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Simulation of nonstationary energetic random processes with the use of stochastic linear processes

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The comparative analysis of both, axiomatic energetic process simulation and constructive energetic process simulation and their use in electrical power engineering is considered. It is demonstrated, that we can find n-dimensional characteristic functions of the simulating process and distribution functions related to them with Poisson hopping spectrum in Kolmogorov form for the processes with discrete or continuous time.

Key words: linear process, axiomatic and constructive approaches, hopping spectrum, simulation, spatiotemporal signals.

Preface

Linear random processes are widely spread in theoretical and application research, notably in the tasks of radio engineering signals modeling, technical diagnostics, hydro acoustics, geophysics, and medical diagnostics [9]. The main advantage of the linear random process is its constructiveness as far as it gives the possibility to use the linear random process for initiating of the signals on computers. In addition, the description of signals and their analysis in terms of multidimensional functions of distribution and characteristic functions are also possible in the linear random process.

The study of random process models and their computer simulation play a vital role while performing a wide range of application tasks. The way of random process simulation is defined by its task. This paper specifies simulation as operation aimed to receive realizations of some processes with particular stochastic features. The functions obtained can be regarded to as models and realizations. It is known that complete description of a stochastic process $\{\xi(t), \quad t \in T\}$, where T represents some parametric multitude (usually time or space) that gives the sequence of finite-dimensional distribution functions expressed as follows:

$$F(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n), \quad \forall t_1, t_2, \ldots \in T, \qquad n = 1,2,\ldots, \quad x_1, \ldots, x_n \in \mathbf{R}^n. \tag{1}$$

It is worth to notice that the elements of the sequence (1) cannot be chosen arbitrarily. The must correspond to specific conditions (invariance and coherency) [4].

Such a process description is called axiomatic one, as far as it is based on the axiom system introduced by A.Kolmogorov [6] The system is used in modern probability theory and the theory of random processes. To describe the simulated process a constructive approach can also be used for it is more demonstrative and practical; moreover, the parameters of its constructions are suitable for describing of physical phenomena.

Nowadays the constructive model of the linear random process is widely used as it is close to convolution of functions and the model of filtration of some initiating process with the filter having deterministic impulse reaction. In this condition two types of the simulated linear process patterns are used, namely, linear processes with discrete time and linear processes with continuous time. In the letter the integral pattern of the simulated linear process is used.

Axiomatic and constructive approaches have a lot in common theoretically, they must be coherent but each of them has some advantages over another. Consider the detailed description of these approaches for the tasks of energetic processes simulation and their use.

Task definition

The purpose of the work is to compare axiomatic and constructive approaches and to define the ways of their solution. The use of linear random process models for simulation of different processes in electric engineering is also considered.

1. *Axiomatic approach* [1]. With the use of this approach in the widest way finite-dimensional sequence of distribution functions is specified. (1) In practice it means that it is necessary to specify the law and general form of the elements of finite-dimensional sequence of distribution that belong to (1). After that to develop the algorithm and the programs with the help of simulation methods that would simulate separate realizations of the random process for any predefined *n*. It is clear that such a task is related to both vast majority of simulated values and the necessity of verification of the received realizations to define their proximity to finite-dimensional distribution functions specified in the task. Although at every fixed *n* such a task can be solved with modern electronic calculating device sometimes this approach gives the answer that is hard to comprehend and analyze.

Concerning the way of accuracy verification of simulation histogram analysis can be used in both multidimensional and one-dimensional case. But even under such conditions histogram analysis well-developed in one-dimensional case is weakly developed in multidimensional case (both in the theory and in the realization of algorithms and programs). Hence, constructive approach is widely used only in terms of two original distribution functions taken from the sequence (1). Moreover, this analysis often comes to the use of L_2-theory (correlation theory).

2. *The use of constructive models* [3,6,8]. Different constructive models are used with axonometric models nowadays. For example, one of the widely used models is multiplicative simulation of two process realizations constituting the product of two random quantities. One of these quantities has inverse sine

distribution another one has Relay distribution. It is known that such a product reproduces the third random value with Gaussian distribution at a concurrent time.

The disadvantage of multiplicative models is the lack of research of their constructions. In certain way the development of multiplicative models along with additive models and boundary transition is the model of dispreading type, from the deeper insight it is the preliminary step of the creation of the linear process model. The use of linear processes widens the class of simulating processes and links constructive axiomatic approaches. It became possible due to theorems of linear processes that state the connection between the elements of the linear process construction and the canonical form of their characteristic functions and characteristic functionals.

It is worth saying that the two approaches can be used in simulating of both stationary and non-stationary random processes and random fields. The constructive model of the linear random process is suitable for simulation of different cyclic phenomena in electric engineering. For example, it can be used for energy load in power transmission systems and shake and noise analysis in the work of power energetic objects.

Consider the definition of the linear process and its relation to axiomatic and constructive models in detail.

Linear random process models

Consider discrete time linear random process and continuous time linear random process. Discrete time linear random process can be described as a discrete convolution of initiating process representing a set of independent values with some numeric sequence. Sometimes such a convolution is called the process of moving total.

The process of moving total [1] can be defined as follows:

Definition 1. *Let* $\ldots\zeta_{-1},\zeta_0,\zeta_1,\ldots$ *be the real sequence of independent and identically distributed random values ,* $\varphi_0,\varphi_1,\ldots,\varphi_q$ *the sequence of* $q+1$ *arbitrary values.*

Then :

$$\xi_t = \varphi_0\zeta_t + \varphi_1\zeta_{t-1} + \ldots + \varphi_q\zeta_{t-q}; \quad t = \ldots,-1,0,1,\ldots \tag{2}$$

is called the process of moving total (moving average with complete limits of summation).

In a special case when $\varphi_0 = \varphi_1 = \ldots = \varphi_q = \dfrac{1}{q+1}$ is arbitrary value ξ_t with arithmetic average used in statistics tasks at data smoothing.

It results form the definition that it is the process with independent increment. If $f_\zeta(u)$ is defined as the characteristic function of the arbitrary element of the sequence of identically distributed random values $\ldots\zeta_{-1},\zeta_0,\zeta_1,\ldots$, then the characteristic function of moving average process (2) can be defined as follows:

$$f_{\xi}(u;t) = \mathbf{M}\exp(iu\xi_t) = \mathbf{M}\exp\left(iu\sum_{j=0}^{q}\varphi_j\zeta_{t-j}\right) =$$
$$= \prod_{j=0}^{q}\mathbf{M}\exp(iu\varphi_j\zeta_{t-j}) = \prod_{j=0}^{q} f(u\varphi_j)\ . \qquad (3)$$

In the sequel $...\zeta_{-1}, \zeta_0, \zeta_1, ...$ sequence is specified as generating sequence of a moving sum (2).

On condition that the characteristic function (3) is infinitely divisible Gilbert one that is $\mathbf{M}\zeta^2 < \infty$, then the logarithm of characteristic function (3) can be represented in Kolmogorov canonical form

$$\ln f_{\zeta}(u;t) = im_{\zeta}u + \int_{-\infty}^{\infty}\left(e^{iux} - 1 - iux\right)\frac{dK_{\zeta}(x)}{x^2}, \qquad (4)$$

at which m_{ζ} is ensemble average ζ and $K_{\zeta}(x)$ is a spectral function of Kolmogorov hopping, m_{ζ} and $K_{\zeta}(x)$ parameters explicitly specify the characteristic function of ζ_t -process (3) and the law of ζ distribution. Vice versa, at the characteristic function (4) of Gilbert infinitely divisible law m_{ζ} and $K_{\zeta}(x)$ parameters are specified explicitly.

Annotation.

1. This process can be specified as follows instead of expression (2) at more general case:

$$\xi_t = \sum_{\tau=-\infty}^{\infty}\varphi_{\tau}\zeta_{t-\tau}U_{\tau}U_{q-\tau}, \quad t \in \left(\overline{-\infty,\infty}\right), \qquad (5)$$

where U_x is Heaviside function $U_x = \begin{cases} 0 & npu\ x < 0, \\ 1 & npu\ x \geq 0 \end{cases}$.

2. Expression (2) can be written as $\xi_t = L\lfloor\zeta_t\rfloor$, where L is the linear operator given by $\varphi_0, \varphi_1, ..., \varphi_q$ sequence that converts $...\zeta_{-1}, \zeta_0, \zeta_1, ...$ sequence into another stationary sequence ξ_t, i.e. it specifies some linear filter.

Notate $F_{\zeta}(x) \equiv F_{\zeta}(x;t)$ distribution function as $\zeta_t, \forall t \in \left(\overline{-\infty,\infty}\right)$.

When

$$\mathbf{M}\zeta_t = \int_{-\infty}^{\infty} x dF_{\zeta}(x) = m_{\zeta} \quad \text{and} \quad \mathbf{M}\{(\zeta_t - m_{\zeta})(\zeta_s - m_{\zeta})\} = \begin{cases} \sigma_{\zeta}^2, & t = s \\ 0, & t \neq s \end{cases},$$

then

$$\mathbf{M}\xi_t = m_\zeta(\varphi_0 + \varphi_1 + ... + \varphi_q) = m_\zeta \sum_{j=0}^{q} \varphi_j ; \mathbf{D}\xi_t = \sigma_\zeta^2 \sum_{j=0}^{q} \varphi_j^2 \quad (6)$$

and at $s \geq 0$

$$R(s) = \mathbf{M}\{(\xi_t - m_\zeta)\cdot(\xi_{t+s} - m_\zeta)\} =$$

$$= \begin{cases} \sigma_\zeta^2(\varphi_0\varphi_s + ... + \varphi_{q-s}\varphi_q) & npu \quad s \in [\overline{0, q}] , \\ 0 & npu \quad s \geq q+1 . \end{cases} \quad (7)$$

where σ_ζ^2 is intensity of generation process, and the process itself is often called white noise [7].

In general case

$$R(s) = R(-s) = \sigma^2 \sum_{j=0}^{q+|s|} \varphi_j \varphi_{j+|s|} U_{|s|-j} ,$$

where U_x – is Heaviside function.

That is the sequence $\{\xi_t , t \in (\overline{-\infty, \infty})\}$, $\xi_t = \sum_{\tau=0}^{q} \varphi_\tau \zeta_{t-\tau}$ is weakly stationary sequence. It is optional to request ζ_t, independence for performing the letter, the demand relating to their non-correlation will be enough. But in that case the expressions for characteristic functions specified in this paper will be odd.

The use of linear random process models in power engineering

Being in (2) or (5) $q \to \infty$ such a moving total process is called the discrete time linear process, and ζ_t is called its generating process. That is in general case discrete time linear process is the process of ∞ order with continuous limits that is represented as:

$$\xi_t = \sum_{\tau=0}^{\infty} \varphi_\tau \zeta_{t-\tau}, \quad t \in (\overline{-\infty, \infty}) \text{ or } \xi_t = \sum_{\tau=-\infty}^{\infty} \varphi_\tau \zeta_{t-\tau}, \quad t \in (\overline{-\infty, \infty}). \quad (8)$$

Such processes are real if

$$\sum_{\tau=0}^{\infty} \varphi_\tau^2 < \infty \text{ or } \sum_{\tau=-\infty}^{\infty} \varphi_\tau^2 < \infty . \quad (9)$$

In this case their correlative function is expressed as

$$R(s) = \sigma^2 \sum_{j=0}^{\infty} \varphi_j \varphi_{j+|s|} ,$$

and when (6) is realized then this function is apparent at all $s \in \left(\overline{-\infty.\infty}\right)$ because $2\left|\varphi_j \varphi_{j+|s|}\right| \le \varphi_j^2 + \varphi_{j+|s|}^2$.

These demands are enough if ζ_t are non-correlated and they have common average and dispersion. Generally, in (8) ζ_t can be represented as dependent ones, in this case these processes will be discrete linear processes apparent if (9) is realized and all ζ_t have constant ensemble average and dispersion. Then the formulas for characteristic functions can be obtained as it is mentioned above but in this case computation becomes more complex.

Poisson spectra of hopping $K_\xi(x)$ and $K_\zeta(x)$ are relative in the way that is shown below (if to assume that ξ_t process is uniform and ζ_t is stationary)

$$K_\xi(x) = \int_{-\infty}^{\infty} R_\varphi(x, y) dK_\zeta(y), \tag{10}$$

where $R_\varphi(x, y)$ is a transformation kernel that doesn't depend on the generating process ζ_t and is definitely specified by the coefficients $\{\varphi_\tau, \quad \varphi_\tau \ne 0, \quad \tau = \overline{0,q}\}$ for expression (2) or in the same way for expression (4) at $\{\varphi_\tau, \quad \tau = \left(\overline{-\infty,\infty}\right)\}$. If a transformation kernel converse to $R_\varphi(x, y)$ is apparent, in this case the converse task can be solved, i. e. calculating $K_\xi(x)$with a given function $K_\zeta(x)$. It is a separate question that overrides the task of this paper.

The processes of moving average are profoundly studied from the point of view of their construction design as a model, but, only Gaussian models are profoundly studied. Nevertheless, the same results can be easily obtained for completely divisible distributions.

Integral pattern of the continuous time linear random process can be used in simulation along with discrete time linear processes. It has the following presentation:

$$\xi(t) = \int_{-\infty}^{\infty} \varphi(\tau, t) d\eta(\tau), \quad t \in T, \tag{11}$$

where T is the field of process definition (11) related to time, $\varphi(\tau, t)$ is a function integrated with the square in τ at $t \in T$, $\eta(\tau)$, $\tau \in (-\infty, \infty)$ is the process with independent increments. The expression (11) is true for both stationary and non-stationary models and has general features. In the stationary process (11) $\varphi(t - \tau)$ must be taken instead of $\varphi(\tau, t)$ and $\eta(\tau)$ must be a homogeneous process with independent increments.

The logarithm of characteristic function of the continuous time linear process (11) in Kolmogorov [7] form is expressed

$$\ln f_{\xi}(u;t) = iu\varkappa_1 \int_{-\infty}^{\infty} \varphi(\tau,t)d\tau + \\ + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(e^{iux\varphi(\tau,t)} - 1 - iux\varphi(\tau,t)\right)\frac{d_x K_{\eta}(x)}{x^2}d\tau, \qquad (12)$$

where $\varphi(\tau,t)$ is the process kernel (11), $K_{\eta}(x)$ is the function of homogeneous generating process with independent increments $\eta(t)$, $\varkappa_1$ is cumulant $\eta(t)$ at $t=1$. Find stochastic characteristics of the process $\xi(t)$.

The expression for n-dimensional characteristics functions continuous component $\xi_c(t)$ at $t_1,\ldots,t_n \in T$ is defined as follows at $L(x,\tau)\equiv 0, \varphi(\tau,t)\equiv\varphi_c(\tau,t)$

$$f_{\xi_c}(u_1,\ldots,u_n;t_1,\ldots,t_n) = \\ \exp\left\{i\sum_{k=1}^{n}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k)d\mu_w(\tau) - \frac{1}{2}\sum_{k,j=1}^{n}u_k u_j\int_{-\infty}^{\infty}\varphi(\tau,t_k)\varphi(\tau,t_j)dD_w(\tau)\right\}$$

where $\mu_w(\tau), D_w(\tau) > 0$ are non-stochastic real functions definitely specified according the process $w(\tau)$.

For the discrete process $\xi(t)$ n-dimensional characteristic function at the moments of time $t_1,\ldots,t_n \in T$ is defined as follows:

$$f_{\xi}(u_1,\ldots,u_n;t_1,\ldots,t_n) = \exp\left\{i\sum_{k=1}^{n}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k)d\mu(\tau) + \right. \\ \left. + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[e^{ix\sum_{k=1}^{n}u_k\varphi(\tau,t_k)} - 1 - \frac{ix\sum_{k=1}^{n}u_k\varphi(\tau,t_k)}{1+x^2}\right]dL(x,\tau)\right\},$$

where $\mu(\tau)$ is non-stochastic real function definitely specified according the process $\pi_1(\tau)$, $L(x,\tau)$ is Poisson spectrum of the process hopping $\pi_1(\tau)$.

Due to separability, the process $\xi(t)$ is completely described by the sequence of its n-dimensional characteristic functions. The known properties of characteristic functions allow to conduct compete stochastic analysis of the process. For example, ensemble average, correlation function and dispersion are respectively defined as

$$\mathbf{M}\xi(\tau)=-i\frac{\partial \ln f_{\xi}(u;t)}{\partial u}\bigg|_{u=0}=\int_{-\infty}^{\infty}\varphi_c(\tau,t)d\mu_c(\tau)+\int_{-\infty}^{\infty}\varphi_b(\tau,t)d\mu_b(\tau)+$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{x^2}{1+x^2}\varphi_b(\tau,t)dL(x,\tau);$$

$$R_{\xi}(t_1,t_2)=-\frac{\partial^2 \ln f_{\xi}(u_1,u_2;t_1,t_2)}{\partial u_1 \partial u_2}\bigg|_{u_1=u_2=0}=$$

$$=\int_{-\infty}^{\infty}\varphi_c(\tau,t_1)\varphi_c(\tau,t_2)dD(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x\varphi_b(\tau,t_1)\varphi_b(\tau,t_2)dL(x,\tau),$$

for $t_1=t_2=t$

$$\mathbf{D}\xi(\tau)=\int_{-\infty}^{\infty}\varphi_c(\tau,t)dD(\tau)+\int_{-\infty}^{\infty}x^2\varphi_b(\tau,t)dL(x,\tau).$$

Thereby, being constructive the model of the linear random process with completely divisible distribution law allows going over axiomatic model and, vise versa, at the sequence of finite-dimensional characteristic functions (axiomatic approach) we can change into the constructive model. It becomes possible due to correlation (11).

Simulation of the processes in power engineering can be represented by the following scheme in the case of use of discrete time linear random process model :

1. Kernel type $\varphi(\tau,t)$ received from the physical model of the simulated process. For example, while simulating the graphs of power load in the particular power system it can be determinate function that is t-cyclic and received in preliminary statistic processing of power load graphs during preliminary time, particularly $\varphi(\tau,t)$ can be rectangular or trapezoid averaged impulse of a separate power consumer.
2. The function of hopping $K_{\eta}(x)$ that describes the stochastic flow of connections and disconnections of power consumers. Simulation of the moments of hopping generating (event flow) is made one of the known methods of stochastic modeling, for example, those described in the paper [2, 5].
3. We receive the component of linear process realization taking into account its construction from the separate ones with the help of corresponding operations (convolution, multiplication).
4. We can evaluate the accuracy of simulation and its reliability by the statistic analysis (for example, moment's estimation, correlation analysis, histogram analysis).

Conclusion

1. Thereby, having Poisson hopping spectra in Kolmohorov form for the processes with discrete and continuous time we can specify n-dimensional characteristic functions of simulating processes, hence, the distribution functions related to them.
2. From the kernel types and hopping functions using the expression (2) or (10) realizations of power processes that are being modulated can be created.
3. Obtaining such realizations and corresponding finite-dimensional distribution functions different tasks related to functioning of power units that work in non-stationary mode can be solved.
4. Thereby, we use all the sequence on finite characteristic functions related to the construction of linear random zone while simulating linear random processes. That is why the use of linear processes in terms of infinitely divisible laws is equivalent to the use of axiomatic approach, that is, obtaining realizations for which their belonging and correspondence to specific systems of infinite distribution functions can be checked.

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