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## MATHEMATICAL MODEL OF FRICTION AT PRESENCE OF ANISOTROPIC ADMIXTURES IN THE COMPOSITION OIL

The mathematical model of process of friction in presence of a lubricant con-taining rigid particles with anisotropic properties is examined. The model is oriented to the numerical experiment. For solving the problem of numerical modeling the modified method of solving of systems of linear equations is applied.

The mathematical modeling of friction phenomena have large value in modern aviation science. On the one hand, the friction is inherent for all moving mechanical parts of the aerial vehicle, such as the parts of the engines. On the other hand, understanding of the nature of the inner friction in the environment would help to build aerodynamic models of the flight.

Boric acid, widely known as an antiseptic tool, possesses unique tribologic characteristics due to its crystalline structure. The solid (crystallized) matter of boric acid consists of separate flat layers in which atoms are located close and densely combined between itself. At the same time, the distance between the nearby levels of this structure is relatively large. Hereupon intermolecular connections between levels, which result from the actions of the Van der Vaals forces – are comparatively weak. Being placed under sliding loadings, these layers can easily slide on each other. Thus the presence of strong connections within the limits of every level prevents a direct contact between sliding parts, reduces a friction and minimizes a wear. Exactly on this effect the use of boric acid is based as a component matter of admixtures to lubricating oils and film coverages of metallic surfaces of friction. A graphite which also is widely utillized in composition lubricating materials has a similar level structure.

As a result of scientific researches new materials are developed and synthesized with the properties similar to those of the graphite and boric acid. Introduction of new materials needs difficult and of long duration process of their research, which includes the study of properties of the matter, development, creation and practical test of the proper technological materials and processes on its basis. Therefore development of mathematical models which will allow the tools of numerical experiment to estimate perspective of certain direction of practical researches is expedient [1].

The purpose of the article is development of mathematical model of process of friction in the presence of liquid lubricating material in the complement of which enter in quality the admixture of particle with anisotropic properties.

As the basis of the offered mathematical model, the classic models of processes of dry and hydrodynamic friction [2] are taken, and also hydrodynamic processes [3]. For the conducting of numerical experiment the method of solving of the systems of linear algebraic equations is utillized [4].

As surfaces of friction are real always have inevenness, the area of contact between them can be divided into areas (fig. 1) in which their co-operation has different character:

- 1) Area of direct contact of materials of the two surfaces (fig.1, position 1). In this area the friction of surfaces takes place without painting (dry friction, solid friction).
- 2) Area of contact of materials through oxidizing film which is present on the spot metal surfaces (fig. 1, position 2).
- 3) Area of contact of materials through the olive film the thickness of which is equal to a few molecules of olive (fig. 1, position 3).
- 4) Area, where materials are parted by thick (comparatively to the characteristic size of molecules) tape of olive (fig. 1, position 4). This area can be named the area of the hydrodynamic greasing.

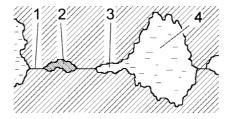


Fig. 1. Chart of contact of the two surfaces: 1 – the area of friction without painting; 2 – the area of contact through oxidizing film; 3 – the area of contact through the olive film of molecular thickness; 4 – the area of the hydrodynamic greasing.

At different terms there can be different friction condition in between the volumes of the corresponding areas. In the first and second areas the olive does not take participation in the process of friction. From the point of view of the hydrodynamic laws we should examine influence of olive sense only in a fourth area, and in the third area inter-molecular interaction plays the primary role.

For development of the basic simplified mathematical model of the process of friction in the fourth area let us consider the two flat and parallel surfaces of friction, one of which moves in parallell to the other (Fig.2). The space between surfaces is filled with liquid lubricating material which contains anisotropic particles in composition. Their anisotropy appears in that force of sliding the parts of a «granule» of admixture in one of directions is insignificant.

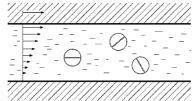


Fig. 2. Between surfaces of friction there is liquid lubricating material with anisotropic particles.

For simplification let us suppose that the anisotropic particles have a bullet form, their density is equal to the density of liquid environment; for every particle the radius of  $R_i$  and the coefficient of s are set, that determines the effort of shift  $S_i$ :

$$S_i = sR_i^2 \ . \tag{1}$$

Given the viscosity of the olive, equal to  $\mu$ , and the coefficients of friction between the olive and the material of surfaces of friction  $f_1$ , and between the olive and the surface of anisotropic particle –  $f_2$ .

For mathematical description of the problem the following relationships are utillized:

1) The Navier-Stokes equations

$$\rho \frac{du}{dt} = \rho X - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \left( 2 \frac{\partial u}{\partial x} - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \right) + \frac{\partial}{\partial y} \left( \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) + \frac{\partial}{\partial z} \left( \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right) ;$$

$$\rho \frac{dv}{dt} = \rho Y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu \left( 2 \frac{\partial v}{\partial y} - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \right) + \frac{\partial}{\partial z} \left( \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right) + \frac{\partial}{\partial x} \left( \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) ;$$

$$\rho \frac{dw}{dt} = \rho Z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left( \mu \left( 2 \frac{\partial w}{\partial z} - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \right) + \frac{\partial}{\partial x} \left( \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right) + \frac{\partial}{\partial y} \left( \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial z} \right) \right) ;$$

where  $\rho$  is the density of the liquid,  $\mu$  is viscosity of the liquid, p is pressure; u,v,w are components of speed of an elementary volume of the liquid; X,Y,Z are the components of force which operates on the elementary volume of the liquid.

2) Maximum terms, which stipulate nonbreakability of the thread of liquid from the hard walls and determine forces of frictions which operate between a hard matter and liquid. Simply enough to set maximum terms for flat surfaces:

$$v = 0$$
,  $F = f_1(V_1 - U)/(V_1 - U)$  (for the top surface),  
 $v = 0$ ,  $F = f_1 U/U$  (for the bottom surface)

where F is the vector of force which operates on the elementary volume of liquid; V1 is a vector of speed of flat topside, U is a vector of speed of elementary volume of liquid, V1 and U are the proper modules of these vectors.

Setting the maximum terms for flowing around the anisotropic particles a liquid it is though possible as equations, however much their use in a numeral experiment causes calculable difficulties because the size of particles is relatively small, and that is why for the correct calculation of conduct of liquid round every particle it is necessary to introduce the calculable net round every particle and considerably to increase the dimension of task. It is therefore expedient to separate equation of motion of liquid round a large spherical particle, oriented in relation to the system properly, and, drawing on the got result as the model problem, to scale it to the sizes of this particle.

3) Correlations which determine effort of shift (slide) and operates on the border of elementary volume of liquid (considering an olive a newtonian liquid):

$$\tau_x = \mu \frac{du}{dy}$$
;  $\tau_z = \mu \frac{dw}{dy}$ .

## **Conclusions**

In the article the mathematical model of process of friction between flat surfaces which move flat and parallel is considered, in the presence of liquid lubricating material in the complement of which introduced the admixture of particles with anisotropic properties. A numerical experiment is conducted from the design of hydrodynamic processes in the oil taking into account influence of anisotropic particles and nanoparticles. As a result of numerical experiment the examples of dependences of equivalent viscosity of olive with admixtures on concentration of admixtures and other parameters are got. The purpose of the conducted numerical experiment was determination of the force of friction. In order to disengage oneself from the geometrical parameters of the experiment, it is expedient to take the relative force of friction to the area of surfaces and to the gradient of speed, and to examine the value  $\mu' = \frac{F}{S \cdot (u/h)}$ , where F is force of friction, S is an area of surfaces of friction, h is distance between them, h is speed of motion of one of the surfaces in relation to other.

Perspective directions of research can be solving the tasks of numeral design of processes of friction in other configurations of surfaces, for example – to the process of hydrodynamic friction.

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