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COMPUTATIONAL MATHEMATICAL METHOD OF FLUID DYNAMICS PROBLEMS SOLVING

A mathematical method of solving of systems of nonlinear equations, produced by the numeral model of hydrodynamic problem, is offered. For the difference scheme on a rectangular calculation mesh it is suggested to utilize the iteration-based method of solving, based on the construction of auxiliary target function, the value of which characterizes the norm of discrepancy of the system.

Software applications and packages which perform various types of hydrodynamic calculations are widely utilized in modern scientific and design practice. For research of turbulent motion of liquids and gases, modern software complexes of hydrodynamic calculation are equipped with the sets of models of turbulence of different levels of complication [1].

Modern approaches for mathematical modeling of laminary turbulent and flows are mainly based on the supposition about acceptability of the Navier-Stokes equations description of flows and prognostication of their characteristics. (For turbulent flows statistical properties of flows ensembles, at identical from the macroscopic point of view external conditions, are examined). Numerous researches have been performed, – for example, described in the sources [2, 3], – which, in the opinion of their authors, confirm the supposition about adequacy of this model.

Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), Reynolds-averaged Navier-Stokes (RANS) and some derived methods are the most common methods of hydrodinamic numerical simulation. There are also combined approaches, which combine those or other features of the DNS, RANS and LES, for example, the Detached Eddy Simulation method (DES).

Direct Numerical Simulation includes solution of complete Navier-Stokes equations, that allows to get instantaneous characteristics of a turbulent flow. In addition, statistics, got as a result of DNS performing, can be utilized for development and research of models of processes of turbulent transfer, development of methods for controlling turbulent streams, and others like that [4, 5].

Because of the limited possibilities of measuring technique, DNS can be used as an additional source of experimental data; for example, at research of such characteristics of flow as pulsations, pressure, vorticity, speed of dissipation of turbulent energy, and also for visualization of instantaneous picture of flow.

Obstacles to the wide use of methods of numeral modeling in practice are related to contradiction between high requirements to the calculation schemes and descriptions of initial and boundary conditions, on one side, and limited computing resources, on the other side. Solving typical modern engineering problems that include aero- and hydrodynamics calculation require the months of work of computer clusters. Application of such methods, as RANS, LES et al, allows to shorten the volume of calculations to a certain extent, in comparison with DNS, but yet does not lead to the ultimate solution of the problem.

Thus, development of such methods and approaches to the solution of the mentioned tasks which would allow to decrease the amount of necessary calculations is the actual scientific problem.

Let us consider a two-dimensional hydrodynamic task, described with the Navier-Stokes equations of the following form:

$$\frac{\partial U_x}{dt}+\frac{\partial}{\partial x}(U_xU_x)+\frac{\partial}{\partial y}(U_yU_x) = -\frac{1}{\rho}\frac{\partial P}{\partial x}+v\left(\frac{\partial^2 U_x}{\partial x^2}+\frac{\partial^2 U_x}{\partial y^2}\right); \qquad (1)$$

$$\frac{\partial U_y}{dt}+\frac{\partial}{\partial x}(U_xU_y)+\frac{\partial}{\partial y}(U_yU_y) = -\frac{1}{\rho}\frac{\partial P}{\partial y}+v\left(\frac{\partial^2 U_y}{\partial x^2}+\frac{\partial^2 U_y}{\partial y^2}\right); \qquad (2)$$

$$\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}=0, \qquad (3)$$

where $U_x = U_x(x,y)$ and $U_y = Uy(x,y)$ are components of the velocity vector along the x and y coordinates, accordingly; $P = P(x,y)$ is pressure; v is the viscosity of the liquid; ρ is the density of the liquid.

For the compactness of record, let us introduce the denotations:

$$U_x(x,y)=f(x,y); \quad U_y(x.y)=g(x,y). \qquad (4)$$

Taking into account the denotations (4), let us write down the difference scheme for the system (1) – (3) on a rectangular mesh:

$$2f_{i,j}\frac{f_{i+1,j}-f_{i,j}}{\Delta_x}+g_{i,j}\frac{f_{i,j+1}-f_{i,j}}{\Delta_y}+f_{i,j}\frac{g_{i,j+1}-g_{i,j}}{\Delta_y}+\frac{1}{\rho}\frac{P_{i+1,j}-P_{i,j}}{\Delta_x}-$$
$$-v\frac{f_{i+1,j}-2f_{i,j}+f_{i-1,j}}{\Delta_x{}^2}-v\frac{f_{i,j+1}-2f_{i,j}+f_{i,j-1}}{\Delta_y{}^2}=0; \qquad (5)$$

$$g_{i,j}\frac{f_{i+1,j}-f_{i,j}}{\Delta_x}+f_{i,j}\frac{g_{i+1,j}-g_{i,j}}{\Delta_x}+2g_{i,j}\frac{g_{i,j+1}-g_{i,j}}{\Delta_y}+\frac{1}{\rho}\frac{P_{i+1,j}-P_{i,j}}{\Delta_x}-$$
$$-v\frac{g_{i+1,j}-2g_{i,j}+g_{i-1,j}}{\Delta_x{}^2}-v\frac{g_{i,j+1}-2g_{i,j}+g_{i,j-1}}{\Delta_y{}^2}=0; \qquad (6)$$

$$\frac{f_{i+1,j}-f_{i,j}}{\Delta_x}+\frac{g_{i,j+1}-g_{i,j}}{\Delta_y}=0, \qquad (7)$$

where $f_{i.j}, g_{i,j}, P_{i,j}$ are values of the functions $f(x,y)$, $g(x,y)$, $P(x,y)$ in the point of calculation mesh under the number (i,j) ; Δ_x and Δ_y are the steps of calculation mesh along the coordinates x and y accordingly. In equations of the system (5)–(7) the order of indexes corresponds to the alphabetical order of variables (the first index corresponds to the x coordinate, the second – to the y coordinate).

It is proposed to utilize the iteration-based method of solving of the system of equations (5)–(7), based on the construction of special-purpose auxiliary function. Let us present the system (5)–(7) in the generalized form:

$$\lambda_1(H) = e_1\,;\ \lambda_2(H) = e_2\,;\quad \ldots\quad \lambda_k(H) = e_k\ , \tag{8}$$

where λ_* – generalized operators which correspond to the left-part functions of the difference equations;

$H = \{a_1, a_2, \ldots, a_n\}$ is a set of the generalized arguments, the elements of which are unknown values of the functions $f_{*,*}, g_{*,*}, P_{*,*}$ in the points of the calculation mesh;

e_* are the values in the right parts of equations which do not depend on the generalized arguments a_* ;

k is the general quantity of equations of the system;

n is the general quantity of the generalized arguments.

Let us build an auxiliary function the value of which characterizes the norm of discrepancy of the system (8) at the intermediate (current in the iterative process) values of generalized arguments, found on the current step of iteration process:

$$V = \sum_{i=1}^{k} \left(\lambda_1(\ldots) - e_1\right)^2\ . \tag{9}$$

Let us introduce auxiliary differential equation which sets a condition on speed of convergence of the iteration process:

$$\dot{V} + cV = 0\ . \tag{10}$$

where $\dot{V} = \dfrac{\partial V}{\partial t}$; c – constant value – a parameter of quality of dynamic process of solution search; t – a relative (calculation) time, related to the amount of iterations of calculation process.

In the process of calculation the generalized arguments a_i change, and, in the case of convergence of the process, they approach the exact solutions of the system of equations (5–7). Therefore, there is an indirect dependence of the function V (9) on the relative time t , as a result of presence of dependences $a_i(t)$. Taking that into account, it is possible to write down:

$$\dot{V} = 2\sum_{i=1}^{k}\left[\left(\lambda_i(\ldots) - e_i\right)\frac{\partial \lambda_i(\ldots)}{\partial t}\right] = 2\sum_{i=1}^{k}\left[\left(\lambda_i(\ldots) - e_i\right)\sum_{j=1}^{l}\left[\frac{\partial \lambda_i(\ldots)}{\partial a_j}\dot{a}_j\right]\right]$$

and the auxiliary equation (10) gets the form:

$$\sum_{i=1}^{k}\left[\left(\lambda_i(...)-e_i\right)\sum_{j=1}^{l}\left[\frac{\partial\lambda_i(...)}{\partial a_j}\dot{a}_j\right]\right]+c\sum_{i=1}^{k}\left(\lambda_1(...)-e_1\right)^2=0 .$$

Also, the auxiliary conditions can be used in the form of a system of equations:

$$\dot{P}_i=-\varepsilon\frac{\partial V}{\partial P_i},\ \varepsilon=cV,\ i=1..k .$$

Conclusions

A mathematical method of solving of systems of nonlinear equations, derived from the numerical model of a hydrodynamic task, was offered. The offered method is based on the use of an auxiliary function and contains the parameter of quality of dynamic process of search of solution, that allows to manage speed of convergence of the system. The offered approach allows the further diminishing of dimension of the search space, and organization of solution calculation problem on the multiprocessor or distributed computing system.

Possible directions of subsequent researches are: study of calculation efficiency of realization of the offered method in the case of two-dimensional and three-dimensional problems; optimization of expressions which are used for reduction of dimension of space of search, with the speed of approaching to the exact decision as a criterion; optimization of information processes of data exchange between blocks in the multiprocessor or distributed system; a study of possibility of application of the proposed method to the problems set on irregular meshes..

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