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ANALYSIS OF MATHEMATICAL METHODS OF IMPLEMENTATION THE COMPARATIVE APPROACH IN REAL ESTATE AVALUATION

АНАЛІЗ МАТЕМАТИЧНИХ МЕТОДІВ РЕАЛІЗАЦІЇ ПОРІВНЯЛЬНОГО ПІДХОДУ В ОЦІНЦІ НЕРУХОМОСТІ

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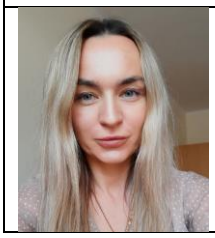
	***Petrenko Nataliia,*** *PhD in Economics, National Transport University, Associate Professor of Department of Transport Construction and Property Management, e-mail:* ***npetrenko79@ukr.net,*** *тел. +380680143841,* *https://orcid.org/0000-0002-9873-2327*

Abstract. The effectiveness of modern mathematical methods of implementation the comparative approach in real estate evaluation is analyzed in the work. Within this approach the following mathematical methods of determining corrective amendments to the value of the valuation object were analyzed: expert method, method based on solving a system of linear equations, calculation method based on normalized distances in the space of price-forming factors, and correlation-regression method. The main computing formulas and the sequence of calculations for each of the researched methods are outlined. The main criteria for choosing one or another method are defined on the basis of their advantages and disadvantages and application restrictions. The results of a computational experiment investigating the degree of sensitivity of mathematical methods regarding ways of formalization (digitization) of the initial qualitative information in determining the value of real estate objects within a comparative approach are presented. It is represented that the method of determining corrective amendments based on the solution of a system of linear equations is the most sensitive to the choice of the method of digitization of the initial data, the method of calculation based on normalized distances in the space of price-forming factors is less sensitive, and the correlation-regression method is not sensitive at all.

Keywords: real estate, valuation of property and real estate, digitization of original data, comparative approach in real estate valuation, value of real estate object.

Introduction. A valuation of property and property rights to real estate objects is an important prerequisite for the functioning and development of civilized market relations. Therefore, solving the problem of determination the true, fair price of real estate is important for the economy of our country and has a scientific and applied nature.

The practice of domestic real estate valuation to some extent follows foreign methods but there is a significant amount of own methodical developments of Ukrainian scientists whose the theory and practice is presented in the scientific studies of Ya. Markus, S. Hrybovskyi, I. Levykina, V. Voronin, B. Grabovetskii,

S. Sivets, N. Maksyshko, V. Horlachuk, Y. Dekhtiarenko, A. Drapikovsky, Y. Kirichenko, M. Lyhogrud, A. Liashchenko, Y. Palekha, L. Perovich, O. Petrakovska, V. Sydorenko, A. Tretiak, P. Cherniaha and many other scientists.

Determining the market value of real estate objects is a time-consuming process based on the analysis of many external and internal economic factors. According to methodological and legislative standpoints the calculation of the value of real estate objects is based on the application of three approaches: cost (based on the concept of costs), income (based on the theory of utility and expectations) and comparative (the theoretical basis of which is the theory of supply and demand) [1].

In practice, the comparative (market) approach to real estate valuation is most often used. It involves determining the value of the appraised object by comparing the prices of recent sales of similar objects on the market.

The main idea of this approach is following: a rational buyer will not pay for a real estate object a price higher than that one at which a similar real estate can be purchased.

The comparative approach can be implemented by means of various methods the application of which depends on the specific conditions and valuation tasks.

Purpose and methods. One of the most difficult stage of implementation a comparative approach in property and real estate valuation is the stage of choosing a definite technique of its implementation. It requires the definition of certain scientifically based criteria for choosing one or another method.

Therefore, it is important to conduct an analysis of modern methods of implementation a comparative approach in the valuation of property and real estate, to estimate the degree of existing methods and models effectiveness and to offer the most effective tools for realization of assessment activities. These issues determined the purpose of this work.

The purpose of the work is to analyze the effectiveness of modern mathematical methods of implementation a comparative approach in real estate valuation and to investigate the degree of their sensitivity to the formalization method (digitization) of the initial qualitative parameters (price-forming factors).

The object of the research is mathematical methods implementing a comparative approach in real estate valuation.

Research methods are mathematical and numerical modeling.

Research was conducted in two main directions:

1. Analysis of existing methods of implementation a comparative approach in real estate valuation, determination of their advantages and disadvantages.

2. Computational experiment from research the sensitivity of mathematical methods of implementation of the comparative approach in real estate valuation regarding the procedure of digitization of initial data (price-forming factors).

Results and explanations. The simplest and most popular method of implementation a comparative approach in property and real estate valuation is the expert method of determining the corrective amendments [2]. It is based on making corrections to the value of similar objects according to certain price-forming factors comparing them with the object of evaluation. Most often, amendments are made to the conditions of sale, location, physical indicators of real estate, and others. At the same time, a certain step is chosen based on the vision of a specific expert conducting the assessment. Most often, such a step is 5%. The final value of the valuation object is output as an arithmetical mean, modal or median value.

It is clear that this method is not formalized in its essence, therefore it depends on the qualification of the expert, the choice of price-forming factors and the calculation step.

A more substantiated and formalized method that implements a comparative approach in the property and real estate valuation is the method of calculation of corrective amendments based on the solution of the system of linear algebraic equations (SLAE) [3, 4].

According to this method, the value of the real estate object is determined by the formula:

$$V_0 = P_i + \sum_{i=1}^{m} \Delta P_{ij} \qquad (1)$$

where V_0 is the value of the object of valuation,

P_i is the value of the *i*-th analog object,

ΔP_{ij} is adjustment of the price of the *i*-th analog object for its difference from the object of evaluation

according to the *j*-th price-forming factor,

m is the number of price-forming factors according to which the adjustment is performed.

According to model (1) a comparison of the evaluated object with each of the n selected analog objects is provided sequentially:

$$\begin{cases} V_0 - \Delta x_{11}\Delta P_1 - \Delta x_{12}\Delta P_2 - ... - \Delta x_{1m}\Delta P_m = P_1 \\ V_0 - \Delta x_{21}\Delta P_1 - \Delta x_{22}\Delta P_2 - ... - \Delta x_{2m}\Delta P_m = P_2 \\ ... \\ V_0 - \Delta x_{n1}\Delta P_1 - \Delta x_{n2}\Delta P_2 - ...\Delta x_{nm}\Delta P_m = P_n \end{cases} \quad (2)$$

where Δx_{ij} is the difference of the values of the *j*-th price-forming factor for the valuation object and the *i*-th analog object: $\Delta x_{ij} = x_{0j} - x_{ij}$;

ΔP_j is contribution to the value of the valuation object of a unit of the *j*-th price-forming factor.

It is most convenient to find the solution of system (2) by the matrix method:

$$V = \Delta X^{-1} \cdot P, \quad (3)$$

$$\Delta X = \begin{pmatrix} 1 & -x_{11} & \cdots & -x_{1m} \\ \vdots & \ddots & & \vdots \\ 1 & -x_{n1} & \cdots & -x_{nm} \end{pmatrix}, \; V = \begin{pmatrix} V_0 \\ \Delta P_1 \\ . \\ \Delta P_m \end{pmatrix}, \quad P = \begin{pmatrix} P_1 \\ P_2 \\ . \\ P_m \end{pmatrix}, \quad (4)$$

where ΔX^{-1} is inverse matrix to the matrix ΔX.

The main disadvantage of the represented method is that the number of analog objects must exceed the number of factors by one ($n = m+1$), which is forced with the specifics of performing arithmetic operations on matrices. This, of course, causes certain difficulties in its implementation.

A more formalized method of weighting factors calculation when determining the value of real estate is the method of normalized distances in the space of price-forming factors, which is based on taking into account the degree of proximity of analog objects to the valuation object.

This method gives good results even if there are significant differences in characteristics between the evaluation object and analog objects.

Within this method, in the *n*-dimensional Euclidean space, center of coordinates is the valuation object and each analog object corresponds to a separate point remote from the center of coordinates (valuation object) by a distance l_i determined by the formula [3 , 4]:

$$l_i = \sqrt{\sum_{j=1}^{m} \left(\Delta k_{ij}\right)^2}, i = \overline{1,n}, \quad (5)$$

$$\Delta k_{ij} = \frac{\Delta x_{ij}}{\sum_{i=1}^{n} \left(\Delta x_{ij}\right)^2}, \quad (6)$$

where Δx_{ij} characterizes the difference between the *i*-th analog object and the valuation object by the *j*-th price-forming factor.

Each value l_i is matched with its inverse value $P_i = \frac{1}{l_i}$, which can be transformed into a weighting factor according to the formula:

$$v_i = \frac{P_i}{\sum_{i=1}^{n} P_i}. \quad (7)$$

The value of the valuation object is defined by the formula:

$$C = \sum_{i=1}^{n} v_i \cdot C_i, \quad (8)$$

where C_i is value of *i*-th analog object.

The advantage of this method is taking into account the degree of proximity of the valuation object to analog objects. Also, this method has no limitations on the number of analog objects and price-forming factors.

If there is a significant amount of information on analog objects then it is most appropriate to determine the market value of the real estate object within multifactor regression analysis, which allows not only to define the current market value of real estate objects, but also to establish causal relationships between price-forming factors, and to predict qualitative and quantitative changes in the real estate market [5, 6].

Within the correlation-regression method, mathematically, the task is reduced to finding an analytical expression reflecting the most optimally the connection of factor characteristics with the outcome measure:

$$Y = f\left(X_1, X_2, X_3, ..., X_n\right). \quad (9)$$

The most difficult problem is the choice of the form of connection between the outcome measure and factor characteristics. However, since any function of many variables can be linearized by logarithm or substitution of variables, multiple regression equations can be expressed in linear form:

$$Y = a_0 + a_1 X_1 + a_2 X_2 + ... + a_n X_n, \quad (10)$$

where Y is calculated value of the outcome measure;

$X_1, X_2, ..., X_n$ are factor price-forming characteristics;

$a_0, a_1, a_2, ..., a_n$ are regression coefficients for the *i*-th factor.

The coefficients of equation (8) are calculated by the method of least squares. In the case of two-factor regression we have:

$$\begin{aligned} \sum Y &= n a_0 + a_1 \sum X_1 + a_2 \sum X_2 \\ \sum Y X_1 &= a_0 \sum X_1 + a_1 \sum X_1^{\,2} + a_2 \sum X_1 X_2 \\ \sum Y X_2 &= a_0 \sum X_2 + a_1 \sum X_1 X_2 + a_2 \sum X_2^{\,2} \end{aligned} \quad (11)$$

Each coefficient of the equation indicates the degree of influence of the corresponding factor on the outcame indicator at a fixed position of the remaining factors.

In order to identify the comparative strength of influence of separate factors and their reserves, the partial elasticity coefficients and beta-coefficients are calculated according to the formulas:

$$\varepsilon_i = a_i \frac{\overline{X_i}}{\overline{Y}}; \quad \beta_i = a_i \frac{\sigma_{X_i}}{\sigma_Y} \quad (12)$$

where $\overline{X}_i$, σ_{X_i} are means and standard deviations of *i*-th factor;

$\overline{Y}$, σ_Y are mean and standard deviation of outcome measure.

Partial coefficients of elasticity represent by how many percent, on average, the outcome measure will change with a change of 1% of each factor with a fixed position of other factors.

The absolute advantage of the correlation-regression method is the ability to take into account the variability of the real estate market, its stochastic nature and multifactoriality.

The value of real estate is known to depend on a number of price-forming factors, which are mostly of a qualitative nature. Such factors may include the location of the object, its technical condition, availability of landscaping, parking spaces, etc. Therefore, one of the main prerequisites for the application of any mathematical method in real estate evaluation is the formalization of the initial information that cannot be quantified, that is, its digitization.

Most often, in practice, the process of formalization (digitization) of qualitative parameters into quantitative ones performs by assigning them certain points on an ordinal (ranking) scale, where the highest point corresponds to the best quality of a certain parameter accordingly, the lowest point corresponds to the worst quality. At the same time, the so-called "1-9" scale (Table 1) is quite popular [3].

Table 1 . Rating scale «1-9»

Gradation of assessment	**Scale value**
The worst factor value	1-2
Insignificant advantage	3-4
Significant advantage	5-6
Clear advantage	7-8
Absolute advantage	9

It is quite obvious that the use of the relative scale "1-9" depends on the subjectivity of the evaluator and the initial ranking of quality parameters.

A logical question arises, are all mathematical methods equally sensitive to the method of digitization of the initial data and how does the choice of one or another score affect the final calculated value of the real estate object? In order to answer these questions, within this work, a series of computational experiments was conducted to investigate the sensitivity of the considered mathematical methods to the method of formalization (digitization) of the initial quality parameters (price-forming factors).

When studying the sensitivity of mathematical methods to ways of formalization (digitization) of the initial parameters it is advisable to evaluate them within the probability and statistical methods, while determining the following characteristics of the sample (series of numerical experiments) [7]:

- the sample mean of the value of the real estate object of n experiments:

$$\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}, \tag{13}$$

X_i is the value of the real estate object obtained in the *i*-th numerical experiment;

- sample variance and standard deviation for the investigated sample:

$$D = \sum_{i=1}^{n} \frac{X^2_i}{n} - \overline{X}^2, \ \delta = \sqrt{D}. \tag{14}$$

For each of the above mathematical methods, a series of 10 computational experiments was carried out, the main idea of which was to vary different methods of initial digitization of the original parameters and to compare the obtained final results of the value of the valuation object. The initial data for the test example are represented in Table 2 (E.O. is valuation object, A1-A5 are analog objects).

The analysis of the initial information (Table 2) allows us to conclude that all price-forming factors affecting the final value of the object of assessment are of a qualitative nature. Among them, the price-forming factor "availability of parking" should be formalized by using a binary variable, which will take the

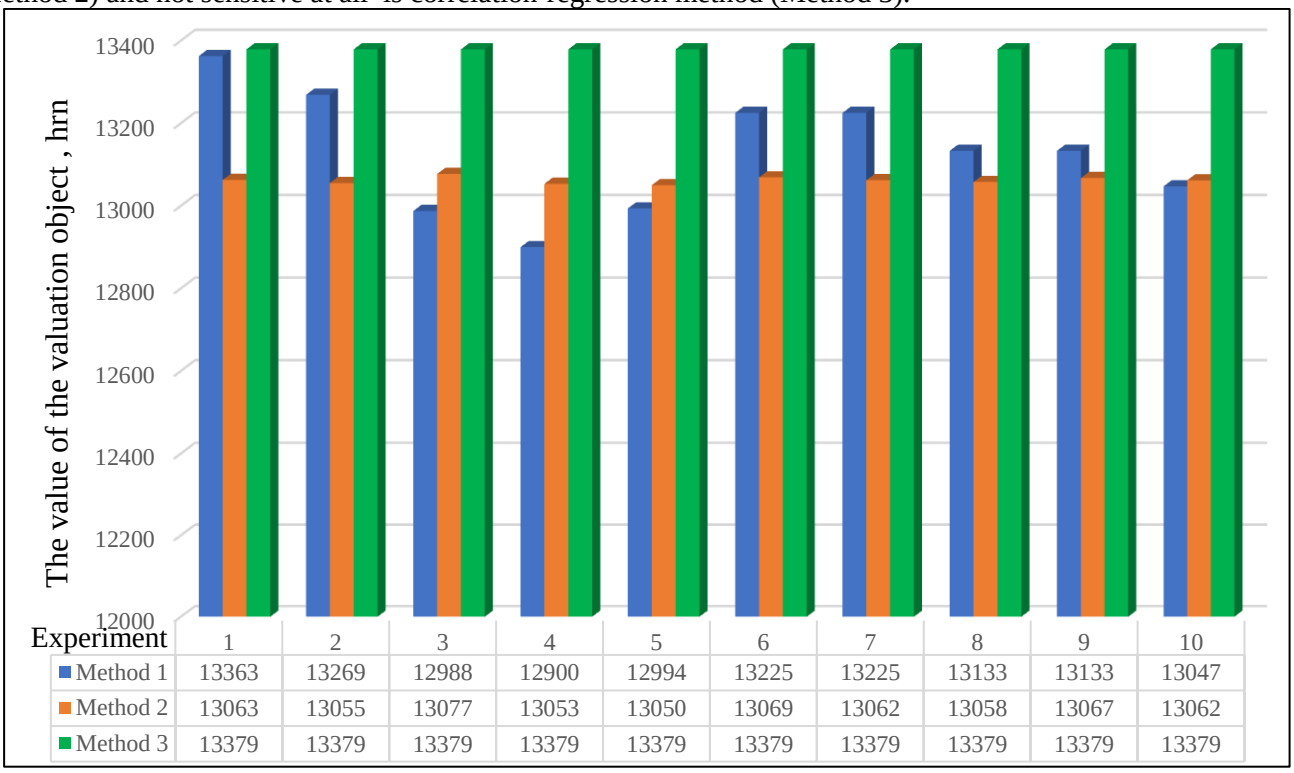
value 1 if there is a parking space and 0 if there is no parking space. The other three price-forming factors (location, technical condition, availability of landscaping) can be logically ranked, which makes it possible to digitize them using a ranking scale. For example, the price-forming factor "technical condition" can be ranked as follows: satisfactory - good - excellent. At the same time, the formalization of this parameter is possible in various ways (for example, 1-2-3; 1-3-5, ...) [8].

Table 2. Initial test data about the valuation object and analog objects

	Technical condition	**Location**	**Availability of landscaping**	**Availability of parking**	**Value 1 м^2**
V.O.	good	central	water supply, lighting	available	
A1	good	central	all good things	available	13500
A2	excellent	median	all good things	available	13500
A3	satisfactory	median	water supply, lighting	no	12900
A4	satisfactory	median	lighting	available	12850
A5	satisfactory	central	without landscaping	available	12900

Figure 1 represents the results of 10 numerical experiments on the calculation of the value of the test real estate object by the method of determining corrective amendments based on the solution of the system of linear equations (Method 1); on the basis of normalized distances in the space of price-forming factors (Method 2); correlation-regression method (Method 3).

Summarizing the results of calculations obtained by various mathematical methods (Table 3), we can conclude that the method of determining corrective amendments based on the solution of a system of linear equations (Method 1) is the most sensitive to the choice of the method of digitizing the initial data, less sensitive is the calculation method based on normalized distances in the space of price-forming factors (Method 2) and not sensitive at all is correlation-regression method (Method 3).



Experiment	1	2	3	4	5	6	7	8	9	10
Method 1	13363	13269	12988	12900	12994	13225	13225	13133	13133	13047
Method 2	13063	13055	13077	13053	13050	13069	13062	13058	13067	13062
Method 3	13379	13379	13379	13379	13379	13379	13379	13379	13379	13379

Figure 1. – The results of a computational experiment on the sensitivity of mathematical methods for the implementation of a comparative approach in real estate valuation in relation to the procedure for digitizing the initial data

Рисунок 1 – Результати обчислювального експерименту із дослідження чутливості математичних методів реалізації порівняльного підходу в оцінці нерухомості щодо процедури оцифрування вихідних даних

Table 3. The results of a computational experiment on the sensitivity of mathematical methods for the implementation of a comparative approach in real estate valuation in relation to the procedure for digitizing the initial data

Таблиця 3 – Результати обчислювального експерименту із дослідження чутливості математичних методів реалізації порівняльного підходу в оцінці нерухомості щодо процедури оцифрування вихідних даних

	Sample mean of the value of the valuated object, hrn./m^2	Standard deviation, hrn
Method of determining corrective amendments based on the solution of the system of linear equations (Method 1)	13127,6	137,8
Method based on normalized distances in the space of price-forming factors (Method 2)	13061,6	7,65
Correlation-regression method (Method 3)	13378,9	0

Conclusions and recommendations

Nowadays, the comparative approach is the most popular at evaluating property and real estate, due to its flexibility and marketability. However, the application of known mathematical methods of implementing a comparative approach is associated with certain limitations and conditions imposed on the initial information about analog objects. Therefore, the evaluator must carefully and attentively approach the choice of one or another calculation method.

A computational experiment of investigation the sensitivity of various mathematical methods to the procedure of formalization (digitization) of the initial qualitative information at determining the value of real estate objects within the comparative approach represents the following. The method of determining corrective amendments based on the solution of a system of linear equations is the most sensitive to the choice of the method of digitization of the initial data, the method of calculation based on normalized distances in the space of price-forming factors is less sensitive, and the correlation-regression method is not sensitive and this fact is its absolute advantage.

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АНАЛІЗ МАТЕМАТИЧНИХ МЕТОДІВ РЕАЛІЗАЦІЇ ПОРІВНЯЛЬНОГО ПІДХОДУ В ОЦІНЦІ НЕРУХОМОСТІ

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Анотація. В роботі проведено аналіз ефективності сучасних математичних методів реалізації порівняльного підходу в оцінці нерухомості. В рамках даного підходу проаналізовано такі математичні методи визначення коригуючих поправок до вартості об'єкта оцінки як: експертний метод, метод на основі розв'язання системи лінійних рівнянь, метод розрахунку на основі нормованих відстаней в просторі ціноутворюючих факторів та кореляційно-регресійний метод. Викладено основні розрахункові формули та послідовність проведення розрахунків для кожного із досліджуваних методів. Визначено основні критерії вибору того чи іншого методу на основі аналізу їх переваг і недоліків, обмежень щодо застосування. Представлено результати обчислювального експерименту із дослідження ступеню чутливості математичних методів щодо способів формалізації (оцифрування) вихідної якісної інформації при визначенні вартості об'єктів нерухомості в рамках порівняльного підходу. Показано, що найбільш чутливим по відношенню до вибору способу оцифрування вихідних даних є метод визначення коригуючих поправок на основі розв'язання системи лінійних рівнянь, менш чутливим є метод розрахунку на основі нормованих відстаней в просторі ціноутворюючих факторів та взагалі не чутливим - кореляційно-регресійний метод.

Ключові слова: нерухоме майно, оцінка майна та нерухомості, оцифрування вихідних даних, порівняльний підхід в оцінці нерухомості, вартість об'єкта нерухомості.

Перелік посилань

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