

PAIRING IN (QUANTUM) CALABI-YAU VARIETIES AND APPLICATIONS

We introduce and investigate pairings and dualities in (quantum) Calabi-Yau (CY) type varieties over finite and local fields and consider applications of the dualities and pairings. By CY variety type variety we understand algebraic variety X over complex numbers with zero canonical class K_X . The one-dimensional CY varieties are elliptic curves and two-dimensional CY varieties are two-dimensional abelian varieties (abelian surfaces) and K3 surfaces. Quantum varieties appears after quantization of sheaves on the varieties.

Most of our considerations refer to these types of CY varieties. Dualities and pairing in elliptic curve and abelian varieties have investigated by A. Weil, I.R. Shafarevich, A. Grothendieck, J.-P. Serre, J. Tate, O.N. Vvedenskii and others. A. Weil pairing, J. Tate pairing, S. Lichtenbaum pairing and another pairings over finite fields found applications in cryptography.

Monodromy pairing has defined by A. Grothendieck. Let A be an abelian variety over complete local field, let A' denote its dual and $\pi(A)$ its fundamental group. Grothendieck has defined in the case of abelian varieties with semi-stable reduction a pairing between $\pi(A)$ and $\pi(A')$ with values in the ring of integer numbers. This type of pairing have investigated and extended to another type of algebraic varieties by R. Coleman, A. Stewart, V. Vologodsky and others.

For elliptic curves, abelian surfaces and K3 surfaces it is possible to construct corresponding formal groups. In this communication we prove several properties of dualities and pairing in the framework of formal groups. Computer algebra aspects of these considerations and applications will be presented.

References

1. Glazunov N.M. Stochastic Properties of Dynamical Systems Arising from (quantum) Spaces and Actions of (quantum) Groups // Chaotic Modeling and Simulation (CMSIM) 2013. – 3. - 395 - 401.
2. Glazunov N.M. On norm maps and "universal norms" of formal groups over integer rings of local fields // Continuous and Distributed Systems. Theory and Applications. Springer. 2014. - P. 73 – 80.