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C3: Time and DOA Estimation Problems

C4: Statistical Signal Processing

C5: Radar Signal Detection and Clutter

Circular Measurement Data Modeling and Statistical Processing in LabView

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Abstract — **The paper represents methods and the software for modeling and statistical proceeding of some common circular data analysis tasks. These tasks include computation of sample circular estimates such as mean direction, median, mode, variance, range, trigonometric moments, skewness, kurtosis and their confidence intervals.**

Keywords-random circle, bootstrap, approximation, Johnson distribution, generation, confidence interval, sample circular characteristic

I. Introduction

According to national and international standards measurement results must include accuracy characteristics. The most widespread of them are expanded uncertainty or confidence interval [1].

The currency of this issue is caused by expanding number of tasks which can be solved by statistical circular analysis methods in such areas as economy, meteorology, geodesy, physics etc. Besides the most well-known packages for engineering calculations such as MathCad, MatLab, LabView do not contain functions for solving problems of statistical circular measurements data processing. Furthermore these program packages do not contain random circular data generators with circular distributions – von Mises, wrapped distributions family etc. For these reasons the possibilities of computer measurement experiments with random circles are limited.

Statistical processing of circular data measurement results software development is the main purpose of the paper.

According to the purpose of the paper, the software must provide:

1. Different probability densities random circular data simulation.

2. Hypothesis tests on von Mises, wrapped normal and uniform distributions.

3. The main circular sample statistical estimates computation.

4. Confidence interval estimation for circular mean and median directions.

II. Simulation

Circular data simulation is the necessary part of computer experiment. The main circular distributions are: uniform, von Mises, wrapped normal, wrapped Cauchy, cardioid, triangular, wrapped Levy etc. Denoted probability densities random circular data can be simulated by inverse function method. The method depends on obtaining the continuous random value with the distribution function $F(x)$ from interval $x \in (-\infty, \infty)$ by functional transformation of the random value with uniform distribution in values $\alpha \in [0,1]$

$$\xi = F^{-1}(\alpha) , \quad (1)$$

where $F^{-1}(\bullet)$ is inverse function $F(x)$. Considered, that $F^{-1}(\alpha)$ exists.

Figure 1 shows the realization of random circle sample from cardioid distribution in polar coordinates.

III. Hypothesis Tests

The proceeding method selection can have obvious difficulties on small sample sizes. Therefore the proposed data proceeding software includes the hypothesis tests of uniformity and symmetry and tests on von Mises and wrapped normal distributions. They are: the Rayleigh test for uniformity [2],

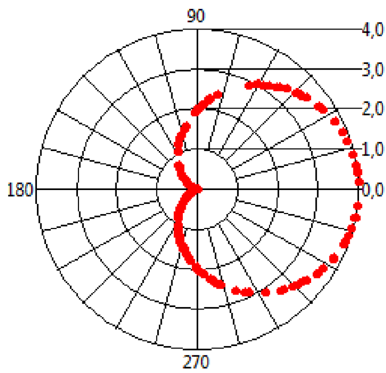

Figure 1. Realization of random circle sample from cardioid distribution in polar coordinates

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Pearson's chi-square tests on von Mises and wrapped normal distributions, Kolmogorov test on wrapped normal distributions.

IV. CIRCULAR SAMPLE STATISTICAL ESTIMATES

The main circular sample statistical estimates are [2, 3]:

- The sample circular mean direction:

$$\bar{\theta} = \{arctg\frac{S}{C} + \frac{\pi}{2}\{2 - (signS)\cdot[1 + signC]\}; \quad (2)$$

- The mean resultant length:

$$r = \sqrt{C^2 + S^2}. \quad (3)$$

- Sample circular variance:

$$V = 1 - r. \quad (4)$$

- Sample standard deviation:

$$\sigma = \sqrt{-2\ln(1-V)} = \sqrt{-2\ln r}. \quad (5)$$

- A sample circular median direction $\bar{\theta}$ of angles $\theta_1,\ldots,\theta_J$ is any angle ϕ such that half of data points lie in the arc $[\phi, \phi+\pi)$, and the majority of the data points are nearer to ϕ than to $\phi+\pi$.
- The circular range is the length of the smallest arc which contain all the observations.
- Mode – angle with the maximum data concentration around.
- Skewness [2, 4]:

$$g_1 = b_3(\bar{\theta}_1)/V^{3/2} = r_2\sin[\bar{\theta}_2(0) - 2\bar{\theta}_1]/V^{3/2}. \quad (6)$$

- Kurtosis:

$$g_2 = [r_2\cos[\bar{\theta}_2(0) - 2\bar{\theta}_1] - (1-V)^4]/V^2. \quad (7)$$

- Trigonometric moment:

$$T_u(\alpha) = \frac{1}{M}\sum_{j=1}^{M} e^{iu(\theta_j - \alpha)}. \quad (8)$$

V. CONFIDENCE INTERVAL ESTIMATION

Wide known methods of confidence interval obtaining are based on preliminary estimation of standard uncertainty and ratio coefficient for known probability density function. Often such distribution based on the results of previous measurements taken close to Gaussian. If the measurement results data is sufficient it checks for Gaussian distribution by one of the known criteria's. However in non priori information case it is impossible to taste sample on distributions and such approach can lead to significant errors in measurement results accuracy estimation.

Developed software includes confidence interval obtaining by traditional method, circular analogue of Chebishev inequality method, and methods with Johnson distribution and bootstrap technology application for circular data.

A. *Traditional method*

Traditional method provides obtaining of sample circular mean direction $\bar{\theta}$, standard deviation $\hat{\sigma}$ and Student coefficient. Confidence interval is obtaining considering the hypothesis of wrapped normal distribution as:

$$\bar{\theta} \pm t_{n,P}\hat{\sigma} \quad (9)$$

B. *Circular analogue of Chebishev inequality method*

Circular analogue of Chebishev inequality method is used when random circle distribution is unknown. Method based on circular analogue of Chebishev inequality application [2]. Applying Chebishev inequality to the random variable $\sin([\theta - \bar{\theta}]/2)$ gives

$$P\left(\left|\sin\left((\theta - \bar{\theta})/2\right)\right| \geq \varepsilon\right) \leq V/2\varepsilon^2. \quad (10)$$

Confidence interval for mean direction can be obtained as:

$$\bar{\theta} \pm 2\arcsin\left(\varepsilon\sqrt{V/2}\right) \quad (11)$$

C. *Method based on Johnson distribution application for random circle confidence interval obtaining*

The Johnson distribution system is based on three possible transformations of a normal random variable. They are known as Johnson SL (lognormal), Johnson SU (unbounded) and Johnson SB (bounded), corresponding to exponential, logistic, and hyperbolic sine transformations. All three can be written as [6]:

$$z = \gamma + \eta\tau(x;\varepsilon,\lambda); \quad \eta > 0, -\infty < \gamma < \infty, \lambda > 0, -\infty < \varepsilon < \infty \quad (12)$$

where z is a standard normal random variable, τ is the transformation, and γ, η, ε and λ are scale and location parameters.

Johnson curves empirical distribution approximation depends on distribution family selection, computation of the parameters estimates and empirical probability function obtaining [7]. Confidence interval obtaining depends on computation of the inverse function for the probability density function $f(\theta)$ and estimation of symmetrical quantiles corresponding to given probability:

$$(\theta_{q/2})\bmod 2\pi, \ (\theta_{1-q/2})\bmod 2\pi, \quad (13)$$

where q is the confidence level.

An empirical probability density function can also be useful for obtaining the mode, median and moments of given orders – the mean direction, standard deviation, skewness, kurtosis.

D. *Bootstrap methods for circular statistical estimates and their confidence intervals obtaining*

A class of computer-intensive statistical procedures called the bootstrap methods has a wide application for assessing the variability of a point estimate in situations where more usual statistical procedures are not valid and/or not available (e.g. the sampling distribution of a statistic is not known). The authors reviewed the parametric and non-parametric bootstrap methods for circular mean direction, median and their confidence interval limits estimation. The base algorithm of non-parametric bootstrap for an empirical distribution quantile estimate obtaining consists of following steps [8]:

1. Sampling with replacement a set of circles $\Theta' = \{\theta_1', \ldots, \theta_n'\}$ from the original $\Theta = \{\theta_1, \ldots, \theta_n\}$, $\theta_i \in [0, 2\pi)$, $n = \overline{1, N}$.
2. Computation of the quantile estimate $\hat{\theta}$ for $\Theta' = \{\theta_1', \ldots, \theta_n'\}$.
3. Repeating steps 1, 2 B times drawing a bootstrap distribution $\hat{\theta}_1, \ldots, \hat{\theta}_B$.
4. Obtaining 2,5th and 97,5th percentiles from $\hat{\theta}_1, \ldots, \hat{\theta}_B$ for 95% confidence interval.

Parametric bootstrap assumes the original circular data sample $\Theta = \{\theta_1, \ldots, \theta_n\}$ from a distribution of a known form, a.g. von Mises $VM(\mu, k)$ distribution, where μ and k are known. With this assumption μ and k are estimated by $\hat{\mu}$ and $\hat{k}$ from $\Theta = \{\theta_1, \ldots, \theta_n\}$ and then each bootstrap sample is obtained by simulation from the $VM(\hat{\mu}, \hat{k})$ distribution [4].

VI. Modules of Software for Circular Measurement Data Modelling and Statistical Proceeding

Based on described methods the software for circular data simulating and statistical proceeding is developed. It consists of four main modules:

- The input module provides an opportunity to keyboard input or loading from the text file and also generation random circles samples in accordance with the probability distribution functions – wrapped normal, von Mises, wrapped Cauchy, cardioid, wrapped Levy etc., – and given parameters (figure 2).
- The module of statistical estimates and their accuracy characteristics obtaining with traditional methods application.
- The module of statistical estimates and their accuracy rates obtaining with Johnson curves approximation of empirical circular distributions application (figure 3).
- The module of statistical estimates and their accuracy characteristics obtaining with bootstrap methods application (figure 4).

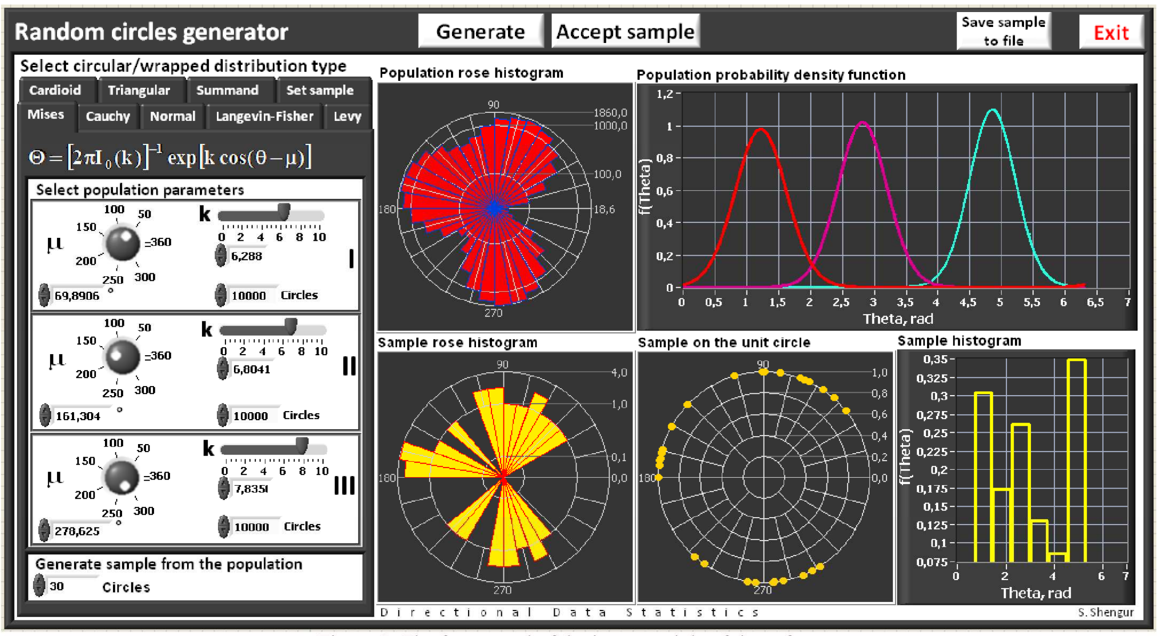

Figure 2. The front panel of the input module of the software

Conclusions

Considered methods of circular data measurement results can be applied in such tasks as radar signals decoding, targets point's center of gravity definition, signal to noise ratio increasing in signal detection tasks.

Software for simulating and statistical processing of measurement circular data is discussed. It can be applied for:

- Probability densities random circular data simulation: von Mises, wrapped normal, wrapped Cauchy, wrapped Levy, cardioid etc.

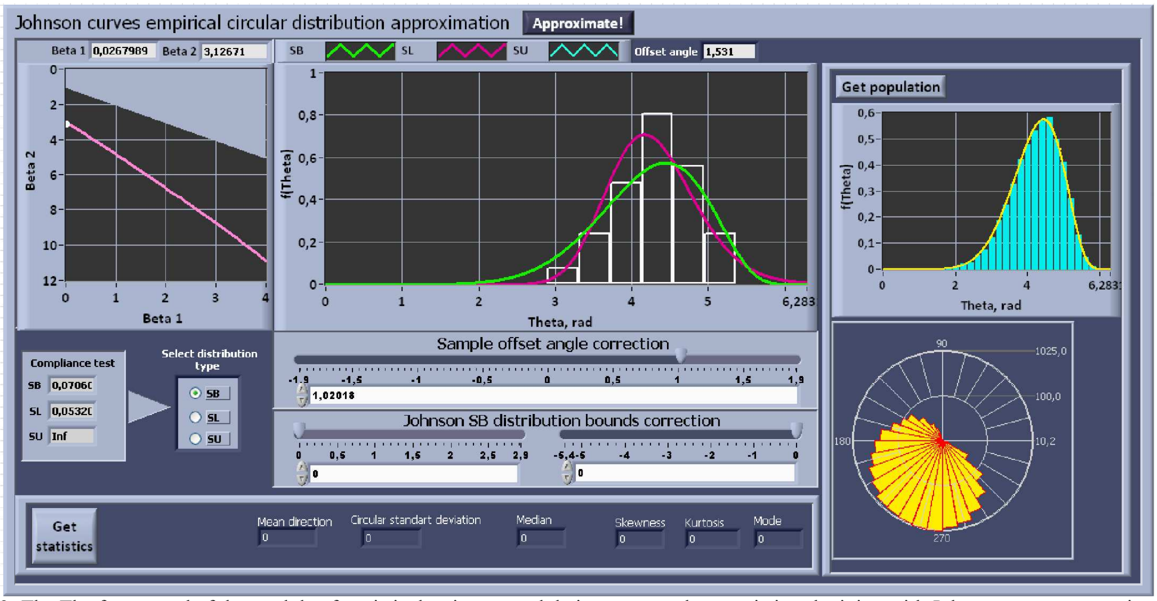

Figure 3. The The front panel of the module of statistical estimates and their accuracy characteristics obtaining with Johnson curves approximation of empirical circular distributions application

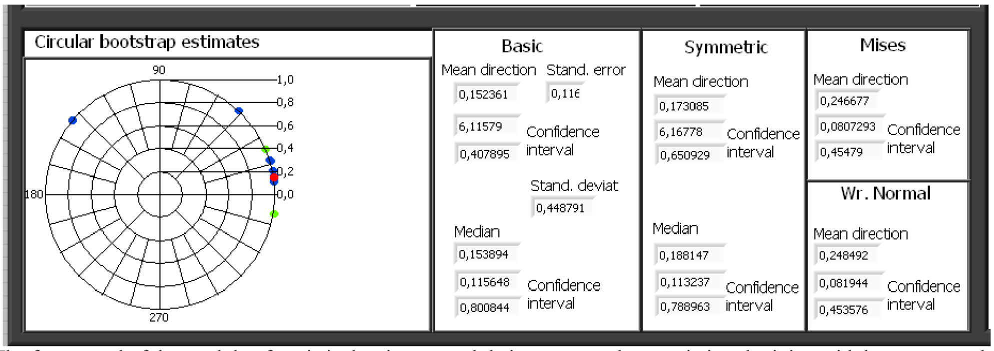

Figure 4. The The front panel of the module of statistical estimates and their accuracy characteristics obtaining with bootstrap methods application

- Obtaining the main statistical estimates – mean direction, mode, median, trigonometric moments, skewness, kurtosis, rate, standard deviation, variance etc.
- Estimation of confidence interval limits for the mean direction and median based on traditional methods, Chebishev and bootstrap methods for unknown circular distribution.
- Estimation of confidence interval limits for random circle based on Johnson curves empirical distribution approximation.
- Displaying the results of simulation and processing on circular diagrams.

Developed software programs have such fields of application as circular measurement data statistical processing and computer experiments with random circles.

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