

## ADAPTIVE ROBUST CONTROL OF LINEAR MIMO STATIC PLANT WITH AN ARBITRARY TRANSFER MATRIX AND BOUNDED NOISE: A GENERALIZATION

Стаття стосується побудови адаптивного робастного регулятора для управління лінійним багатозв'язним статичним об'єктом з можливо виродженою передатною матрицею за наявності завад, межі яких априорі невідомі.

*Ключові слова:* робастний адаптивний регулятор; багатозв'язний статичний об'єкт; обмежена завада; вироджена передатна матриця; обмеженість

Статья касается построения адаптивного робастного регулятора для управления линейным многосвязным статическим объектом с возможно вырожденной передаточной матрицей при наличии помех, границы которых априори неизвестны.

*Ключевые слова:* робастний адаптивний регулятор; многосвязный статический объект; ограниченная помеха; вырожденная передаточная матрица; ограниченность.

The paper deals with designing an adaptive robust controller for the control of linear interconnected static plants with possibly singular transfer matrices in the presence of noises whose bounds are a priori unknown.

*Keywords:* robust adaptive controller; interconnected static plant; bounded noise; singular transfer matrix, boundedness.

The problem of adaptive control in the presence of noise stated several decades ago [1] remain actual up to now. In particular, new important results in this research direction were achieved in [2, 3].

Adaptive control systems containing MIMO (multi-input multi-output) plants have been studied in [1, 3]. In these works, the basic assumption is that they are stably invertible; see [1, Assumption 6.3.M.(4)] and also [3, Subsect. 4.2]. Recently, such an assumption has been removed in [4, 5].

This paper extends and generalizes the approach advanced in [4, 5].

The plant to be controlled is assumed to be a static linear time-invariant interconnected square system [1] described by

$$y_n = Bu_n + v_n, \quad (1)$$

where  $y_n = [y_n^{(1)}, \dots, y_n^{(N)}]^T$  is the  $N$ -dimensional output vector to be measured at  $n$ th time instant;

$u_n = [u_n^{(1)}, \dots, u_n^{(N)}]^T$  is the  $N$ -dimensional control vector at  $n$ th time instant;

$v_n = [v_n^{(1)}, \dots, v_n^{(N)}]^T$  is the  $N$ -dimensional vector of noises at  $n$ th time instant;

$B$  is an arbitrary transfer  $N \times N$  matrix given as

$$B = \begin{pmatrix} b_{11} & \dots & b_{1N} \\ \dots & \dots & \dots \\ b_{N1} & \dots & b_{NN} \end{pmatrix}.$$

The matrix  $B$  may be singular, in principle, and it is essential. As in [2–5], it is assumed that there are finite intervals

$$\underline{b}_{ik} \leq b_{ik} \leq \bar{b}_{ik}, \quad i, k = 1, \dots, N \quad (2)$$

to which its elements,  $b_{ik}$  belong.

Similar to [3], we suppose that  $v_n^{(i)}$ s are all upper bounded in modulus according to

$$|v_n^{(i)}| \leq \varepsilon_i < \infty \quad \forall i = 1, \dots, N, \quad (3)$$

where  $\varepsilon_i$ s are unknown constants.

The problem is to design an adaptive robust controller to be able to stabilize the plant (1) at a neighborhood of the desired output vector  $y^0$  such that

$$\limsup_{n \rightarrow \infty} (\|y_n\| + r \|u_n\|), \quad r > 0. \quad (4)$$

Key idea advanced first in [4] is the transaction from the adaptive control of the true plant (1) having the singular transfer matrix  $B$  to the adaptive identification of a fictitious plant with the nonsingular transfer matrix  $\tilde{B}$  of the form

$$\tilde{B} = B + \delta_0 I, \quad (5)$$

where  $I$  denotes the identity matrix;

$\delta_0$  is a fixed quantity.

Within this approach, the fact that each eigenvalues of  $B$  lie in one of closed regions of the complex plane consisting of all the Gersgorin discs [4, 5] is utilized. This remarkable fact allows to go to the transfer matrix of  $\tilde{B}$  via the suitable shift of these discs. It turns out that they can be shifted left or right by exploiting the *a priori* information with respect to the bounds (2). More certainty, we are able to find some number  $\delta_0$  specifying the size of such a shift leading to (5).

Noting that the fictitious plant whose equation has the form

$$\tilde{y}_n = \tilde{B}u_n + v_n, \quad (6)$$

is subjected by the same noises  $v_n^{(i)}$  as the true plant (1), we observe that its output vector,  $\tilde{y}_n$  can be calculated as (due to (5), (6))

$$\tilde{y}_n = y_n + \delta_0 u_n.$$

Now, the adaptive control law is described by

$$u_{n+1} = u_n + \tilde{B}_n^{-1}(y^0 - \tilde{y}_n), \quad (7)$$

in which  $\tilde{B}_n$  is the current estimate of  $\tilde{B}$  in (6) updated by exploiting the same recursive procedure as in [4, 5]. (Before choosing (7) we previously exploited the fact that  $\tilde{B}$  which remains unknown in (6) is nonsingular, i.e.,  $\det \tilde{B} \neq 0$ .)

The main results consist in establishing the facts that the proposed adaptation algorithm converges at a finite time, and the control objective (4) is achieved.

## References

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