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GRADUATION WORK
(EXPLANATORY NOTES)

FOR THE DEGREE OF BACHELOR
SPECIALTY 173 'AVIONICS'

Theme: «Nonorthogonal Gyroscopic Instrument for Measuring Vector of Aircraft Angular Rate»

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Kyiv 2022

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ
ФАКУЛЬТЕТ АЕРОНАВІГАЦІЇ, ЕЛЕКТРОНІКИ ТА
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КАФЕДРА АВІОНІКИ

ДОПУСТИТИ ДО ЗАХИСТУ

Завідувач випускової кафедри

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ДИПЛОМНА РОБОТА

(ПОЯСНЮВАЛЬНА ЗАПИСКА)

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БАКАЛАВР ЗА СПЕЦІАЛЬНІСТЮ 173

«АВІОНІКА»

Тема: «Неортогональний гіроскопічний вимірювач вектора кутової швидкості літака»

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Київ 2022

NATIONAL AVIATION UNIVERSITY

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APPROVED

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' ____ ' _____ 2022

TASK for execution graduation work

V.S. Chorna

Theme: «Nonorthogonal Gyroscopic Instrument for Measuring Vector of Aircraft Angular Rate», approved by order №352/CT of the Rector of the National Aviation University of 04 April 2022.

1. Duration of which is from 16.05.2022 to 19.06.2022.
2. Input data of graduation work: The study has a theoretical and practical nature, based on the analysis of literary and Internet sources.
3. Content of explanatory notes: List of conditional terms and abbreviations; Introduction; Chapter 1 construction of nonorthogonal rate vector sensor; Chapter 2 research on reliability of excessive meters; Chapter 3 technological features and examples of modern mems gyroscopes; Conclusion.
4. The list of mandatory graphic material: figures, charts, graphs.

5. Planned schedule

№	Task	Duration	Signature of supervisor
1.	Validate the rationale of graduation work theme	16.05-20.05	
2.	Carry out a literature review	21.05-24.05	
3.	Develop the first chapter of diploma	25.05-29.05	
4.	Develop the second chapter of diploma	30.05-02.06	
5.	Develop the third chapter of diploma	03.06-05.06	
6.	Develop the fourth chapter of diploma	06.06-08.06	
7.	Tested for anti-plagiarism and obtaining a review of the diploma	09.06-19.06	

6. Date of assignment: ‘ __ 16 ’ __ 05 ____ 2022

Supervisor

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The task took to perform

_____ (signature)

(surname, name, patronymic)

ABSTRACT

The explanatory notes to the graduate work ‘‘Non-orthogonal gyroscopic angular velocity vector meter on an airplane’ contained 54 pages, 19 drawings, 2 tables, 0 flow-charts, 13 information source.

Keywords: ANGULAR VELOCITY VECTOR, GYROSCOPIC METER, AIRPLANE, SENSOR, SPATIAL POSITION, NONORTHOGONAL .

The purpose of the graduate work is to develop the non-orthogonal gyroscopic meter of the angular velocity vector on the aircraft, taking into account the peculiarities of its operation.

The object of the research is the process of measuring the angular velocity vector on an airplane.

The subject of the research is non-orthogonal gyroscopic angular velocity vector meter on an airplane.

Research Method - comparative analysis, theory of gyroscopes, statistical processing, methods of functional redundancy.

The scientific novelty of the research - it is recommended to use the materials of the thesis when designing the new airborne measuring instruments with improved characteristics of accuracy and reliability. The increased reliability and accuracy are provided due to the redundant number of inertial sensors and using of their nonorthogonal configurations. The failure of the measuring channel will only lead to an increase in the measurement error. Thus, it is possible to develop an algorithm that will adapt to possible faults. In case of failures, it will not even be necessary to modify the algorithms (rebuild the direction cosine matrix), it will be enough to set too high a dispersion value for the failed channel in the covariance matrices. The proposed approach will be especially useful for application in control systems of unmanned aerial vehicles.

The importance of the graduate work - in this thesis, the advantages and disadvantages of design, the design of micro-mechanical gyroscopes were considered. The issue of redundancy of non-orthogonal measuring devices to

increase accuracy and reliability is considered. An accuracy study will also be performed experimentally and theoretically to determine the optimal type of sensor configuration.

INTRODUCTION

The advantages of a strap-down inertial navigation system are relative cheapness (it is possible to use sensors from the “low cost” category), small dimensions and weight, and low power consumption. These advantages are especially clearly seen in SINS built on micromechanical navigation sensors (gyroscopes - MMG, accelerometers - MMA).

Blocks of linear micromechanical accelerometers are used as sensors of the apparent acceleration of a moving object as part of inertial navigation systems. They do not have high accuracy characteristics (if we consider inexpensive MMA). However, there are tasks for which they are well suited. For example, in small aircraft, small size and weight, coupled with a low power consumption of MMA units, bring many benefits. The same applies to the automotive industry, where they are integrated with satellite systems (GPS, GLONASS, etc.).

One of the serious problems of low-cost micromechanical sensors is random zero drift. In simple terms, drift is when the sensor indicates that the object is rotating (MMG drift), when in fact there is no rotation. Drift has a constant component, which can be compensated, and a random component, which is difficult to compensate. There are different ways to deal with random errors. One of them is the construction of a non-orthogonal SINS with information redundancy.

This thesis has made a number of significant contributions to the field of aviation reliability and flight safety.

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LIST OF ABBREVIATIONS

SINS - strap-down inertial navigation system

MEMS - Micromechanical sensors

CONSTRUCTION OF NONORTHOGONAL RATE VECTOR SENSOR

1.1. General characteristic of a gyroscopic rate sensor.

I could start off by writing that a rate gyro includes six gauge with noncomplanar arrangement of sensitivity axes (measuring axes).

In particular, all six measuring axes ($\bar{a}_1, \dots, \bar{a}_6$) at nominal arrangement is placed in parallel to regular hexahedron base edges, added in a cone of rotation with an angle of semisolution ϕ , equaled 0,9553 rad, and having a symmetric arrangement of edges by a circle of a cone base with angular step θ , equaled to 1,04 rad. [1]

1. As instrumental co-ordinate system is accepted right orthogonal $Ox_p y_p z_p$, materialized by planting places on rate gyro case. Orientation of rate gyro sensitivity axes concerning axes of instrumental co-ordinate system is as follows:

$Ox_p y_p z_p$ – rate gyro instrumental co-ordinate system;

$\bar{a}_1, \dots, \bar{a}_6$ - positive sensitivity axes directions of rate gyro ($A_1, A_2, A_3, A_4, A_5, A_6$ measuring instruments, accordingly).

In the same way ensitivity axes \bar{a}_1 and \bar{a}_4 is parallel to the $x_p O y_p$ plane. In fig.1.3. positive angles directions of axes deviation of sensitivity measuring instruments concerning a nominal position, where:

$\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4, \bar{a}_5, \bar{a}_6$ - nominal sensitivity axes positions of $A_1, A_2, A_3, A_4, A_5, A_6$ measuring instrument accordingly;

$\Delta\theta_1, \Delta\phi_1, \Delta\theta_2, \Delta\phi_2, \dots, \Delta\theta_6, \Delta\phi_6$ - positive deviation angles of axes concerning a nominal position.

2. At rate gyro rotation round a sensitivity axis $\bar{a}_1(\bar{a}_2, \bar{a}_3, \bar{a}_4, \bar{a}_5, \bar{a}_6)$ in a positive direction (counter-clockwise, looking from vectors end) output information from measuring instrument $\bar{a}_1(\bar{a}_2, \bar{a}_3, \bar{a}_4, \bar{a}_5, \bar{a}_6)$ corresponds to positive value of parameter and on the contrary.

3. Relative orientation of instrumental coordinate system axes and construction co-ordinate system of a product is that, that axis x_p coincides with a negative direction of an axis z_{prod} ; y_p axis with a positive direction of an axis x_{prod} ; z_p coincides with a negative direction of an axis y_{prod} . [2]

Furthermore with rate gyro, output information in discrete form is given from each measuring instrument ($A_1, A_2, A_3, A_4, A_5, A_6$) in the form of a unitary code - pulse link, transmitted in onboard digital computing system (ODCS) by electrically non-connected channels. Each information channel has two functional communication lines; on one line pulses is distributed, corresponded to a positive projection, and on other line, corresponding to a negative projection of rate to an axis of measuring instrument sensitivity are given.

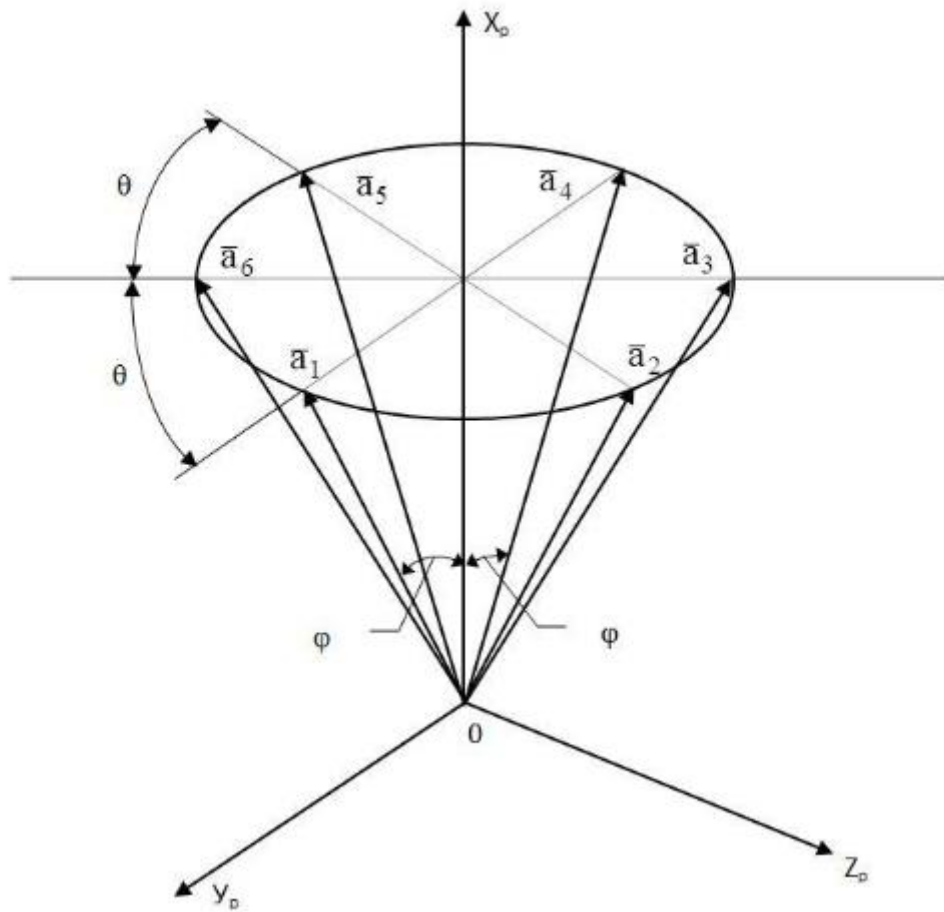


Fig.1.1. A rate gyro includes six gauge with noncomplanar arrangement of sensitivity axes (measuring axes)

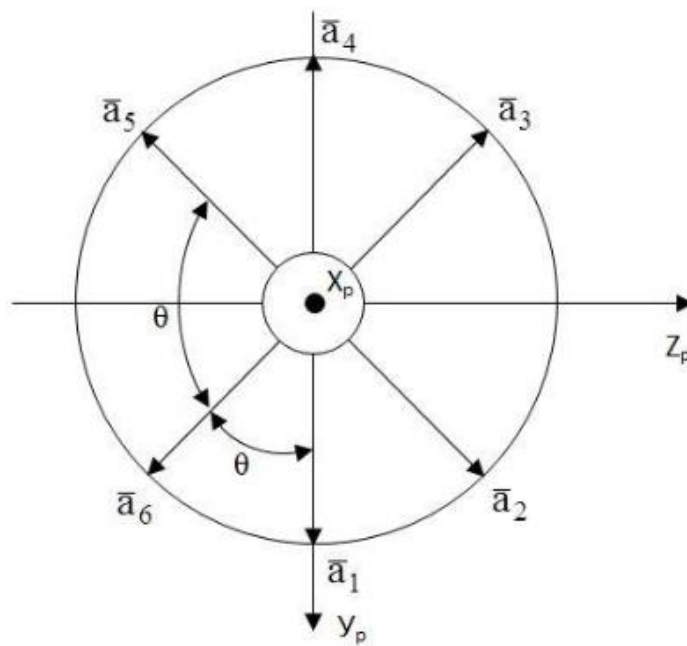


Fig.1.2. Rate gyro sensitivity axes orientation concerning axes of instrumental coordinate system

where: $\Delta\phi_I$ corresponds to $\Delta\phi_1 \div \Delta\phi_6$; \bar{a}_i corresponds to $\bar{a}_1 \div \bar{a}_6, i = 1, 2, \dots, 6$.

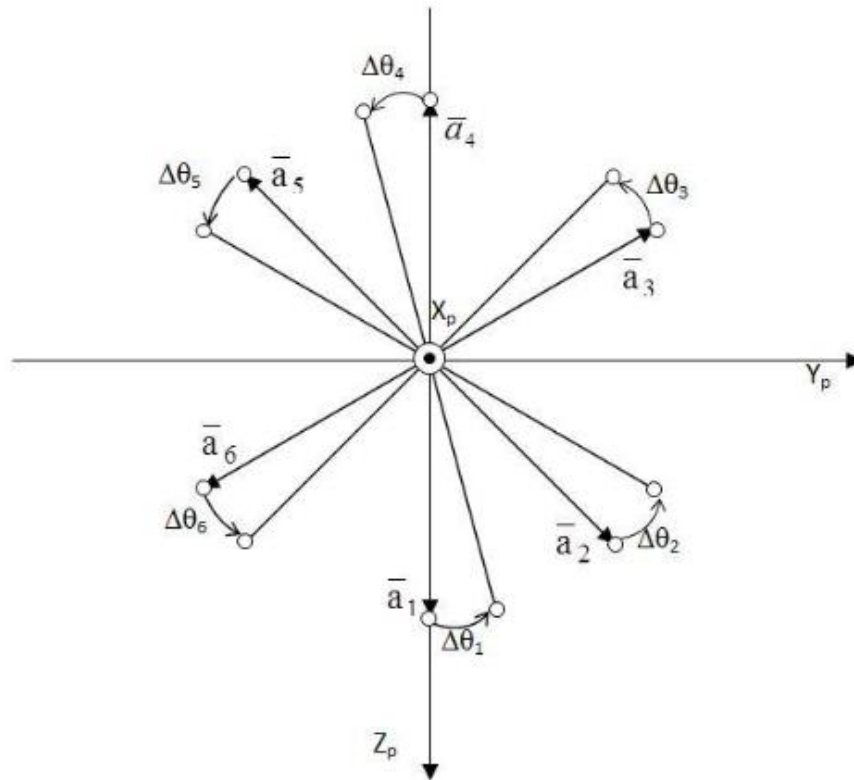


Fig.1.3. Positive angles directions of a sensitivity axes deviation of measuring instruments concerning a nominal position

1.2. Redundancy basis application features

The first think I should mention is that for measurement of any vector, it is necessary to realize some measuring basis. Set of measured vectors forms vector space.

Nonredundant vector measuring instrument intended for nonredundant measurement of vector space, is called the measuring instrument for which dimensions of measuring and control bases coincides, that is $n = n_0$ [3].

In a case $n < n_0$ received information are not enough for aircraft's control, and at $n > n_0$ information is redundant.

As to acceleration measurement, redundant vector accelerometer is formed by three single-stage accelerometers which axes of sensitivity are noncollinear.

As it is known from the literature [2], as a redundant vector measuring instrument understand a measuring instrument which consists of the single-stage measuring instruments which quantity exceeds the minimum number of the measuring instruments necessary for vector measurement.

One of basic attribute of such measuring instrument is the parameter of redundancy which is determined by the formula:

$$m=n-n_0 \tag{1.2.1.}$$

where n - total number of measuring instruments;

n_0 - the minimum number of measuring instruments, necessary for vector's measurement.

From expression (1.2.1.) it is visible that the minimum number of redundant measuring instruments is equal to one.

Under construction of a redundant vector measuring instrument the primary goal consists in effective redundancy using.

Also, orientation of sensitivity axis of each single-stage measuring instrument is characterized by single vector; therefore, for set of vectors the vector space which is linear vector space is formed.

Practical interest represents interrelation of the vector space created by a redundant vector measuring instrument and vector space, created by non-redundant vector measuring instrument. Considering that linear vector space can have some bases, we will call the space, having some bases, as a redundant vector space [1]. Thus, the vector space, under formation of redundant vector measuring instrument, is redundant vector space concerning vector space, under formation of a vector measuring instrument and bases of these spaces are, accordingly, redundant and non-redundant.

Characteristic feature of an aircraft control systems redundancy application consists in all components of efficiency improving allowance, namely: accuracy, readiness, reliability. At control system designing to minimize an error of the acceleration measured by the vector accelerometer, it is possible by optimum

selection of a measuring instrument orientation and optimum processing of redundant information [2].

Most effectively redundancy can be used, if linear independence of all single vectors connected with orientation of sensitivity axes of single-stage measuring instruments is supplied, that is at use of so-called functional redundancy. In this case there is a refused vector measuring instrument redundancy replacement capability, to an operable one, or a capability of formation of the non-redundant vector measuring instruments greatest number.

Under consideration of redundant measuring instruments orientation ways there are two primary goals. One of them is connected with selection of set of parameters, which unequivocally determines orientation of each measuring instrument concerning the main (control) basis. The second problem consists in obtaining of transformation formulas of vector coordinates from every redundant measuring basis in the main.

Problem of vector's coordinate transformation is caused by discrepancy of measuring and control bases.

Selection of this or another way of orientation, and also set of the parameters characterizing orientation of redundant measuring instruments, is conducted taking into account all problems arising in redundant aircraft control systems. Except a direct problem of orientation it is necessary to consider convenience and efficiency of coordinate's transformation of the vectors measured, simplicity of detection of failures and reorganization of a control system structure[4].

1.3. Orientation schemes of redundant measuring instrument

For that time for construction of a redundant measuring instrument four ways of orientation of sensitive element's redundant measuring instruments with use as a figure of symmetry of a cone [5] are known:

1st way – all n measuring instruments are guided by the generatrix of cone, a symmetry axis - on measured vector or in an optimum direction for a case of variable

in a direction measured vector. Thus sensitivity axes of measuring instruments take place on generatrix of cone through equal angles $\alpha = \frac{2\pi}{n}$ (fig. 1.3.)

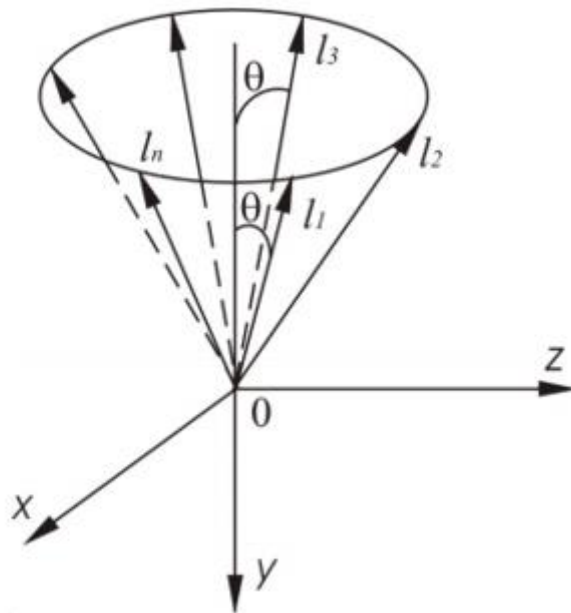


Fig. 1.3. Orientation of redundant measuring instruments by the generatrix of cone over equal angles $\alpha = 2\pi / n$

2nd way - one of measuring instruments is disposed on an axis of symmetry of a cone, others $(n - 1)$ measuring instruments are disposed on the generatrix of cone over equal angles $\alpha = \frac{2\pi}{n - 1}$ (fig. 1.4.)

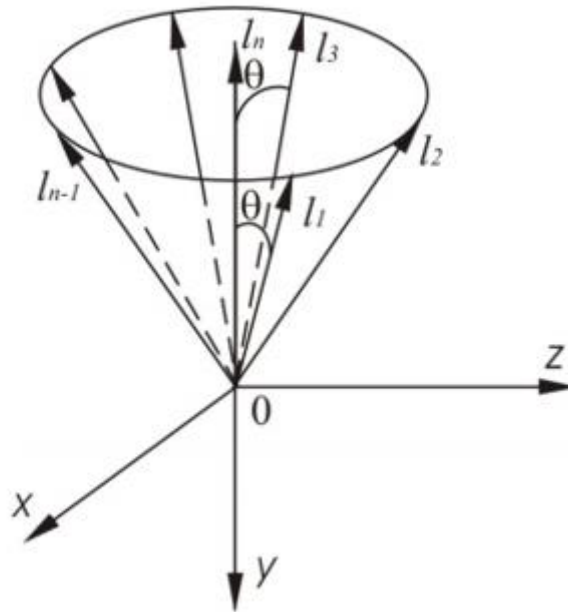


Fig. 1.4. Orientation of redundant measuring instruments by generatrix of cone over equal angles

3rd way - multicone arrangement with unified for all cones symmetry axis;

4th way - multicone arrangements with various for each of cones symmetry;

In fig. 1.3. and 1.4. sensitivity axes of measuring instruments are designated as l_i .

One of possible ways of orientation consists in identical orientation of sensitivity axes concerning edges of regular polyhedron, for example, perpendicularly to their edges [5]. In some regular polyhedrons separate edges can be parallel each other and consequently the number of independent measuring instruments (for example, accelerometers) at the given way of orientation does not coincide with number of edges. It is known that exists only five regular polyhedrons, which characteristics, and also the maximum numbers of redundant accelerometers are resulted in table 1.1.

Table 1.1.

Redundant accelerometers of five regular polyhedrons

<i>Polyhedron name</i>	<i>Number of tops</i>	<i>Number of edges</i>	<i>Facets number</i>	<i>The maximum number A</i>
Cube	8	12	6	3
Tetrahedron	4	6	4	4
Octahedron	6	12	8	4
Dodecahedron	20	30	12	6
Icosahedron	12	30	20	10

One of variants of such approach that found enough extended coverage in the domestic and foreign technological literature represents orientation of six measuring instruments to perpendicularly edges of a dodecahedron. Choosing orientation of a dodecahedron with relation to control basis, it is possible to obtain symmetric orientation of sensitivity axes of measuring instruments concerning axes of this basis and related to each other (fig.1.5)

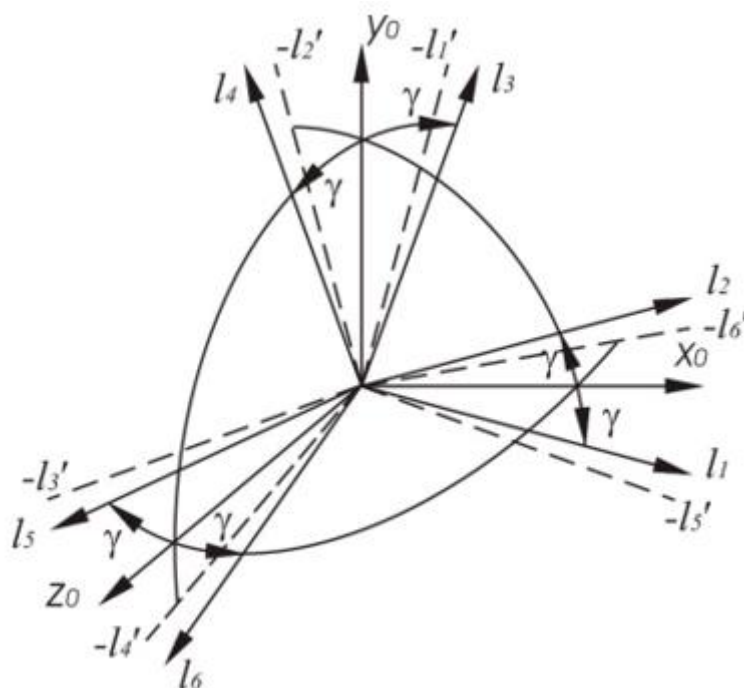


Fig.1.5. Symmetric orientation of axes of sensitivity of redundant measuring instruments

In the presented scheme of a sensitivity axis of all measuring instruments are orientated concerning the nearest axes of control basis on angle $\gamma = 31^\circ 43'$. Orientation of two measuring instruments in one plane corresponds to the minimum number of measuring instruments for obtaining of the information about the vector, disposed in a plane. The single vectors directed on sensitivity axes of measuring instruments, form vector space with linearly-independent making, thus, its dimension is equal to number of measuring instruments. [6]

In the given project 7 possible variants of construction schemes of a redundant vector measuring instrument are considered:

- I. 4 sensitive elements are located by generatrix of cone;
- II. 3 sensitive elements are located by generatrix of cone and 1 sensitive element by the symmetry axis;
- III. 5 sensitive elements located by generatrix of cone;
- IV. 4 sensitive elements are located by generatrix of cone and 1 sensitive element by the symmetry axis;
- V. 6 sensitive elements are located by generatrix of cone;
- VI. 5 sensitive elements are located by generatrix of cone and 1 sensitive element by the symmetry axis;
- VII. 6 sensitive elements are located over the dodecahedron edges.

For the analysis of each of schemes we enter following designations:

vector of measuring coordinate system. Dimension of this vector coincides with quantity of sensitive elements measuring instruments;

ω - state vector, i.e. vector of basic coordinate system:

$$\omega = [\omega_x \ \omega_y \ \omega_z]^T;$$

H - matrix of transformation of basic coordinate system into measuring coordinate system, than, in the matrix form we can write:

$$l = H\omega$$

(1.3.1.)

whence

$$\omega = H^{-1}l \tag{1.3.2.}$$

where

$$l = [l_1 l_2 \dots l_n]^T ; \omega = [\omega_x \omega_y \omega_z]^T . \tag{1.3.3.}$$

n - quantity of sensitive elements in measuring instrument. Dimension of matrix H is $n \times 3$.

Let's consider each of the offered schemes and according to (1.3.1.) we will obtain a matrix of direction cosines.

Variant of 4 sensitive elements location by the generatrix of cone.

Space orientation of a redundant vector measuring instrument which consists from 4 sensitive elements placed by generatrices of cone, is presented in fig. 1.6., where OXYZ - basic coordinate system, $O l_1 l_2 l_3 l_4$ - measuring coordinate system, an angle $\theta = 54^\circ 44'$.

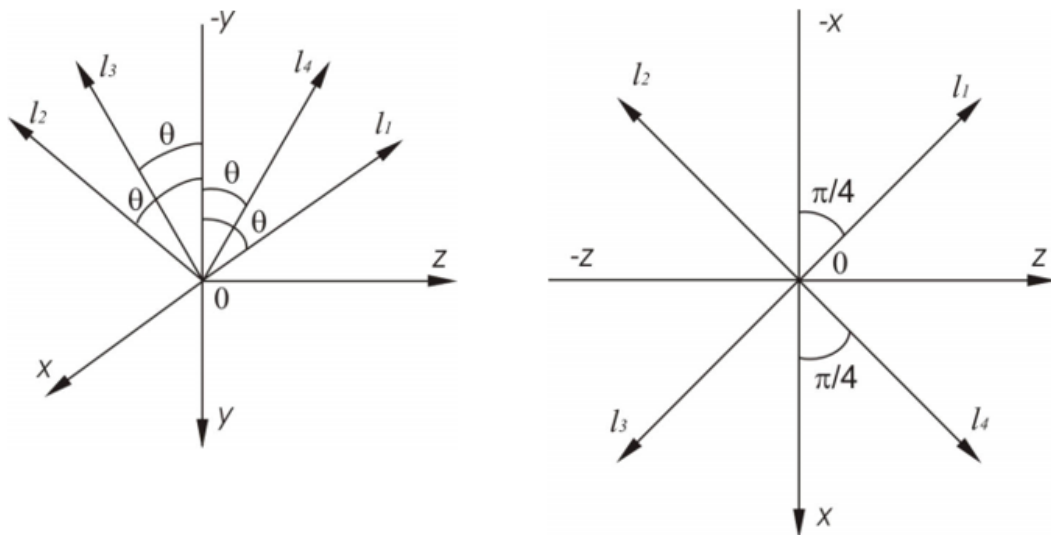


Fig. 1.6. Orientation of the 4 sensitive elements by the generatrix of cone

Taking into account fig. 3.2.2.4 it is possible to create a matrix of directing cosines:

$$\mathbf{H} = \begin{bmatrix} -\cos \pi / 4 \sin \theta & -\cos \theta & \cos \pi / 4 \sin \theta \\ -\cos \pi / 4 \sin \theta & -\cos \theta & -\cos \pi / 4 \sin \theta \\ \cos \pi / 4 \sin \theta & -\cos \theta & -\cos \pi / 4 \sin \theta \\ \cos \pi / 4 \sin \theta & -\cos \theta & \cos \pi / 4 \sin \theta \end{bmatrix}$$

(1.3.4.)

Taking into account value of an angle θ it is possible to rewrite (1.2.2.4) in a following kind:

$$\mathbf{H} = \begin{bmatrix} -\sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \\ -\sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \end{bmatrix}.$$

(1.3.5.)

Variant of 3 sensitive elements location by the generatrix of cone and 1 sensitive element over a symmetry axis.

Orientation of a vector measuring instrument is presented in fig. 1.7. which consist of 3 sensitive elements placed by the generatrix of cone and 1 sensitive element which are located over a symmetry axis, where Oxyz – basic coordinate system, $Ol_1l_2l_3l_4$ - measuring coordinate system, an angle $\theta = 54^\circ 44'$.

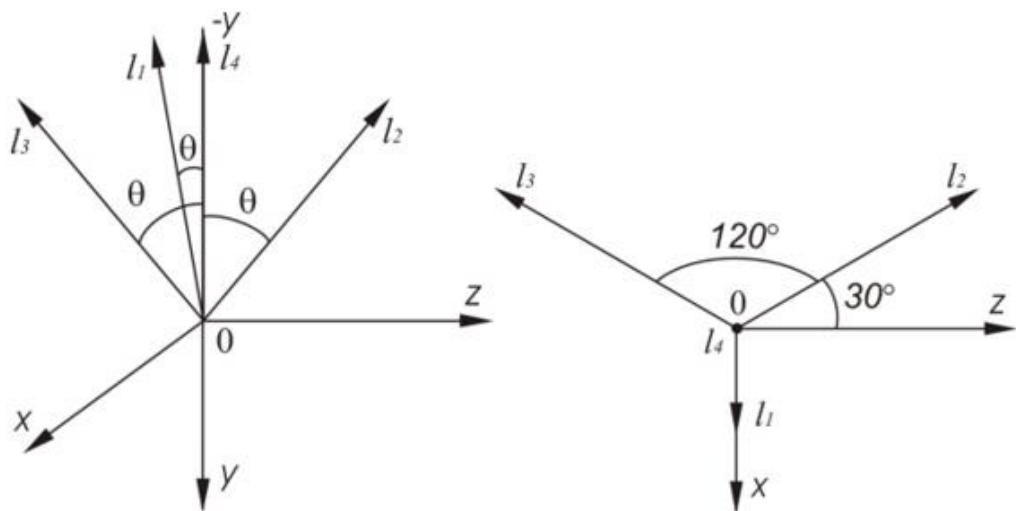


Fig.1.7. sensitive elements orientation by the generatrix of cone and 1 sensitive element over a symmetry axis

Taking into account fig. 1.7. it is possible to create a matrix of directing cosines:

$$\mathbf{H} = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ -\cos \pi/3 \sin \theta & -\cos \theta & \cos \pi/6 \sin \theta \\ -\cos \pi/3 \sin \theta & -\cos \theta & -\cos \pi/6 \sin \theta \\ 0 & -1 & 0 \end{bmatrix}.$$

(1.3.6.)

Taking into account value of an angle θ , matrix (1.2.2.6) can be overwritten in a following view:

$$\mathbf{H} = \begin{bmatrix} \sqrt{6}/3 & -\sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & -\sqrt{2}/2 \\ 0 & -1 & 0 \end{bmatrix}$$

(1.3.7.)

Variant of 5 sensible elements location by generatrix of cone.

Orientation of redundant vector measuring instrument which consists of 5 sensitive elements located by generatrix of cone is presented in fig. 1.8., where Oxyz - basic coordinate system, $Ol_1l_2l_3l_4l_5$ - measuring coordinate system, an angle $\theta = 54^\circ 44'$.

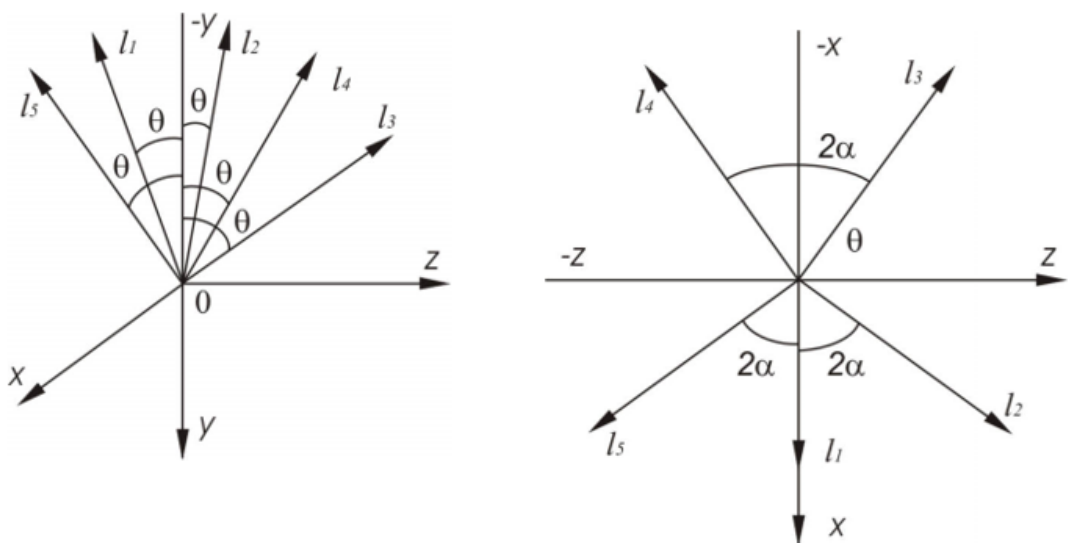


Fig. 1.8. Orientation of 5 sensitive elements by the generatrix of cone

Taking into account fig. 1.8., matrix of directing cosines becomes:

$$\mathbf{H} = \begin{bmatrix} -\cos \pi / 4 \sin \theta & -\cos \theta & \cos \pi / 4 \sin \theta \\ -\cos \pi / 4 \sin \theta & -\cos \theta & -\cos \pi / 4 \sin \theta \\ \cos \pi / 4 \sin \theta & -\cos \theta & -\cos \pi / 4 \sin \theta \\ \cos \pi / 4 \sin \theta & -\cos \theta & \cos \pi / 4 \sin \theta \\ 0 & -1 & 0 \end{bmatrix} \quad (1.3.8.)$$

Taking into account value of an angle θ , matrix (1.3.8.) can be overwritten in a following view:

$$\mathbf{H} = \begin{bmatrix} -\sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \\ -\sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \\ 0 & -1 & 0 \end{bmatrix} \quad (1.3.9.)$$

Variant of 6 sensitive elements location by generatrix of cone.

Orientation of redundant vector measuring instrument which consist of 6 sensitive elements located by generatrix of cone, is presented in fig. 1.9., where Oxyz - basic coordinate system; $O l_1 l_2 l_3 l_4 l_5 l_6$ - measuring coordinate system, an angle $\theta = 54^\circ 44'$.

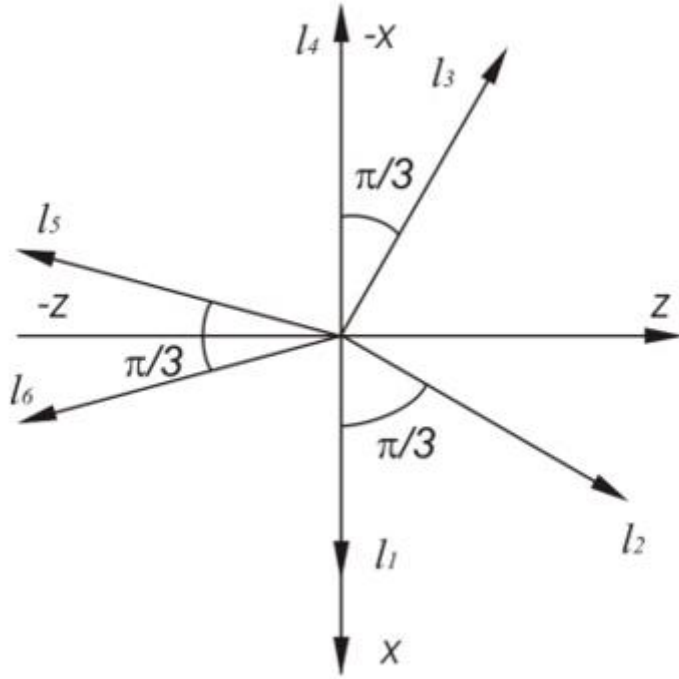


Fig.1.9. sensitive elements orientation over generatrix of cone

The matrix of directing cosines for the given scheme of sensitive elements orientation looks like:

$$\mathbf{H} = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ \cos \pi / 3 \sin \theta & -\cos \theta & \sin \pi / 3 \sin \theta \\ -\cos \pi / 3 \sin \theta & -\cos \theta & \sin \pi / 3 \sin \theta \\ -\sin \theta & -\cos \theta & 0 \\ -\cos \pi / 3 \sin \theta & -\cos \theta & -\sin \pi / 3 \sin \theta \\ \cos \pi / 3 \sin \theta & -\cos \theta & -\sin \pi / 3 \sin \theta \end{bmatrix}$$

(1.3.10.)

Taking into account value of an angle θ , matrix (1.2.2.11) can be overwritten in a following kind:

$$\mathbf{H} = \begin{bmatrix} \sqrt{6}/3 & -\sqrt{3}/3 & 0 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \\ -\sqrt{6}/3 & -\sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & -\sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & -\sqrt{2}/2 \end{bmatrix}$$

(1.3.11.)

Variant of 5 sensitive elements location by generatrix of cone and 1 sensitive element over a symmetry axis.

Orientation of redundant vector measuring instrument which consist of 5 sensitive elements which is located over generatrix of cone and 1 sensitive element which is located over the symmetry axis is presented in fig. 1.10., where Oxyz - basic coordinate system, $O l_1 l_2 l_3 l_4 l_5 l_6$ - measuring coordinate system, angles $\theta=54^\circ44'$, $\alpha=36^\circ$.

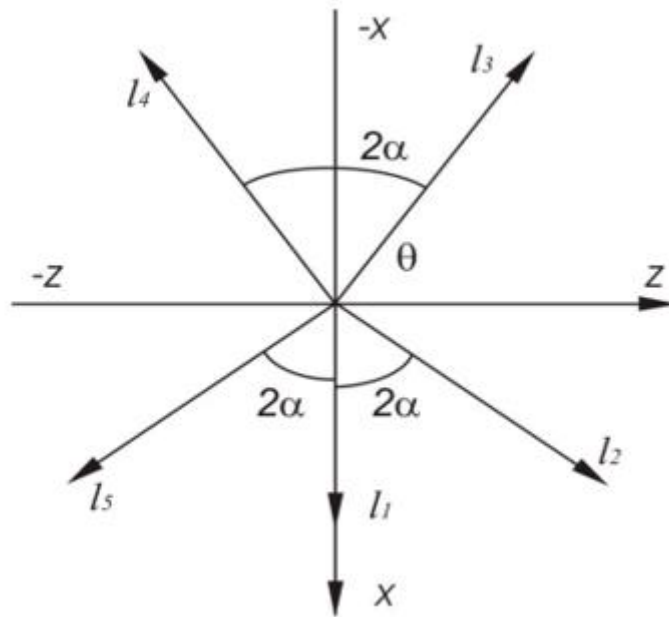


Fig. 1.10. Orientation of 5 sensitive elements over generatrix of cone and 1 sensitive element over a symmetry axis

Directing cosines matrix for the given scheme of sensitive elements orientation has the following view:

$$H = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ \cos 2\alpha \sin \theta & -\cos \theta & \sin 2\alpha \sin \theta \\ -\cos \alpha \sin \theta & -\cos \theta & \sin \alpha \sin \theta \\ -\cos \alpha \sin \theta & -\cos \theta & -\sin \alpha \sin \theta \\ \cos 2\alpha \sin \theta & -\cos \theta & -\sin 2\alpha \sin \theta \\ 0 & -1 & 0 \end{bmatrix}$$

(1.3.12.)

Variant of 6 sensitive elements location over a dodecahedron.

Space orientation of redundant vector measuring instrument is presented in fig. 1.5. Projections of vector of the measuring coordinate system (1.3.2.), consisting of 6 sensitive elements which are located on edges of a dodecahedron in the scalar form it is possible to write taking into account (1.3.4.) in a following view:

$$l_1 = \omega_x \cos \gamma - \omega_y \sin \gamma ,$$

$$l_2 = \omega_x \cos \gamma + \omega_y \sin \gamma ,$$

$$l_3 = \omega_y \cos \gamma - \omega_z \sin \gamma ,$$

$$l_4 = \omega_y \cos \gamma + \omega_z \sin \gamma ,$$

$$l_5 = \omega_z \cos \gamma - \omega_x \sin \gamma ,$$

$$l_6 = \omega_z \cos \gamma + \omega_x \sin \gamma ,$$

(1.3.13.)

where $\gamma = 31^\circ 43'$.

Taking into account (1.3.13.) it is possible to write down a directing cosines matrix \mathbf{H} , which looks like:

$$\mathbf{H} = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \cos \gamma & \sin \gamma & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \cos \gamma & \sin \gamma \\ -\sin \gamma & 0 & \cos \gamma \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix}$$

(1.3.14.)

Symmetric location of measuring instruments supplies identical accuracy of measurement with each measuring instrument that follows from (1.3.1.), (1.3.13.) and (1.3.14.)

1.4. Determination of the redundant measuring equipment optimal orientation

Optimum orientation of redundant measuring instruments should be determined, proceeding from a condition of obtaining of redundant value of the chosen criterion.

Conclusive at criterion selection is necessity of obtaining of the best accuracy (minimization of measurement errors of vector) and reliability [6]. One of stages of the solution of a problem of selection of optimum orientation of redundant measuring instruments consists in obtaining of communications of the chosen criterion with parameter, which is optimized. At obtaining of such communications it is necessary to consider not only orientation of measuring instruments, but also a data processing way, that is, as a whole optimization of a redundant vector measuring instrument should implement simultaneously, as on the parameters, determining orientation, and on parameters of processing of surplus information [7]. One of exhaustive characteristics of quality (accuracy) of measurement is the correlation matrix of errors. At the organization of processing of surplus information and selection of corresponding methods and algorithms it is necessary to start with satisfaction of following basic requirements:

- a) maximum accuracy and reliability;
- b) simplicity of control of a measuring instrument condition, problem diagnostics and reorganizations of structure or adaptation of algorithms of a data processing;
- c) minimum expenses of machine time for a data processing.

Accuracy of estimations depends on quantity and quality of the primary information (measurements) and a way of processing of surplus information.

Under processing of the redundant information [6] it is possible to use a least squares method. For the given method of a data processing at independent statistical characteristics with a zero mathematical expectation as a criterion of optimality we will use the minimum trace of a correlation matrix of errors [8].

Trace of a correlation matrix of errors, is determined by expression:

$$D=[H^T H]^{-1} \quad (1.4.1.)$$

where H^T - the transposed matrix of the directing cosines;

H – a directing cosines matrix that is the sum of its diagonal elements of matrix D , i.e. dispersions of errors:

$$trD = \sum_{i=1}^3 d_{ii} \quad (1.4.2.)$$

where d_{ii} - diagonal elements of matrix D .

It is possible to present algorithm of a least squares method in the form of following matrix expression:

$$W = DH^T E \quad (1.4.3.)$$

or

$$W = ME \quad (1.4.4.)$$

where W - vector of estimations of state vector;

E - vector of measurements;

M - transformation matrix of vector E indications of an redundant measuring instrument in indications of vector W :

$$M = DH^T \quad (1.4.5.)$$

Thus, from the resulted expressions follows that the correlation matrix of errors of a vector redundant measuring instrument depends on orientation of measuring instruments, as is confirmed by results of work.

1.5. Conclusion

Motion properties in control systems are determined using accelerometers or angular velocity meters. The main parameters of such meters include the redundancy orator of the measuring instrument, which is designed to evaluate the effectiveness of the use of redundancy. The use of redundancy allows to reduce the measurement error and also to increase the reliability, because if one of the sensors fails, the measuring device will continue its work with an increased error instead of complete failure.

To build a vector space, there are four variants of positioning sensing elements, the figure of symmetry for which a cone is used, their orientation occurs along the forming cones. Sensing elements are located on the forming cones at intervals equal to the value of a certain angle, and can also be located on and off the axis of symmetry.

One of variants of construction of an excessive vector gauge is its construction with use of a figure of symmetry - a cone. To create such vector space there are four variants of arrangement of sensors, orientation of which takes place according to the forming cones. Sensitive elements are located on the formative cones with an interval equal to the value of a certain angle, and can also be located on and off the axis of symmetry.

CHAPTER 2 RESEARCH ON RELIABILITY OF EXCESSIVE METERS

2.1. Identification of failures of sensitive elements of potential elements

I would like to start with the fact that for ease of detection and localization of failure of the sensitive element of the redundant gauge we will use the criterion of maximum probability density of measurement errors. The accuracy characteristic for this can be the variance of estimates.

The identification task is to determine the number (number in case of several sensors failures) of the failed measurand. The solution of this problem is closely connected to the problem of identifying the failure state of redundant gauges. For all methods of failure state detection based on pairwise comparison, the possibility of detecting the failure state of redundant meters and the maximum number of identifiable failures, depending on the degree of redundancy used, can be summarized in the table. 2.1.

Table. 2.1.

Ability to detect faulty backup meters and the maximum number of identified failures depending on the degree of used backup

Features	Redundancy					
	0	1	2	3	4	5
Ability to detect the state of failure of excessive meters	-	+	+	+	+	+
Maximum number of meter failures	0	0	1	2	3	4

Using the over-information comparison method to identify failed meters makes it possible to identify failures of all excessive meters and, in addition, to fix

the failure of an excessive vector meter based on comparison with the threshold value of the sensing element.

This paper proposes two methods to identify the failure of sensitive elements.

Each sensor measures the signal along its sensitivity axis. The signal of each sensitive element has some error p_i . If in the analytical analysis of the study we neglect such traditional errors as instability of the scale factor and zero drift (zero signal), then the error of measurement of the state vector projections by the given sensing element can be considered equal to zero.

In the presence of these errors of the sensitive element it is possible to obtain the error of measurement of the state vector by the following two methods:

The 1st method consists in finding two maximum values of errors of sensitive elements and disconnecting these sensitive elements as "suspicious" for failure. This method of identification will be called the method of identification without correction.

The 2nd method - method of identification with correction consists of two stages:

- a) determination of the first maximum value;
- b) recalculation of error readings of all sensing elements when disconnecting the sensing element suspected of failure.

Consequently, after applying one of the two methods of identification you can get the numbers of the sensitive element with the maximum values of errors, i.e. suspicious for failure, which must be temporarily disconnected to check their operability. The essence of the check consists in obtaining a state vector (estimation of the state vector) with disabled sensitive elements of the redundant meter. After that, having received an estimate of the state vector according to Table 2.1, we find an estimate of the vector of measurements of the excessive vector meter and directly compare the values and obtained estimates of the "suspicious" for failure sensitive element meters.

If the modulus of difference of value and evaluation of these sensitive elements does not go beyond the set failure threshold value, then this sensitive element is considered operable, but further more detailed check is required to confirm its state.

If the modulus of the difference between the value and the evaluation exceeds the limit value of the operability evaluation (greater than the threshold value of the sensitive element failure), then the given gauge is considered to be incapable of work.

The advantages of these identification methods are as follows:

- these methods are simple to understand;
- they are easily implemented on a computer;
- take little time to perform the identification procedure;
- exclude from the identification algorithm such operations as division, reduction to a degree, etc., that cause certain difficulties for computer implementation;
- allow you to use only the summation operation with a change of scale.

Let us determine the error of meter readings for the above considered schemes of redundant meters.

In summary we have considered how we determine the localization and determination of failure of the sensitive element. Next, I propose to consider the determination of the error of meter readings for the above considered schemes of redundant meters.

2.2. Orientation scheme of 6 sensitive elements on the dodecahedron

Consider pairs of triples of meters symmetric with respect to the faces of the dodecahedron orientation of the base SC (Oxyz):

- 1) $(l_1 \ l_3 \ l_5)$ i $(l_2 \ l_4 \ l_6)$;
- 2) $(l_1 \ l_4 \ l_5)$ i $(l_2 \ l_3 \ l_6)$;
- 3) $(l_1 \ l_3 \ l_6)$ i $(l_2 \ l_4 \ l_5)$;

$$4) (l_1 \ l_4 \ l_6) \quad i \quad (l_2 \ l_3 \ l_5).$$

Let us determine the error of the state vector $\Delta\omega$ for the given pairs of triplets of meters:

$$\Delta\omega = \omega_1 - \omega_2, \tag{2.2.1}$$

where ω_1 is the state vector found with the help of 1st triple meters;

ω_2 is the state vector obtained with the help of 2nd triple meters.

The error vector of the coordinate measuring system is equal to:

$$\mathbf{p} = \mathbf{H}\Delta\omega, \tag{2.2.2}$$

where $\mathbf{p} = [p_1, p_2, \dots, p_n]^T$, n number of meters.

To find the vector \mathbf{p} , it is not important which pair of triple meters we will use. To do this, we find $\Delta\omega$ for the 1st pair of triple meters and, for example, the 4th pair of triple meters.

For the three meters $(l_1 \ l_3 \ l_5)$ we have:

$$\mathbf{H}_{1,3,5} = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ -\sin \gamma & 0 & \cos \gamma \end{bmatrix}, \tag{2.2.3.}$$

For a triple of meters $(l_2 \ l_4 \ l_6)$, the matrix of guiding cosines of this triple of meters $\mathbf{H}_{2,4,6}$ has the form:

$$\mathbf{H}_{2,4,6} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ 0 & \cos \gamma & \sin \gamma \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix} \tag{2.2.4.}$$

The error of the state vector in the projections on the axis of the basic coordinate system has the form:

$$\begin{aligned} \Delta\omega_{xi} = & \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_1 + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_3 + \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_5 - \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 + \\ & + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_4 - \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_6; \end{aligned}$$

$$\begin{aligned}
\Delta \boldsymbol{\omega}_{y1} &= \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_1 + \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_3 + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_5 - \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 - \\
&- \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_4 + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_6; \\
\Delta \boldsymbol{\omega}_{z1} &= \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_1 + \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_3 + \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_5 + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 - \\
&- \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_4 - \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_6.
\end{aligned}
\tag{2.2.5.}$$

Consider the next pair of triple meters, namely, the fourth ($l_1 \ l_4 \ l_6$).

$$\mathbf{H}_{1,4,6} = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ 0 & \cos \gamma & \sin \gamma \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix}
\tag{2.2.6.}$$

For the three meters ($l_2 \ l_3 \ l_5$) we obtain:

$$\mathbf{H}_{2,3,5} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ -\sin \gamma & 0 & \cos \gamma \end{bmatrix}
\tag{2.2.7.}$$

Find the error of the state vector in the projections on the axis of the basic coordinate system:

$$\begin{aligned}
\Delta\omega_{x2} &= \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_1 + \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_4 - \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_6 - \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 + \\
&+ \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_3 + \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_5; \\
\Delta\omega_{y2} &= \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_1 + \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_4 - \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_6 - \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 - \\
&- \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_3 - \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_5; \\
\Delta\omega_{z2} &= \frac{-\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_1 - \frac{\sin^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_4 + \frac{\cos^2 \gamma}{\cos^3 \gamma - \sin^3 \gamma} l_6 - \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 + \\
&+ \frac{\sin^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_3 - \frac{\cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_5.
\end{aligned} \tag{2.2.8.}$$

We show that the vector $\Delta\omega_1$ and the vector $\Delta\omega_2$ coincide.

$$\Delta\omega_1 = \Delta\omega_2 \Leftrightarrow \Delta\omega_1 - \Delta\omega_2 = \mathbf{0} \tag{2.2.9.}$$

Find the projections of the state vector and get the relationship between the projections of the vectors l_3 and l_4, l_5, l_6 :

$$\begin{aligned}
\omega_y &= \frac{1}{2 \sin \gamma \cos^2 \gamma} [2 \sin \gamma \cos \gamma l_4 - \sin^2 \gamma l_5 - \sin^2 \gamma l_6] = \frac{1}{2 \cos^2 \gamma} [2 \cos \gamma l_4 - \sin \gamma (l_5 + l_6)] \\
\omega_x &= \frac{1}{2 \sin \gamma (l_6 - l_5)} \\
\omega_z &= \frac{1}{2 \cos \gamma (l_5 + l_6)}
\end{aligned} \tag{2.2.10.}$$

Then, substituting the obtained expressions for $\omega_x, \omega_y, \omega_z$ in relation, we obtain:

$$l_3 = l_4 - \operatorname{tg} \gamma (l_5 + l_6) \tag{2.2.11.}$$

Then,

$$\begin{aligned} \sin \gamma \cos \gamma (l_3 - l_4) + \sin^2 \gamma (l_5 + l_6) &= \sin \gamma \cos \gamma ((l_4 - t g \gamma (l_5 + l_6)) - l_4) + \sin^2 \gamma (l_5 + l_6) = \\ &= -\sin^2 \gamma (l_5 + l_6) + \sin^2 \gamma (l_5 + l_6) = 0. \end{aligned} \quad (2.2.12.)$$

So from the transformations we get that $\Delta \omega_{x1} = \Delta \omega_{x2}$.

Consider the expressions for the projections $\Delta \omega_{y1}$ and $\Delta \omega_{y2}$. Perform this procedure by analogy with finding $\Delta \omega_{x1}$ and $\Delta \omega_{x2}$, that is, show that

$$\Delta \omega_{y1} - \Delta \omega_{y2} = 0 \quad (2.2.13.)$$

Then, substituting the obtained expressions for ω_x , ω_y , ω_z in relation, we obtain:

or what is the same

$$\left[(l_3 - l_4) \cos^2 \gamma + (l_5 + l_6) \sin \gamma \cos \gamma \right] \cdot \left[\frac{1}{\cos^3 \gamma - \sin^3 \gamma} - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} \right] = 0 \quad (2.2.14.)$$

Expression (2.2.14) is satisfied if there is a relation:

$$(l_3 - l_4) \cos^2 \gamma + (l_5 + l_6) \sin \gamma \cos \gamma = 0 \quad (2.2.15.)$$

Substituting the expression (2.2.11.) In the relation (2.2.15.), We obtain:

$$\begin{aligned} (l_3 - l_4) \cos^2 \gamma + (l_5 + l_6) \sin \gamma \cos \gamma &= (l_4 - t g \gamma (l_5 + l_6) - l_4) \cos^2 \gamma + (l_5 + l_6) \sin \gamma \cos \gamma = \\ &= \sin \gamma \cos \gamma [-(l_5 + l_6) + (l_5 + l_6)] = \Phi. \end{aligned} \quad (2.2.16.)$$

So, we get that $\Delta \omega_{y1} = \Delta \omega_{y2}$.

Consider the performance of relations $\Delta \omega_{z1} = \Delta \omega_{z2}$.

or what is the same:

$$\begin{aligned} \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} 2l_1 + 2l_2 \frac{\sin \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} + \left[\frac{1}{\cos^3 \gamma - \sin^3 \gamma} - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} \right] \times \\ \times (\sin^2 \gamma (l_3 + l_4) + \cos^2 \gamma (l_5 - l_6)) = 0. \end{aligned} \quad (2.2.17.)$$

Let's express indications of sensitive elements l_1, l_2, l_3 through projections l_4, l_5, l_6 . In this case we get:

$$l_3 = l_4 - tg\gamma(l_5 + l_6);$$

$$l_1 = \omega_x \cos \gamma - \omega_y \sin \gamma = \frac{\cos \gamma}{2 \sin \gamma} (l_6 - l_5) - \frac{1}{2 \cos^2 \gamma} [2 \cos \gamma \sin \gamma l_4 - \sin^2 \gamma (l_5 + l_6)];$$

$$l_2 = \omega_x \cos \gamma + \omega_y \sin \gamma = \frac{\cos \gamma}{2 \sin \gamma} (l_6 - l_5) + \frac{1}{2 \cos^2 \gamma} [2 \cos \gamma \sin \gamma l_4 - \sin^2 \gamma (l_5 + l_6)];$$

(2.2.18.)

Substituting expression (2.2.17) in relation (2.2.18), we obtain:

$$2 \sin \gamma \cos \gamma \left(\frac{\frac{\cos \gamma}{2 \sin \gamma} (l_6 - l_5) - tg\gamma l_4 - 0.5tg^2\gamma(l_5 + l_6)}{\cos^3 \gamma - \sin^3 \gamma} + \frac{\frac{\cos \gamma}{2 \sin \gamma} (l_6 - l_5) + tg\gamma l_4 - 0.5tg^2\gamma(l_5 + l_6)}{\cos^3 \gamma + \sin^3 \gamma} \right) +$$

$$+ \left[\frac{1}{\cos^3 \gamma - \sin^3 \gamma} - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} \right] (2l_4 \sin^2 \gamma - \sin^2 \gamma tg\gamma(l_5 + l_6)) + \left[\frac{1}{\cos^3 \gamma - \sin^3 \gamma} + \frac{1}{\cos^3 \gamma + \sin^3 \gamma} \right] \times$$

$$\times \cos^2 \gamma (l_5 - l_6) = \frac{\cos^2 \gamma (l_6 - l_5) - 2 \sin^2 \gamma l_4 + \sin^2 \gamma tg\gamma(l_5 + l_6) + 2 \sin^2 \gamma l_4 - \sin^2 \gamma tg\gamma(l_5 + l_6) + \cos^2 \gamma (l_5 - l_6)}{\cos^3 \gamma - \sin^3 \gamma} +$$

$$+ \frac{\cos^2 \gamma (l_6 - l_5) + 2 \sin^2 \gamma l_4 - \sin^2 \gamma tg\gamma(l_5 + l_6) - 2 \sin^2 \gamma l_4 + \sin^2 \gamma tg\gamma(l_5 + l_6) + \cos^2 \gamma (l_5 - l_6)}{\cos^3 \gamma + \sin^3 \gamma} = 0.$$

(2.2.19.)

Therefore, the above ratios show that $\Delta\omega_{z1} = \Delta\omega_{z2}$. Thus, based on the above, to find the error of the state vector $\Delta\omega$, we can consider one of a pair of triples of sensitive elements symmetrically located relative to the axes of the basic coordinate system Oxyz.

To find the vector p in (2.2.2) we will use the error of the vector of the state of expression (2.2.18).

Taking into account the matrix of guiding cosines H for the vector p in the projections on the sensitivity axis of the meters, we can write the following expressions:

$$\begin{aligned}
p_1 &= \Delta\omega_x \cos\gamma - \Delta\omega_y \sin\gamma \\
p_2 &= \Delta\omega_x \cos\gamma + \Delta\omega_y \sin\gamma \\
p_3 &= \Delta\omega_y \cos\gamma - \Delta\omega_z \sin\gamma \\
p_4 &= \Delta\omega_y \cos\gamma + \Delta\omega_z \sin\gamma \\
p_5 &= \Delta\omega_z \cos\gamma - \Delta\omega_x \sin\gamma \\
p_6 &= \Delta\omega_z \cos\gamma + \Delta\omega_x \sin\gamma
\end{aligned}$$

(2.2.20.)

Substitute (2.2.8) in (2.2.20) and find the projections of the vector p - the error of the meters as a function of the vector l of the sensitive element:

$$\begin{aligned}
p_1 &= l_1 - \frac{\cos^3 \gamma - \sin^3 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_2 + 2 \frac{\sin \gamma \cos^2 \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_4 - 2 \frac{\sin^2 \gamma \cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} l_6 = l_1 - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} \times \\
&\times [(\cos^3 \gamma - \sin^3 \gamma)l_2 - 2 \sin \gamma \cos \gamma (l_4 \cos \gamma - l_6 \sin \gamma)]; \\
p_2 &= \frac{\cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} (\cos^2 \gamma l_1 + \sin \gamma \cos \gamma l_3 + \sin^2 \gamma l_5) - \frac{\cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} (l_2 \cos^2 \gamma - \sin \gamma \cos \gamma l_4 + \sin^2 \gamma l_6) + \\
&+ \frac{\sin \gamma}{\cos^3 \gamma - \sin^3 \gamma} (\sin^2 \gamma l_1 + \cos^2 \gamma l_3 + \sin \gamma \cos \gamma l_5) - \frac{\sin \gamma}{\cos^3 \gamma + \sin^3 \gamma} (\sin^2 \gamma l_2 + \cos^2 \gamma l_4 - \sin \gamma \cos \gamma l_6) = \\
&= -l_2 + \frac{1}{\cos^3 \gamma - \sin^3 \gamma} [(\cos^3 \gamma + \sin^3 \gamma)l_1 + 2 \cos \gamma \sin \gamma (l_3 \cos \gamma + l_5 \sin \gamma)]; \\
p_3 &= \frac{\cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} (\sin^2 \gamma l_1 + \cos^2 \gamma l_3 + \sin \gamma \cos \gamma l_5) - \frac{\cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} (l_2 \sin^2 \gamma + l_4 \cos^2 \gamma - l_6 \sin \gamma \cos \gamma) - \\
&- \frac{\sin \gamma}{\cos^3 \gamma - \sin^3 \gamma} (\sin \gamma \cos \gamma l_1 + l_3 \sin^2 \gamma + l_5 \cos^2 \gamma) + \frac{\sin \gamma}{\cos^3 \gamma + \sin^3 \gamma} (-\sin \gamma \cos \gamma l_2 + \sin^2 \gamma l_4 + \cos^2 \gamma l_6) = \\
&= l_3 - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} [(\cos^3 \gamma - \sin^3 \gamma)l_4 - 2 \sin \gamma \cos \gamma (l_6 \cos \gamma - l_2 \sin \gamma)]; \\
p_4 &= \frac{\cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} (\sin^2 \gamma l_1 + \cos^2 \gamma l_3 + \sin \gamma \cos \gamma l_5) - \frac{\cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} (l_2 \sin^2 \gamma + \cos^2 \gamma l_4 - \sin \gamma \cos \gamma l_6) + \\
&+ \frac{\sin \gamma}{\cos^3 \gamma - \sin^3 \gamma} (l_1 \sin \gamma \cos \gamma + \sin^2 \gamma l_3 + l_5 \cos^2 \gamma) - \frac{\sin \gamma}{\cos^3 \gamma + \sin^3 \gamma} (-\sin \gamma \cos \gamma l_2 + \sin^2 \gamma l_4 + \cos^2 \gamma l_6) = \\
&= -l_4 + \frac{1}{\cos^3 \gamma - \sin^3 \gamma} [(\cos^3 \gamma + \sin^3 \gamma)l_3 + 2 \sin \gamma \cos \gamma (l_1 \sin \gamma + l_5 \cos \gamma)]; \\
p_5 &= \frac{\sin \gamma}{\cos^3 \gamma - \sin^3 \gamma} (l_1 \cos^2 \gamma + \sin \gamma \cos \gamma l_3 + \sin^2 \gamma l_5) + \frac{\sin \gamma}{\cos^3 \gamma + \sin^3 \gamma} (l_2 \cos^2 \gamma - \sin \gamma \cos \gamma l_4 + \sin^2 \gamma l_6) + \\
&+ \frac{\cos \gamma}{\cos^3 \gamma - \sin^3 \gamma} (l_1 \sin \gamma \cos \gamma + l_3 \sin^2 \gamma + l_5 \cos^2 \gamma) - \frac{\cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} (-\sin \gamma \cos \gamma l_2 + \sin^2 \gamma l_4 + \cos^2 \gamma l_6) = \\
&= l_5 - \frac{1}{\cos^3 \gamma + \sin^3 \gamma} [(\cos^3 \gamma - \sin^3 \gamma)l_6 - 2 \sin \gamma \cos \gamma (l_2 \cos \gamma - l_4 \sin \gamma)]; \\
p_6 &= \frac{\sin \gamma}{\cos^3 \gamma - \sin^3 \gamma} (l_1 \cos^2 \gamma + \sin \gamma \cos \gamma l_3 + l_5 \sin^2 \gamma) - \frac{\sin \gamma}{\cos^3 \gamma + \sin^3 \gamma} (l_2 \cos^2 \gamma - \sin \gamma \cos \gamma l_4 + \sin^2 \gamma l_6) + \\
&+ \frac{\cos \gamma}{\cos^3 \gamma + \sin^3 \gamma} (-\sin \gamma \cos \gamma l_2 + \sin^2 \gamma l_4 + \cos^2 \gamma l_6) = -l_6 + \frac{1}{\cos^3 \gamma - \sin^3 \gamma} \times \\
&\times [(\cos^3 \gamma + \sin^3 \gamma)l_5 + 2 \sin \gamma \cos \gamma (l_1 \cos \gamma + l_3 \sin \gamma)]
\end{aligned}$$

(2.2.21)

Therefore, in (2.2.21) we characterize the vector p of the errors of the meters in the projections on the axis of the measuring coordinate system for the scheme of orientation of the sensitive elements along the dodecahedron.

2.3. Scheme of orientation of 6 sensing elements by forming cones

For this scheme of orientation of the sensitive element we will consider a pair of triple meters, also symmetrically located relative to the axes of the basic coordinate system, taking into account the property of independence of finding the error of the state vector $\Delta\omega$.

Consider a pair of triple meters $(l_1 \ l_3 \ l_5)$ and $(l_2 \ l_4 \ l_6)$.

Determine $\Delta\omega$ according to (2.2.1). We will use the matrix of guiding cosines H (2.2.8):

$$\mathbf{H}_{1,3,5} = \begin{bmatrix} \sqrt{6}/3 & -\sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \\ -\sqrt{6}/6 & -\sqrt{3}/3 & -\sqrt{2}/2 \end{bmatrix} \quad (2.2.22)$$

Projections of the vector $\Delta\omega$ on the axis of the basic coordinate system:

$$\begin{aligned} \Delta\omega_x &= \sqrt{6}/6(2l_1 - l_3 - l_5 + 2l_4 - l_2 - l_6); \\ \Delta\omega_y &= \sqrt{3}/3(l_2 + l_4 + l_6 - l_1 - l_3 - l_5); \\ \Delta\omega_z &= \sqrt{2}/2(l_3 - l_5 - l_2 + l_6). \end{aligned} \quad (2.2.23)$$

Now, taking into account (1.2.8) and (2.2.23.) The projection of the vector p on the axis of the measuring coordinate system have the following form:

$$\begin{aligned}
p_1 &= \sqrt{6}/3\Delta\omega_x - \sqrt{3}/3\Delta\omega_y = l_1 + \frac{1}{3}[l_4 - 2(l_2 + l_6)]; \\
p_2 &= \sqrt{6}/6\Delta\omega_x - \sqrt{3}/3\Delta\omega_y + \sqrt{2}/2\Delta\omega_z = -\left[l_2 + \frac{1}{3}(l_5 - 2(l_1 + l_3))\right]; \\
p_3 &= -\sqrt{6}/6\Delta\omega_x - \sqrt{3}/3\Delta\omega_y + \sqrt{2}/2\Delta\omega_z = l_3 + \frac{1}{3}[l_6 - 2(l_2 + l_4)]; \\
p_4 &= -\sqrt{6}/3\Delta\omega_x - \sqrt{3}/3\Delta\omega_y = -\left[l_4 + \frac{1}{3}(l_1 - 2(l_3 + l_5))\right]; \\
p_5 &= -\sqrt{6}/6\Delta\omega_x - \sqrt{3}/3\Delta\omega_y - \sqrt{2}/2\Delta\omega_z = l_5 + \frac{1}{3}[l_2 - 2(l_4 + l_6)]; \\
p_6 &= \sqrt{6}/6\Delta\omega_x - \sqrt{3}/3\Delta\omega_y - \sqrt{2}/2\Delta\omega_z = -\left[l_6 + \frac{1}{3}(l_3 - 2(l_1 + l_5))\right].
\end{aligned}
\tag{2.2.24}$$

Consequently, the set of expressions (2.2.24) characterizes the vector p of errors of meters in projections on the axes of the measuring coordinate system for the orientation scheme of 6 sensitive elements along the formative cones.

2.4. Scheme of orientation of 5 sensing elements along the forming cones and 1 sensing element along the axis of symmetry

Consider a pair of triples of gauges $(l_1 \ l_3 \ l_5)$ and $(l_2 \ l_4 \ l_6)$ to find the vector , according to expression (2.2.1) and the matrix of guiding cosines[1]:

$$\mathbf{H}_{1,3,5} = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ -\cos \alpha \sin \theta & -\cos \theta & \sin \theta \sin \alpha \\ \cos 2\alpha & -\cos \theta & -\sin 2\alpha \sin \theta \end{bmatrix}
\tag{2.2.25}$$

Where $\theta = 54^\circ 44'$, $\alpha = 36^\circ$.

Find the inverse matrix

$$\det_1 = \sin^2 \theta \cos \theta (\sin 2\alpha + \sin \alpha + \sin 2\alpha \cos \alpha - \sin \alpha \cos 2\alpha) = \sin^2 \theta \cos \theta (\sin 2\alpha + 2 \sin \alpha);$$

$$\begin{aligned}
\mathbf{H}_{1,3,5}^{-1} &= \frac{1}{\sin^2 \theta \cos \theta (\sin 2\alpha + 2 \sin \alpha)} \times \\
&\times \begin{bmatrix} (\sin 2\alpha + \sin \alpha) \sin \theta \cos \theta & -\sin 2\alpha \sin \theta \cos \theta & -\sin \alpha \sin \theta \cos \theta \\ -\sin \alpha \sin^2 \theta & -\sin 2\alpha \sin^2 \theta & -\sin \alpha \sin^2 \theta \\ 0 & \sin \theta \cos \theta (1 - \cos 2\alpha) & -\sin \theta \cos \theta (1 + \cos \alpha) \end{bmatrix}
\end{aligned}
\tag{2.2.25}$$

$$\mathbf{H}_{6,4,2} = \begin{bmatrix} 0 & -1 & 0 \\ -\cos \alpha \sin \theta & -\cos \theta & -\sin \alpha \sin \theta \\ \cos 2\alpha \sin \theta & -\cos \theta & \sin 2\alpha \sin \theta \end{bmatrix}$$

(2.2.26)

$$\det_2 = \sin^2 \theta (-\sin 2\alpha \cos \alpha + \sin \alpha \cos 2\alpha) = -\sin \alpha \sin^2 \theta$$

$$\mathbf{H} = -\frac{1}{\sin \alpha \sin^2 \theta} \begin{bmatrix} -(\sin 2\alpha + \sin \alpha) \sin \theta \cos \theta & \sin 2\alpha \sin \theta & \sin \alpha \sin \theta \\ \sin \alpha \sin^2 \theta & 0 & 0 \\ (\cos \alpha + \cos 2\alpha) \sin \theta \cos \theta & -\cos 2\alpha \sin \theta & -\cos \alpha \sin \theta \end{bmatrix}$$

(2.2.27)

Using relations (2.2.25) and (2.2.27), find the vector projections on the axes of the reference coordinate system Oxyz:

$$\begin{aligned} \Delta \omega_x &= \frac{1}{\sin \theta} \left[\frac{(2 \cos \alpha + 1)l_1 - 2 \cos \alpha l_3 - l_5}{2(\cos \alpha + 1)} - (2 \cos \alpha l_4 + l_2 - l_6 \cos \theta (2 \cos \alpha + 1)) \right]; \\ \Delta \omega_y &= -\frac{1}{\cos \theta} \left[\frac{l_1 + l_5 + 2 \cos \alpha l_3}{2(\cos \alpha + 1)} - l_6 \cos \theta \right]; \\ \Delta \omega_z &= \frac{1}{\sin \theta \sin \alpha} \left[\frac{(1 - \cos 2\alpha)l_3 - (1 + \cos \alpha)l_5}{2(1 + \cos \alpha)} + l_6 (\cos \alpha + \cos 2\alpha) \cos \theta - \cos 2\alpha l_4 - \cos \alpha l_2 \right]. \end{aligned}$$

(2.2.28)

Find the projections of the vector p on the axes of the measuring system of coordinates:

$$\begin{aligned} p_1 &= l_1 + l_6 \cos \theta (2 \cos \alpha - 2 \cos \alpha l_4 - l_2); \\ p_2 &= -\left[l_2 (4 \cos^2 \alpha - 1) - \frac{l_1 (\cos 2\alpha (2 \cos \alpha + 1) + 1) + 4 \cos \alpha (1 - \cos 2\alpha) l_3 - 2(\cos \alpha + \cos 2\alpha) l_5}{2(\cos \alpha + 1)} \right]; \\ p_3 &= l_3 + l_4 - l_1 \frac{\cos \alpha (2 \cos \alpha + 1) + 1}{2(\cos \alpha + 1)} + 2l_6 \cos \theta (\cos 2\alpha + \cos \alpha); \\ p_4 &= l_4 (4 \cos^2 \alpha - 1) + l_5 + l_2 (2 \cos \alpha - l_6 \cos \theta (4 \cos^2 \alpha + 2 \cos \alpha) + \\ &+ \frac{l_1 (1 - \cos \alpha (2 \cos \alpha + 1)) + 2l_3 (\cos 2\alpha + \cos \alpha)}{2(1 + \cos \alpha)}); \\ p_5 &= l_5 + l_2 - 2l_6 \cos \theta + l_1 \frac{1 + \cos 2\alpha (2 \cos \alpha + 1)}{2(\cos \alpha + 1)}; \\ p_6 &= -\left[l_6 - \frac{l_1 + l_5 + 2 \cos \alpha l_3}{\cos \theta (2(\cos \alpha + 1))} \right]. \end{aligned}$$

(2.2.29)

Consequently, the expression (2.2.29) characterizing the vector p in projections on the axes of the measuring coordinate system is obtained. It is not difficult to notice the connection with the readings of the most sensitive elements in these expressions.

2.5. Scheme of orientation of the 5 sensing elements along the forming cones

Consider a pair of triples of gauges in which one of the gauges is "common," for example (l_1, l_2, l_3) i l_1, l_4, l_5 .

Let's show that if there is a common gauge in the triplet, two of the three projections of the state vector error $\Delta\omega$ on the base coordinate system axes will be proportional, which corresponds to the gauge proportionality theorem.

Given the matrix of directional cosines, find the inverse matrices $\mathbf{H}_{1,2,3}^{-1}$ i

$\mathbf{H}_{1,4,5}^{-1}$:

$$\mathbf{H}_{1,2,3} = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ \cos 2\alpha \sin \theta & -\cos \theta & \sin 2\alpha \sin \theta \\ -\cos \alpha \sin \theta & -\cos \theta & \sin \alpha \sin \theta \end{bmatrix} \quad (2.2.30)$$

$$\mathbf{H}_{1,4,5} = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \\ -\cos \alpha \sin \theta & -\cos \theta & -\sin \alpha \sin \theta \\ \cos 2\alpha \sin \theta & -\cos \theta & -\sin 2\alpha \sin \theta \end{bmatrix} \quad (2.2.31)$$

Where $\theta = 54^\circ 44'$; $\alpha = 36^\circ$.

Find the inverse matrices:

$$\det_1 = \det_2 = \det = \sin^2 \theta \cos \theta (\sin 3\alpha + \sin 2\alpha - \sin \alpha)$$

$$\mathbf{H}_{1,2,3}^{-1} = \frac{1}{\det} \begin{bmatrix} (-\sin \alpha + \sin 2\alpha) \sin \theta \cos \theta & \sin \alpha \sin \theta \cos \theta & -\sin 2\alpha \sin \theta \cos \theta \\ -\sin 3\alpha \sin^2 \theta & \sin \alpha \sin^2 \theta & -\sin 2\alpha \sin^2 \theta \\ -(\cos \alpha + \cos 2\alpha) \sin \theta \cos \theta & \sin \theta \cos \theta (1 + \cos \alpha) & -\sin \theta \cos \theta (1 - \cos 2\alpha) \end{bmatrix} \quad (2.2.32)$$

$$\mathbf{H} = \frac{1}{\det} \begin{bmatrix} (-\sin \alpha + \sin 2\alpha) \sin \theta \cos \theta & -\sin 2\alpha \sin \theta \cos \theta & \sin \alpha \sin \theta \cos \theta \\ -\sin 3\alpha \sin^2 \theta & -\sin 2\alpha \sin^2 \theta & \sin \alpha \sin^2 \theta \\ (\cos \alpha + \cos 2\alpha) \sin \theta \cos \theta & \sin \theta \cos \theta (1 - \cos 2\alpha) & -\sin \theta \cos \theta (1 + \cos \alpha) \end{bmatrix} \quad (2.2.33)$$

Find the projections of the vector on the axes of the reference coordinate system:

$$\begin{aligned} \Delta \omega_x &= \frac{\sin \theta \cos \theta}{\det} [l_2 \sin \alpha - l_3 \sin 2\alpha + l_4 \sin 2\alpha - l_5 \sin \alpha]; \\ \Delta \omega_y &= \frac{\sin^2 \theta}{\det} [l_2 \sin \alpha - l_3 \sin 2\alpha + l_4 \sin 2\alpha - l_5 \sin \alpha]; \\ \Delta \omega_z &= \frac{\sin \theta \cos \theta}{\det} [-2l_1 (\cos 2\alpha + \cos \alpha) + (1 + \cos \alpha)(l_2 + l_5) - (1 - \cos 2\alpha)(l_3 + l_4)]. \end{aligned} \quad (2.2.34)$$

It is easy to see that in the relation (2.2.34.): $\Delta \omega_y = \operatorname{tg} \theta \cdot \Delta \omega_x$, the two projections ω_x , ω_y are proportional.

By its physical nature, the vector p characterizes the deviation of the vector l , that is, some error in the readings of the sensitive element l_i .

In other words, if the readings of the sensing element have no error, then $p_i = 0$. It is not difficult to express the readings of one sensitive element because of the readings of other sensitive elements of the chosen orientation scheme. Then the error of the given sensitive element will be characterized by the difference between the signal of the sensitive element and the function from other sensitive elements of the given scheme. Let us consider the proposed approach of finding the errors p_i in more detail. We will consider the projection on the z-axis, i.e. ω_z , as a finding of the state vector projection ω_z , since the relation (2.2.34.) is valid for this circuit.

For the expression when the common sensitive element is the first element, we get:

$$\omega_z = f_1(l_1, l_2, l_3) = f_2(l_1, l_4, l_5) \quad (2.2.35)$$

From where:

$$l_1 = \frac{1}{2(\cos 2\alpha + \cos \alpha)} [(l_3 + l_4)(1 - \cos 2\alpha) - (l_2 + l_5)(1 + \cos \alpha)] = f(l_3, l_4, l_2, l_5) \quad (2.2.36)$$

Then:

$$p_1 = l_1 - f(l_3, l_4, l_2, l_5) \quad (2.2.36)$$

For the expression when the common sensitive element is the second element, we get:

$$\omega_z = f_1(l_2, l_3, l_4) = f_2(l_2, l_5, l_1) \quad (2.2.37)$$

From where:

$$\begin{aligned} l_2 &= l_5 + (l_3 - l_4)2\cos \alpha, \\ p_2 &= l_2 - f(l_5, l_3, l_4) \end{aligned} \quad (2.2.38)$$

For the expression when the common sensitive element is the third element, we get:

$$\omega_z = f_1(l_3, l_4, l_5) = f_2(l_3, l_1, l_2) \quad (2.2.39)$$

From where:

$$\begin{aligned} l_3 &= \frac{1}{\sin 3\alpha + \sin 2\alpha + \sin \alpha(1 - \cos 2\alpha)} \times \\ &\times [l_4(\sin 3\alpha + \sin 2\alpha - \sin \alpha) + 2\sin \alpha(l_2(1 + \cos \alpha) - l_1(\cos \alpha + \cos 2\alpha))], \\ p_3 &= l_3 - f(l_4, l_2, l_1) \end{aligned} \quad (2.2.40.)$$

For the expression when the common sensitive element is the fourth element, we get:

$$\omega_z = f_1(l_4, l_5, l_1)f_2(l_4, l_2, l_3) \quad (2.2.41.)$$

From where:

$$l_4 = \frac{1}{\sin 3\alpha + \sin 2\alpha + \sin \alpha(1 - \cos 2\alpha)} \times$$

$$\times [l_3(\sin 3\alpha + \sin 2\alpha - \sin \alpha) + 2\sin \alpha(l_5(1 + \cos \alpha) - l_1(\cos \alpha + \cos 2\alpha))]$$

$$p_4 = l_4 - f(l_3, l_5, l_1)$$
(2.2.42.)

For an expression where the common sensitive element is the fifth element, we get:

$$\omega_z = f_1(l_5, l_1, l_2) = f_2(l_5, l_3, l_4)$$
(2.2.43.)

From where:

$$l_5 = l_2 + 2\cos \alpha(l_4 - l_3),$$

$$p_5 = l_5 - f(l_2, l_4, l_3)$$
(2.2.44.)

So, we got expressions (2.2.36), (2.2.38), (2.2.40), (2.2.42), (2.2.44) and (2.2.46), characterizing a vector of gauges errors in projections on measuring system coordinate axes.

2.6. Conclusion

Motion properties in control systems are determined using accelerometers or angular velocity meters. The main parameters of such meters include the redundancy orator of the measuring instrument, which is designed to evaluate the effectiveness of the use of redundancy. The use of redundancy allows to reduce the measurement error and also to increase the reliability, because if one of the sensors fails, the measuring device will continue its work with an increased error instead of complete failure.

To build a vector space, there are four variants of positioning sensing elements, the figure of symmetry for which a cone is used, their orientation occurs

along the forming cones. Sensing elements are located on the forming cones at intervals equal to the value of a certain angle, and can also be located on and off the axis of symmetry.

One of variants of construction of an excessive vector gauge is its construction with use of a figure of symmetry - a cone. To create such vector space there are four variants of arrangement of sensors, orientation of which takes place according to the forming cones. Sensitive elements are located on the formative cones with an interval equal to the value of a certain angle, and can also be located on and off the axis of symmetry.

CHAPTER 3

TECHNOLOGICAL FEATURES AND EXAMPLES OF MODERN MEMS GYROSCOPES

3.1. Analog Devices

Analog Devices is a leader in the development of MEMS gyroscopes, the main representatives of which are chamberton gyroscopes of different types.

Its sensors are represented in the market by iMEMS® and iSensor™ Gyros technologies, which constitute the main part of MEMS gyroscopes for angular velocity measurement. The iMEMS technology concentrates on the placement of the micromechanical sensor structure and processing circuitry on a single chip [9].

Gyroscopes using this technology are represented by the ADXRS family, representative images are shown in Fig. 3.1.1.

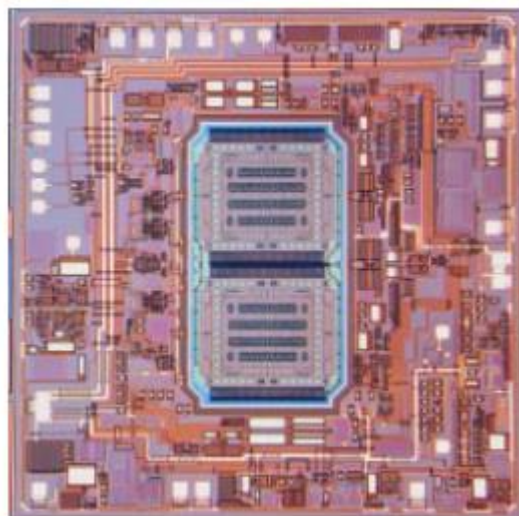


Fig. 3.1.1. AXDRS type gyrosopic speed meter manufactured by Analog Devices

3.2. iSensor Gyros

The special feature of the iSensor Gyros technology is the built-in signal control and processing. It is implemented in the ADIS Gyros sensor line.

Both technologies are performed on a single crystal, which houses the integrated circuit for the specific implementation and the mechanical part. Due to

the fact that the mass is connected to a polycrystalline frame, resonance occurs in the same direction. Capacitive sensors measure the displacement of the resonating mass and the frame, which is due to the Coriolis force.

All gyroscopes in the ADXRS family are designed to measure angular velocity. They are used for safety, navigation, stabilization and inertial measurements. These gyroscopes feature a capacitive measurement capability of $12 \cdot 10^{-21} \text{F}$ with a deviation of $16 \cdot 10^{-4} \text{E}$. This family is also characterized by increased resistance to vibration and overloads up to 1 g. Two independent resonators are used for angular velocity measurements, which makes it possible to ignore external accelerations. The measurement results are two signals of different polarity, the difference of which is proportional to the angular velocity.

The AIDS iSensor line of gyroscopes are designed to improve the performance of the ADXRS series by adding signal processing functionality such as programming, power management, adding digital input or output, adding additional ASIC interfaces, etc. These sensors have a wide range of applications in navigation, security, unmanned systems, autopilot systems, communication systems, etc.

One of the representatives of this line is ADIS16060, the scheme of which is shown in Fig. 3.2.1. In which the mechanical signal after demodulation is fed to the SPI interface that provides digital data proportional to angular velocity. The unified construction of the sensor and signal processing means allows to increase the resistance of the sensor to external noise. The SPI interface also provides external temperature measurement data. [10].

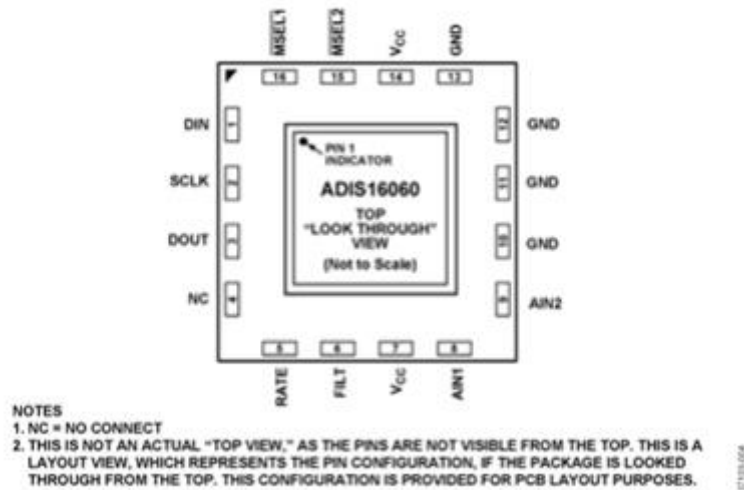


Fig. 3.2.1. ADIS16060 gyro circuit with SPI interface

3.3. BEI Systron Donner

BEI Systron Donner is the main developer and supplier of gyroscopes for automobile transport. The sensors are produced on the basis of all-quartz inertial sensor (Fig. 3.3.1.). These MEMS gyroscopes are based on vibrating tuning forks and piezoelectric processing principles. Due to the use of piezoelectric quartz material, there is a simplification of the sensor element capable of operating in wear-free modes, thus providing increased reliability, stability and durability.

Illustrations of the sensing element (Fig.3.3.1), the finished enclosure (Fig.3.3.2) and the modular version (Fig.3.3.3) are shown below[11].



Fig.3.3.1. Systron Donner quartz sensing element

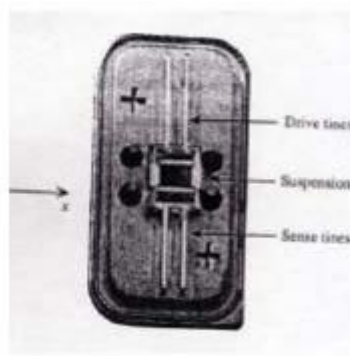


Fig.3.3.2. Enclosure design



Fig.3.3.3. Modular design

3.4. Silicon Sensing Systems

Silicon Sensing Systems gyroscopes use the Silicon Vibrating Structure Gyroscope (SiVSG) technology to create a vibration ring whose mode changes due to the Coriolis force (Fig. 3.4.1). The ring resonator is supplemented with a central magnet to create electromagnetic resonance. A current is transmitted to the supports of the ring which creates resonance. The motion of the ring is determined by the voltage applied to the supports[12].

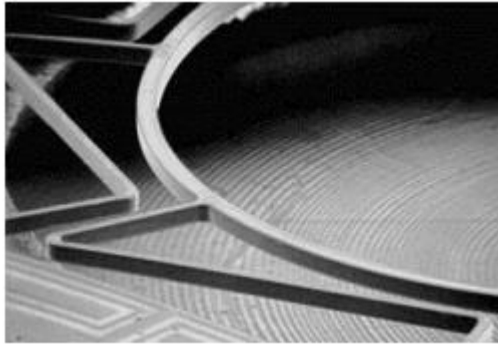


Fig. 3.4.1. Resonating ring volume, gyroscope manufactured by Silicon Sensing Systems



Fig. 3.4.2. A gyroscope design with a permanent magnet mounted on top of the sensor

3.5. Melexis

The MLX90609 gyroscope from Melexis.

This sensor consists of a silicon micromechanical part and a signal processing circuit. The manufacturing is based on a process called SOI (Silicon on Insulator). Each part of the gyro structure consists of a two frame silicon based gyroscope. A special feature of this gyroscope is the ability to generate two types of signal: digital and analog, proportional to angular velocity. The gyroscope meter has the following applications: navigation, stabilization and robotics[13].



Fig. 3.5.1. The MLX90609 gyroscope is manufactured by Melexis

3.6. Conclusions on the structure and characteristics of modern MEMS gyroscopes

Gyroscopes occupy a significant portion of the MEMS market in the integrated sensor industry. Applications do not end with navigation and are widely used in medical, industrial, security, communications, and stabilization systems.

The MEMS gyro consists of a resonator mounted on the sensing axis and a micro sensor mounted on the measuring axis. The characteristics of the gyroscope are mostly dependent on possible deviations during manufacturing and design of the layout, linear acceleration and temperature.

Micromechanical sensors, discussed in the previous subsection, form the basis for the development of new modifications of MEMS gyroscopes. The most relevant for the present are capacitive vibration gyroscopes, the operation of which is based on the Coriolis effect.

CONCLUSION

In the course of the thesis an analysis of implementations of micromechanical gyroscopes with angular velocity sensors was carried out. Traditional resonant gyroscopes occupy most of the market, since their cost is the most optimal, given their characteristics.

Manufacturers of gyroscopes offer a wide choice of layout of elements of micromechanical sensors. Thus, Analog Devices offers gyroscopes with placement of both the sensor and the system for processing its data on one chip, such layout allows to avoid external noise.

A surplus of micro mechanical gyroscopes is necessary to increase the reliability and accuracy of the measurement, this is done by increasing the number of sensors. Thus, if one of the sensors fails, the measuring device will still work, but with less accuracy.

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