.0	1.05	1.1	1.15	1.25	1.35	1.4

As a rule in general purpose speed reducers the working temperature of bearings is less than 100 ºC.

11.6. Determine the radial force factor X and axial force factor Y for both supports. For this purpose it is necessary to find ratio $\frac{F_{ai}}{V \cdot F_{ri}}$ for every support and compare with axial load parameter e.

If $\frac{F_{ai}}{V \cdot F_{ri}} \le e$ then X_i =1 and Y_i = 0. Otherwise X_i and Y_i are determined according to table 11.1.

In our case: V =1; K_s =1.3; K_t =1; $\frac{F_{a1}}{V \cdot F_{r1}} = \frac{2614.4}{1 \cdot 7491.6} = 0.349 > e$, then X_1 =0.45 and Y_1 = 1.81; $\frac{F_{a2}}{V \cdot F_{r2}} = \frac{2961.1}{1 \cdot 9870.2} = 0.3 = e$, then X_2 =1 and Y_2 = 0.

11.7. Determine the equivalent radial loads for both supports

$$P_{r1} = (X_1 \cdot V \cdot F_{r1} + Y_1 \cdot F_{a1}) \cdot K_s \cdot K_t = (0.45 \cdot 1 \cdot 7491.6 + 1.81 \cdot 2614.4) \cdot 1.3 \cdot 1 = 10530 \text{ N},$$

$$P_{r2} = (X_2 \cdot V \cdot F_{r2} + Y_2 \cdot F_{a2}) \cdot K_s \cdot K_t = (1 \cdot 1 \cdot 9870.2 + 0 \cdot 2961.1) \cdot 1.3 \cdot 1 = 12831 \text{ N}.$$

11.8. Determine the rated life in million revolutions for the most loaded support

$$L = \left(\frac{C_r}{P_{rmax}} \right)^m = \left(\frac{80200}{12831} \right)^3 = 244.2 \,,$$

where m = 3 for ball bearings and $m = \frac{10}{3}$ for roller bearings.

11.9. Determine the rated life in hours

$$L_h = \frac{L \text{Ч} 10^6}{60 \text{Ч} n} > L_{h\,\min} \,.$$

For general purpose speed reducers $L_{h\,\min}$=12000 hours.

If the last inequality is not carried out it is necessary to reselect the bearing of more heavy series and make all calculations once more.

In our case $L_h = \frac{244.2 \text{Ч} 10^6}{60 \text{Ч} 93.6} = 43482.5$ hours $> L_{h\,\min}$. Condition is satisfied and we can use this type of bearings.

12. Designing the gear speed reducer

12.1. Design supports of the speed reducer shafts

Normal operation of bearings depends upon the arrangement of bearings on the shaft as well as the method of fixation of the shaft in the axial direction.

According to fixation of the shaft in the axial direction there are fixed support and floating support.

Depending upon arrangement of bearings we will distinguish between inward (face to face) arrangement and arrangement with the fixed and the floating supports. Sometimes outward (back to back) arrangement of bearings is used too (for example, for bevel pinion shafts).

Let us consider features of all arrangements.

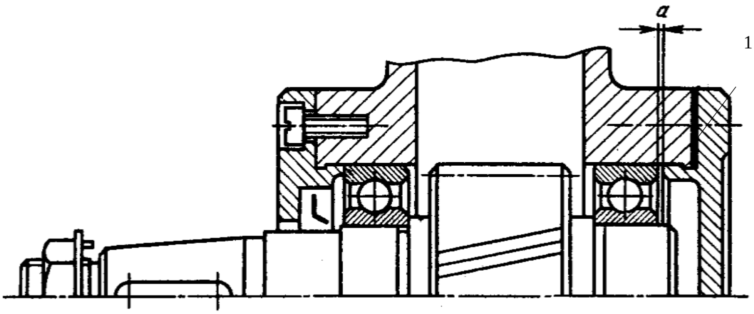
Inward arrangement of bearings.

This is the most popular arrangement according to which a shaft is fixed in the axial direction in both supports. Every support withstands one-sided axial load. The inner ring of both bearings rests against the shoulder made on the shaft or against the end face of other component installed on the shaft. In its turn, the outer ring of bearings rests against the end face of a cap or other part secured in the casing (Fig. 12.1).

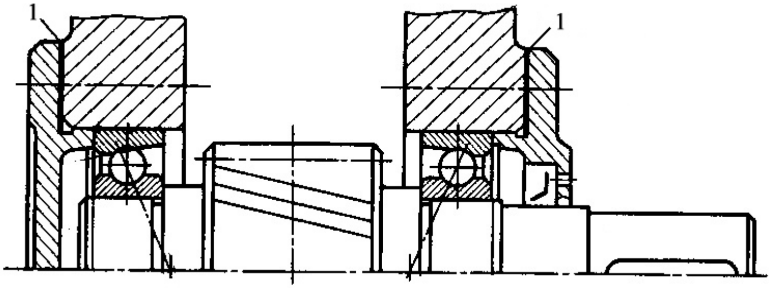
The inward arrangement is simple in construction and permits easy adjustment of bearings. But there exists a risk that the shaft can be seized at supports. It is explained by the fact that during operation of bearings, shafts and casings heat up. As a result, radial clearances in bearings decrease. Also, the shaft length increases on heating, which reduces the axial clearance in bearings.

For safeguard against seizure we should provide small clearance ***a*** at one end of the shaft between the outer ring of the bearing and the end face of the cap (Fig. 12.1, *a*). This clearance is very small (0.2…0.5 mm). That is why it is not shown in the drawing. In order to adjust this clearance we use either thin metal shims **1** placed between the casing and the flanged cap at one end of the shaft or a spacer ring installed between the end face of the embedded cap and the outer ring of the bearing.

In radial-thrust bearings (angular contact ball bearings and tapered roller bearings) there exist radial clearances between rolling elements and both rings that may be adjusted in assembly. We can achieve adjustment of these clearances by means of thin metal shims1 mounted between the casing and flanges of caps at both ends of the shaft (Fig. 12.1, *b*). If we have embedded caps needed adjustment is achieved by screw 1 and intermediate washer 2 (Fig. 12.2).



a)



b)

Fig 12.1 Inward arrangement of bearings: *a* – radial bearings; *b* – radial thrust bearings

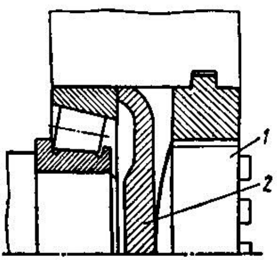


Fig.12.2. Adjustment of bearings by screw 1 and intermediate washer 2

In this case the threaded element simplifies adjustment because caps need not be removed to replace the shims but the bearing assembly becomes more elaborate.

Inward arrangement of bearings is used for shafts of straight spur gears, helical spur gears, bevel gear and worm gear.

Bearings of the worm shaft are arranged as fixed and floating supports (Fig. 12.3).

The fixed support consists of two inward facing radial thrust bearings (as a rule tapered roller bearings) mounted in the bearing housing. Outer and inner rings of the fixed support should be fixed in both axial directions. That is why the fixed support can withstand double-sided axial load. In order to adjust radial clearances in tapered roller bearings we use slotted nut (table 12.1.) and lock-washer (table 12.2).

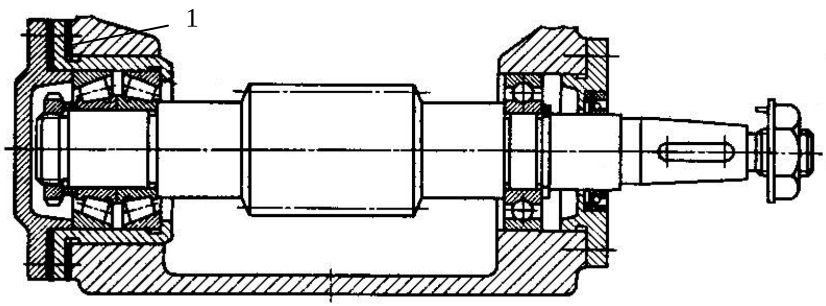


Fig.12.3. Arrangement with the fixed and floating supports of the worm shaft

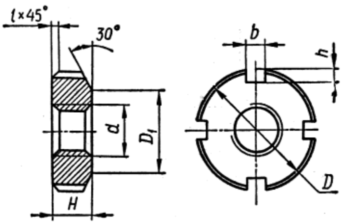
Floating support is used to compensate thermal deformations of the shaft and manufacturing errors. We use as floating support a radial ball bearing which inner ring is fixed in both axial directions and outer ring is left free. For fixation of the inner ring we employ either end plates (table 13.3, 13.4), or slotted nuts with lock washers, or spring ring.

The rightness of the worm engagement is adjusted by means of metal shims **1** (Fig. 12.3) mounted between the casing and the flange of the housing.

Supports of the bevel pinion shaft are installed in the housing according to outward arrangement (Fig. 12.4). In this case we obtain the minimum bending moment developing on the shaft. In order to adjust radial clearances in bearings we should use a slotted nut with a lock-washer. Rightness of the bevel gear engagement is adjusted by metal shims **1** (Fig. 12.4) mounted between the casing and the housing flange.

Table 12.1

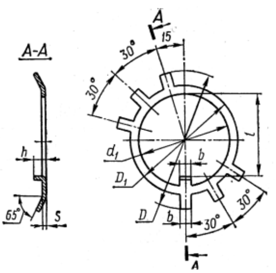
Standard slotted nuts (GOST 11871-88)



Thread diameter, d	Thread pitch	D	D_1	H	b	h
20	1.5	34	27	8	5	2.5
22		39	30	10		
24		42	33			
27		45	36			
30		48	39			
33		52	42			
36		56	45		6	3
39		60	48			
42		65	52			
45		70	56			
48		75	60	12	8	4
52		80	65			
56	2	85	70			
60		90	75			
64		95	80			
68		100	85	15	10	5
72		105	90			
76		110	95			
80		115	100			

Table 12.2

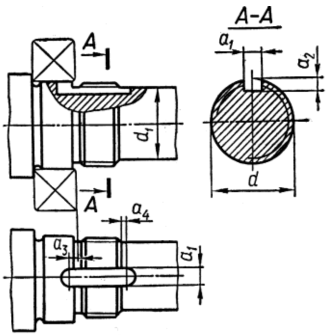
Standard lock washers (GOST 11872-80)



Thread diameter	d_1	d_2	d_3	b	S	l	h	
							Not less	Not more
20	20.5	37	27	4.8	1.0	17	3.5	6.0
22	22.5	40	30			19		
24	24.5	44	33			21		
27	27.5	47	36			24	4.5	8.0
30	30.5	50	39			27		
33	33.5	54	42	5.8	1.6	30		
36	36.5	58	45			33		
39	39.5	62	48			36		
42	42.5	67	52			39		
45	45.5	72	56			42		
48	48.5	77	60	7.8		45		
52	52.5	82	65			49	5.5	10.0
56	57	87	70			53		
60	61	92	75			57		
64	65	97	80			61		
68	69	102	85	9.5		65	6.5	13.0
72	73	107	90			69		
76	77	112	95			73		
80	81	117	100			76		

Table 12.3

Dimensions of Slots Receiving Lock Washer Tabs, mm



Thread diameter *d, mm*	a_1	a_{2min}	a_{3min}	a_{4min}	d_{1min}
20	6	2	3.5	1.0	16.5
22	6	2	3.5	1.0	18.5
27	6	3	4.0	1.5	23.5
30	6	3	4.0	1.5	26.5
33	6	3	4.0	1.5	29.5
36	6	3	4.0	1.5	32.5
39	6	3	4.0	1.5	35.5
42	8	3	5.0	1.5	38.5

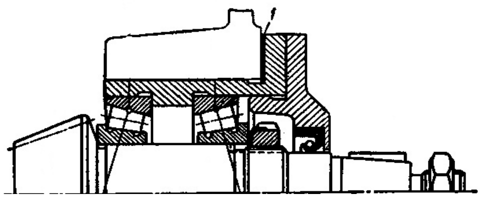


Fig 12.4. Outward arrangement of bearings of the bevel pinion shaft

12.2. Determine dimensions of elements that are a part of the support assemblies.

12.2.1. Bearing caps.

Bearing caps may be made with screws or embedded in the casing. Construction of the screw cap and its dimensions are given in Fig. 12.5 *a*, where d_k is determined according to table 12.4. If bearings are held in place with nuts or spring washers, convex caps are used (Fig. 12.5, c).

If the length of the cap sleeve permits, a groove is made in the sleeve, in which a round-section packing ring of gasoline- or oil-resistant rubber is fitted (Fig. 12.5, b). The groove profile is shown in Fig. 12.5, b and the dimensions of its basic elements are taken as follows: b = 5.6 mm and $d_1 = D$ – 7.4 mm. The cross-section diameter is assumed as d = 4.6 mm.

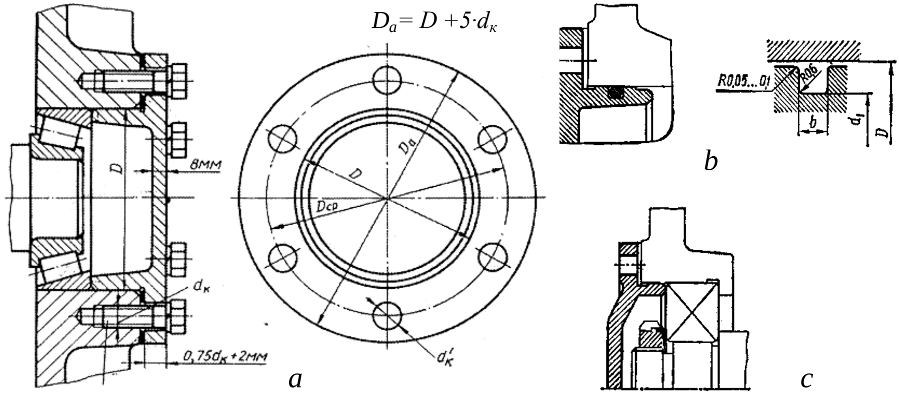


Fig.12.5. Bearing screw caps

Table 12.4

Screws for fastening bearing caps

D, mm	d_k	Number of screws
To 75	M8	4
80…95	M10	4
100…140	M10	6
150…215	M12	6
225…360	M16	6

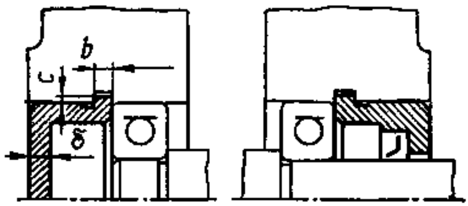


Fig.12.6. Embedded caps

Constructions of embedded caps are given in Fig. 12.6 where δ is determined according to table 12.5, $b \approx \delta$, $c \approx 0.5 \cdot \delta$. These caps require no special attachment to the casing, so no holes in the caps, threaded holes in the casing, or screws are needed. However, caps of this construction may only be used where the joint between the casing and the cover lies in the plane of the shaft axis. Besides, bearing assemblies with embedded caps become more elaborate in shape. It is explained by necessity to adjust radial clearances in the bearing.

Table12.5

Thickness of the embedded cap

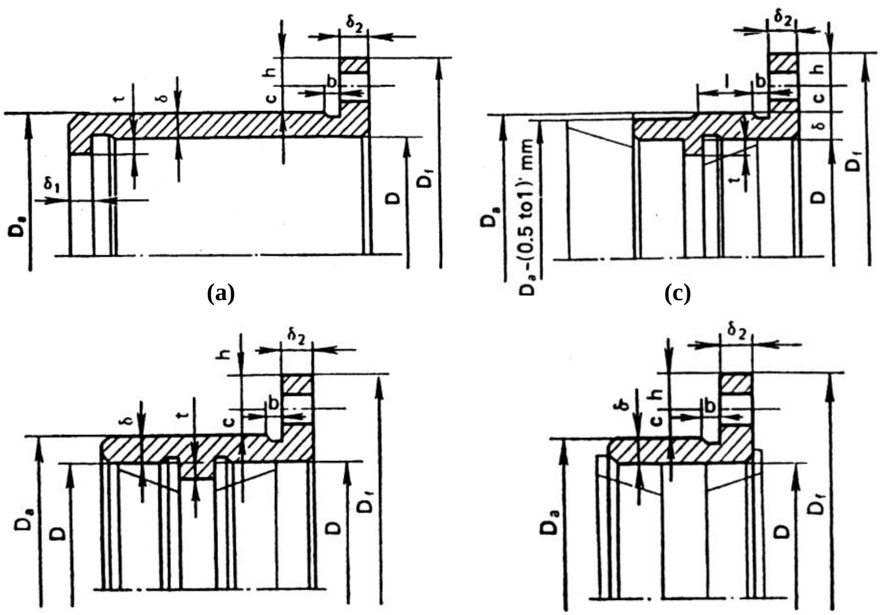
D, mm	50…62	63…95	100…145	150…220
δ, mm	5	6	7	8

12.2.2.Bearing housing

Constructions of bearing housings and their dimensions are given in table 12.6

Table 12.6

Bearing housings



(a) **(c)** **(b)** **(d)**

D, mm	<50	50…80	80…120	120…170
δ, mm	4…5	6…8	8…10	10…12.5
b, mm	3	5	5	8

$D_1 = D_a + 5d_h$; $c \approx d_h$; $h = (1.0…1.2)d_h$; $\delta_1 \approx \delta$; $\delta_2 \approx 1.2\delta$, where d_h is diameter of the hole for a screw from table 12.7

Table 12.7

Diameters of holes for bolts, mm

Bolt diameter	M10	M12	M16	M20	M24	M30	M36
Hole diameters for anchor bolts	-	15	19	24	28	35	42
Hole diameter for tie bolts	11	14	18	22	26	33	-

12.3. Designing casings.

Casings are used to support torque-transmitted elements in design position and take up loads developed in speed reducers during operation. The main material of casings is cast-iron.

A casing must be rigid to prevent shaft misalignment under internal and external loads. This can be achieved by using stiffening ribs that also carry out the function of cooling fins.

Casing may have split or single-peace construction. For split constructions the joint between a casing and a cover is usually provided in the plane parallel to the base and that passes through the axis of rotation of corresponding gears.

The main dimensions of the casing (Fig. 12.7):

12.3.1. Casing and cover wall thickness:

- for single stage spur gear speed reducer $\delta = 0.025 \cdot a + 1 \geq 8$ mm;
- for single stage bevel gear speed reducer $\delta = 0.05 \cdot R_e + 1 \geq 8$ mm;
- for single stage worm gear speed reducer $\delta = 0.04 \cdot a + 4 \geq 8$ mm;
- for double stage gear speed reducer $\delta = 0.025 \cdot a_L + 3 \geq 8$ mm,

where a is the centre distance, a_L is the center distance of the low-speed transmission, R_e is outer cone distance.

12.3.2. Flange thickness at the joint plane

$$b = 1.5 \cdot \delta.$$

12.3.3. Thickness of the flange for connection to a frame

$$p = 2.35 \cdot \delta.$$

12.3.4. Thickness of the rib

$$m = (0.8 \ldots 1.0) \cdot \delta.$$

12.3.5. Diameter of the anchor bolt

$$d_1 = (0.03 \ldots 0.036) \cdot a_L + 12,$$

$$d_1 = 0.072 \cdot R_e + 12.$$

Obtained magnitude of d_1 should be rounded off to the nearest greater side according to standard series given in table 12.9.

12.3.6. Number of anchor bolts

$$z = 0.005 \cdot (L_0 + B_0) \geq 4,$$

where B_0 is the width of the speed reducer base; L_0 is the length of the speed reducer base. It is necessary to note that the number of anchor bolts should be always even.

12.3.7. Diameter of the tie-bolt near bearings

$$d_2 = (0.7 \ldots 0.75) \cdot d_1.$$

Obtained value of d_2 should correspond to standard value (table 12.9).

12.3.8. Diameter of tie bolts that connect the casing and the cover flanges

$$d_3 = (0.5 \ldots 0.6) \cdot d_1.$$

Round off obtained value of d_3 to the greater side according to standard series (table 12.9).

12.3.9. Distance between tie-bolts of diameter d_3

$$l = (10 \ldots 12) \cdot d_3.$$

12.3.10. Disposition of bolts on the flange (Fig)

$$x = (1 \ldots 1.2) \cdot d_{\text{hole}},$$

$$y = \delta + e,$$

where d_{hole} is the diameter of the hole for fitting the bolt (table 12.7);
e is the distance that allows to grip the bolt head by a spanner (table12.8).

Table 12.8

Value of *e*

Diameter of the bolt	M10	M12	M16	M20	M24
e, mm	13	14	16	20	24

12.3.11. Height of the boss is determined from structural consideration taking into account the following requirement

$$q \geq 0.5 \cdot d_2 + d_k,$$

where d_k is the diameter of the bolt, that connects the bearing cap with the casing (table12.4).

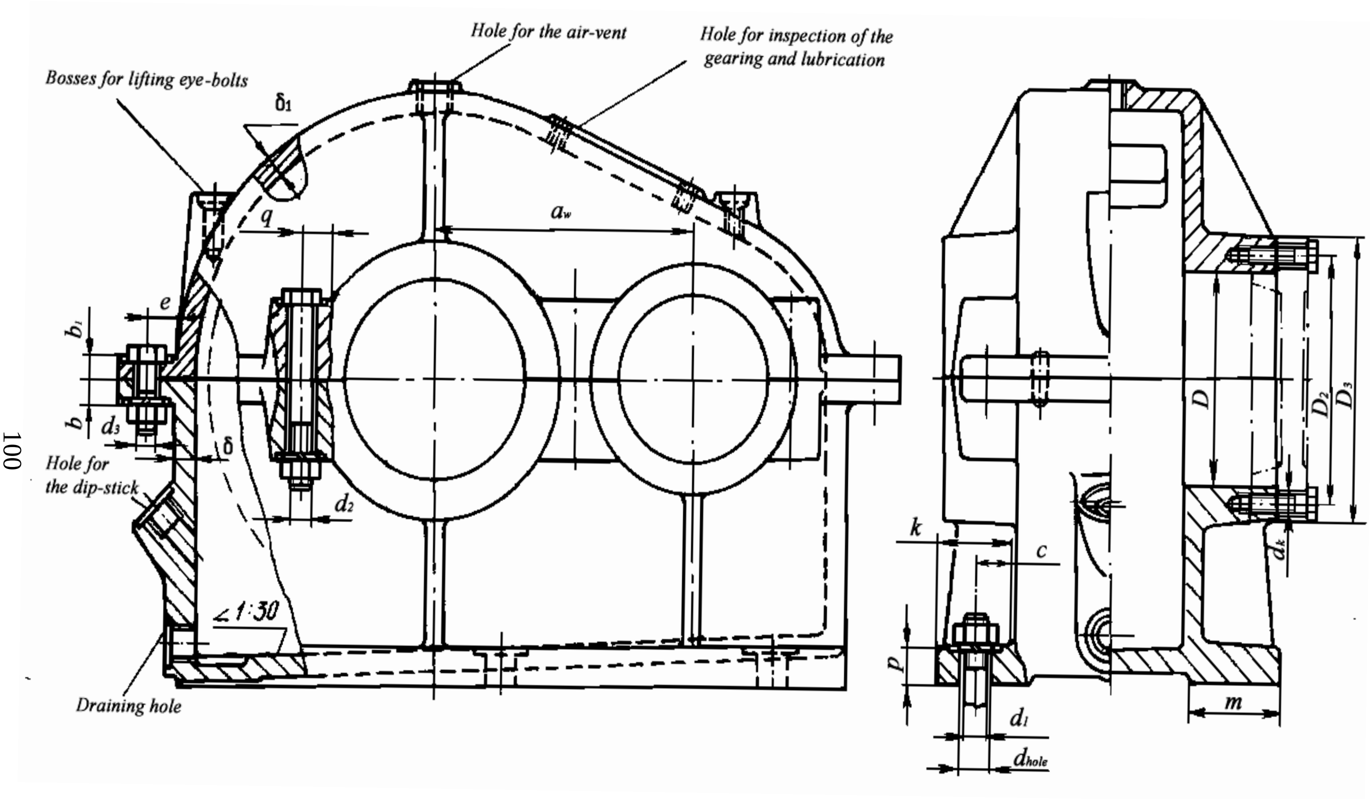
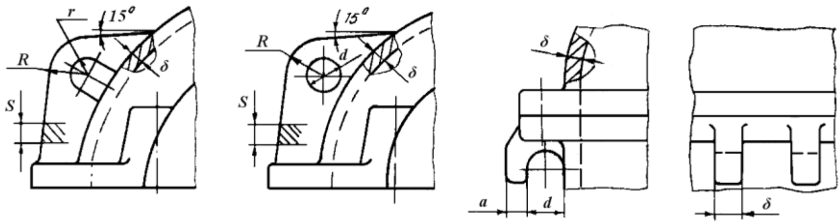


Fig.12.7.Constructive elements and their dimensions of the casing

12.3.12. Dimensions of bolts, nuts and washers are given in tables 12.9 -12.11. The length and threaded length of the bolt are given in table 12.12. Engaged length of the bolt, the depth of the hole and threaded length of the hole are determined according to table 12.13.

12.3.13. In order to hold together the casing and the cover and to prevent their relative movement in axial direction two pins are used. They are placed along the flange diagonal. The diameter of the pin is determined as $d_{pin} = 0.5 \cdot d_1$.

12.3.14. For transition and installation the cover and the casing should be made with lifting eye-bolts (table 12.14) or eyes and load hooks (Fig. 12.8).



$r = 1.5 \cdot \delta$; $R = 3 \cdot \delta$; $d = 3 \cdot \delta$; $a = (1.2 \ldots 1.5) \cdot \delta$; $s = (2 \ldots 3) \cdot \delta$

Fig.12.8. Load eyes and hooks

12.3.15. For gear inspection and lubrication an inspection hole is provided in the upper portion of the cover. The cover of the inspection hole is determined according to table 12.15.

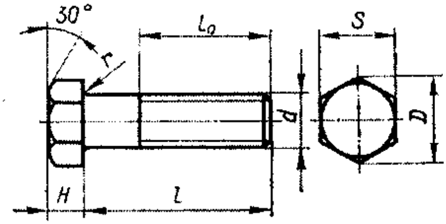
12.3.16. During operation, pressure inside a casing increases due to heating of the oil and air. As a result the lubricant is ejected outside through seals and joints. To avoid this fact air-vents are provided in the top portion of the casing for communication between the inner space and the environment. Possible constructions of air-vents are given in tables 12.16 and 12.17.

12.3.17. For draining the oil with grit and other debris a drain hole is made in the casing bottom, which is stopped with a plug with either straight or taper thread (table 12.18).

12.3.18. The level of the oil contained in the speed-reducer casing is checked with oil gauges of various constructions. Among them there are dip sticks (table 12.19) or transparent tube gauges. (table 12.20).

Table 12.9

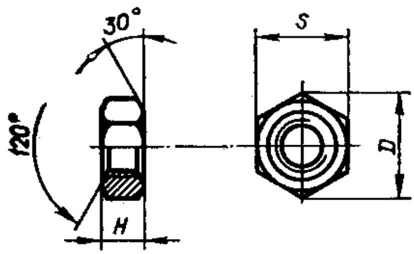
Standard bolts (GOST 7798-70)



Parameter	Thread diameter *d*, mm						
	6	8	10	12	16	20	24
Thread pitch, mm	1.0	1.25	1.5	1.75	2.0	2.5	3.0
Radius *r*, mm not more	0.6	1.1	1.1	1.6	1.6	2.2	2.2
Diameter *D*, mm	11	14.5	19	21	27	33.5	40.5
Span *S*, mm	10	13	17	19	24	30	36
Height *H*, mm	4.5	5.5	7	8	10	13	15

Table 12.10

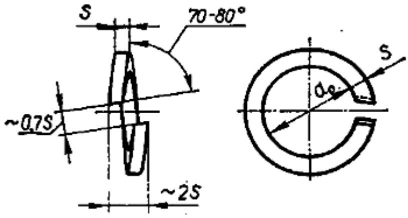
Standard nuts (GOST 5915-70)



Parameter	Thread diameter *d*, mm								
	6	8	10	12	16	20	24	30	36
Thread pitch, mm	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Diameter *D*, mm near	11	14.5	19	21	27	33.5	40.5	51.5	62
Span *S*, mm	10	13	17	19	24	30	36	46	55
Height *H*, mm	5	6.5	8	10	13	16	19	24	29

Table 12.11

Standard spring washers (GOST 6402-70)



Parameter	Thread diameter d, mm								
	6	8	10	12	16	20	25	30	36
Diameter d_0, mm	6.1	8.2	10.2	12.2	16.3	20.5	24.5	30.5	36.5
Thickness s, mm	1.4	2.0	2.5	3.0	3.5	4.5	5.5	6.5	8.0

Table 12.12

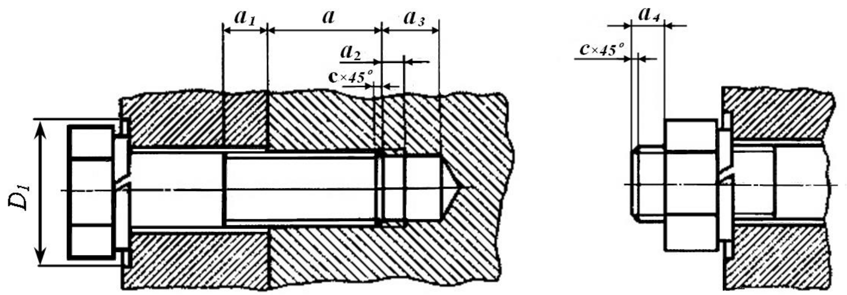
Bolt and screw length, mm

d	L / l_0	d	L / l_0
8	$\frac{8...25}{l_0}; \frac{30...100}{22}$	16	$\frac{20...40}{l_0}; \frac{48...150}{38}; \frac{160...300}{44}$
10	$\frac{10...30}{l_0}; \frac{35...150}{26}; \frac{160...200}{32}$	20	$\frac{25...50}{l_0}; \frac{55...150}{46}; \frac{160...300}{52}$
12	$\frac{14...30}{l_0}; \frac{35...150}{30}; \frac{160...260}{36}$	24	$\frac{35...60}{l_0}; \frac{65...150}{54}; \frac{160...300}{60}$

Bolt length should be chosen from the following standard series, mm: 8, 10, 12, 14, 16, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 190, 200, 220, 240, 260, 300.

Table 12.13

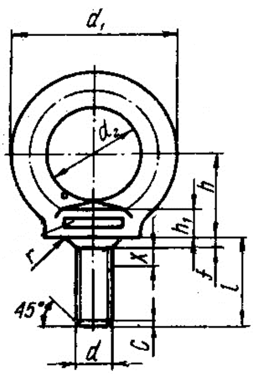
Dimensions of bolt joints



Parameter	Thread diameter d, mm						
	6	8	10	12	16	20	24
a, not less	1.25·d						
a_1, not less	3.5	4	4.5	5.5	6	7	8
a_2, not less	2	2.5	3	3.5	4	5	6
a_3, not less	6	8	9	11	12	15	18
a_4	1.5…	1.5	2…	2…	2.5…	2.5…	3…
c	1	1.6	1.6	1.6	2	2.5	2.5
D_1		15	18	22	30	35	
D_2		18	22	25	30	38	

Table 12.14

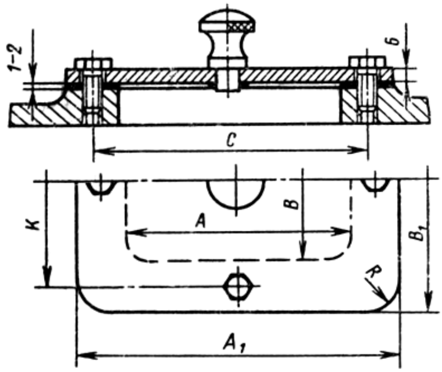
Lifting eye-bolts, mm



d	d_1	d_2	h	h_1	l	f	c	x
M8	36	20	18	6	18	2	1.2	2.5
M10	45	25	22	8	21		1.5	3
M12	54	30	26	10	25		1.8.	3.5
M16	63	35	30	12	32		2	4

Table 12.15

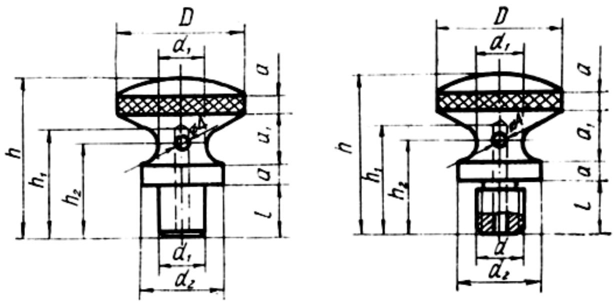
Inspection hole cover



A mm	*B* mm	A_1 mm	B_1 mm	*C* mm	C_1 mm	*K* mm	*R* mm	Dimensions of screw	Number of screws
100	75	150	100	125	-	100	12	M8x22	4
150	100	200	150	175	-	125	12	M8x22	4
200	150	250	200	230	130	182	15	M10x22	6

Table 12.16

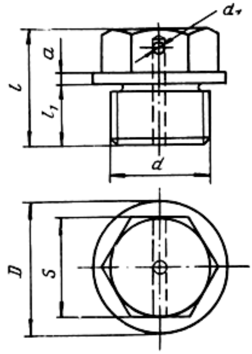
Air vent, mm



D	d_1	d_2	*D*	*h*	*l*	*a*	H_1	H_2	a_1
M12x1.75	12	20	32	40	12	5.5	29	24	13
M16x2	16	25	40	50	16	7	35	30	16

Table 12.17

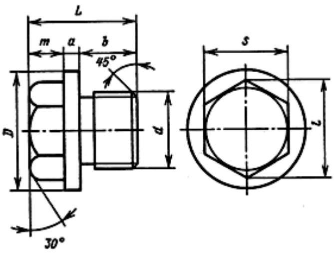
Air vent, mm



D	D	l	l_1	d_1	a	s
M12x1.25	16	19	10	4	2	17
M16x1.5	22	23	12	5	2	22
M20x1.5	30	28	15	6	4	22

Table 12.18

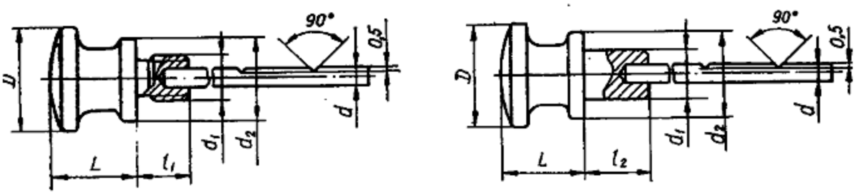
Standard plugs for drain holes, mm



<table>
<tr><th>D</th><th>b</th><th>m</th><th>a</th><th>L</th><th>D</th><th>s</th><th>l</th></tr>
<tr><td>M12x1.25</td><td>12</td><td>8</td><td rowspan="3">3</td><td>23</td><td>26</td><td>1.7</td><td>19.6</td></tr>
<tr><td>M16x1.5</td><td rowspan="2">15</td><td>9</td><td>28</td><td>30</td><td rowspan="2">22</td><td rowspan="2">24.4</td></tr>
<tr><td>M20x1.5</td><td>10</td><td>29</td><td>32</td></tr>
</table>

Table 12.19

Standard dip sticks, mm



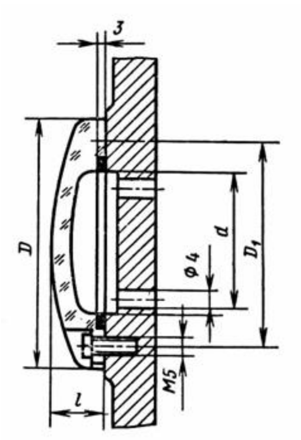
a) *b)*

a – with thread; *b* – without thread

d	d_1	Thread pitch	d_2	D	l_1	l_2	L
3	10	1.0	16	20	10	12	16
4	12	1.25	20	25	12	15	20
6	16	1.5	25	32	15	15	25

Table 12.20

Transparent tube gauge, mm



d	D	D_1	L
30	60	48	12
50	82	70	14.5

12.4. Designing lubrication system of the speed reducer.

Lubrication serves to decrease frictional loses, to offer protection against corrosion and to improve speed reducer operation. The usual lubricants for meshing elements, bearings of the speed reducer are mineral and synthetic oils and greases.

The basic parameter of any oil is its viscosity that characterizes the ability of fluid layers to resist flow. The oil viscosity is chosen depending upon the expected peripheral speed, load and tooth materials. It should be raised with increasing load and decreasing speed. The oil viscosity is determined according to table 12.21and 12.22. The oil is chosen by table 12.23.

Table 12.21

Recommended viscosity of oils for lubricating gearing when t=50°C

Contact stresses , σ_H, MPa	Kinematic viscosity, 10^{-6} at peripheral speed V, m/sec		
	Up to 2	Over 2 to 5	Over 5
Up to 600	34	28	22
Over 600 to 1000	60	50	40
Over 1000 to 2000	70	60	50

Table 12.22

Recommended viscosity of oils for lubricating worm gears when t=100°C

Contact stresses , σ_H, MPa	Kinematic viscosity, 10^{-6} at peripheral speed V, m/sec		
	Up to 2	Over 2 to 5	Over 5
Up to 200	25	20	15
Over 200 to 250	32	25	18
Over 250 to 300	40	30	23

There are two methods of lubrication:

- immersion lubrication, when toothed wheels are immersed into the oil bath;
- stream lubrication, when oil feeds the contact area of toothed wheels by means of special nozzles.

Table 12.23

Recommended oil grades for general-purpose speed reducer

Oil	Oil grade	Kinematic viscosity, 10^{-6}м^2/с
Industrial at 50°C	И-12А	10-14
	И-20А	17-23
	И-25А	24-27
	И-30А	28-33
	И-40А	35-45
	И-50А	47-55
	И-70А	65-75
	И-100А	90-118
Aviation at 100°C	МС-14	14
	МК-22	22
	МС-20	20.5

Immersion lubrication is effective when peripheral speed is less than 10 m/sec.

The oil bath lubricates the larger wheel. The recommended depth of immersion of high-speed spur gear is ranged form m to $5 \cdot m$ but not less than 10 mm (m is the module of the gearing). Low-speed spur gear should be immersed not more than 100 mm. Bevel gears are immersed along the entire length of a tooth.

If toothed wheels cannot be lubricated by immersing the additional pinions, rings or other devices are used (Fig. 12.9).

In worm-down speed reducers the oil level should not exceed the thread height of the worm. But in this case the oil level should not rise above the centre of the lower rolling element of the worm shaft bearings. If the worm is not immersed in oil the additional rings with blades are used (Fig. 12.10)). In worm-up arrangement the worm gear should be immersed not more than 1/3 of the worm gear radius.

Let us determine the volume of the oil bath.

Needed volume is chosen to ensure removing the heat generated in the engagement of a gearing to casing walls. Recommended volume of the oil bath is chosen as 0.6 to 0.8 liter of the oil per 1 kilowatt of transmitted power. Consequently, the needed volume $V = (0.6\ldots0.8) \cdot P_{\text{motor}}$.

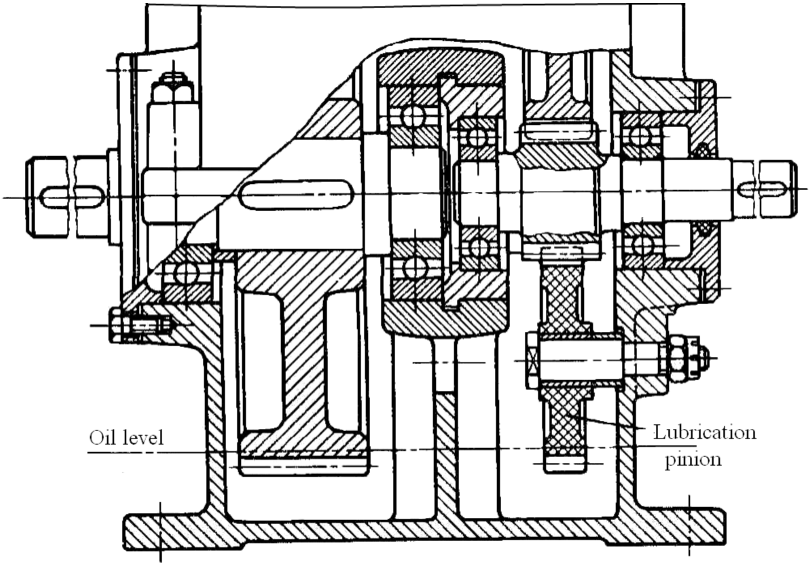


Fig. 12.9. Lubrication pinion made of textolite

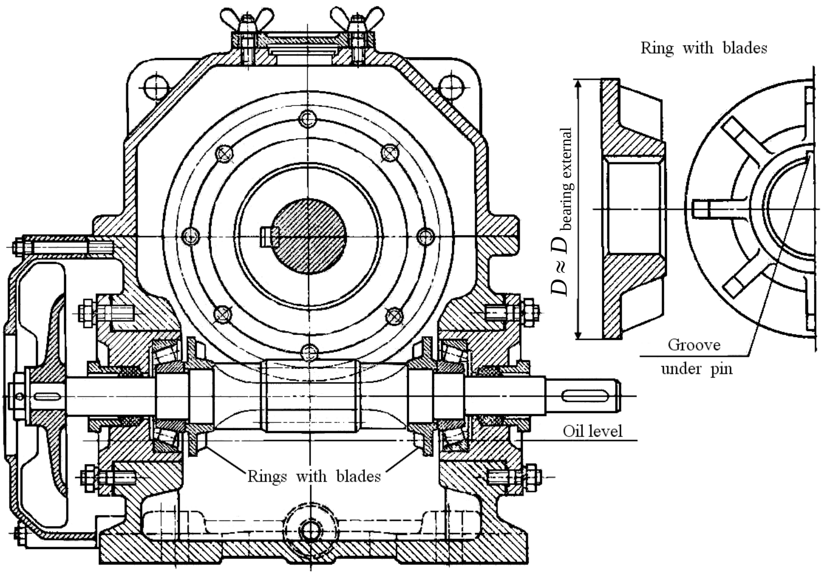


Fig. 12.10. Ring with blades

The larger volume, the longer oil life and better lubrication conditions. That is why the volume of the oil bath is only limited by the maximum permissible oil level in the casing.

The distance between the oil level and the speed reducer bottom is determined as $H = \frac{V}{S}$, where $S = L{\cdot}B$ is the area of the speed reducer inner space in dm^2; L and B are correspondingly the length and the width of the speed reducer inner space in dm. It is necessary to note that the minimum distance between tops of teeth of the larger gear and the speed reducer bottom is 20 mm.

Lubrication of bearings.

Bearings may be lubricated with the same oil as used for the mashing parts (when the peripheral speed is greater than 3 m/sec) or individually with greases.

Splash lubrication is used when the bearings are installed in cases which are not insulated from the general system of lubrication unit. Rotating parts (gears, wheels etc.) come into contact with oil which is fill in into the housing then under rotation sprays oil, which falls on the rolling bodies and bearing tracks.

To protect the bearings from the heavy jets of oil (which create high-speed helical pinions or worms) and getting into them products of wear the shields (protective washers) are installed (Fig. 12.11).

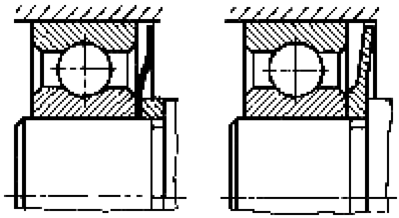


Fig. 12.11. Bearings with shields

Pressure lubrication through nozzles is used for reducers, working long time without interruption, as well as for the bearings of high-speed transmission, which is necessary to provide intensive heat removal.

Oil fog lubrication is used for high-speed understressed bearings. With help of special nozzles under pressure in the unit supplied a jet of air, which carries oil particles.

This method allows to penetrate the oil in the bearings, located in inaccessible places, creates a flow with minimal lubrication oil consumption provides a good cooling of bearings and the pressure protects the assembly from contamination.

Greases offer better protection against corrosion than oils and it's used when environment contains harmful impurities or the temperature of the assembly sharply changes. In this case grease-retaining rings (Fig 12.12) are used to isolate the bearing cavity from the inner space of the casing. The ring periphery should extend over the end face of the bearing housing for 1 or 2 mm (Fig. 12.13). The gap between the ring periphery and the housing should be about 0.2 mm. The ring rotates together with the shafts and it has from two to four grooves. In order to feed the grease inside the bearing without removing the cap grease cups are used (Fig.12.13). The lubricant is injected under pressure by means of a grease gun.

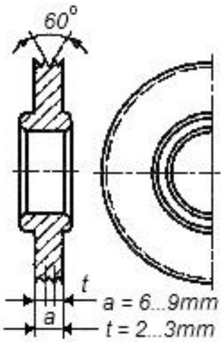


Fig. 12.12. Grease-retaining ring

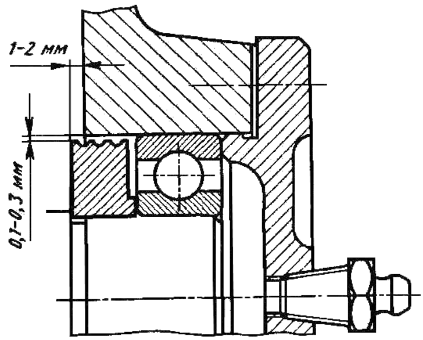


Fig. 12.13. Bearing assembly with grease cup

12.5. Analysis of keyed joints.

Dimensions of keys are chosen according to table 12.24 depending upon the shaft diameter. The length of the key should be less than the hub length by 5…10 mm and correspond to the standard series.

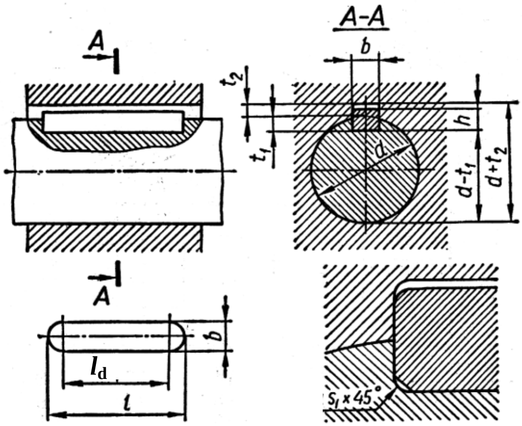
In general-purpose speed reducer, keyed joints are usually analyzed to prevent bearing stresses.

$$\sigma_{bear} = \frac{2 \cdot T}{d \cdot (h - t_1) \cdot l_d} \le [\sigma_{bear}],$$

where T is the torque in N·mm; d is the diameter of the shaft in mm; h is the height of the key in mm; t_1 is the depth of the slot in the shaft; l_d is the design length of the key in mm (for keys with round sides $l_d = l - b$; for keys with square sides $l_d = l$, where l is the length of the key; b is the width of the key); $[\sigma_{bear}]$ is the allowable bearing stress (for cast-iron hubs $[\sigma_{bear}]$=60…80 MPa; for steel hubs $[\sigma_{bear}]$=100…120 MPa).

Table 12.24

Standard Sunk Keys



Shaft diameter *d*, mm	Key cross section, mm		Keyseat depth, mm		Length *l*
	b	*h*	shaft, t_1	hub, t_2	
Over 17 to 22	6	6	3.5	2.8	Over 14 to 70
Over 22 to 30	8	7	4	3.3	Over 18 to 90
Over 30 to 38	10	8	5	3.3	Over 22 to 110
Over 38 to 44	12	8	5	3.3	Over 28 to 140
Over 44 to 50	14	9	5.5	3.8	Over 36 to 160
Over 50 to 58	16	10	6	4.3	Over 45 to 180
Over 58 to 65	18	11	7	4.4	Over 50 to 200
Over 65 to 75	20	12	7.5	4.9	Over 56 to 220
Over 75 to 85	22	14	9	5.4	Over 63 to 250
Over 85 to 95	25	14	9	5.4	Over 70 to 280
Over 95 to 110	28	16	10	6.4	Over 80 to 320
Over 110 to 130	32	18	11	7.4	Over 90 to 360

Note: The length of the key is chosen according to the following series: 6; 8; 10; 12; 14; 16; 18; 20; 25; 28; 32; 35; 40; 45; 50; 56; 63; 70; 80; 90; 100; 110; 125; 140; 160; 180; 200.

13. Designing the mechanical drive

A mechanical drive is drawn in two projections (as a rule, front view and top view) to scale 1: 4 or 1:5.

13.1. Draw the main structural units of the mechanical drive. Among them there are electrical motor (table 9.1), coupling with rubber-bushed studs (tables 9.7), belt drive, speed reducer, chain coupling (table 9.8), flanged coupling (table 9.9), chain drive. Dimensions of all mentioned above elements and units have been found.

13.2. Design the output shaft of the mechanical drive (Fig.13.1).

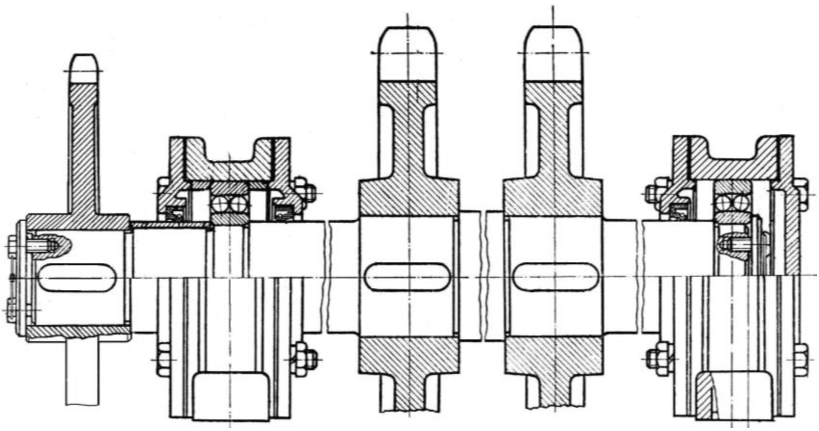


Fig.13.1. Output shaft of the mechanical drive

Determine the minimum diameter of the output shaft. For this purpose we use the same formula as for the other speed reducer shafts

$$d_{\min} = \sqrt[3]{\frac{T}{0.2 \cdot [\tau]}} .$$

If the output shaft of the mechanical drive is joined with the speed reducer shaft by the coupling, the minimum diameter of the mechanical drive output shaft is equal to the minimum diameter of the speed reducer output shaft.

On the cantilever portion of the output shaft half coupling, sprocket or gear may be mounted. They should be fixed in the axial direction. That is why it is necessary to provide the shoulder on the shaft. Then the diameter of the second portion of the shaft is determined as $d_2 = d_1 + 2 \cdot t_1$ (t_1 is chosen from table 9.2).

The second portion of the output shaft is for mounting a bearing. That is why the diameter of this portion should be ended by 0 or 5.

The next portion of the output shaft is necessary for installation of the drum or conveyer sprockets. Diameter of this portion is $d_3 = d_2 + 2 \cdot t_1$.

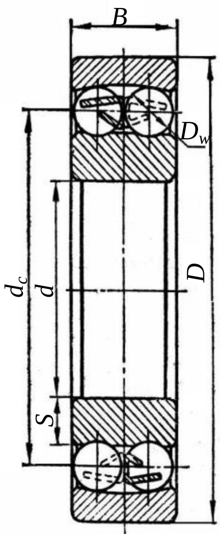
The last portion of the output shaft is for mounting the second bearing. The diameter of this portion is d_2.

13.3. Design supports of the output shaft.

As a rule, supports of the output shaft are mounted in different housings. In order to compensate inaccuracy and misalignment in assembly self-aligning double-row spherical radial ball bearings of the light–weight series are used (table 13.1). Owing to the race spherical surface of the outer ring, these bearings can handle the shaft misalignment of up to 2 or even 3.

Table 13.1

Double-row radial ball bearings. Lightweight series, mm



$d_c = 0.5 \cdot (D + d)$
$D_w = 0.25 \cdot (D - d)$
$S = 0.17 \cdot (D - d)$

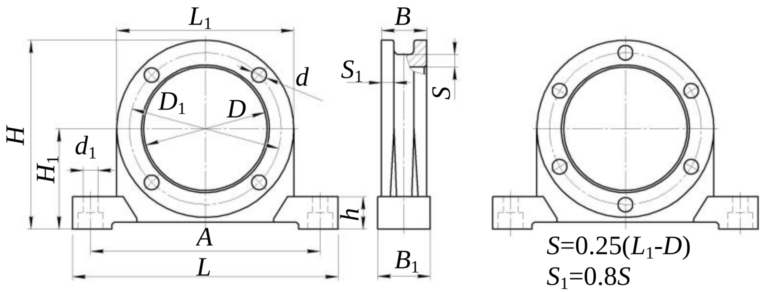
Type designation	d	D	B	r	C_r, kN	C_0, kN	Y for			
							e	$\frac{F_a}{F_r} < e$	$\frac{F_a}{F_r} > e$	Y_0
1204	20	47	14	1.5	9.95	3.18	0.27	2.31	3.57	2.42
1205	25	52	15	1.5	12.1	4.0	0.27	2.32	3.6	2.44
1206	30	62	16	1.5	15.6	5.8	0.24	2.58	3.99	2.7
1207	35	72	17	2	15.9	6.6	0.23	2.74	4.24	2.87
1208	40	80	18	2	19.0	8.55	0.22	2.87	4.44	3.01
1209	45	85	19	2	21.6	9.65	0.21	2.97	4.6	3.11
1210	50	90	20	2	22.9	10.8	0.21	3.13	4.85	3.28
1211	55	100	21	2.5	26.5	13.3	0.2	3.23	5.0	3.39
1212	60	110	22	2.5	30.2	15.5	0.19	3.41	5.27	3.57
1213	65	120	23	2.5	31.2	17.2	0.17	3.71	5.73	3.68
1214	70	125	24	2.5	34.5	18.7	0.18	3.51	5.43	3.88
1215	75	130	25	2.5	39.0	21.5	0.18	3.6	5.57	3.77
1216	80	140	26	3	39.7	23.5	0.16	3.94	6.11	4.13
1217	85	150	28	3	48.8	28.5	0.17	3.69	5.71	3.87
1218	90	160	30	3	57.2	32.0	0.17	3.76	5.82	3.94
1220	100	180	34	3.5	63.7	37.0	0.17	3.68	5.69	4.81
1221	105	190	36	3.5						
1222	110	200	38	3.5						
1224	120	215	42	3.5						

Bearings of the output shaft are arranged as the fixed and floating supports located in bearing housings (table 13.2). The fixed support can take up double-sided axial load. For that the inner and outer rings of the bearing are fixed in both axial directions. The floating support should compensate deformation of the shaft. In this case the inner ring of the bearing is fixed in both directions. For that we use boundary plates (tables 13.3 and 13.4) or spring rings (table 13.5). The outer ring is left free.

Table 13.2

Standard bearing housings (GOST 13218.3-67), mm

УМ 110-150 УМ 160-400



Bore diameter *D*, mm	D_1	*d*	d_1	*A*	*B*	B_1	*L*	L_1	*H*	H_1	*h*
120	145	M12	17	210	48	48	260	175	179.5	92	32
125	150	M12	17	220	48	48	270	180	188	98	34
130	155	M12	17	225	50	54	280	185	190.5	98	34
140	165	M12	22	235	52	58	295	195	199.5	102	35
150	180	M12	22	255	55	64	315	210	215	110	40
160	190	M12	22	260	60	64	315	220	230	120	40
170	200	M12	22	280	63	66	335	230	240	125	40
180	210	M12	22	290	68	75	355	240	250	130	40
190	220	M16	22	295	70	82	360	250	260	135	40
200	230	M16	22	300	75	82	365	260	270	140	40
215	250	M16	22	325	85	90	385	285	292.5	150	45
225	260	M16	26	340	90	95	410	295	307.5	160	48

13.4. Design the drum or sprockets of the conveyer.

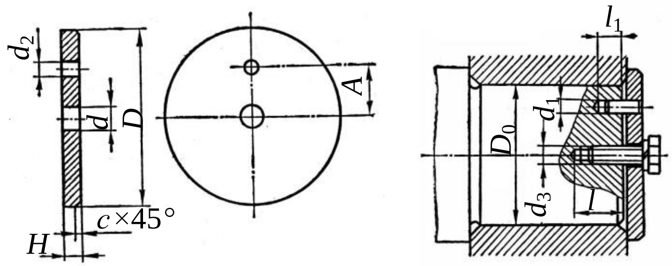
Diameter of the drum (sprocket) is given in the specification for the course paper.

The drum (sprockets) should be mounted on the output shaft by distance 100 or 200 mm relative to supports.

There exist cast drums and welded drums. Welded drums are used more frequently.

Table13.3

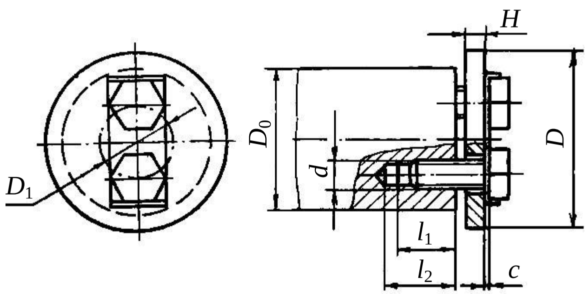
Boundary plate (ГОСТ 14734–69), mm



Designation	D	H	A	d	d_2	c	D_0	d_3	d_1	l	l_1	Screw (ГОСТ 7798–70)	Pin (ГОСТ 3128–70)
7019–0623	32	5	9	6,6	4,5	1,0	24...28	M6	4	18	12	M6×16	4*m*8×12
7019–0625	36		10				28...32						
7019–0627	40		10				32...36						
7019–0629	45		12				36...40						
7019–0631	50		16				40...45						
7019–0633	56		16				45...50						
7019–0635	63	6	20	9,0	5,5	1,6	50...55	M8	5	22	16	M8×20	5*m*8×16
7019–0637	67		20				55...60						
7019–0639	71		25				60...65						
7019–0641	75		25				65...70						
7019–0643	85		28				70...75						

Table 13.4

Boundary plate with two screws and retaining plate, mm



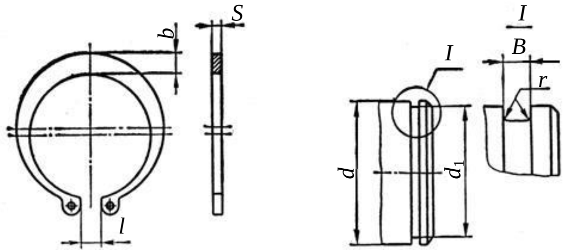
D_0	D	H	D_1	c	l_1	l_2	Screw d×l
From 35 to 40	50	6	20	0,5	20	24	M8×20
above 40 to 45	55		20				
from 45 to 50	60		25				

End of the table 13.4

from 50 to 60	70	8	30		30	35	м12×30
from 60 to 70	80		35				
from 70 to 80	90	10	40				
from 80 to 90	110		45				
from 90 to 100	120	12	50		36	42	м16×36
from 100 to 110	125		55				
from 110 to 120	140		60				
from 120 to 130	150		65				

Table 13.5

Spring snap rings and grooves for them (ГОСТ 13942-86), mm



Shaft diameter d	Grooves		Rings		
	d_1	B	s	b	l
20	18.6	1.4	1.2	3.2	3.0
25	23.5			3.6	
30	28.5			4.0	
35	33.0	1.9	1.7	4.9	6.0
40	37.5			5.5	
45	42.5				
50	47.0	2.2	2.0	6.0	
55	52.0				
60	57.0			6.5	
65	62.0	2.8	2.5	7.0	
70	67.0				
75	72.0			8.0	
80	76.5				

Possible constructions of welded drums are given in Fig.13.2. The rim and disks of the drum are made of steel sheet of thickness 8 mm. Ribs of the drum are produced from the strip of width 40 mm and thickness 6 mm.

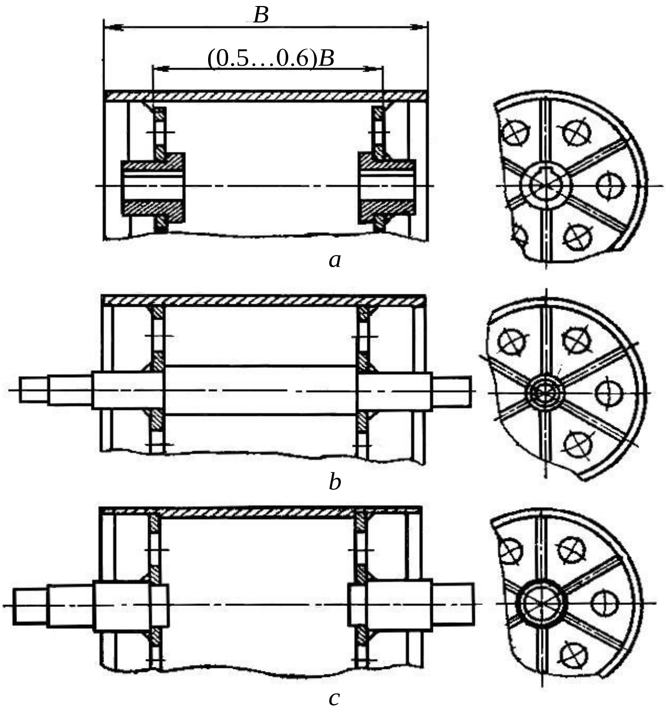


Fig. 13.2. Welded drums

For determination sizes of the conveyer sprocket it is necessary to choose a pull chain (table 13.6).

Dimensions of the pull chain sprocket are determined in the following way:

– addendum circle diameter $D_a = D + 0.25 \cdot D_1 + 6$ mm;

– dedendum circle diameter $D_f = D - D_1$;

– web thickness $b = 0.9 \cdot B_{\text{bush}}$;

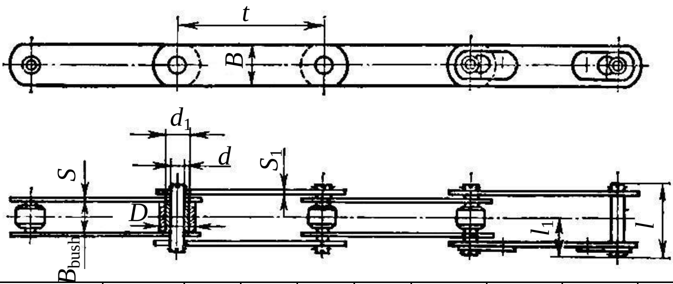
where D is the nominal pitch circle diameter of the sprocket (look through the specification for the course paper)); D_1 is the diameter of the roller; B_{bush} is the bush width.

13.5. Design the welded frame.

Welded frames are used for mounting assembly units. They should correspond to certain requirements of accuracy to provide needed relative disposition of assembly units. Besides, the welded frame should have high rigidity.

Table 13.6

Standard pull chains (GOST 588 – 64), mm



t	B_{bush}	B	S	S_1	d	d_1	D_1	l	l_1
100	32	36	5	4	14	21	30	64	35
100	38						36	70	38
125	44	50	7	7	20	30	44	100	54

Welded frames are obtained as a result of welding steel rolling elements such as channel bars, angle (*L*) bars, strips, sheets and others. Channel bars are used more frequently.

The height of the channel bar is determined as

$$H \approx 0.1 \cdot L_{max},$$

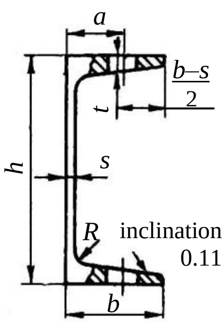
where L_{max} is the maximum length of the mechanical drive.

Obtained value of H should be rounded off to the greater side according to standard series given in table 13.7.

In order to simplify fixation of assembly units to the frame channel bars are mounted with outside flanges. For aligning the base surface for the bolt head we use skew plates (table13.8)

Table 13.7

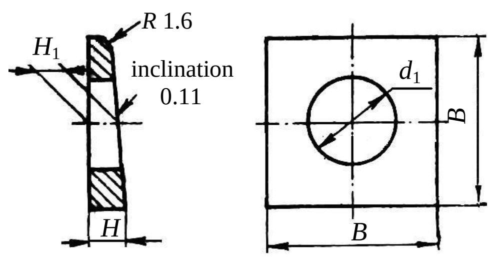
Channels (GOST 8240-72), mm



Number of channel	Dimensions, mm: h	b	s	t
14	140	58	4.9	8.0
16	160	64	5.0	8.4
18	180	70	5.1	8.7
20a	200	80	5.2	9.0
22a	220	87	5.4	9.5
24a	240	95	5.6	10.7
27	270	95	6	10.5
30	300	100	6.5	11
33	330	105	7	11.7

Table 13.8

Standard skew plates (GOST 10906 – 78)



Skew plates parameters, mm				
Bolt diameter	d_1	H_1	H	B
12	13	5.7	6	30
16	17	5.7	6	30
18	19	6.2	7	40
20	22	6.2	7	40
22	24	6.2	7	40
24	26	6.8	9	50

If assembly units of the mechanical drive are located at different levels we may use constructions of frames given in Fig.13.3, *a–e*.

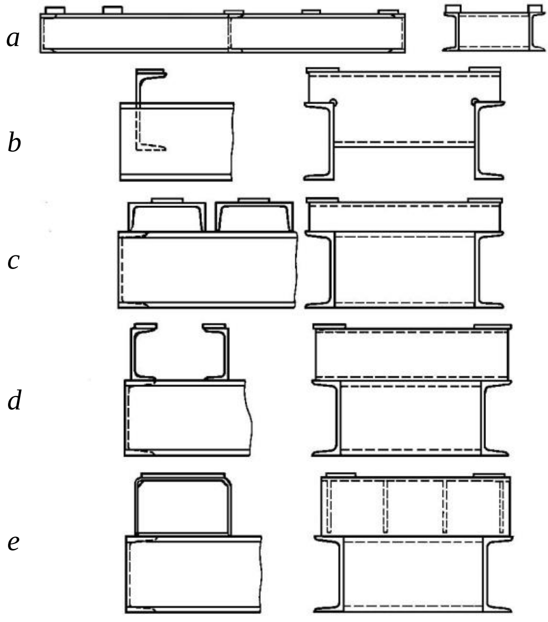
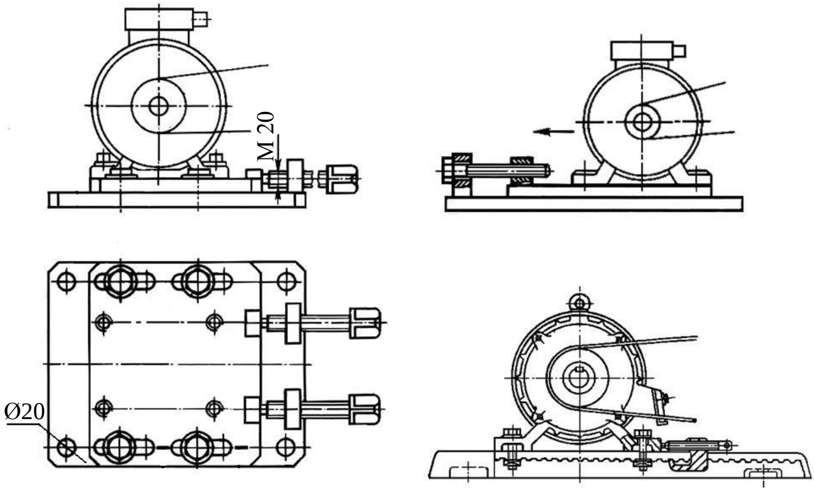


Fig.13.3. Alternatives of frames, when assembly units are located at different levels.

During operation of the belt drive the belt stretches. In order to provide tension of the belt at the required level belt tension adjusters are used. Possible constructions of bet tension adjusters are given in Fig 13.4, *a, b* and table 13.9.

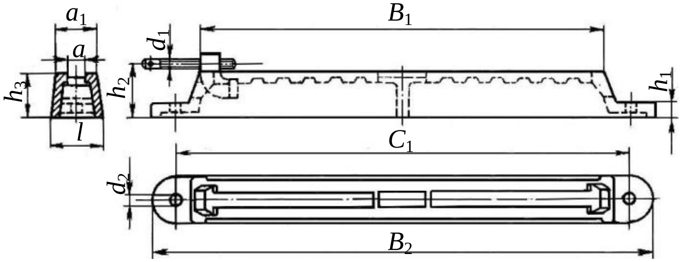


a *b*

Fig.13.4. Constructions of belt tension adjusters.

Table13.9

Sledges for electric motor



Type	Dimensions, mm											Mass of set, kg	Screws for engine mounting
	a	a_1	B_1	B_2	C_1	d_1	d_2	h_1	h_2	h_3	l		
C–3	16	38	370	440	410	M12	12	15	44	36	42	3,8	M10×35
C–4	18	45	430	540	470	M12	14	18	55	45	50	5,3	M12×40
C–5	25	65	570	670	620	M16	18	22	67	55	72	12,5	M16×55

14. Shop drawings of speed reducer elements

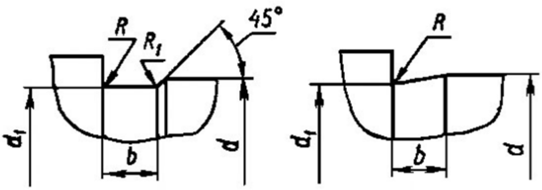
According to the specification for the course paper it is necessary to make two shop drawings such as the shop drawing of the speed reducer output shaft and the shop drawing of the toothed wheel.

Shop drawing of the output shaft

The shop drawing is carried out to scale 1:1 or 1:2. First of all, it is necessary to plot the output shaft according to the dimensions of assembly drawing. The shaft must be drawn in the position of installing on a machine, i.e. the axis of the shaft should be parallel to the main inscription. Transition from one installing diameter to the other is carried out by means of fillets or grooves. Fillets are used when the installing surface is not ground. Shaft portions where bearings are installed should be ground. As a space required for grinding wheel grooves are used. Possible alternatives of grooves are shown in table 14.1. Dimensions of grooves are given in table 14.1. In the shop drawing grooves should be represented to the increased scale separately.

Table 14.1

Grooves for a grinding wheel in shafts and their dimensions, mm



d	b	d_1	R	R_1
Over 10 to 50	3	d – 0.5	0.1	0.5
Over 50 to 100	5	d – 0.5	1.6	0.5
Over 100	8	d – 1.0	2.0	1.0

The next step is dimensioning the drawing. The number of dimensions should be minimal but enough to produce the shaft. Chamfers and grooves width cannot be included to the total dimensions chain.

There exist three methods of dimensioning drawings:

- chain method that provides the accuracy of disposition of every following element relative to the previous. In this case the accuracy relative to certain base is decreased;
- coordinate method according to which dimensioning of a drawing is carried out with respect to base *A*;
- combined method that consists of the chain and coordinate methods.

For the shop drawing the recommended method is combined method. Dimensioning in the axial direction is carried out under an element drawing.

As it is known dimension must be held between two limits. The difference of these limits is called tolerance. Tolerance limits of relatively low accuracy dimensions are not marked on a drawing. In this case it is necessary to make the following inscription:

"*Dimensional tolerances: holes H14, shafts h14, other elements* $\pm\frac{\text{IT14}}{2}$ *(medium accuracy class)*".

The nature of elements connections is called a fit. Fits may provide clearance or interference. There exists also transition fits that may have either clearance or interference. Fits are marked by a letter of Roman alphabet. Letters *a*-*h* corresponds to clearances, *js*-*n* – transition fits, *p*-*z* - interference. A numeral near a letter shows the quality grade. There exist 19 quality grades. For mechanical engineering the most typical are quality grades 5 through 12. Quality grades 6 through 8 refer to critical parts and units.

Example of the shaft drawing – Fig. 14.1, example of the gear construction – Fig. 14.2.

Fits of main elements

1. Straight spur gear on the shaft - $\frac{H7}{p6}$;

2. Helical spur gear and worm gear on the shaft - $\frac{H7}{r6}$;

3. Bevel gear on the shaft - $\frac{H7}{s6}$;

4. Coupling and gear located on a cantilever portion of the shaft - $\frac{H7}{k6}$;

5. Pulleys or sprockets - $\frac{H7}{h6}$;
6. Commercial seals - *h*11;
7. Inner ring of a bearing - *k*6;
8. Outer ring of a bearing - *H*7;
9. The width of a keyseat in the shaft - *P*9;
10.The width of a keyseat in the hole of a hub - J_S9.

During treatment of the shaft besides errors of linear dimensions errors of geometrical shapes and errors of surfaces disposition arise.

Possible errors of geometrical shapes are non-cylindrical surfaces, non-rounding, non-flatting.

For shaft portions and gear holes tolerances of cylindrical surface should be taken into account. Sign /○/ marks tolerance of cylindrical surface. The magnitude of this tolerance is determined as $0.3{\cdot}t{\cdot}10^{-3}$, where *t* is diametrical tolerance range in micrometers (table 14.2).

Table 14.2

Tolerance ranges in micrometers

Dimensions in mm	Quality grades											
	3	4	5	6	7	8	9	10	11	12	13	14
Over 3 to 5	2.5	4	5	8	12	18	30	48	75	120	180	300
Over 6 to 10	2.5	4	6	9	15	22	36	58	90	150	220	360
Over 10 to 18	3	5	8	11	18	27	43	70	110	180	270	430
Over 18 to 30	4	6	9	13	21	33	52	84	130	210	330	520
Over 30 to 50	4	7	11	16	25	39	62	100	160	250	390	620
Over 50 to 80	5	8	13	19	30	46	74	120	190	300	460	740
Over 80 to 120	6	10	15	22	35	54	87	140	220	350	540	870
Over 120 to 180	8	12	18	25	40	63	100	160	250	400	630	1000
Over 180 to 250	10	14	20	29	46	72	115	185	290	460	720	1150

Possible errors of surface dispositions are non-perpendicularity relative to a base, misalignment, non-symmetry, non-parallelism.

For shafts it is necessary to use total tolerances that take into account tolerance of shape and tolerance of surface disposition: radial run-out that allows for non-rounding and misalignment and end-play that takes into account non-flatting and non-perpendicularity.

The magnitude of radial run-out ↗ depends upon the peripheral speed and is determined according to table 14.3.

The magnitudes of end play ↗ may be found according to table 14.4.

Table 14.3

Tolerances of radial run out of shaft portions for fitting toothed wheels, pulleys, couplings

Peripheral speed of elements mounted on a shaft, m/sec	< 2	2…6	6…10	> 10
Tolerance of radial run out	$2.0 \cdot t \cdot 10^{-3}$	$1.4 \cdot t \cdot 10^{-3}$	$1.0 \cdot t \cdot 10^{-3}$	$0.7 \cdot t \cdot 10^{-3}$

Table 14.4

Tolerances of end play of gearings, hubs, of toothed wheels and shaft shoulders

Degree of accuracy	For gearing at *d*=100 mm, and at face width of		For toothed wheel hub and shaft shoulder at bore diameter (shaft diameter of)		
	< 55	55…110	< 55	55…80	> 80
6	0.017	0.009	0.02	0.03	0.04
7	0.021	0.011	0.02	0.03	0.04
8	0.026	0.014	0.03	0.04	0.05
9	0.034	0.018	0.03	0.04	0.05

As a base the shaft axis of rotation is used for surfaces where bearings are installed. In order to eliminate misalignments of elements that are installed on the shaft we will use as a base of all other shaft surfaces the surfaces where bearings are located.

Possible errors of a keyseat are non-parallelism and non-symmetry. Parallelism tolerance is marked as // and is equal to $0.6 \cdot t_{ks} \cdot 10^{-3}$, where t_{ks} is keyseat width tolerance range in micromeres (table 14.2). Symmetry tolerance is marked as ÷ and is $4 \cdot t_{ks} \cdot 10^{-3}$.

The next step is marking surfaces roughness.

Surface roughness may be evaluated by average deviation of profile R_a or height of profile irregularities R_z by ten points. Marking surface roughness by R_a is more preferable. The magnitude of surface roughness depends upon the surface treatment and the quality grade.

Main surfaces roughness

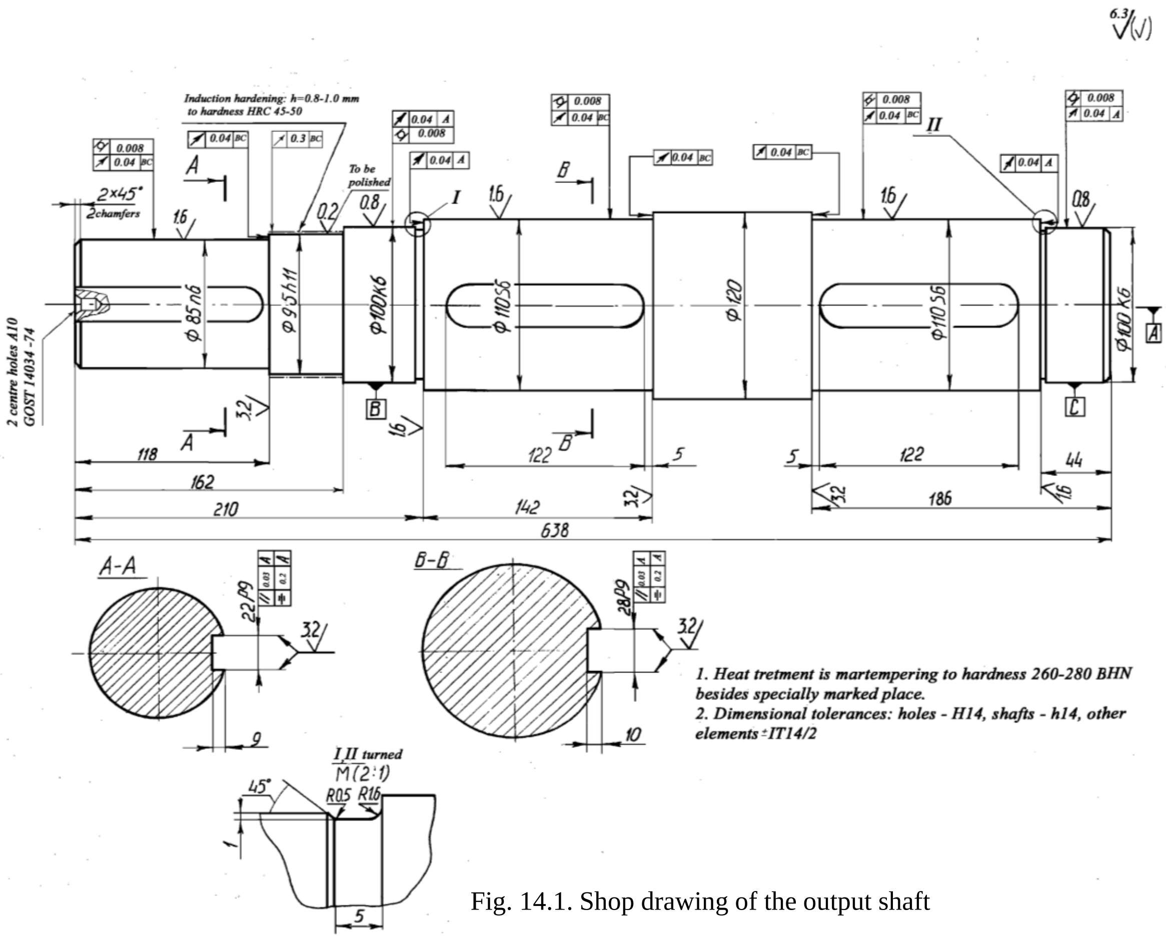
- shaft portions for installing bearings – 0.8;
- shaft portions for installing gears, half-couplings, sprockets, pulleys:

if *d* < 55 mm – 0.8, if 55 < *d* < 120 – 1.6;

- shoulder face end for bearings fixation – 1.6;
- shoulder face end for fixation of gears, sprockets, pulleys, half-couplings – 3.2;
- shoulder potion for installing seals – 0.2;
- working surfaces of a keyseat made in the shaft – 1.6;
- holes of gears that are installed on the shaft
  if *d* < 55 mm – 0.8;
  if 55mm < *d* < 120mm – 1.6;
- gear face end that is fixed by the shaft shoulder or the face end of other element – 3.2;
- free gear face end – 6.3;
- tooth profile

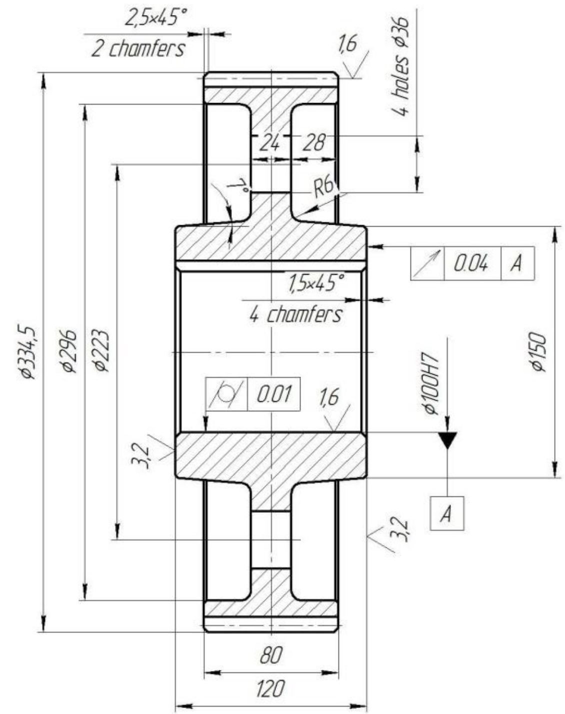
for 6th degree of accuracy – 0.4;
for 7th degree of accuracy – 0.8;
for 8th degree of accuracy – 1.6;
for 9th degree of accuracy – 3.2;

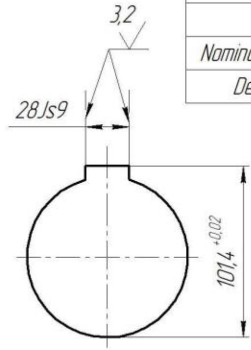
- working surfaces of a keyseat made in the hub– 3.2.



1. Heat tretment is martempering to hardness 260-280 BHN besides specially marked place.
2. Dimensional tolerances: holes - H14, shafts - h14, other elements ±IT14/2

Fig. 14.1. Shop drawing of the output shaft





6,3

Module	m	4
Number of teeth	z	98
Helix angle	β	11.48°
Offset factor	x	0
Nominal pith circle diameter	d	326.5
Degree of accuracy		9

1. Heat tretment is martempering to hardness 269–302 BHN
2. Dimensional tolerances: holes – H14, shafts – h14, other elements ±IT14/2

Fig. 14.2. Spur gear construction

References

1. Баласанян Р. А. Атлас деталей машин / Баласанян Р. А. – Х.: Основа, 1996. – 256 с.
2. Berezovsky Yu. Machine Design / Berezovsky Yu., Chernilevsky D., Petrov M. – M.: Мир, 1983. – 456 с.
3. Деталі машин: Конспект лекцій / [Павлов В. М., Крижановський А. С., Цимбалюк А. А. та ін.] – К.: НАУ, 2003. – 159 с.
4. Иванов М. П. Детали машин / Иванов М. П. – М.: Высшая школа, 1982. – 351 с.
5. Курсовое проектирование деталей машин / [Чернавский С. А., Боков К. Н., Чернин И. М. и др.] – М.: Машиностроение, 1987. – 416 с.
6. Павлище В. Т. Основи конструювання та розрахунок деталей машин / Павлище В. Т. – К.: Вища школа, 1993. – 556 с.
7. Шейнблит А. Е. Курсовое проектирование деталей машин: Учеб. Пособие / Шейнблит А. Е. — Калининград: Янтар. сказ., 2002. — 454 с.
8. Цехнович Л. Н. Атлас конструкций редукторов / Л. Н. Цехнович, И. П. Петриченко. – К.: Высшая школа, 1979. – 151 с.

CONTENTS