Calculation and Analysis of Initial Conditions at Takeoff of UAV with Springboard

N.F. Tupitsin
Aviation Computer-Integrated Complexes Dept.
National Aviation University
Kyiv, Ukraine
nift@mail.ru

S.A. Malakhov
Aviation Computer-Integrated Complexes Dept.
National Aviation University
Kyiv, Ukraine
serg_malahov@ukr.net

L.S. Shcherbina
Aviation Computer-Integrated Complexes Dept.
National Aviation University
Kyiv, Ukraine
liza.shcherbina777@gmail.com

***Abstract*—In article the initial constructive part of springboard is proposed. Also, the calculation and analysis of unmanned aerial vehicle initial conditions at takeoff with spring-loaded final part of springboard are executed. The two technical contrivances as spring-loaded final part were considered: elastically restrained plate and spring-loaded bridge.**

Keywords—unmanned aerial vehicle; elastically restrained plate, spring-load bridge, take-off; springboard.

I. INTRODUCTION

Development of new methods and devices, which are directed to decreasing runway, is actuality for most modern types of unmanned aerial vehicles (UAVs). Takeoff and landing of the UAVs on the short runway expand the opportunities of their use. For example, the dimensions of flight deck are restricted by the vessel's size, often comparatively small, and the width is also limited by the carrier installations.

A great number of works is dedicated to the development and research of the new methods and devices, which are directed to decreasing runway [1]–[4], [6].

For example, in the work [1], during the research of the initial conditions of the takeoff with the springboard, there were obtained the dependences (Fig. 1) of the loads on the wheels of the nose ($R_1 = P_k^n$) and main landing gears ($R_2 = P_k^0$) from the time (*t*). The dependence of the speed value of the aircraft (*V*) from the time is also present.

The analysis of the obtained dependences (Fig. 1) illustrates that by the increasing of the aircraft speed on the final part of springboard from 100 to 120 km/h (just about from 28 to 33 m/s), summary load of the 9 tons weight aircraft on the landing gear decreases on the almost 50 %. Such decrease of the load on the landing gear can be explained by the increase of aircraft lift force. When the velocity is changing from 100 to 122 (m / s), the ram air is increased from 12490 to 18590 (n / m2). Time interval between the unstuck of the nose and main landing gear is just about 0.03 s, what can allow to make a conclusion about the distance between this wheels (≈ 10 m), under the given speed value.

Determining of the initial conditions for the task of the aircraft dynamics after its descent from the elastically restrained plate (ERP), situated under the θ angle to the horizon, is shown in [2]. UAV takeoff from the elastically plate is represented on Fig. 2

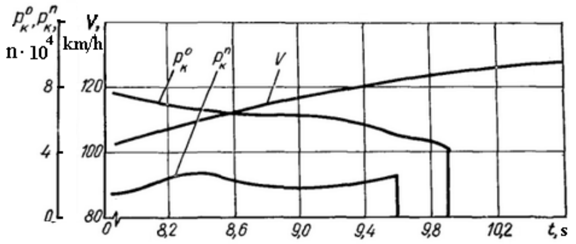

Fig. 1. The dependence of speed value of the aircraft and the loads on the wheels of the nose and main landing gears from time.

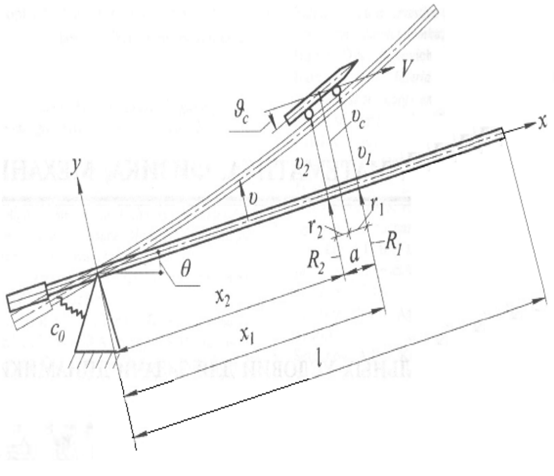

Fig. 2. The UAV takeoff from the elastically plate.

Action of the gear legs (UAV bearing) on the plate is presented as two reactions R_1 and R_2. Values υ_c and ϑ_c defines cross movement in the UAV centre of gravity and its rotation angle, and c_0 – rigidity of the bearing spring.

The scheme of forces, which are acting on UAV at takeoff from elastic beam (or spring-load bridge) is shown on Fig. 3. Calculation of the cross nonstationary oscillations of such plate allows to determine the initial conditions of the UAV takeoff. The plate of such type can be set in the final part of

978-1-4673-6566-6/15/$31.00 © 2015 IEEE

the springboard [2], [3] or used for the improvement of the initial conditions of the takeoff from the horizontal runway.

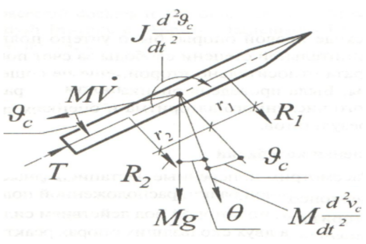

Fig. 3. The scheme of forces, which are acting on UAV at takeoff in vertical plane.

II. PROBLEM STATEMENT

The aim is to: 1) choose the form of the initial design of the springboard; 2) perform calculations and analysis of the effectiveness of using two technical contrivances, namely, an elastically restrained plate and spring-loaded directing bridge, which can provide certain benefits during takeoff of the UAV and thus reduce the runway length.

III. THE CHOICE OF SHAPE OF THE STRUCTURE OF AN INITIAL SPRINGBOARD

Let's represent the cross-section of the initial part of springboard design by a triangle *ABC* (Fig. 4) with an inclined plane at an angle θ_{sp} to the horizontal plane, or a curved working part (arc *AB*). Neglecting the work of forces of rolling friction at the sector of descent, we assume that the potential energy of the UAV, which it has at the point *A* transforms into kinetic point *B*, i. e. $m(y_0 - y_1)g = mV_B^2/2$. With this assumption, the minimum descent time ($t_{\min}$) from point *A* to point *B* is determined by solving the simplest variational problem [5], and the arc *AB*, which defines the curve of descent of the UAV for the minimum time, is a cycloid. It is not difficult to calculate that the value $t_{\min}$, in seconds, can be determined from the following expression

$$t_{\min} = \pi\sqrt{\frac{(y_0 - y_1)}{2g}}, \tag{1}$$

where (y_0-y_1) is the height of the springboard initial part in meters; g is the gravity acceleration.

However, the descent with the minimum time for the cycloid is possible only at a certain ratio of values $(y_0 - y_1) = H$ and $(x_1 - x_0) = L_1$, namely:

$$L_1^* / H^* = \pi / 2. \tag{2}$$

Let us compare the time (t_{AB}) of descent of the UAV down an inclined plane, where the line *AB* is located, and value $t_{\min}$. The time of the UAV descent down an inclined plane can be determined from the following expression

$$t_{AB} = \left(\frac{1}{\sin\theta_{sp}}\right)\sqrt{\frac{2(y_0 - y_1)}{g}}, \tag{3}$$

where θ_{SP} is the angle between the inclined and horizontal planes.

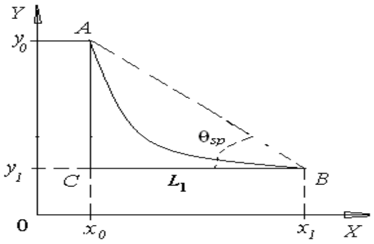

Fig. 4. Proposed scheme of springboard initial constructive part for UAV takeoff.

With help of (1), (3), we can construct dependences of time of the UAV descent for two types of trajectories on the springboard at change of the length of the horizontal section (Fig. 5). From calculations it can be stated, during the change of the value L_1 from 3 to 9 m the dependence $t_{\min} / t_{AB}$ remains constant and is approximately 0.84, and value $t_{\min/}$ changes from 0.98 s to 1.7 s. The velocity of UAV in point *B*, only due to the transition of the potential energy in point *A* to kinetic in point *B*, changes from 6.1 to 10.5 m / s.

During motion on a cycloid the velocity vector of the UAV at point *B* is parallel to the horizontal plane, while moving down an inclined plane the velocity vector has a vertical component, which next part of springboard needs to be fend off.

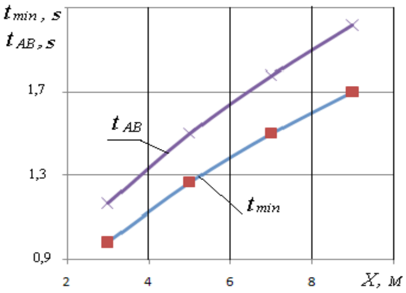

Fig. 5. The dependence of the time of descent of the UAV on the trajectory of its movement on the springboard.

In the case of non-compliance of the ratio (2) the problem of choosing the form of an initial construction of the springboard is more difficult.

Lets consider variant of the design of the springboard initial part for which $L_1 = (x_1 - x_0^x) < L_1^*$. Thus we can assume that the trajectory of the UAV descent with the minimum time consists of line A^xD and part of the cycloid *DB* (Fig. 6).

If $L_1 > L_1^*$, then, obviously, the descent trajectory of the UAV must include the cycloid and motion on the horizontal section.

IV. THE CALCULATION OF THE UAV MOTION FOR FINAL SEGMENT OF SPRINGBOARD

Besides elastic restrained plate on the final segment of runway it may be installed a spring loaded bridge, which can also create certain advantages for initial conditions of the UAV takeoff.

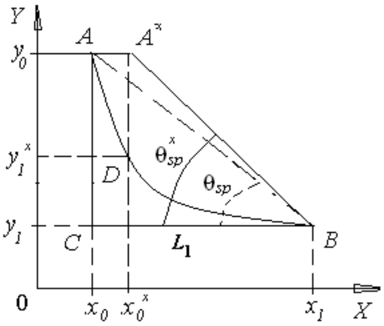

Fig. 6. The estimated trajectory of the UAV on the springboard at $L_1 < L_1^*$.

An example of spring loaded bridge is shown on Fig. 7, in which: 1 is movable panel (MP); 2 is fixed panel – base (an optional element); 3 are springs, each with rigidity c_0 ; 4 is a hinge. Let us assume that the MP has a rectangular shape with the same cross-section (thickness) and is characterized by three dimensions: length L_M , width b_M and thickness δ_M of the MP.

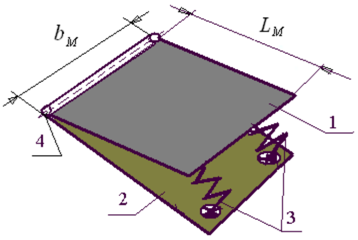

Fig. 7. A spring loaded bridge.

Generally speaking, the principal difference between a spring loaded bridge and a elastic restrained plate consists in the fact that: 1) location of the bridge's spring can be varied along its length (L_M); 2) an elastic plate does not have a clearly defined transverse hinge such as in a spring loaded bridge.

The equations of motion of the aerial vehicle [2] in projections on Ox and Oy ($|\vartheta_C| \ll 1$, $\cos\vartheta_C \approx 1$, $\sin\vartheta_C \approx \vartheta_C$) are written in the form (see Fig. 2).

$$\begin{aligned} &M\dot{V} = T - Mg\sin\theta; \\ &\mathrm{M}\frac{d^2 y_c}{dt^2} = T\vartheta_c - Mg\cos\theta - R_1 - R_2; \qquad (4) \\ &J\frac{d^2\vartheta_c}{dt^2} = -R_1 r_1 + R_2 r_2, \end{aligned}$$

here M is the mass of aerial vehicle; T is thrust; J is the moment of inertia of the aerial vehicle with respect to its center of gravity; r_1 and r_2 is the distance between the center of gravity of the UAV and landing gear legs, while $r_1 + r_2 = a$. Initial conditions $(t = 0)$:

$$V_c = 0, \quad y_c = 0, \quad \vartheta = 0, \quad \dot{\vartheta}(0) = 0. \qquad (5)$$

In work [2] system of differential equations (4) is integrated numerically for the given initial conditions (5) and kinematic parameters of the takeoff UAV are obtained. The Ritz method was used, and deflection of the beam $y(x,t)$ was written in the form

$$\mathrm{y}(x,t) = y_0(t) + \sum_{m=0}^{s} q_m(t) X_m(x), \qquad (6)$$

where $y_0(t)$ is given initial bending of the beam (plate) based on its static deflection by the force of gravity at $t = 0$; $X_0 = x$ specifies a value of beam rotation with respect to the elastic hinge as a rigid body; $X_m(x), m = 1,2,...,s$; $q_m(t)$ specify own waveforms of a homogeneous cantilever beam and generalized coordinates, satisfying the initial conditions $q_m(0) = 0$, $\dot{q}_m(0) = 0$.

Let us find an approximate solution of the system (4).

At the minor plate deflection $|\theta| \ll 1$, in the 1st equation (4) can neglect the second summand in right part. At constant values T and M the 1st equation is independently from the others. So, we receive uniformly accelerated motion of UAV

$$x_1(t) = \frac{T}{M}t^2 + V_0 t, \qquad (7)$$

where x_1 is the distance, which the UAV passes by the condition of the attached motion; V_0. is the velocity of UAV at $t = 0$.

In case of approximate approach to solving (4) the motion of UAV and the oscillations of MP we will consider separately.

Let's consider more detailed the dependences of the loads on the landing gear at the aircraft (aerial vehicle) run.

As we can see from the Fig. 1, mean load on the wheels of the nose landing gear during the run time on the springboard is just about $R_1 = P_k^n = 2 \cdot 10^4\, n$.

The mean load on the wheels of the main landing gear, which is located closer to the centre of mass, changes about the linear law

$$R_2 = P_k^0 = P_k^1 - P_k^2 t, \qquad (8)$$

And besides at the initial time moment (at $V = 0$)

$$P_k^n + P_k^1 = Mg. \qquad (9)$$

From (8) we can determine the value

$$P_k^1 = Mg - P_k^n. \qquad (10)$$

And at the moment of aircraft unstuck from the plate

$$P_k^0 = Mg - Y, \qquad (11)$$

where Y is the lift force of the aircraft, but in [2] all calculations are executed without account of aerodynamic forces. So, if we know the value of aircraft lift force at its motion, can calculate the meaning R_2 with help of equations (7)–(11).

As shown in [2], the motion on final part of MP may divide on four sections. Obviously, that the most important third section, when there was a separation between the nose landing gears wheels and MP. Namely, the kinematic parameters of UAV in the end of this section are determined the initial conditions of UAV motion in air [6].

Peculiarity of the UAV motion over spring-loaded bridge is that at motion on this bridge, as the compression force its springs are used the UAV weight.

V. AN EXAMPLE OF THE CALCULATION OF THE UAV MOTION

Suppose that hypothetical UAV of the “Heavenly patrol” type has the similar to aircraft scheme of the landing gear disposition

At the plate oscillations, during the UAV motion on the landing gear, condition of motion without separation is difficult to execute. In such way ski landing gear is more suitable.

The oscillation period of system, which is involved MP and moving on its UAV, is determined by time of the UAV motion on MP, i. e.

$$T_1 = (-V_0 + \sqrt{V_0^2 + 4L_{br}T/M})/(2T/M) = 2\pi/\nu_1, \qquad (12)$$

where T_1 is the oscillation period of the MP with help by one spring; L_{br} is the length of spring-loaded bridge; ν_1 is the frequency of system oscillation.

As known, the frequency of free oscillation dependences from the value of M and the rigidity of spring k_1

$$\nu_1^2 = k_1/m. \qquad (13)$$

At using two springs (see Fig. 7) the oscillation period is

$$T_2 = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}. \qquad (14)$$

If the value k_1 is equal $k_2 = k$, then $T_2 = T_1/\sqrt{2} \approx 0.707\ T_1$.

Evaluate necessary materials’ mass for the moving panel, which is consist of the duralumin (D16) and the steel (st3). Obviously that for the chosen form of MP, its mass is proportional to the values L_M, b_M and δ_M. The dependence between materials’ mass for moving panel and its lengths and width at the width $b_M = 2$ m is shown on the Fig. 8.

Under action of UAV weight itself system will oscillate independent from our springs. To make this system more rigid, it is necessary install the panel of stiffeners, but in this work the weight stiffness will not be taken into consideration.

Dependence of free frequency oscillation from rigidity of springs, when the summary mass MP + UAV is equal 250 kg, is shown on Fig. 9. At such high frequencies it is difficult to calculate desirable conditions the UAV takeoff from the MP.

Besides, the impulses from oscillations can unfavorably influence upon landing gear, the fuselage and wings of UAV.

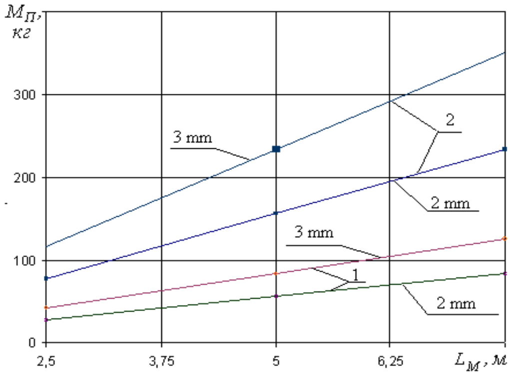

Fig. 8. The dependence between materials’ mass for moving panel and its lengths and width: 1- duralumin D16; 2-steel (st3).

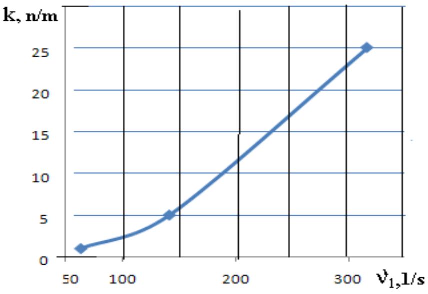

Fig. 9. The dependence of free frequency oscillation from rigidity of springs.

VI. CONCLUSION

In article the times of UAV descent on the initial constructive part of springboard are considered. The calculation and analysis of UAV motion at takeoff with spring-loaded final part of springboard are executed.

REFERENCES

[1] V.F. Pavlenko, Ships aircrafts, Moskow: Military Publishing, 1990, (in Russian).

[2] S.V. Russkikh, “Defenition of initial conditions for the problem of aircraft dynamics after its take-off from elastic rail beam”.Vestnik MAI vol. 21, no. 2, 2014. pp. 129-137. (in Russian)

[3] I. N. Tupitsin and N. F. Tupitsin, “The device takeoff and landing of aerial vehicle at the springboard”. Patent of Ukraine no. 94251, 2014. 6 p. (in Ukrainian)

[4] N.F. Tupitsin, B.I. Dmitrenko, A.A. Ziganshin and I.A. Stepanenko, “The method of takeoff and landing UAV with springboard”. Electronics and Control Systems. 2014. no. 3 (41). pp. 91-97.

[5] E. Giusti, Direct methods in the calculus of variations. Singapore: World Scientific, 2003 (2005). 403 p. Ref.: p.377-398. Ind.: pp. 399-403.

[6] V.I. Silkov, A.V. Samkov and N.F. Tupitsin, “Technique calculation of UAV kinematical parameters at take-off with springboard”. IEEE 2nd International Conference “Actual Prob-lems of Unmanned Air Vehicles Developments” Ukraine, Kyiv, October, 15-17, 2013. pp. 217-220.