MINISTRY OF EDUCATION AND SCIENCE
NATIONAL AVIATION UNIVERSITY

THE THEORY OF MECHANISMS AND MACHINES

Guide to Laboratory Work for students of specialities 272 «Aviation Transport» and 134 «Aviation and Aerospace Technology»

Kyiv
2018

UDC 531.8(076.5)
ББК К412 я 7
T44

Authors:
A. Kornienko – PhD, associate professor – laboratory works 1-4, 8, 9
O. Bashta – PhD, associate professor 6, 7
O. Tisov – PhD, associate professor 5, 10

Reviewers by:
M. Sisoev – PhD, associate professor (National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”);
A. Khimko – PhD, associate professor (National Aviation University).

The English language advisor: L. Budko, assistant professor, Department of Foreign Languages for Professional Purposes.

Approved by the Methodological-Editorial Board of National Aviation Univercity (Minutes № ___ of ___________, 2018)

Містить загальні методичні рекомендації до виконання лабораторних робіт з дисципліни «Теорія механізмів і машин». До кожної роботи наведено мету, порядок виконання, форму звіту, приклади оформлення.

Для студентів спеціальностей 272 «Авіаційний транспорт» та 134 «Авіаційна та ракетно-космічна техніка».

The Theory of Mechanisms and Machines. Guide to Laboratory Work
T44 Authors: A. Kornienko, O. Bashta, O. Tisov. – K.: NAU, 2018. – 35 p.

The guide includes general methodical recommendations for carrying out laboratory works on “Theory of Mechanisms and Machines”. There have been given the purpose of the laboratory works, the order of carrying them out, the forms of reports and samples of works.

For students of specialities 272 «Aviation Transport», 134 «Aviation and Aerospace Technology».

INTRODUCTION

The guide to laboratory work is an integral part of the course «Theory of mechanisms and machines». Its aim is to elucidate theoretical knowledge obtained in the course of lectures and practical classes, to work out the skills to perform tasks on mechanisms and machines analysis. While performing laboratory works students acquire practical skills in drawing mechanism diagrams, investigating the motion character of separate links and finding mechanism freedom degrees, master methods of mechanisms division into structural links groups, methods of finding basic geometrical parameters of toothed wheels. They get acquainted with theoretical and experimental research methods of toothed wheels and cam mechanisms operation, with principles of tooth involute profile generation while producing gears, and also with rotating links balancing including aviation engines rotors and propellers.

Laboratory works are performed under the supervision of teachers according to «The rules of carring out laboratory works in "Theory of mechanisms and machines" (appendix 1) in specially equipped laboratories of engineering department.

The guide contains protocols of all laboratory works according to the plan of 272 «Aviation Transport» and 134 «Aviation and Aerospace Technology».

LABORATORY WORK # 1

DETERMINATION OF THE NUMBER OF FREEDOM DEGREES OF PLAIN MECHANISMS

The purpose of the work is getting acquainted with classification of kinematic pairs of plain and space mechanisms; getting practical skills of drawing diagrams of plain mechanisms; studying schematic symbols of kinematic pairs and links of plain mechanisms; investigating the character of separate links motion and finding number of freedom degrees of plain mechanisms.

The number of freedom degrees of plain mechanism can be found by Chebyshev formula:

$$W = 3n - 2\,p_1 - 1\,p_2\ .$$

For example, let’s find number of freedom degrees of helicopter engine gas distribution mechanism (Fig. 1.1).

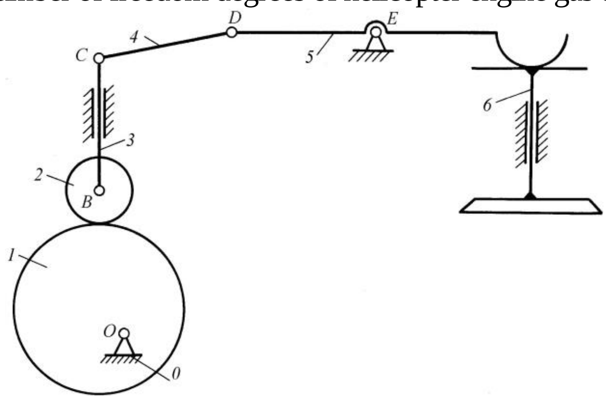


Fig. 1.1. Helicopter engine gas distribution mechanism

This problem is solved in the following:

1. Number all mechanism links and find their total number. Figure number *1* stands for the link which receives the forced motion, and the last figure stands for the fixed link. As you can see in Fig. 1.1, the mechanism consists of 6 movable links, so n = 6.

2. Define kind, type and the number of kinematic pairs, formed by the mechanism links. Mark with letters the centers of rotation of turning kinematic pairs.

Links *0* and *1, 2* and *3, 3* and *4, 4* and *5, 5* and *0* form the 1st kind kinematic pairs (turning). Links *3* and *0, 6* and *0* form the 1st kind kinematic pairs (sliding). Links *1* and *2, 5* and *6* form the 2nd kind kinematic pairs (of TS-type). Thus, the mechanism consists of 7 the 1st kind kinematic pairs and of 2 the 2nd kind kinematic pairs ($p_1 = 7, p_2 = 2$).

3. The solution of the problem is represented in Table 1.1. It is made in a column in the same order as those groups of links were enumerated.

Table 1.1

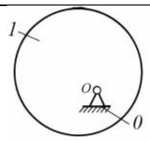
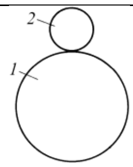
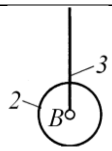
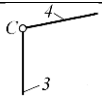
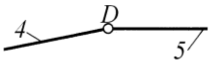
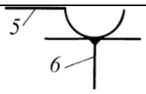
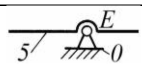
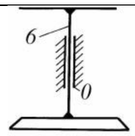
Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation
0 and *1* turning		1 / 5	contact by the surfaces (lower KP)	*O*
1 and *2* TS		2 / 4	contact in the lines (higher KP)	–
2 and *3* turning		1 / 5	contact by the surfaces (lower KP)	*B*

Table 1.6. Continuation

Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation
3 and *4* turning		1 / 5	contact by the surfaces (lower KP)	*C*
3 and *0* sliding		1 / 5	contact by the surfaces (lower KP)	–
4 and *5* turning		1 / 5	contact by the surfaces (lower KP)	*D*
5 and *6* TS		2 / 4	contact in the lines (higher KP)	–
5 and *0* turning		1 / 5	contact by the surfaces (lower KP)	*E*
6 and *0* sliding		1 / 5	contact by the surfaces (lower KP)	–

1. Determine the number of freedom degrees of mechanism by Chebyshev formula:

$$W = 3n - 2\,p_1 - 1\,p_2 = 3\cdot 6 - 2\cdot 7 - 1\cdot 2 = 2.$$

The result shows, that all links of mechanism being analyzed will perform strictly definite motion in case if 2 links mechanism are given the forced motion. At the same time, analyzing the diagram of investigating operation, we will see, that position of link *6* (valve) is fully determined by the position of link *1* (cam) - any position of link *1* corresponds a definite position of link *6.* So, this mechanism will operate as if it has the only 1 freedom degree *(W* = 1). It is due to the fact that a roller (link *2)* is round and its rotational axis coincides with geometrical axis. That's why rotation of roller does not influence the valve position, and is an additional freedom degree and it is referred to as the ***passive*** link. This mechanism is provided with a roller to decrease friction and wear of links. If the roller is not round or being round its rotational axis would not coincide with its geometrical axis, the position of valve *6* depends on both the position of cam *1* and the position of roller *2.* In this case to get fully defined motion of link *6,* it would be necessary to preset forced motion to 2 links (the roller and the cam).

Work Order

Task 1

1.1. Study the mechanism operation and investigate the character of its links motion.

1.2. Draw mechanism diagram and enumerate the links.

The mechanism diagram

1.3. Determine type and kind of kinematic pairs, formed by mechanism links. Mark with letters the rotation centers of turning kinematic pairs. The results should be entered into table (table1.2).

Table 1.2

Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation

1.4. Determine the total number of the 1st and the 2nd kind kinematic pairs. Calculate the number of freedom degrees of mechanism. The results should be entered into table 1.3.

Table 1.3

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Number of freedom degrees by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3n - 2p_1 - p_2 =$

Task 2

1.1. Study the mechanism operation and investigate the character of its links motion.

1.2. Draw mechanism diagram and enumerate the links.

1.3. Determine type and kind of kinematic pairs, formed by mechanism links. Mark with letters the rotation centers of turning kinematic pairs. The results should be entered into table (table1.4).

1.4. Determine the total number of the 1st and the 2nd kind kinematic pairs. Calculate the number of freedom degrees of mechanism. The results should be entered into table 1.5.

The mechanism diagram

Table 1.4

Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation

Table 1.5

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Number of freedom degrees by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3n - 2p_1 - p_2 =$

Task 3

1.1. Study the mechanism operation and investigate the character of its links motion.

1.2. Draw mechanism diagram and enumerate the links.

1.3. Determine type and kind of kinematic pairs, formed by mechanism links. Mark with letters the rotation centers of turning kinematic pairs. The results should be entered into table (table1.6).

1.4. Determine the total number of the 1st and the 2nd kind kinematic pairs. Calculate the number of freedom degrees of mechanism. The results should be entered into table 1.7.

The mechanism diagram

Table 1.6

Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation

Table 1.6.Continuation

Links, forming KP / Type of KP	Schematic representation of KP	Kind / Class	Nature of contact	Center of rotation

Table 1.7

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Number of freedom degrees by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3n - 2p_1 - p_2 =$

Date __________________ Grade ___________________ Signature___________________

Test questions and tasks.

1. Basic definitions: machine, mechanism, link, part, kinematic pair.
2. What is the classification of kinematic pairs according to kinds (according to Dobrovolsky)?
3. What is the classification of kinematic pairs according to classes (according to Artobolevsky)?
4. What is the classification of kinematic pairs according the type of contact (according to Relo)?
5. Schematic representation of kinematic pairs and links of plain mechanisms.
6. Determining the mechanism degrees of freedom.

LABORATORY WORK 2

DETERMINING THE PLAIN MECHANISMS STRUCTURE

The purpose of the work is getting skills in plotting plain mechanisms diagrams and determining their degree of freedom; learning the method of replacing the higher kinematic pairs by the lower ones and plotting structural diagrams; gaining practical skills to determine the structure of plain mechanism with the 1st and the 2nd kind kinematic pairs.

Determining *mechanism structure* is to find out what links groups the given mechanism consists of and the order they are connected.

Only the mechanisms in which the links form the 1st kind kinematic pairs (rotational and translational) can be divided into the Assur's groups. Mechanism with the 2nd kind KP can be divided into structural groups, only after replacing it with the equivalent mechanism with the 1st kind KP. That's why, all the 2^{nd} kind kinematic pairs are replaced by the 1^{st} kind KP. As a result of such a replacement the so called mechanism structural diagram is obtained (replaced mechanism).

Division of mechanism into groups of links is carried out by consequent disconnection of the most remote (from the initial link) Assur's groups. After disconnection of each Assur's group, the remained simpler mechanism will have the same degree of freedom as the initial mechanism.

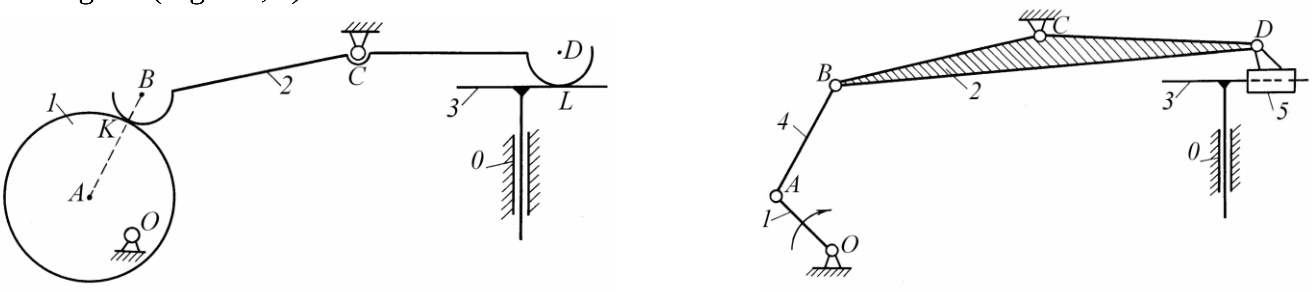
Recording the mechanism structure is carried out by enumerating all groups of links in order of their disconnection from the mechanism.

Let's consider the example how to determine the cam mechanism structure of a diaphragm fuel pump.

The problem should be solved in the following order.

1. Draw the mechanism diagram, enumerate all the mechanism links and mark with letters the centers of rotation of turning kinematic pairs (Fig. 2.1, *a).*

2. Replace the 2^{nd} kind kinematic pairs by the 1^{st} kind pairs and draw mechanism structural diagram (Fig. 2.1, *b).*



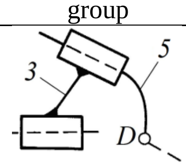
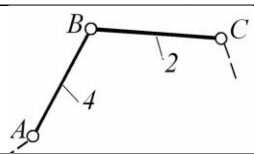
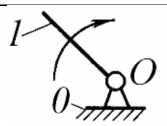
a *b*

Fig. 2.1. Cam mechanism of diaphragm fuel pump: *a* - mechanism diagram; *b*- mechanism structural diagram

Links *1* and *2* form the 2^{nd} kind kinematic pair in the mechanism and contact in *K* point with curvilinear profiles. An additional link *5* is obtained while replacing. It has 2 turning pairs *A* and *B*, which centers of rotation are in curvature centers of curvilinear profiles. Links *2* and *3* also form the 2^{nd} kind kinematic pair and contact in *L* point by rectilinear and curvilinear profiles. While replacing, additional link *5* with turning and sliding kinematic pairs is obtained. Rotational center *D* of turning pair will be in center of curvature of link *2* curvilinear profile. Link *3* rectilinear profile is a guide of the sliding kinematic pair.

Link *1* (cam), which is a crankshaft, is represented by *OA* line on the structural diagram (Fig. 2.1, *b,* table 2.1). Link *2,* being a part of three turning kinematic pairs *B, C* and *D,* which are not on the same straight line, is represented as a hatched triangle.

Table 2.1

Links in order of their disconnection	KP order according to type of motion	The group structure	Graphical representation of the structural group
Links *3* and *6*	- 2 translational and 1 rotational (SST)	Assur's group, II class, 2^{nd} order, dyad # 4	
Links *2* and 5	- 3 rotational (TTT)	Assur's group, II class, 2^{nd} order, dyad # 1	
Links *1* and *4*	-	I class (initial) mechanism	

3. Determine the number freedom degrees of mechanism in accordance with the mechanism structural diagram shown on Fig. 2.1, *a*. The mechanism consists of 6 movable links, so n = 6. Determine also the number of kinematic pairs: p_1 = 7; p_2 = 0. Using Chebyshev's formula, we get:

$W = 3n - 2\ p_1 - 1\ p_2 = 3 \cdot 5 - 2 \cdot 7 - 1 \cdot 0 = 1$. So, the mechanism has 1 initial link (link *1*). Its rotational motion is transformed into translational motion of link *3* (valve). The initial link is marked with an arrow on the diagram.

4. Determine mechanism structure. The Assure's group, consisting of links *3* and *5* and three kinematic pairs (turning *D* and 2 translational) is the most remote from the initial link. This group of links is dyad #5. The next Assur's group consists of links *2* and *4* and three turning pairs *A*, *B* and *C*, and is dyad #1. Links *1* and *4* form the first class (initial) mechanism.

Thus, **the mechanism structure writing down**: I class (*0*, *1*) − II class, dyad 1 (*4*, *2*) − II class, dyad 5 (*3*, *5*).

Work order

Task 1. (Mechanism #1 from the laboratory work #1).

1.1. Draw mechanism diagram and enumerate the links. The results should be entered into table 2.2

The mechanism diagram The mechanism structural diagram

Table 2.2

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Degree of freedom by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3\ n - 2p_1 - p_2 =$

1.2. Divide the mechanism into Assur's groups and initial mechanism. Draw groups of links in order of their disconnection from the mechanism and make up mechanism structure. The results should be entered into table 2.3

Table 2.3

Links in order of their disconnection	KP order according to type of motion	The group structure	Graphical representation of the structural group

The mechanism structure writing down:

Task 2. (Mechanism #2 from the laboratory work #1).

2.1 Draw mechanism diagram and enumerate the links. The results should be entered into table 2.4

2.2. Divide the mechanism into Assur's groups and initial mechanism. Draw groups of links in order of their disconnection from the mechanism and make up mechanism structure. The results should be entered into table 2.5

The mechanism diagram The mechanism structural diagram

Table 2.4

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Degree of freedom by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3\,n - 2p_1 - p_2 =$

Table 2.5

Links in order of their disconnection	KP order according to type of motion	The group structure	Graphical representation of the structural group

The mechanism structure writing down:

Task 3.

3.1 Draw mechanism diagram and enumerate the links. The results should be entered into table 2.6

3.2 Divide the mechanism into Assur's groups and initial mechanism. Draw groups of links in order of their disconnection from the mechanism and make up mechanism structure. The results should be entered into table 2.7

The mechanism diagram The mechanism structural diagram

Table 2.6

Number of links	Number of the 1st kind KP	Number of the 2nd kind KP	Degree of freedom by Chebyshev formula
$n =$	$p_1 =$	$p_2 =$	$W = 3\,n - 2p_1 - p_2 =$

Table 2.7

Links in order of their disconnection	KP order according to type of motion	The group structure	Graphical representation of the structural group

The mechanism structure writing down:

Date ____________________Grade _____________________Signature_____________________

Test questions and tasks

1. The structure of the plain mechanisms. A first class (initial) mechanism. Assur's groups.
2. Two and three arm Assur's groups.
3. Classification of Assur's groups.
4. Synthesis of four link mechanisms.
5. What is the order of replasment of the higher kinematic pairs of plain mechanisms by the lower ones?
6. How is the plain mechanisms structure determined?

LABORATORY WORK 3

KINEMATIC ANALYSES OF THE MECHANISMS

The purpose of the work is gaining practical skills in kinematic analyses of mechanisms

Task: Calculate aircraft air compressor mechanism (Fig. 3.1, table. 3.1).

Initial data: Links dimensions, mm: l_{OA} = mm, l_{AB}= mm, l_{AS2}= mm. Rotational speed: n_1 = rpm. Mechanism position for the analyses –

Work order

1. Plot kinematic diagram of the mechanism (taking into account scale factor μ_l) according to the task. Determine mechanism structure.

Determine degree of freedom of the mechanism: $W = 3n - 2p_1 - p_2 =$

The mechanism structure writing down:

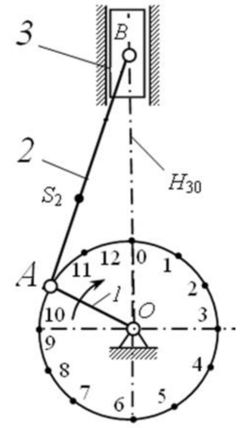


Рис. 3.1. Aaircraft air compressor mechanism diagram

Table.3.1

Data for calculating aircraft air compressor mechanism

Sizes of links, mm: l_{OA} l_{AB} l_{AS2}	 40 150 50
Rotational speed of link *1* n_1, rpm	1200
Mass of links, kg: m_2 m_3	0,50 0,40
Moment of inertia of links I_{S2}, kg·m^2	0,0048
Air force F_3, kN	0,5
Mechanism position for kinematic analysis	1-5, 7-11

Mechanism diagram plotting begins from link *1* using corresponding scale factor in chosen position. On a diagram link *1* is represented by a segment $\overline{OA}$ = mm.

Determine scale factor of the mechanism diagram $\mu_l = \dfrac{l_{OA}}{\overline{OA}}$, where l_{OA} – real length of link *1*, m; $\overline{OA}$ – length of segment *OA* in mm, measured on mechanism diagram, μ_l – scale factor, m/mm.

Then mechanism diagram scale factor $\mu_l = \dfrac{l_{OA}}{\overline{OA}} = \underline{\qquad\qquad} =$ m/mm.

A segment which corresponds to link *2* on mechanism diagram equals:

$$\overline{AB} = \frac{l_{AB}}{\mu_l} = \underline{\qquad\qquad} = \qquad \text{mm.}$$

Determine length of segment $\overline{AS}_2$: $\overline{AS}_2 = \dfrac{l_{AS2}}{\mu_l} = \underline{\qquad\qquad} =$ mm.

2. Plot velocity diagram using scale factor μ_V. Determine velocities of all mechanism points.

Kinematic analyses is carried out starting from 1 class mechanism in order of Assours groups sequence. The crank rotates around the axes which passes through point *O*, its linear velocity $V_O = 0$.

Determine velocity of point *A*, which belongs to initial link (*I* class mechanism):

$$V_A = \omega_1 l_{OA} = \frac{\pi n_1}{30} \cdot l_{OA} = \underline{\qquad\qquad} \cdot \qquad = \qquad \text{m/sec.}$$

Draw the velocity of point *A* on the velocity diagram by a segment $\overline{pa}$ = mm. Then scale factor of velocity diagram: $\mu_V = \dfrac{V_A}{\overline{pa}} = \underline{\qquad\qquad} =$ m·sec/mm.

Arbitrary chose point *p* (pole of velocity diagram) and lay off normally to segment *OA* segment $\overline{pa}$ in direction of crank rotation. It will correspond to velocity of point *A*.

To determine velocities of point *B* (belongs to Assour group of *II* class, dyad # 2) set up vector equation: $\underline{\overline{V_B}} = \underline{\underline{\overline{V_A}}} + \underline{\overline{V_{BA}}}$, where $\overline{V_A} \perp OA$; $\overline{V_{BA}} \perp BA$; $\overline{V_B} \parallel H_{30}$.

Solve this equation by graphical method. For this pass through point *a* on velocity diagram a straight line normal to *AB*, and a line parallel to guide H_{30} through point *p*. At their intersection set point *b*. Segment $\overline{pb}$ corresponds to velocity of point *B* (V_B), segment $\overline{ab}$ – to velocity of point *B* relative to point *A* (V_{BA}). Vector $\overline{V_B}$ is directed from pole *p* to point *b*, vector $\overline{V_{BA}}$ – from point *a* to point *b*.

Numeric values of these velocities are:

$$V_B = \mu_v \cdot \overline{pb} = \qquad\qquad = \qquad \text{m/sec;}$$

$$V_{BA} = \mu_v \cdot \overline{ab} = \qquad\qquad = \qquad \text{m/sec.}$$

To determine velocity of center of crank gravity S_2 use similarity theorem. Lay off segment $\overline{as_2}$ on segment *ab*.

$$\overline{as_2} = \overline{ab}\frac{\overline{AS_2}}{\overline{AB}} = \quad \times \frac{\qquad}{\qquad} = \qquad \text{mm.}$$

Connect point S_2 with pole *p*. Determine velocity of point S_2:

$$V_{S2} = \mu_V \cdot \overline{ps_2} = \qquad \text{m/sec.}$$

3. Determine value and direction of angular velocity of mechanism links.

$$\omega_2 = \frac{V_{BA}}{l_{AB}} = \frac{\qquad}{\qquad} = \qquad \text{sec}^{-1};$$

Directions of angular velocities are determined according to direction of relative velocity of link corresponding point. Determine the direction of angular velocity select link *2*. Point *B* together with link 2 moves around relatively fixed point *A* in the direction of relative velocity $\overline{V_{BA}}$.

4. Plot acceleration diagram using scale factor μ_a. Determine acceleration of all points of the mechanism.

Determine acceleration of point *A*, which rotates along the circle with radius l_{OA}: $\overline{a_A} = \overline{a_A^n} + \overline{a_A^t}$, where a_A^n – normal acceleration, directed along line *AO* from point *A* to the center of rotation – point *O*; a_A^t – tangential acceleration, which is normal to *AO* and directed to the side of link 1 angular acceleration ε_1.

These accelerations are determined as follows:

$$a_A^n = \omega_1^2 \cdot l_{OA} \text{ and } a_A^t = \varepsilon_1 \cdot l_{OA}$$

In our case ω_1= const, angular velocity ε_1 =0, so $a_A^t = \varepsilon_1 \cdot l_{OA} = 0$.

Then acceleration of point *A* equals to: $a_A = a_{A0}^n = \omega_1 \cdot l_{OA} = \left(\frac{\pi n_1}{30}\right)^2 \cdot l_{OA}$

$$a_A = a_{A0}^n = \omega_1 \cdot l_{OA} = \left(\frac{\pi n_1}{30}\right)^2 \cdot l_{OA} = \frac{\qquad}{\qquad} \times \qquad = \qquad \text{m/sec}^2.$$

Arbitrary choose point p' (acceleration diagram pole) and lay off a segment $\overline{p'a'}$ = mm parallel to *OA* directed to point *O*:

$$\mu_a = \frac{a_A}{\overline{p'a'}} = \frac{\qquad}{\qquad} = \qquad \text{m·sec/mm.}$$

To determine acceleration of point *B* (belongs to *II* class Assour group, dyad #2) set up a vector equation: $\overline{a_B} = \underline{\underline{\overline{a_A}}} + \underline{\underline{\overline{a_{BA}}}} = \underline{\underline{\overline{a_A}}} + \underline{\underline{\overline{a_{BA}^n}}} + \underline{\overline{a_{BA}^t}}$,

where $\overline{a_B} \parallel H_{30}$; a_{BA}^n – normal acceleration of point *B* relative to point *A;* $\overline{a_{BA}^n} \parallel BA$; a_{BA}^t – tangential acceleration of point *B* relative to point *A;* $\overline{a_{BA}^t} \perp AB$.

Determine a_{BA}^n by the formula

$$a_{BA}^n = \frac{V_{BA}^2}{l_{AB}} = \frac{\qquad}{\qquad} = \qquad \text{m/sec}^2.$$

Solve the vector equation by graphical method. For this pass a line through point a' which is parallel to *AB*, and lay off on it a vector $\overline{a'b''}$ directed from B to A. It will correspond to normal acceleration a_{BA}^n.

Length of the segment: $\overline{a'b''} = \frac{a_{BA}^n}{\mu_a} = \frac{\qquad}{\qquad} = \qquad$ mm.

Through point b'' pass a straight line normal to *AB* and another line through the pole p' – parallel to the guide H_{30}. At the point of their intersection set point b'. Segment $\overline{p'b'}$ corresponds to the acceleration of point *B* (a_B), segment $\overline{b'b''}$ – corresponds to acceleration a_{BA}^t. Their values are determined as follows:

$$a_B = \mu_a \cdot \overline{p'b'} = \qquad \text{m/sec}^2;$$

$$a_{BA}^t = \mu_a \cdot \overline{b'b''} = \qquad \text{m/sec}^2.$$

Acceleration of point S_2 (a_2) can be determined using the similarity theorem. Position of point S_2' on a velocity diagram cam be determined from the ratio:

$$\overline{a's_2'} = \overline{a'b'}\,\frac{\overline{AS_2}}{\overline{AB}} = \qquad \times \rule{2cm}{0.4pt} = \qquad \text{mm.}$$

On a segment $\overline{a'b'}$ of acceleration diagram from point a' lay off a segment $\overline{a's_2''}$, set point S'$_2$. As on mechanism diagram, it is located between points *a'* and *b'* ($A \to S_2 \to B$ and $a' \to s' \to b'$).

Connect point S_2' with pole p'. Obtained segment $\overline{p's_2'}$, corresponds to acceleration of point S_2'.

$$a_2 = \mu_a \cdot \overline{p's_2'} = \qquad \text{m/sec}^2;$$

5) Determine value and direction of angular acceleration of link #2:

$$\varepsilon_2 = \frac{a_{BA}^t}{l_{AB}} = \rule{2cm}{0.4pt} = \qquad \text{sec}^{-2}.$$

To determine the direction of angular acceleration choose link *2*. Point B moves together with link *2* around relatively fixed point in the direction of tangential acceleration a_{BA}^t.

MECHANISM DIAGRAM (Fig. 3.2)

$\mu_l =$ m/mm.

$\overline{OA} =$ mm;

$\overline{AB} =$ mm;

$\overline{AS}_2 =$ mm.

$\omega_1 =$ rpm

$\mu_V =$ m·sec/mm.

$V_A =$ m/sec; $\overline{pa} =$ mm;

$V_B =$ m/sec; $\overline{pb} =$ mm;

$V_{BA} =$ m/sec; $\overline{ab} =$ mm;

$V_{S2} =$ m/sec; $\overline{ps_2} =$ mm.

$\mu_a =$ m/sec^2 /mm.

$a_A =$ m/sec^2; $\overline{p'a'} =$ mm;

$a_B =$ m/sec^2; $\overline{p'b'} =$ mm;

$a_{BA}^n =$ m/sec^2; $\overline{a'b''} =$ mm;

$a_{BA}^t =$ m/sec^2; $\overline{b'b''}$ mm;

$a_{S2} =$ m/sec^2; $\overline{p's'_2}$ mm.

Date ___________________ Grade ____________________Signature___________________

Test questions and tasks

1. What are main kinematic characteristics of the mechanism?
2. Name tasks and methods of kinematic analysis.
3. Compare the methods, point on their advantages and disadvantages.
4. What is velocity (acceleration) diagram? What are scale factors?
5. Name main properties of velocity and acceleration diagrams.

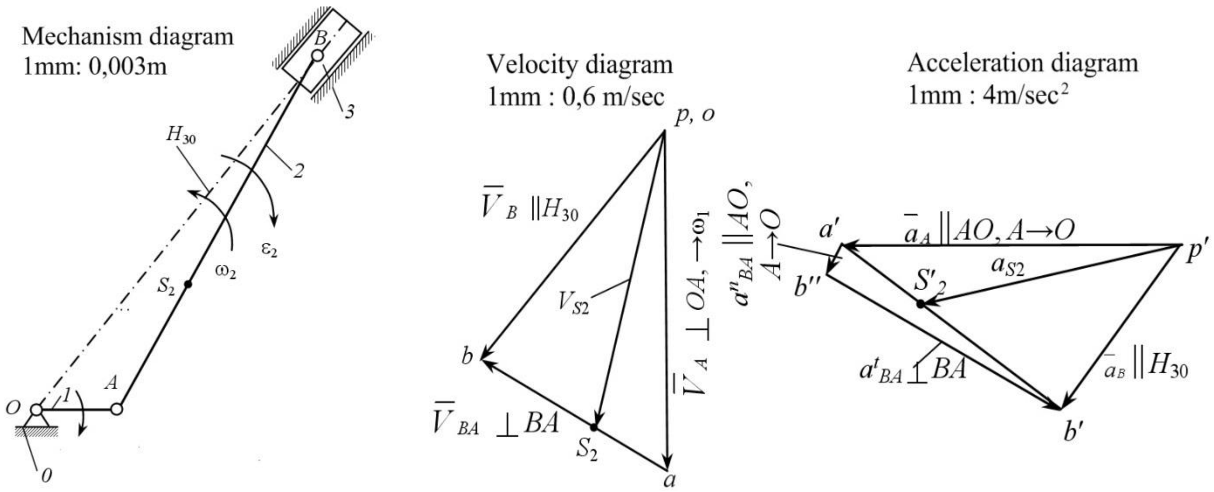


Fig. 3.2. Example of mechanism diagram, velocity and acceleration diagrams

LABORATORY WORK 4

FORCE ANALYSIS OF MECHANISMS

The purpose of the work is gaining practical skills in force analysis of mechanisms

Work order

Data for calculating are taken from the laboratory work «KINEMATIC ANALYSES OF THE MECHANISMS» (table 3.1).

Lenth of link AB, $l_{AB}=$ m;
Acceleration of point B, $a_B =$ m/sec^2;
Acceleration of point S_2, $a_{S2} =$ m/sec^2;
Angular acceleration $\varepsilon_2=$ m/sec^2.
Moment of inertia of con rod: $J_{S2} =$ кг·м^2.
Masses of the mechanism links, $m_2=$ kg; $m_3=$ kg.
Compressed air pressure: $F_3=$ kN.

1. Magnitudes of links forces of inertia applied to corresponding centres of mass.

$$F_{in2} = m_2 \cdot a_{s2} =$$

$$F_{in3} = m_3 \cdot a_B =$$

Forces of inertia are oppositely directed to accelerations of corresponding centres of link mass (Fig. 3.1).

Magnitude of moment of a couple of inertia forces:

$$M_{in2} = J_{s2} \cdot \varepsilon_2 =$$

The direction of the moment of a couple of inertia forces is opposite to the angular acceleration of the corresponding link.

2. *Determination of reacting forces in kinematic pairs of Assur's group formed by links 2 and 3*

Plot Assur's group formed by links 2 and 3 at a given mechanism position taking into account the scale factor of length $\mu\ell = \frac{\text{m}}{\text{mm}}$ and apply all forces acting on the mechanism links (Fig.4.1, *a*).

Link 3 is loaded by four forces such as pressure F_3, inertia force F_{in3}, force R_{03} that acts from the side of the fixed link *0* and force R_{23} that acts from the side of link 2. Link 2 is under the action of one moment of a couple of inertia forces M_{in2} and three forces: inertia force F_{in2}, force R_{12} from the side of link 1 and force R_{32} from the side of link 3. Unknown forces are: reacting force in the sliding kinematic pair R_{03}, forces $\overline{R_{23}} = -\overline{R_{32}}$ in internal turning kinematic pair B and force R_{12} that develops in external turning pair A.

Force R_{03} is perpendicular to guide of motion H_{30}, but the point of application of this force is unknown. That is why we mark the line of action of this force with a dotted line perpendicular to H_{30} that passes at certain distance h_{03} relative to point B (Fig. 4.1, *a*). Arm h_{03} should be found.

Unknown force R_{12} is resolved into two components: normal force R^{n}_{12} that is parallel to AB and tangential force R^{t}_{12} that is perpendicular to AB (Fig. 4.1, *a*). The direction of R^{t}_{12} is chosen arbitrarily.

Consider the state of equilibrium of link 2. As this link is in a state of equilibrium, the sum of moments of all acting forces relative to any point must be equal to zero. Let us set up an equation of moments with respect to point B:

$$-R^{t}_{12}\cdot l_{AB}+F_{\text{in}2}\cdot h_2+M_{\text{i}2}=0.$$

Forces R^{n}_{12} and R_{32} do not form moments because their arms relative to point B are equal to zero. Arm h_2 is determined from the Assur's group diagram (Fig. 4.1, *a*). For that, we should multiply the corresponding segment length in millimeters by scale factor $\mu\ell$.

$R^{t}_{12}=(F_{\text{in}2}\cdot h_2+M_{\text{in}2})/\,l_{AB}=$

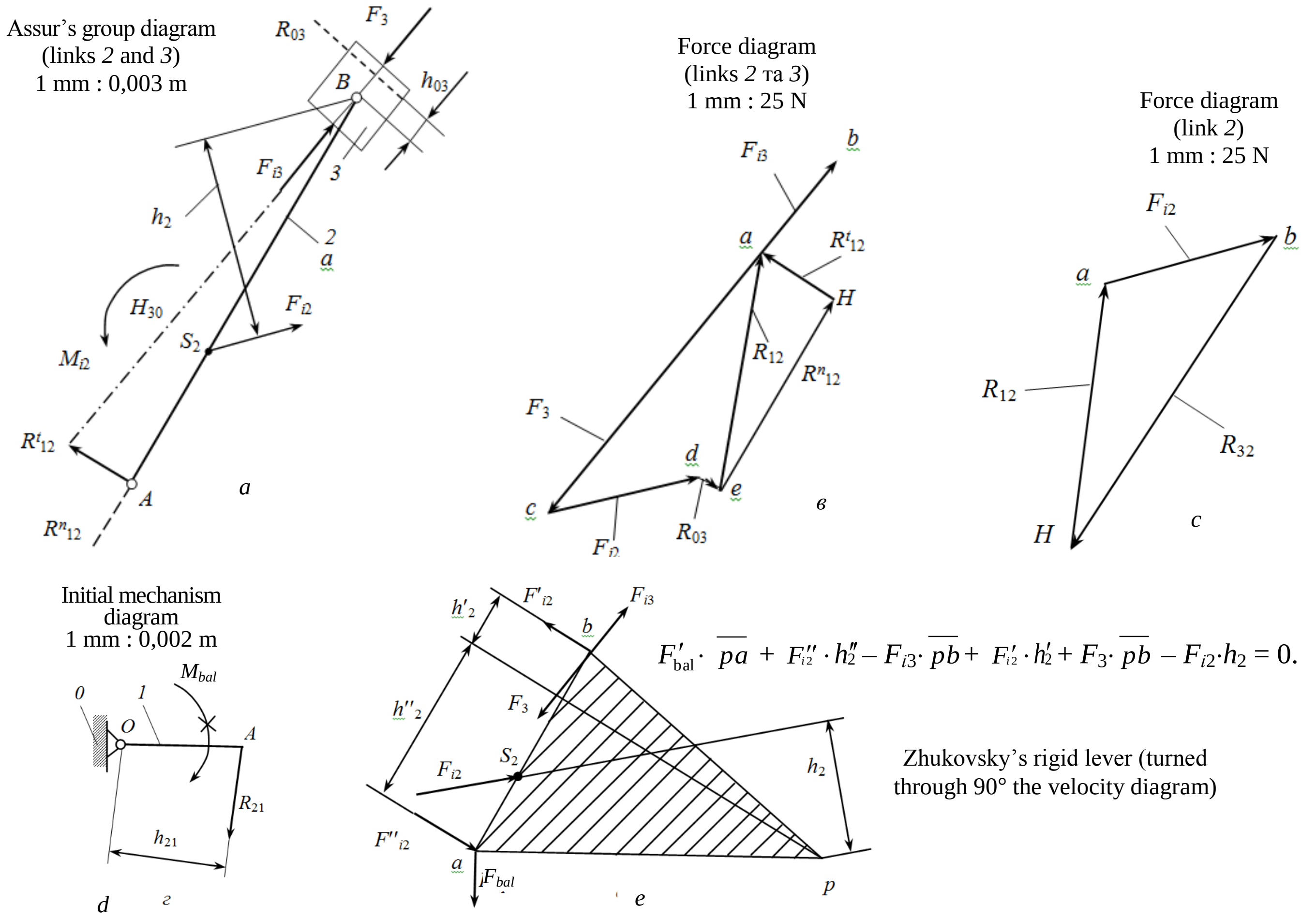


Fig. 4.1. Force analysis

Let us consider equilibrium of the whole of Assur's group. As Assur's group is in a state of equilibrium the vector sum of all forces acting on the group of links must be equal to zero:

$$\underline{\underline{\overline{R^{t}_{12}}}}+\underline{\underline{\overline{F_{in3}}}}+\underline{\underline{\overline{F_{3}}}}+\underline{\underline{\overline{F_{in2}}}}+\underline{\overline{R_{03}}}+\underline{\overline{R^{n}_{12}}}=0.$$

The vector equation has two unknown parameters. We will solve this equation by plotting a force diagram. For that, we arbitrarily choose the scale factor $\mu_F=\dfrac{\text{N}}{\text{mm}}$ of the force diagram and lay off forces R^{t}_{12}, F_{in2}, F_3, F_{in3} in succession marking vector ends with letters *a*, *b*, *c*, and *d* correspondingly (Fig. 4.1, *b*). Through obtained point *d* we draw a line parallel to R_{03} and through pole *H* we pass a line parallel to R^{n}_{12}. The point of intersection of these lines is marked as *e*. Segments $\overline{de}$, $\overline{aH}$ and $\overline{ea}$ represent corresponding forces R_{03}, R^{n}_{12}, R_{12} in scale. To determine the magnitudes of these forces, we should multiply corresponding segments by the scale factor. The direction of these forces is determined according to the rule of vector composition. As a result we obtain

$R_{03} = \mu_F \cdot \overline{de} =$

$R^{n}_{12} = \mu_F \cdot \overline{eH} =$

$R_{12} = \mu_F \cdot \overline{ea} =$

Consider the state of equilibrium of either link 2 or link 3 and set up a vector equation of forces, acting on the link ($\sum \overline{F_i} = 0$). For example, let us consider the state of equilibrium of link 2:

$$\underline{\underline{\overline{R_{12}}}} + \underline{\underline{\overline{F_{in2}}}} + \underline{\overline{R_{32}}} = 0.$$

This vector equation has two unknowns (the direction and the magnitude of force R_{32}). We solve this equation by plotting a force diagram. Taking into account the scale factor we lay off R_{12}, F_{in2} in succession marking corresponding vectors ends with letters *a* and *b*. After connecting point *b* with pole *H* we obtain segment $\overline{bH}$ that represents force R_{32} (Fig. 4.1, *c*).

$R_{32} = \mu_F \cdot \overline{bH} =$

Consider the state of equilibrium of link 3. The sum of moments of all acting forces with respect to point *B* must be equal to zero

$$-R_{03} \cdot h_{03} = 0.$$

Moments of forces R_{23}, F_3 and F_{in3} are equal to zero because their arms relative to point B are equal to zero, too.

As force $R_{03} \neq 0$ consequently arm $h_{03} = 0$.

Force analysis of the initial mechanism

Draw the initial mechanism (links *1* and *0*) at the given position taking into account the scale factor $\mu\ell = \dfrac{\text{m}}{\text{mm}}$ and apply all forces that act on crank *1* (Fig.4.1, d).

The crank is under the action of one balancing moment M_{bal} that balances the action of all forces applied to the mechanism links and two forces: R_{21} from the side of link 2 and R_{01} from the side of the fixed link 0

Let us consider equilibrium of crank 1. The vector sum of all forces acting on this link should be equal to zero:

$$\underline{\underline{\overline{R_{21}}}} + \overline{R_{01}} = 0. \quad \text{Consequently } \overline{R_{01}} = -\overline{R_{21}}.$$

Determine balancing moment M_{bal}. For that, we set up an equation of moments of all forces relative to point O.

Moment of force R_{01} is equal to zero due to the fact that its arm with respect to point O is zero. Arm h_{21} = mm, is determined from the diagram of the initial mechanism. For that, we multiply the corresponding segments in millimeters by $\mu\ell$ (Fig.4.1, d).

$$R_{21} \cdot h_{21} \cdot \mu_l - M_{\text{bal}} = 0.$$

$M_{\text{bal}} = R_{21} \cdot h_{21} \cdot \mu_l =$ N·m.

Determination of the balancing moment by Zhukovsky's method

For the given mechanism position we plot the velocity diagram turned through 90° to an arbitrary scale. After that we will transfer all external forces that act on the mechanism links from the mechanism diagram to the corresponding points of the velocity diagram (Fig. 4.1, e).

The known moment M_{in2} is represented by a couple of forces F'_{in2} and F''_{in2} that are applied at points A and B perpendicular to AB (Fig. 4.1, e). The magnitude of these forces is determined by the formula

$F'_{in2} = F''_{in2} = M_{in2} / l_{AB} =$

Set up an equation of moments of all forces with respect to the pole of the velocity diagram and determine F_{bal} and M_{bal}.

Arms of all forces are substituted in the equation in millimeters. The lengths of these arms are determined from Fig. 4.1, e. As a result we obtain:

$$F_{bal} \cdot \overline{pa} \qquad F''_{in2} \cdot h''_2 \qquad F_{in3} \cdot \overline{pb} \qquad F'_{in2} \cdot h'_2 \qquad F_3 \cdot \overline{pb} \qquad F_{in2} \cdot h_2 = 0.$$

$M_{bal} = F_{bal} \cdot l_{OA} =$ N·m.

Determine the difference between the balancing moments obtained by the two methods (combined static and inertia force analysis and Zhukovsky’s method):

The error must be not more than 5 %.

$\Delta M_{bal} =$

Date __________________Grade __________________Signature___

Test questions and tasks

1. Tasks and methods of force analysis
2. What forces act in mechanisms and machines?
3. Determination of links forces of inertia and riacting forces.
4. What is the order of determining the balancing moment by Zhukovsky’s method?

LABORATORY WORK 5
BALANCING ROTATING LINKS

The purpose of the work is gaining theoretical and practical knowledge of balancing mechanisms and machines; familiarizing with facilities for static and dynamic balancing; mustering methods of rotating links (rotors) balancing.

Work Order

1. Familiarize with the laboratory installation (Fig. 5.1). Perform the laboratory work installation of TMMIM type is used for dynamic balance consisting of rotor 1, installed in rolling bearings 2, permanently connected with pendulum frame 4. The pendulum frame is connected to the installation bed plate and can swing in the vertical plane around the horizontal axis O.

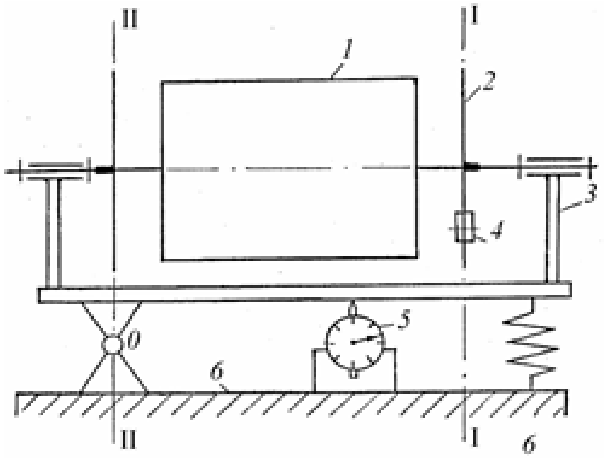


Fig. 5.1. Installation diagram

Balance link (rotor) 1 is accelerated by the electric motor with a friction disk to higher than resonant rotational speeds. After that the drive is turned off and the balanced link begins to run out. With good rolling bearings the rotational speed of balanced link decreases slowly. Therefore the balanced link rotates quite long with the rotational speeds close to resonant ones, and the amplitude of oscillations of the pendulum frame is able to reach its greatest value. The amplitude of oscillations of the pendulum frame is measured by the indicator 5 of hour type. As follows from conditions of dynamic and complete balancing of the rotatory link, it can be balanced completely or dynamically by two counterweights located in two planes, chosen out of constructive reasons.

In the places of location of the chosen planes of balancing (correction) I-I and II-II on the shaft of rotor 1 two disks with slots are mounted, in which additional weights *4* can be mounted at various distances from the axis of rotation. At counterbalance of the rotating link it is mounted on the pendulum frame so that balancing plane II-II passed through the axis *O* of the frame rolling.

2. Selecting control weight G_k and calculating Δ_{sk} disbalance:

G_k = sN; $2 \cdot G_k$ = sN;

r_k = sm; $\Delta_{sk} = G_k \cdot r_k$ = sNsm

3. Determining oscillations amplitude of pendulum frame. The results should be entered into table 5.1.

Table 5.1

The experiment results

Measurement number	Amplitude of pendulum frame oscillations, mm		
	A_1	A_2	A_3
1			
2			
3			
Mean amplitude, mm			

4. Determining oscillations amplitude A_k, caused only by disbalance Δ_{sk} of control weight G_K:

Draw the oscillation amplitude of the frame mark two segments on a straight line according to the selected scale $\overline{ab} = \overline{bc} = \overline{A}_2$ (Fig. 5.2). Having drawn an arc with radius A_3 from point *a* and an arc with radius A_1 from point *c* we will receive point *d* at the intersection of these arcs. The segment $\overline{bd}$ is the amplitude A_k in the selected scale.

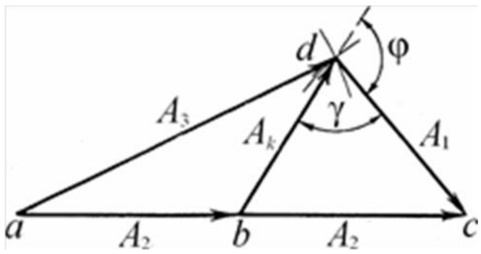


Fig. 5.2. Determine the oscillations amplitude of pendulum frame A_k, caused by disbalance Δ_{sk} of control weight G_K only

a) by an graphical method: A_k= mm

b) by an analytical method: $A_k = \sqrt{\frac{A_1^2+A_3^2-2A_2^2}{2}} =$ mm

5. Calculating angle γ between the oscillations amplitudes A_1 and A_K radius vectors:

a) by an graphical method: $\gamma =$ ^{o}

b) by an analytical method: $\gamma =$ ^{o}

$$cos\gamma = \frac{A_1^2+A_k^2-A_2^2}{2A_1A_k} = \quad \text{mm}$$

6. Determining the coefficient of proportionality μ and calculating the rotor disbalance Δ_{S1} without control weight:

$$\mu = \frac{A_k}{\Delta_{sK}} = \quad \frac{\text{mm}}{\text{sNsm}}; \qquad \Delta_{s1} = \frac{A_1}{\mu} = \quad \text{sNsm}$$

7. Calculating counterweight radius r_{cw}:

G_{cw} = sN; $r_{cw1} = \frac{\Delta_{s1}}{G_{cw1}} =$ sm; G_{cw2} = sN; $r_{cw2} = \frac{\Delta_{s1}}{G_{cw2}} =$ sm.

8. Rotor balancing results:

a) Determining the residual amplitude of pendulum frame oscillations (A'). The results should be entered into table 5.2.

Table 5.2

Results of experiment

Number of measurement	1	2	3	Mean amplitude value
Amplitude of oscillations A', mm				

b) Determine the amplitude of residual oscillations:

$$\Delta'_s = \frac{A'}{\mu} = \qquad sNsm \qquad \delta = \frac{\Delta'_s}{\Delta_{s1}} \cdot 100\% =$$

Date ___________________Grade ___________________Signature___________________

Test questions and tasks

1. What is the purpose of rotating links balancing?
2. Static and inertia moments.
3. Types of disbalance, depending on mutual disposition of rotational axis and principal central axis of inertia.
4. What is the difference between static balancing and dynamic balancing?
5. Determination of counterweights mass and their radii of installation by a graphical method.

LABORATORY WORK 6
DETERMINATION OF TOOTHED WHEEL MAIN PARAMETERS

The purpose of the work is summering up knowledge on toothed wheels geometry; familiarizing with the simplest methods of toothed wheel module determination; gaining practical skills in determining main geometrical parameters of straight spur gear.

Work Order

1. Count the number of gear teeth $z =$

Choose the number of teeth being measured according to table 6.1 $n =$

Table 6.1

z	12–18	19–27	28–36	37–45	46–54	55–63
n	2	3	4	5	6	7

2. Use caliper to measure distances L_n and L_{n+1} (Fig. 6.1). The results should be entered into table 3.2.

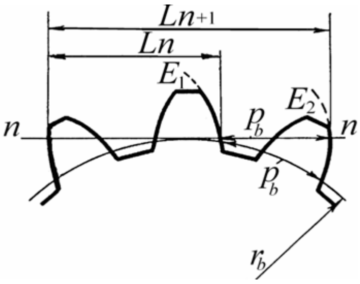


Fig. 6.1 Toothed wheel measurement diagram

Table 6.2

The measurement number	The segment length, mm	
	L_n	L_{n+1}
1		
2		
3		
Mean value of segment length, mm		

3. Determine toothed wheel pitch $p_в$ measured on the base circle.

$$p_в = L_{n+1} - L_n =$$

4. Calculate the module of toothed wheel, measured round the nominal pitch circle and round out the result to the closest standard value. Pressure angle α = 20º (for standard gears) cos 20º = 0,94.

Module of the gear measured around the nominal pitch circle:

$$m = \frac{p_b}{\pi \cdot cos\alpha} = \frac{}{2.953} = \qquad \text{mm.}$$

Value of module according to ГОСТ 9563-60 $m =$ mm.

5. Calculate the nominal pitch circle diameter $d = mz =$ mm.

6. Using a caliper to measure dedendum d_f and addendum d_a circles diameters. The results should be entered into table 6.3.

Table 6.3

Number of measurement	Segment length, mm	
	d_a	d_f
1		
2		
3		
Mean value of diameter, mm		

7. Determine the base circle diameter $d_b = mz\cos\alpha =$
   pitch measured on the pitch circle $p = \pi m =$
   space width $S = p/2 = \pi m/2 =$
   tooth thickness $e = p/2 = \pi m/2 =$
   angular pitch $\tau = 360/z =$

8. Determine the height of tooth addendum and dedendum parts
   height of tooth addendum part $h_a = (d_a - d)/2 =$
   height of tooth dedendum part $h_f = (d - d_f)/2 =$

9. Determine the coefficient of tooth addendum h^*_a and dedendum h^*_f. parts height (for standard gears h^*_a = 1,0; h^*_f = 1,25). If results of calculations differ greatly from the given values, gear is modified (manufactured with an altitude or angular correction) or has stub teeth.
   coefficient of tooth addendum part height $h^*_a = h_a/m =$
   coefficient of tooth dedendum part height $h^*_f = h_f/m =$

10. Determine the measurement error:

$$\varepsilon_a = \left|\frac{h^*_a - 1}{1}\right| \cdot 100\% \le 5\% \qquad \varepsilon_a =$$

$$\varepsilon_f = \left|\frac{h^*_f - 1.25}{1.25}\right| \cdot 100\% \le 5\% \qquad \varepsilon_f =$$

11. Draw on Fig. 6.2 the calculated basic toothed wheel geometric parameters.
12. Mark on Fig. 6.2 the pitch circle and the magnitude.

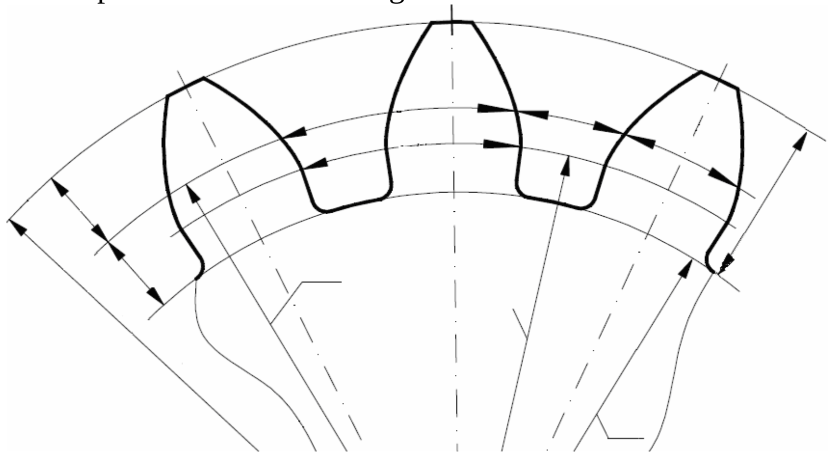


Fig. 6.2. Geometrical parameters of straight spur gear

Date _________________ Grade _________________ Signature _________________

Test questions and tasks

1. Tooth involute profile. Involute of circle and its equation.
2. Why are involute curves used in gears?

3. Involute properties.
4. What are geometrical parameters of standard involute straight spur gear?
5. Formulas for determination of geometrical parameters of toothed wheels.
6. Base circle, pitch and nominal pitch circle of toothed wheel.
7. What are parameters of the involute gearing?

LABORATORY WORK 7

CONSTRUCTING TOOTH INVOLUTE PROFILE OF TOOTHED WHEELS BY THE GENERATING METHOD

The purpose of the work is familiarizing with the principle of tooth involute profile forming while producing toothed wheels by generating method; determination of the influence of tool position while cutting the gear on its geometrical parameters; summing up knowledge of modified toothed wheels parameters determination.

Work Order

1. Get aquainted with the installation operation (Fig. 7.1) and its initial parameters.

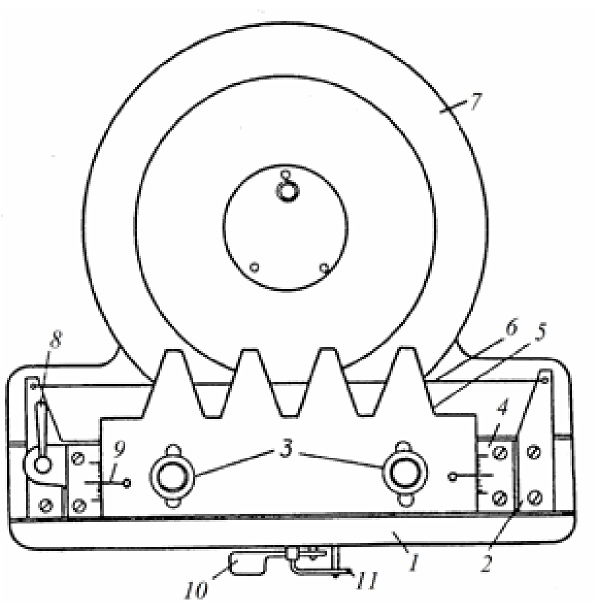


Fig. 7.1. Toothed wheel profile forming installation diagram

1 – fixed base
2 – carriage
3 – fixing fasteners
4 – scales
5 – toothed rack
6 – flexible wire
7 – rotating disk
8 – lever
9 – mark on a rack against the scale zero grade
10 – lever
11 – handle

The description of laboratory - scale plant. The plant consists of stationary base 1, translated carriage 2 and turning disk 7 (рис. 7.1). A paper circle simulating a blank of a toothed wheel being cut is set onto the disk. Rack cutter 5 is fixed on carriage 2 by screws 3, that can be moved towards and away from the disk axis. Rack cutter offsetting is measured by scale 4. The carriage and disk are connected by flexible wire 6. The wire covers disk round the circle, the diameter of which is equal to the nominal pitch circle diameter of the toothed wheel being cut. This connection allows to reproduce the generating motion: at linear motion of the rack the disk rotates in such a way as if in engagement with the rack there was an available toothed wheel.

For profiling wheel teeth it is necessary:

1) to turn counterclockwise handle 11 of the carriage free course and lever 8 of the carriage coupling with the disk against stop;

2) to set the carriage with rack cutter to the extreme right position;

3) to turn clockwise handle 11 and lever 8 against stop;

4) to set a paper circle on disk 7;

5) to set the rack in the required position with the help of screws 3. If hairline 9, marked on the rack, is against the zero division of scale 4, a wheel will be cut without offsetting;

6) to draw in pencil round the profiles of all teeth and spaces of the rack on the paper circle;

7) to press lever 10 down against stop (then the rack will move a little to the left, and the disk

will turn by 1-2°);

8) to draw round the profiles of all teeth and rack spaces with a pencil, and then to press again lever 10; to repeat these two operations until the carriage with the rack will move to the extreme left position.

2. Determine installation initial parameters for cutting a gear.

Module	$m =$
Nominal pitch circle diameter	$d =$
Pressure angle	$\alpha = 20^{\circ}$ ($\cos\alpha = 0{,}94$, $\mathrm{tg}\,\alpha = 0{,}36$)
Coefficient of tooth addendum	$h^{*}_{\alpha} = 1$.

3. Calculate main parameters of standard wheel (without offset). The results should be entered into table 7.1.

Table 7.1

Toothed wheels parameters

Parameter	Standard gear	Modified gear
Number of teeth	$z = d/m$	$z = d/m$
Offset factor	–	$x = (17 - z)/17$
	–	
Rack offset, mm	–	$в = x \cdot m$
	–	
Nominal pitch circle diameter, mm	$d = m \cdot z$	$d = m \cdot z$
Base circle diameter, mm	$d_в = m \cdot z \cdot \cos\alpha$	$d_в = m \cdot z \cdot \cos\alpha$
Addendum circle diameter, mm	$d_a = m \cdot (z + 2)$	$d_a = m \cdot (z + 2 + \cdot 2x + \Delta y)$
Dedendum circle diameter, mm	$d_f = m \cdot (z - 2{,}5)$	$d_f = m(z - 2{,}5 + 2x)$
Pitch measured round the nominal pitch circle, mm	$p = \pi m$	$p = \pi m$
Pitch measured round the base circle, mm	$p_b = \pi m \cos\alpha$	$p_b = \pi m \cos\alpha$
Tooth thickness measured round the nominal pitch circle, mm	$s = \pi m/2$	$s = m \cdot (\pi/2 + 2 \cdot x \cdot \mathrm{tg}\,\alpha)$
Space width measured round the nominal pitch circle, mm	$e = \pi m/2$	$e = m \cdot (\pi/2 - 2 \cdot x \cdot \mathrm{tg}\,\alpha)$

4. Calculate main parameters of a positive offset toothed wheel. The results should be entered into table 7.1.

5. Remove paper circle off the disc and outline the circles d, $d_в$, d_a, d_f using compass. Use a ruler to measure parameters of standard and positive offset wheels and compare them (see the example Fig. 7.2). The results should be entered into table 7.2.

Table 7.2

Results of measurements of profiled toothed wheels

Parameter	Standard toothed wheel	Modified toothed wheel
Tooth thickness measured round the nominal pitch circle, mm	$s =$	$s =$
Space width measured round the nominal pitch circle, mm	$e =$	$e =$
Pitch measured round the nominal pitch circle, mm	$p =$	$p =$
Tooth thickness measured round the base circle, mm	$s_в =$	$s_в =$
Pitch measured round the base circle, mm	$p_в =$	$p_в =$
Tooth thickness measured round the addendum circle, mm	$s_a =$	$s_a =$

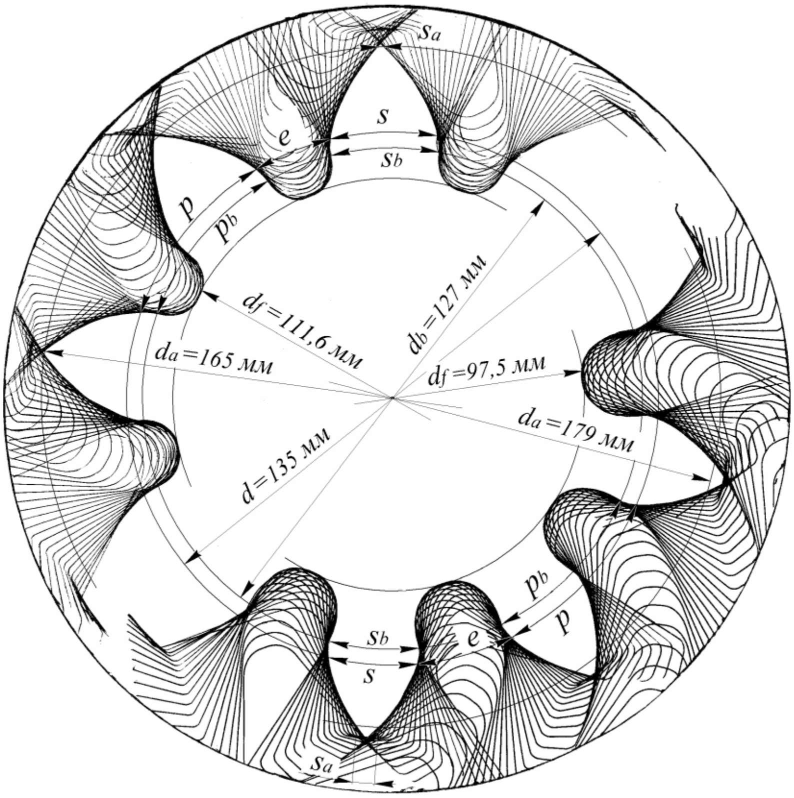


Fig. 7.2. Example of teeth profiling of standard and modified toothed wheel

Date ___________________Grade ____________________Signature____________________

Test questions and tasks

1. Methods of gear cutting.
2. Which types of modified gears are used in gear trains?
3. The height correction and its properties.
4. The angular correction and its properties.
5. What is the phenomenon of tooth undercutting and conditions of its appearance?
6. Usage of modified gears.

LABORATORY WORK 8

DETERMINATION OF VELOCITY RATIOS OF GEARINGS

The purpose of the work is gaining practical skills for plotting gear mechanisms diagrams; acquiring knowledge to determin gearings velocity ratios; gaining skills in kinematic analysis of gearings by experimental method

Toothed wheels of gearings in common generally rotate with different angular velocities ω_1 and ω_2. The *velocity ratio* of the gearing is the ratio of angular velocities of toothed wheels. The velocity ratio is designated by the letter u supplied by the appropriate indices (numbers of links):

$$u_{12} = \frac{\omega_1}{\omega_2} \text{ and } u_{21} = \frac{\omega_2}{\omega_1}.$$

Thus, the magnitudes represent the velocity ratios of the same transmission, but only in the first case magnitude of u_{12} represents the velocity ratio from toothed wheel *1* to toothed wheel *2*, and magnitude u_{21} represents the velocity ratio from toothed wheel *2* to toothed wheel *1*. The velocity ratio may be negative $u_{12} < 0$, if the toothed wheels rotate in the different directions. In this case of engagement we deal with gearing with *external toothing*. If both wheels rotate in the same direction, the velocity ratio is considered positive $u_{12} > 0$ and we obtain gearing with *internal toothing*.

The smaller toothed wheel of the gear train is called *a pinion*, and bigger - *a gear*. Gear train with one degree of freedom is usually named a *gearing*.

Work order

Task 1

1.1. Get acquainted with the mechanism operation. Analyze type of its links motion and determine the mechanism type.

1.2. Plot the mechanism diagram, enumerate the links and mark toothed wheels. Count the number of teeth of toothed wheels.

1.3. Determine velocity ratio of the mechanism. The results should be entered into table 8.1.

Table 8.1

Mechanism diagram	Number of teeth	Velocity ratio of the mechanism	Velocity ratio of the mechanism determined experimentally
	$z_1 =$ $z_2 =$	$u_{12} = \pm\frac{z_2}{z_1} =$	$u_{12} = \frac{n_1}{n_2^*} =$ $n_2^* = 1$

Task 2

2.1. Get acquainted with the mechanism operation. Analyze type of its links motion and determine the mechanism type.

2.2. Plot the mechanism diagram, enumerate the links and mark toothed wheels. Count the number of teeth of toothed wheels.

2.3. Determine velocity ratio of the mechanism. The results should be entered into table 8.2.

Task 3

3.1 Get acquainted with the mechanism operation. Analyze type of its links motion and determine the mechanism type.

3.2 Plot the mechanism diagram, enumerate the links and mark toothed wheels. Count the number of teeth of toothed wheels.

3.3 Determine velocity ratio of the mechanism. The results should be entered into table 8.3.

Table 8.2

Mechanism diagram	Number of teeth	Velocity ratio of the mechanism	Velocity ratio of the mechanism determined experimentally
	$z_1 =$ $z_2 =$	$u_{12} = \pm\frac{z_2}{z_1} =$	$u_{12} = \frac{n_1}{n_2^*} =$ $n_2^* = 1$

Table 8.3

Mechanism diagram	Number of teeth	Velocity ratio of the mechanism	Velocity ratio of the mechanism determined experimentally
	$z_1 =$ $z_2 =$	$u_{12} = \pm\frac{z_2}{z_1} =$	$u_{12} = \frac{n_1}{n_2^*} =$ $n_2^* = 1$

Task 4

4.1. Plot the double-stage gearing mechanism diagram by combining diagrams from Task 1, 2 and 3.

4.2. Enumerate the links and mark toothed wheels.

4.3. Determine velocity ratio of the mechanism. The results should be entered into table 8.4.

Mechanism diagram	Number of teeth	Velocity ratio of the mechanism
	$z_1 =$ $z_2 =$ $z'_2 =$ $z_3 =$	u_{13}=

Date ____________________Grade ____________________Signature____________________

Test questions and tasks

1. What is the classification of gearings?
2. The velocity ratio of the gearing. What is the defference between speed reducer and multiplier?
3. Main types of complex gearings.
4. Multistage gearings. Gearings with intermediate wheels.

LABORATORY WORK 9

DETERMINATION OF VELOCITY RATIOS OF PLANETARY GEARINGS

The purpose of the work is getting practical skills in planetary gearings diagrams plotting; summing up knowledge on determination of planetary gearings velocity ratios; gaining skills in kinematic analysis of planetary gearings by experimental method

Fig.9.1 shows four diagrams of planetary gearings that have found the widest application in the mechanical engineering.

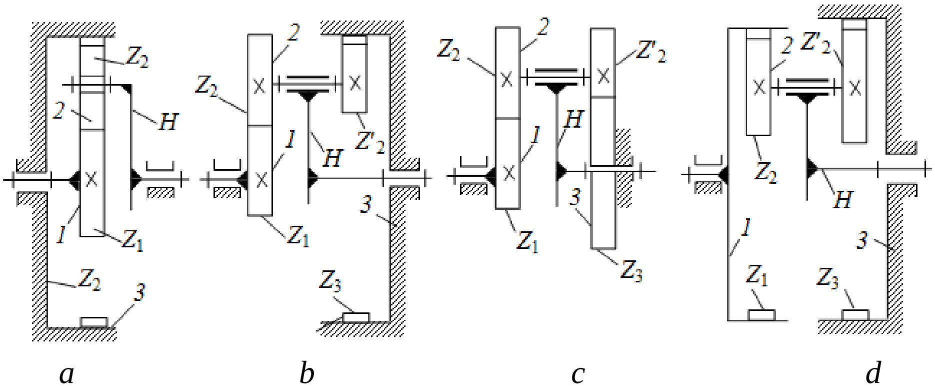


Fig. 9.1. Typical diagrams of planetary gearing:
a — single planet pinion; *b* — planet pinions with external and internal toothing; *c* — double planet pinions with external toothing; *d* — double planet pinions with internal toothing

Every of planetary gearings has one degree of freedom and consists of four links. Toothed wheels z_1 and z_3 whose geometrical axes coincide with the main axis of the mechanism are called *sun gears*. Toothed wheel z_2 having a movable axis is named a *planet pinion*. The shaft of the planet pinion rotates in the bearing, which is mounted on link 4, named a *driver* or *carrier*, and together with this link, rotates around the main axis of the planetary gear train. The planetary gearing may contain single (Fig.9.1, *a*) or double planet pinions (Fig.9.1, *b*, *c*, *d*).
In order to carry out kinematic analysis of the planetary gear train we should use the method of reversed motion. According to this method it is necessary to add the rotational speed of the driver *H* with opposite sign to rotational speeds of all mechanism links. In this case rotational speeds of links *1, 2, 3* and *H* of planetary gearing (Fig. 9.1) are correspondingly $(n_1 - n_H)$, $(n_2 - n_H)$, $(n_3 - n_H) = -n_H$, $(n_H - n_H) = 0$. As a result, driver 4 becomes immovable and a planetary gear train is transformed into ordinary gear train with fixed axes of toothed wheels (reversed mechanism) (Fig.9.2).

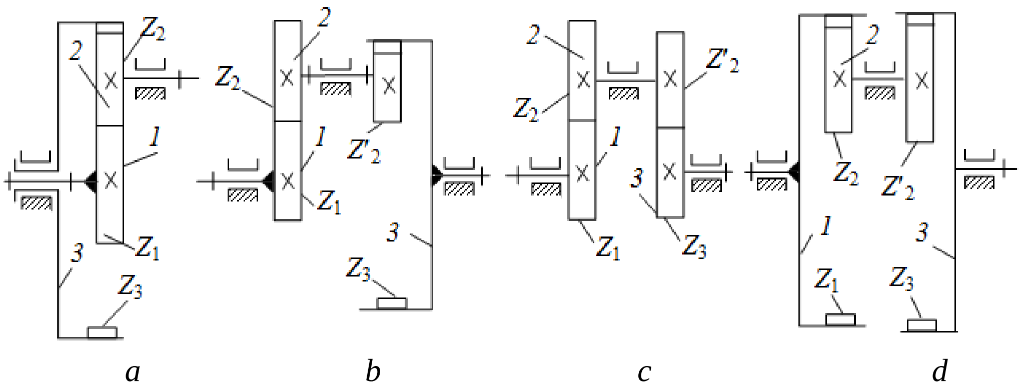


Fig. 9.2. Diagrams of reversed mechanisms:
a — single planet pinion; *b* — planet pinions with external and internal toothing; *c* — double planet pinions with external toothing; *d* — double planet pinions with internal toothing

The velocity ratio of the reversed mechanism from link *1* to link *3* is designated as $u_{13}^{(H)}$:

$$u_{13}^{(H)} = \frac{n_1 - n_H}{n_3 - n_H} = \frac{n_1 - n_H}{-n_H} = -\frac{n_1}{n_H} + 1 = -u_{1H} + 1, \text{ then } u_{1H} = 1 - u_{13}^{H}.$$

The obtained formula allows to determine the velocity ratio of the planetary gearing (Fig. 9.1). For this purpose it is necessary to reverse a mechanism and to find its velocity ratio.

The velocity ratio of reversed mechanism $u_{13}^{(H)}$ is determined through the known numbers of wheel teeth and is substituted in the formula with its sign. In general, the formula is $u_{13}^{H} = u_{12}^{H} u_{23}^{H}$. To determine the velocity ratios of typical planetary gearings by the given formula, we will compile a table.

Formulas for determining velocity ratios of typical planetary gearings

Velocity ratio	Type *a*	Type *b*	Type *c* and *d*
$u_{13}^{(H)}$	$-\dfrac{z_3}{z_1}$	$-\dfrac{z_2 z_3}{z_1 z_2'}$	$\dfrac{z_2 z_3}{z_1 z_2'}$
$u_{31}^{(H)}$	$-\dfrac{z_1}{z_3}$	$-\dfrac{z_1 z'}{z_2 z_3}$	$\dfrac{z_1 z_2'}{z_2 z_3}$
u_{1H}	$1+\dfrac{z_3}{z_1}$	$1+\dfrac{z_2 z_3}{z_1 z_2'}$	$1-\dfrac{z_2 z_3}{z_1 z_2'}$
u_{H1}	$\dfrac{1}{1+\dfrac{z_3}{z_1}}$	$\dfrac{1}{1+\dfrac{z_2 z_3}{z_1 z_2'}}$	$\dfrac{1}{1-\dfrac{z_2 z_3}{z_1 z_2'}}$

Work order

Task 1

1.3. Get acquainted with the mechanism operation. Analyze type of its links motion and determine the mechanism type.

1.4. Plot the mechanism diagram, enumerate the links and mark toothed wheels. Count the number of teeth of toothed wheels.

1.3. Determine velocity ratio of the planetary mechanism. The results should be entered into table 9.1.

The planetary gearing diagram — The reversed mechanism diagram

Table 9.1

Toothed wheels numbers and the number of gears teeth	Velocity ratio of reversed mechanism	Velocity ratio of planetary gearing	Velocity ratio of planetary gearing determined experimentally

Task 2

2.1 Get acquainted with the mechanism operation. To analyze type of its links motion and determine the mechanism type.

2.2 Plot the mechanism diagram, enumerate the links and mark toothed wheels. Count the number of teeth of toothed wheels.

2.3 Determine velocity ratio of the planetary mechanism. The results should be entered into table 9.2.

The planetary gearing diagram | The reversed mechanism diagram

Table 9.2

Toothed wheels numbers and the number of gears teeth	Velocity ratio of reversed mechanism	Velocity ratio of planetary gearing	Velocity ratio of planetary gearing determined experimentally

Date __________________Grade ___________________Signature___________________

Test questions and tasks

1. What is the planetary gearing?
2. What is the method of reversing movement?
3. What is the reversed planetary mechanism?
4. Determine the planetary gearing velocity ratio.

LABORATORY WORK 10

INVESTIGATION OF PLAIN CAM MECHANISM

The purpose of the work is developing skills in plotting diagrams of a plain cam mechanism; gaining practical skills in plotting diagrams of cam mechanism follower path; mastering the method of plotting the cam profile according to the predetermined diagram of the follower path.

Work Order

1. Familiarize with the laboratory installation (Fig. 10.1), Determine the motion type of mechanism links and to determine its type.
2. Determine principal mechanism dimensions, which are necessary for plotting a diagram of the follower path and cam profiling.

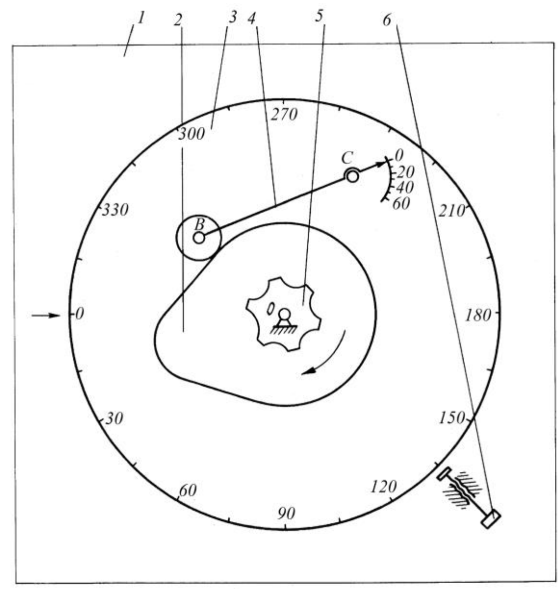


Fig. 6.1. Laboratory installation:
1 – baseplate; *2* – cam; *3* – movable disk with axis, co-axial with the cam (point O);
4 – follower; *5, 6* – back set

General geometrical dimensions

Eccentricity $e =$

The minimal coordinate of the follower roller center

$x_0 =$

The minimal angle of the rotating follower deflection

$\psi_0 =$

The roller radius r = 12,5 mm

The length of rotating follower $\ell =$

Center distance (for mechanisms with a rotative follower)

$L =$

3. Perform mechanism reversing and determine the follower displacement at different angles of cam rotation. The results should be entered into table 10.1.

4. Plot the diagram of the follower path (Fig. 10.2) and determine the values of phase angles of cam rotation. (To begin plotting with marking x_0 or ψ_0 moving along X axis)

5. Ddetermine phase angles of cam rotation according to the diagram of the follower path and to draw the cam profile.

Table 10.1

Angle of cam rotation φ, deg.			0	10	20	30	40	50	60	70	80	90
Displacement of a follower Δx, mm or $\Delta\psi$, deg.												
100	110	120	130	140	150	160	170	180	190	200	210	220
230	240	250	260	270	280	290	300	310	320	330	340	350

Phase angles of cam rotation

1. Angle of departure $\varphi_1 =$
2. Upper position angle $\varphi_2 =$
3. Approach angle $\varphi_3 =$
4. Lower position angle $\varphi_4 =$

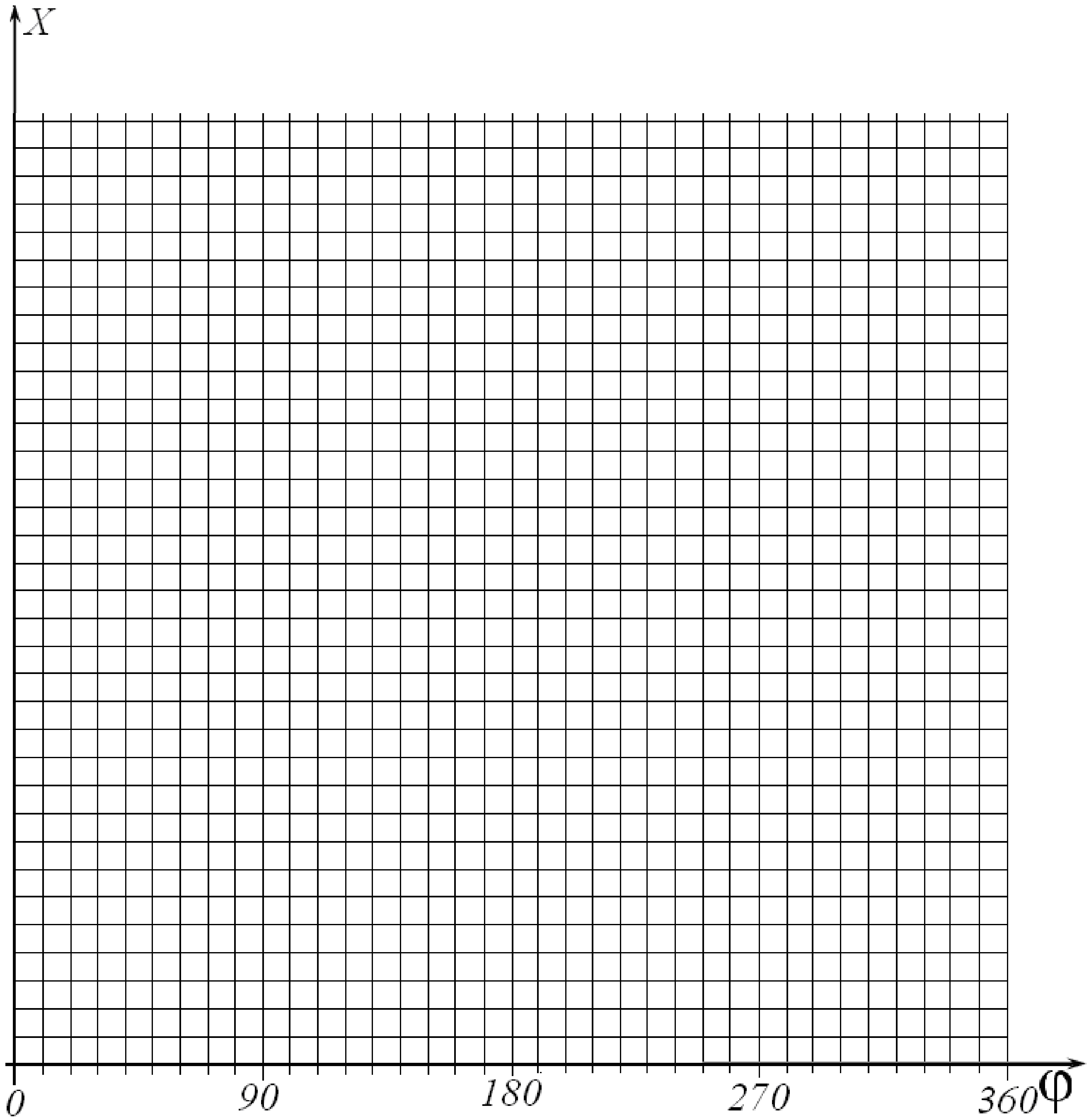


Fig 10.2. Diagram of the follower path

Date ___________________Grade ____________________Signature____________________

Test questions and tasks

1. Cam mechanisms. Advantages and disadvantages.
2. Closing the 2nd kind kinematic pair in cam mechanism.
3. What is the classification of cam mechanisms?
4. Cam mechanism diagram. Diagram of the follower path.
5. What is the method of reversing movement?
6. Designing cam mechanism diagrams and diagram of the follower path for main types of cam mechanisms.
7. Choosing the roller radius and pressure angle in cam mechanisms. Choosing the law of follower motion.

Plotting the cam profile

References

1. *Кіницький Я.Т.* Теорія механізмів і машин. – К.: Наук. думка, 2002. – 660 с.
2. *Budynas, Richard G.* Shigley's mechanical engineering design / Richard G. Budynas, J. Keith Nisbett. — 9th ed. – New York, McGraw-Hill, 2011. – 1109 p.
3. *Баранов Г.Г.* Курс теории механизмов и машин / Г.Г. Баранов – М.: Машиностроение, 1975. – 494 с.
4. *Артоболевський И.И.* Теория механизмов и машин / И.И.Артоболевський – М.: Наука. 1988. – 640 с.
5. *Воронкін М.Ф.* Основи теорії механізмів і машин: конспект лекцій / М.Ф. Воронкін, А.А. Цимбалюк – К.: КМУЦА, 2000.-208 с.
6. *Воронкін М. Ф.*, Кононихін Ю. Г., Крижановський А.С. Theory of mechanisms and machines. Laboratory works / М.Ф. Воронкін, Ю.Г. Кононихін, А.С. Крижановський . – К.: НАУ, 2003. – 57 с.
7. *Крижановський А.С.* Theory of mechanisms and machines: Kinematic and force analyses of leverages. A method guide / А.С. Крижановський, Ю. Г. Кононихін. – К.: НАУ, 2003. – 44 с.
8. *Крижановський А.С.* Theory of mechanisms and machines. Gear trains. A method guide / А. С. Крижановський, Є. М. Бабенко, А. О. Корнієнко. – К.: Вид-во Нац. авіац. ун-ту "НАУ-друк", 2009. – 44 p.
9. *Бабенко Є.М.* Теорія механізмів і машин. Розв'язання задач. Навчальний посібник / [Є. М. Бабенко, А. О. Корнієнко, О.В. Башта та ін.] – К.: Вид-во Нац. авіац. ун-ту "НАУ-друк", 2011. – 120 с.

Appendix 1

THE RULES OF CARRING OUT LABORATORY WORKS IN "THEORY OF MECHANISMS AND MACHINES"

1. Laboratory works are performed under the teacher's supervision in equipped laboratories of engineering department.

2. A student should prepare theoprtical material for the work at home.

3. A student finds the corresponding work in this guide, studies its theoretical part using lecture notes and familiarizes with the work order.

4. Students are permitted to carry out the laboratory work if they are provided with:

– synopsis and laboratory manual;

– copybook with ready-to use protocol (containing calculated formulas, diagrams, etc.);

– necessary a pencil, a ruler, a compass, etc;

5. Should learn and can explain theoretical material of the particular lab, methods of performing the lab and processing the results.

6. The estimate of student's readiness for the work is performed by questioning them before the class begins. Unprepared students aren't alowed to carry out the work.

7. While performing the work, a student writes a report due to the form and gives it to the teacher to be checked. After checking it, the teacher makes a mark which is the permission to defend it.

8. The defense of the work is done in the form of discussion or testing at the end of the class or at the tutorial. In case of getting a positive mark, it is put into the protocol with the teacher's signature.

9. In case of re-defense, the mark is lowered by one point.

10. A student is allowed to defend the work if the previous works have been done.

11. In the case of missing a laboratory work, it must be carried out by the student during tutorials. It is marked in the report and the student can defend it.

12. The marks, given for each work make up the current module rating mark of each "Theory of mechanisms and machines" module.

13. Students, who haven't fulfilled all works, aren't allowed to take module or semester tests.

14. Students, who have done all types of works included in the course and have got positive marks according to national estimating rating system, are allowed to take an examination (credit) test.