СЕКЦІЯ „АНАЛІЗ ТА МОДЕЛЮВАННЯ СКЛАДНИХ СИСТЕМ І ПРОЦЕСІВ”

ANALYTIC DESIGN OF THE OPTIMUM CONTROL SYSTEM FOR FIVE-DEGREE-OF-FREEDOM STAND SIMULATOR OF THE SPACECRAFT MOTION

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Introduction. The problem of professional training of operators to control aviation and space objects is definitely very important [1]. An operator is the only who takes the most difficult and responsible decisions on the object control, and not only the fulfillment of assigned mission depends on the accuracy of his actions for in-time finding and implementing the appropriate solution, but also the integrity of the object itself and safety of people in certain cases. Operator's enhanced role in controlling complex dynamic objects raises the problem of upgrading methodical and technical means of operator training. Trainers and flight simulators with motion systems, which are widely used not only in aviation and astronautics but also in other fields, are among the most effective means of operator training, considering real operating conditions. Modern dynamic flight simulators and disturbed flight trainers are complex multidimensional systems intended for operation in conditions of stochastic influences. One of the main purposes of such systems is providing dynamic conditions of real stochastically disturbed flight during ground tests and analysis of airborne control systems.

Normally, the scaled-down simulating systems combine the functions of simulators and trainers that make it possible to use the system as a flight simulator or a trainer as appropriate.

Problem statement. Five-degree-of-freedom stand simulator of (manned) spacecraft real motions is a necessary and complex test system for preflight training of astronauts. As a rule, such stand simulates translational motions of the object height and one of linear horizontal coordinates, as well as rotary motions of the object heading, pitch and roll. Figure 1 below shows kinematic scheme of the spacecraft stand simulator suspension. Translational coordinates of the stand motion are shown as z and y, and rotary motions are shown as ψ, ϑ and γ.

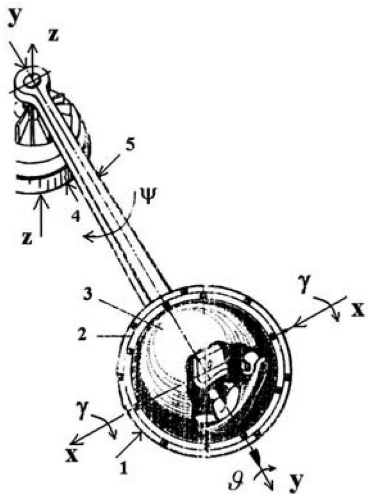

Fig. 1. Kinematic scheme of the suspension of five-degree-of-freedom stand simulator of the spacecraft flight

1. The axis of rotation $\gamma\gamma$ coincides with the axis xx.
2. The axis of rotation $\vartheta\vartheta$ is directed along the axis $\gamma\gamma$.
3. The cabin also rotates about the axis $\gamma\gamma$.
4. The fifth motion is performed by rotation of the arm 5 together with the gimbal suspension and the cabin about the vertical axis zz.

It should be noted that the action of all five coordinates of the stand simulator motions as perceived by the astronaut during training are actually concentrated in the point of sitting located on the inner platform of the gimbal suspension and are perceived by the astronaut in complex. This circumstance actually defines the nature of perception of the stand simulator motion actions by the astronaut.

Control paths for any of five coordinates of the stand are similar to a considerable extent. It is expedient to upgrade any of them as follows [2]. Figure 2 below shows flow chart of any of five upgraded paths, ζ for example.

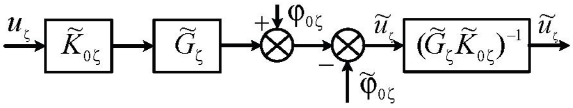

Fig. 2. Flow chart of upgraded path for setting the stand simulator motion program u_ζ (index «~» marks the elements to be adjusted during the path testing)

Fourier-transformed differential equation that describes transformation of the input signal u_ζ into the output signal $\tilde{u}_\zeta$ has the following form:

$$\tilde{u}_\zeta = (\tilde{G}_\zeta \tilde{K}_{0\zeta})^{-1} (\tilde{G}_\zeta \tilde{K}_{0\zeta} u_\zeta + \varphi_{0\zeta} - \tilde{\varphi}_{0\zeta}) \approx u_\zeta + (\tilde{G}_\zeta \tilde{K}_\zeta)^{-1} \tilde{\theta}_{\varphi_{0\zeta}} \quad (1)$$

The following elements are introduced into the equation (1): $\tilde{K}_{0\zeta}$ is frequency response of the setter of program u_ζ for the stand simulator motion along the coordinate ζ; $\tilde{G}_\zeta$ is frequency response of servo actuator in the examined path of the stand simulator motion; $\varphi_{0\zeta}$ is frequency response of the disturbance signal for setting program u_ζ; $\tilde{\varphi}_{0\zeta}$ is the evaluation of frequency response of the disturbance signal $\varphi_{0\zeta}$ as per results of the path testing; $\tilde{\theta}_{\varphi_{0\zeta}}$ is the difference in frequency response of disturbance signals $\varphi_{0\zeta}$ and $\tilde{\varphi}_{0\zeta}$. Frequency response of the vector of signals $\tilde{u}_0$ for the stand simulator motion program setting has the following form [3]:

$$\tilde{u}_0 = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} (\tilde{G}_1 \tilde{K}_{01})^{-1} \cdot \tilde{\theta}_{\varphi_{01}} \\ (\tilde{G}_2 \tilde{K}_{02})^{-1} \cdot \tilde{\theta}_{\varphi_{02}} \\ (\tilde{G}_1 \tilde{K}_{03})^{-1} \cdot \tilde{\theta}_{\varphi_{03}} \\ (\tilde{G}_1 \tilde{K}_{04})^{-1} \cdot \tilde{\theta}_{\varphi_{04}} \\ (\tilde{G}_1 \tilde{K}_{05})^{-1} \cdot \tilde{\theta}_{\varphi_{05}} \end{bmatrix} \approx u_0 + (\tilde{G}_0 \tilde{K}_0)^{-1} \cdot \tilde{\theta}_{\tilde{\varphi}_0}. \quad (2)$$

The system of Fourier-transformed and linearized differential equations of the stand simulator motion has the following form:

$$P_0 \tilde{x}_0 = M_0 \tilde{v}_0 + \psi_0,, \quad (3)$$

it defines the flow chart of the spacecraft motion stand simulator as follows (Fig. 3):

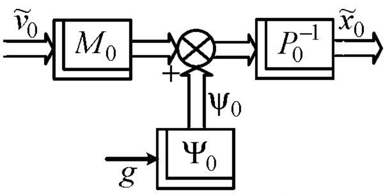

Fig. 3. Flow chart of the spacecraft motion stand simulator as the control object

(here $g = 1$ if action $\overline{\psi}_0$ is determinate and $g = \Delta$ (white noise) if action $\overset{\circ}{\psi}_0$ is random stationary)

It is expedient to introduce the following designations:

$$\Phi_{10} = P_0^{-1} M_0, \quad \psi_0 = \Psi_0 g, \quad \Phi_{20}, = P_0^{-1} \Psi_0 \quad (4)$$

where Φ_{10} and Φ_{20} are frequency response matrices of the stand simulator for control and disturbance.

Considering the designations (4), the system of differential equations (3) can be written in the following form:

$$\tilde{x}_0 = \Phi_{10} \tilde{v}_0 + \Phi_{20} g = (\Phi_{10} \Phi_{20}) \begin{pmatrix} \tilde{v}_0 \\ g \end{pmatrix} = V_0 \tilde{z}. \quad (5)$$

Flow chart of the stand simulator control system considering the upgraded input paths of the motion program setting and the upgraded paths of measurements of the output signal vectors of the control object is shown in Figure 4.

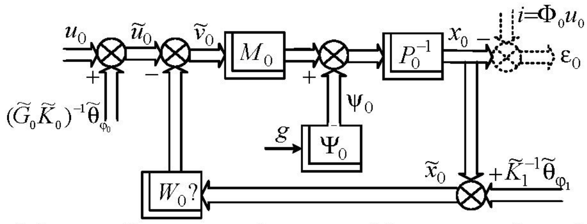

Fig. 4. Flow chart of the coordinate control system of the spacecraft motion stand simulator

Frequency response of the equivalent vector of control system disturbance when reducing it to the stabilization system is as follows:

$$\eta_0 = M_0[u_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\theta}_{\varphi_0}] + \Psi_0 g + P_0\tilde{K}_1^{-1}\tilde{\theta}_{\varphi_1};$$
$$\eta_{0*} = [u_{0*} + \tilde{\theta}_{\varphi*}(\tilde{G}_0\tilde{K}_0)^{-1}]M_{0*} + g_*\Psi_{0*} + \tilde{\theta}_{\varphi_1 *}\tilde{K}_{1*}^{-1}; \qquad (6)$$
$$\eta_{00} = (P_0^{-1})\eta_0 = \Phi_{10}[u_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\theta}_{\varphi_0}] + \Phi_{20}g + \tilde{K}_1^{-1}\tilde{\theta}_{\varphi_1}.$$

The equivalent flow chart of the stand simulator motion stabilization system will have the following form (Fig. 5):

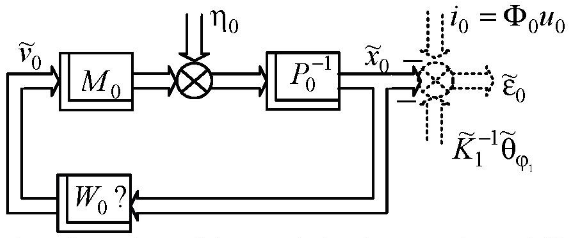

Fig. 5. Equivalent flow chart of the stand simulator motion stabilization system

It is expedient to introduce the following designations:

$$\tilde{v}_0 = F_{\tilde{v}_0}\eta_0; \quad \tilde{x}_0 = F_{\tilde{x}_0}\eta_0; \quad W_0 = F_{\tilde{v}_0}(F_{\tilde{x}_0})^{-1};$$

and also the equation of constraints of matrices:

$$F_{\tilde{x}_0} = \Phi_{10}F_{\tilde{u}_0} + P_0^{-1}; \qquad (7)$$

and the expression:

$$W_0 = F_{\tilde{v}_0}(\Phi_{10}F_{\tilde{v}_0} + P_0^{-1})^{-1}. \qquad (8)$$

Frequency response of the simulation error signal vectors under determinate actions will have the following form:

$$\tilde{\bar{\varepsilon}}_0 = \tilde{\bar{x}}_0 - \Phi_0\bar{u}_0 - \tilde{K}_1^{-1}\tilde{\bar{\theta}}_{\varphi_1} =$$
$$= \Phi_{10}\bar{F}_{\bar{v}_0}\bar{\eta}_0 + \Phi_{10}[\bar{u}_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}] + \Phi_{20}g - \Phi_0\bar{u}_0 = \qquad (9)$$
$$= \Phi_{10}\bar{F}_{\bar{v}_0}\bar{\eta}_0 + \Phi_{10}[\bar{u}_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}] - \Phi_0\bar{u}_0,$$

$$\bar{\varepsilon}_{0*} = \bar{\eta}_{0*}\bar{F}_{\bar{v}_{0*}}\Phi_{10*} + [\bar{u}_{0*} + (\tilde{G}_0\tilde{K}_0)_*^{-1}\tilde{\bar{\theta}}_{\varphi_{0*}}]\Phi_{10*} - \bar{u}_{0*}\Phi_{0*}.$$
$$\bar{v}_0 = \bar{F}_{\bar{v}_0}\bar{\eta}_0; \quad \bar{v}_{0*} = \bar{\eta}_{0*}\bar{F}_{\bar{v}_{0*}}. \qquad (10)$$

The simulation quality index (quality functional) under determinate actions has the following form:

$$\bar{I} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr(\tilde{\bar{\varepsilon}}_0\tilde{\bar{\varepsilon}}_{0*}\bar{R} + \tilde{\bar{v}}_0\tilde{\bar{v}}_{0*}\bar{C})\,ds, \quad s = j\omega. \qquad (11)$$

The quality functional of the spacecraft motion simulation under random stationary actions has the following form:

$$\mathring{e} = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\,(S'_{\tilde{\mathring{\varepsilon}}_0 \tilde{\mathring{\varepsilon}}_0}\mathring{R} + S'_{\tilde{\mathring{v}}_0 \tilde{\mathring{v}}_0}\mathring{C})\,ds, \tag{12}$$

where $S'_{\mathring{\varepsilon}_0 \mathring{\varepsilon}_0}$ and $S'_{\mathring{v}_0 \mathring{v}_0}$ are spectral density matrices of vectors $\mathring{\varepsilon}_0$ and $\mathring{v}_0$.

By analogy with the expression (9), frequency response of signal vectors $\tilde{\mathring{\varepsilon}}_0$ and $\tilde{\mathring{v}}_0$ has the following form:

$$\begin{aligned}
\tilde{\mathring{\varepsilon}}_0 &= \Phi_{10}\mathring{F}_{\mathring{v}_0}\mathring{\eta}_0 + \Phi_{10}[\mathring{u}_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\mathring{\theta}}_{\varphi_0}] - \Phi_0\mathring{u}_0;\\
\tilde{\mathring{\varepsilon}}_{0*} &= \mathring{\eta}_{0*}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + [\mathring{u}_{0*} + \tilde{\mathring{\theta}}_{\varphi_0 *}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} - \mathring{u}_{0*}\Phi_{0*},
\end{aligned} \tag{13}$$

and the transposed matrices of spectral densities of signal vectors $\tilde{\mathring{\varepsilon}}_0$ and $\tilde{\mathring{v}}_0$ are as follows:

$$\begin{aligned}
&S'_{\tilde{\mathring{\varepsilon}}_0 \tilde{\mathring{\varepsilon}}_0} = <\tilde{\mathring{\varepsilon}}_0 \tilde{\mathring{\varepsilon}}_{0*}> = <\{\Phi_{10}\mathring{F}_{\mathring{v}_0}\mathring{\eta}_0 + \Phi_{10}[\mathring{u}_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\mathring{\theta}}_{\varphi_0}] - \Phi_0\mathring{u}_0\}\\
&\{\mathring{\eta}_{0*}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + [\mathring{u}_{0*} + \tilde{\mathring{\theta}}_{\varphi_0 *}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\,\Phi_{10*} - \mathring{u}_{0*}\Phi_{0*}\}> = \Phi_{10}\mathring{F}_{\mathring{v}_0} S'_{\mathring{\eta}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} +\\
&+\Phi_{10}\mathring{F}_{\mathring{v}_0}[S'_{\mathring{\eta}_0\mathring{u}_0} + (\tilde{G}_0\tilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}\mathring{\varphi}_0\mathring{\varphi}_0}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\,\Phi_{10*} - \Phi_{10}\mathring{F}_{\mathring{v}_0}\tilde{S}'_{\mathring{\eta}_0\mathring{u}_0}\mathring{F}_{\mathring{v}_{0*}}\Phi_{0*} +\\
&+\Phi_{10}[S'_{\mathring{u}_0\mathring{\eta}_0} + (\tilde{G}_0\tilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}\mathring{\varphi}_0\tilde{\mathring{\theta}}\mathring{\varphi}_0}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + \Phi_{10}[S'_{\mathring{u}_0\mathring{u}_0} + 0 + 0 +\\
&+(\tilde{G}_0\tilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}\mathring{\varphi}_0\tilde{\mathring{\theta}}\mathring{\varphi}_0}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} - \Phi_{10}(S'_{\mathring{u}_0\mathring{u}_0} + 0)\Phi_{0*} - \Phi_0\tilde{S}'_{\mathring{u}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} -\\
&-\Phi_0(S'_{\mathring{u}_0\mathring{u}_0} + 0)\,\Phi_{10*} + \Phi_0 S'_{\mathring{u}_0\mathring{u}_0}\Phi_{0*};
\end{aligned} \tag{14}$$

$$S'_{\mathring{v}_0\mathring{v}_0} = <\mathring{v}_0\mathring{v}_{0*}> = <\mathring{F}_{\mathring{v}_0}\mathring{\eta}_0\mathring{\eta}_{0*}\mathring{F}_{\mathring{v}_{0*}}> = \mathring{F}_{\mathring{v}_0}S'_{\mathring{\eta}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}. \tag{15}$$

Synthesis of the optimized structure of the regulator in the equivalent stabilization system of the spacecraft motion stand simulator under determinate actions

Substitution of expressions (9) and (10) into the functional (11) presents the functional in the following form:

$$\begin{aligned}
&\bar{I} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr(\{\Phi_{10}\bar{F}_{\bar{v}_0}\bar{\eta}_0 + \Phi_{10}[\bar{u}_0 + (\tilde{G}_0\tilde{K}_0)^{-1}\bar{\tilde{\bar{\theta}}}_{\varphi_0}] - \Phi_0\bar{u}_0\}\\
&\{\bar{\eta}_{0*}\bar{F}_{\bar{v}_{0*}}\Phi_{10*} + [\bar{u}_{0*} + \bar{\tilde{\bar{\theta}}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} - \bar{u}_{0*}\Phi_{0*}\}\bar{R} +\\
&+\bar{F}_{v_0}\bar{\eta}_0\bar{\eta}_{0*}\bar{F}_{v_{0*}}\bar{C})\,ds = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr((\{\Phi_{10}\bar{F}_{\bar{v}_0}\bar{\eta}_0\bar{\eta}_{0*}\bar{F}_{v_{0*}}\Phi_{10*} +\\
&+\Phi_{10}\bar{F}_{\bar{v}_0}[\bar{\eta}_0\bar{u}_{0*} + \bar{\eta}_0\bar{\tilde{\bar{\theta}}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} - \Phi_{10}\bar{F}_{\bar{v}_0}\bar{\eta}_0\bar{u}_{0*}\Phi_{0*} +\\
&\Phi_{10}[\bar{u}_0\bar{\eta}_{0*} + (\tilde{G}_0\tilde{K}_0)^{-1}\bar{\tilde{\bar{\theta}}}_{\varphi_0}\bar{\eta}_{0*}]\bar{F}_{\bar{v}_{0*}}\Phi_{10*} + \Phi_{10}[\bar{u}_0\bar{u}_{0*} + (\tilde{G}_0\tilde{K}_0)^{-1}\bar{\tilde{\bar{\theta}}}_{\varphi_0}\bar{u}_{0*} +\\
&\bar{u}_0\bar{\tilde{\bar{\theta}}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)^{-1}_{*} + (\tilde{G}_0\tilde{K}_0)^{-1}\bar{\tilde{\bar{\theta}}}_{\varphi_0}\bar{\tilde{\bar{\theta}}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} - \Phi_{10}[\bar{u}_0\bar{u}_{0*} +\\
&+(\tilde{G}_0\tilde{K}_0)^{-1}\bar{\tilde{\bar{\theta}}}_{\varphi_0}\bar{u}_{0*}]\Phi_{0*} - \Phi_0\bar{u}_0\bar{\eta}_{0*}\bar{F}_{\bar{v}_*}\Phi_{10*} - \Phi_0[\bar{u}_0\bar{u}_{0*} + \bar{u}_0\bar{\tilde{\bar{\theta}}}_{\varphi_0 *}(\tilde{G}_0\tilde{K}_0)^{-1}_{*}]\Phi_{10*} +\\
&+\Phi_0\bar{u}_0\bar{u}_{0*}\Phi_{0*}\}\bar{R} + \bar{F}_{v_0}\bar{\eta}_0\bar{\eta}_{0*}\bar{F}_{v_{0*}}\bar{C}))ds.
\end{aligned} \tag{16}$$

The problem of synthesis of the optimized (or optimum) structure of the regulator in the equivalent stabilization system of the stand simulator motions under determinate actions can be solved using Wiener-Kolmogorov method [4]. The first variation of the functional (16) has the following form:

$$\delta\bar{I}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}tr\,((\{\Phi_{10*}\bar{R}\Phi_{10}+\bar{C})\bar{F}_{\bar{v}_0}\bar{\eta}_0\bar{\eta}_{0*}+$$
$$+\Phi_{10*}\bar{R}\Phi_{10}[\bar{u}_0\bar{\eta}_{0*}+(\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}\bar{\eta}_{0*}]-$$
$$-\Phi_{10*}\bar{R}\Phi_0\bar{u}_0\bar{\eta}_{0*}\}\delta\bar{F}_{\bar{v}_{0*}}\delta\bar{F}_{\bar{v}_0}\{\bar{\eta}_0\bar{\eta}_{0*}\bar{F}_{v_{0*}}\times \quad (17)$$
$$\times(\Phi_{10*}\bar{R}\Phi_{10}+\bar{C})+[\bar{\eta}_0\bar{u}_{0*}+\bar{\eta}_0\tilde{\bar{\theta}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)_*^{-1}]\times$$
$$\times\Phi_{10*}\bar{R}\Phi_{10}-\bar{\eta}_0\bar{u}_{0*}\Phi_{0*}\bar{R}\Phi_{10}\}))ds.$$

The following designations are introduced into the variation (17):

$$\bar{\Gamma}_*\bar{\Gamma}=\Phi_{10}\bar{R}\Phi_{10}+\bar{C};\quad \bar{D}\bar{D}_*\approx\bar{\eta}_0\bar{\eta}_{0*},\quad \text{and } |\bar{\eta}_0\bar{\eta}_{0*}|=0;$$

$$\bar{T}=\bar{T}_0+\bar{T}_++\bar{T}_-\approx\Phi_{10}\bar{R}\{\Phi_{10}[\bar{u}_0+(\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}]-\Phi_0\bar{u}_0\}\bar{\eta}_{0*}(\bar{D}_*)^{-1}. \quad (18)$$

Considering the designation (18), the variation (17) will take the following form:

$$\delta\bar{I}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}tr\,[\bar{\Gamma}_*(\bar{\Gamma}\bar{F}_{\bar{v}_0}\bar{D}+\bar{T})\bar{D}_*\delta\bar{F}_{\bar{v}_{0*}}+\delta\bar{F}_{\bar{v}_0}\bar{D}(\bar{D}_*\bar{F}_{\bar{v}_{0*}}\bar{\Gamma}_*+\bar{T}_*)\bar{\Gamma}]\,ds,$$

and the condition of approximate equality of the variation to zero will be as follows:

$$\bar{\Gamma}\bar{F}_{\bar{v}_0}\bar{D}\approx-(\bar{T}_0+\bar{T}_+).$$

The synthesis algorithm for the optimum structure of matrix $\hat{\bar{F}}_{\bar{v}_0}$ has the following form:

$$\hat{\bar{F}}_{\bar{v}_0}\approx-\bar{\Gamma}^{-1}(\bar{T}_0+\bar{T}_+)\bar{D}_*^{-1},$$

The equation of constraints of matrices $\hat{\bar{F}}_{\bar{v}_0}$ and $\bar{F}_{\bar{x}_0}$ is as follows:

$$\bar{F}_{\bar{x}_0}=\Phi_{10}\bar{F}_{\bar{v}_0}+P_0^{-1},$$

and the optimized structure $\hat{W}_0$ of the regulator in the stabilization system will have the following form:

$$\hat{\bar{W}}_0=\hat{\bar{F}}_{\bar{v}_0}(\hat{\bar{F}}_{\bar{x}_0})^{-1}\approx\hat{\bar{F}}_{\bar{v}_0}(\Phi_{10}\hat{\bar{F}}_{\bar{v}_0}+P_0^{-1})^{-1}. \quad (19)$$

Thereby, the set problem of synthesis of the optimized structure of the regulator is solved.

Partial version of the problem of synthesis of the optimum structure of the regulator in the equivalent stabilization system of the spacecraft stand simulator motions under determinate actions

Let all features of signal vector $\bar{\eta}_0$ lie in only left half plane of complex variable $s=j\omega$. Then the first variation (17) can be rewritten as follows:

$$\delta\bar{I}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}tr\,((\Phi_{10*}\bar{R}\Phi_{10}+\bar{C})\bar{F}_{\bar{v}_0}\bar{\eta}_0+\Phi_{10*}\bar{R}\{[\Phi_{10}\bar{u}_0+(\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}]-$$
$$-\Phi_0\bar{u}_0\}\bar{\eta}_{0*}\delta\bar{F}_{\bar{v}_{0*}}+\delta\bar{F}_{\bar{v}_0}\bar{\eta}_0\{\bar{\eta}_{0*}\bar{F}_{\bar{v}_{0*}}\times(\Phi_{10*}\bar{R}\Phi_{10}+\bar{C})+\{[\bar{u}_{0*}\Phi_{10*}+ \quad (20)$$
$$+\tilde{\bar{\theta}}_{\varphi_{0*}}(\tilde{G}_0\tilde{K}_0)_*^{-1}]-\bar{u}_{0*}\Phi_{0*}\}\bar{R}\Phi_{10}))ds.$$

The following designations are introduced into the variation (20):

$$\bar{\Gamma}_*\bar{\Gamma}=\Phi_{10*}\bar{R}\Phi_{10}+\bar{C};$$
$$\bar{T}=\bar{T}_0+\bar{T}_++\bar{T}=(\bar{\Gamma}_*)^{-1}\Phi_{10*}\bar{R}\{[\Phi_{10}\bar{u}_0+(\tilde{G}_0\tilde{K}_0)^{-1}\tilde{\bar{\theta}}_{\varphi_0}]-\Phi_0\bar{u}_0\}. \quad (21)$$

Considering the designations (21), the variation (20) will take the following form:

$$\delta\bar{I}=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}tr\,[\bar{\Gamma}_*(\bar{\Gamma}\bar{F}_{\bar{v}_0}\bar{\eta}_0+\bar{T})\bar{\eta}_{0*}\delta\bar{F}_{\bar{v}_{0*}}+\delta\bar{F}_{\bar{v}_0}\bar{\eta}_0(\bar{\eta}_{0*}\bar{F}_{\bar{v}_{0*}}\bar{\Gamma}_*+\bar{T}_*)\bar{\Gamma}]\,ds$$

and the condition of equality of the variation (20) to zero will be as follows:

$$\overline{\Gamma}\overline{F}_{\bar{v}_0}\overline{\eta}_0 = -(\overline{T}_0 + \overline{T}_+)$$

The optimum structure of matrix $\hat{\overline{F}}_{\bar{v}_0}$ will have the following form:

$$\hat{\overline{F}}_{\bar{v}_0} = -(\overline{\Gamma})^{-1}(\overline{T}_0 + \overline{T}_+)\overline{\eta}_0^{\#}, \tag{22}$$

where sign «#» is the symbol of the vector pseudo-inversion.

As per Gantmaher [2], pseudo-inversed vector $\overline{\eta}_0^{\#}$ complies with the following expression:

$$\overline{\eta}_0^{\#} = A^+ = [C^*(CC^*)^{-1}(B^*B)^{-1}B^*]$$

where $B = \overline{\eta}_0$; $C = (1,0)$, $B^* = \overline{\eta}_0'$; $C^* = \begin{pmatrix}1\\0\end{pmatrix}$, $CC^* = 1{,}0$;

$(B^*B)^{-1}B^* = \overline{\eta}_0^{-1}(\overline{\eta}_0')^{-1}\overline{\eta}_0' = \overline{\eta}_0^{-1}$, т.е. $\overline{\eta}_0^{\#} = A^+ = \overline{\eta}_0^{-1}$.

Thereby, the optimum structure of matrix $\overline{F}_{\bar{v}_0}$ should be written as follows:

$$\hat{\overline{F}}_{\bar{v}_0} = -\overline{\Gamma}^{-1}(\overline{T}_0 + \overline{T}_+)\overline{\eta}_0^{-1}. \tag{23}$$

The equation of constraints of matrices $\overline{F}_{\bar{x}_0}$ and $\overline{F}_{\bar{v}_0}$ will have the following form:

$$\overline{F}_{\bar{x}_0} = \Phi_{10}\overline{F}_{\bar{v}_0} + P_0^{-1},$$

and the optimum structure of the regulator $\hat{\overline{W}}_0$ in the equivalent stabilization system of the stand simulator motion has the following form:

$$\hat{\overline{W}}_0 = \hat{\overline{F}}_{\bar{v}_0}(\hat{\overline{F}}_{\bar{x}_0})^{-1} = \hat{\overline{F}}_{\bar{v}_0}(\Phi_{10}\hat{\overline{F}}_{\bar{v}_0} + P_0^{-1})^{-1}. \tag{24}$$

Synthesis of the optimum structure $\hat{\overline{W}}_0$ of the regulator in the equivalent stabilization system of the spacecraft stand simulator motions under random stationary actions

The problem of synthesis of the optimum structure of the regulator in this version can be also solved using Wiener-Kolmogorov method. The matrices (14) and (15) shall be substituted into the stabilization quality functional (12). After this the functional (12) will take the following form:

$$\begin{aligned}\mathring{e} = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\,((\{&\Phi_{10}\mathring{F}_{\mathring{v}_0} S'_{\mathring{\eta}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + \Phi_{10}\mathring{F}_{\mathring{v}_0}[S'_{\mathring{\eta}_0\mathring{u}_0} + (\widetilde{G}_0\widetilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}_{\varphi_0}\tilde{\mathring{\theta}}_{\varphi_0}} \times \\ &\times(\widetilde{G}_0\widetilde{K}_0)_*^{-1}]\Phi_{10*} - \Phi_{10}\mathring{F}_{\mathring{v}_0}S'_{\mathring{\eta}_0\mathring{u}_0}\Phi_{0*} + \Phi_{10}[S'_{\mathring{u}_0\mathring{\eta}_0} + (\widetilde{G}_0\widetilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}_{\varphi_0}\tilde{\mathring{\theta}}_{\varphi_0}}(\widetilde{G}_0\widetilde{K}_0)_*^{-1}]\times \\ &\times\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + \Phi_{10}[S'_{\mathring{u}_0\mathring{u}_0} + (\widetilde{G}_0\widetilde{K}_0)^{-1}\times S'_{\tilde{\mathring{\theta}}_{\varphi_0}\tilde{\mathring{\theta}}_{\varphi_0}}(\widetilde{G}_0\widetilde{K}_0)_*^{-1}]\Phi_{10*} - \Phi_{10}S'_{\mathring{u}_0\mathring{u}_0}\Phi_{0*} - \\ &-\Phi_0 S'_{\mathring{u}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}\Phi_{10*} + \Phi_0 S'_{\mathring{u}_0\mathring{\eta}_0}\Phi_{0*}\}\mathring{R} + \mathring{F}_{\tilde{v}_0}S'_{\mathring{\eta}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}\mathring{C}))\,ds.\end{aligned} \tag{25}$$

The first variation of the functional (25) has the following form:

$$\begin{aligned}\delta\mathring{e} = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\,((\{&[(\Phi_{10*}\mathring{R}\Phi_{10} + \mathring{C})\mathring{F}_{\mathring{v}_{0*}}S'_{\mathring{\eta}_0\mathring{\eta}_0} + \Phi_{10*}\mathring{R}\Phi_{10}[S'_{\mathring{u}_0\mathring{\eta}_0} + (\widetilde{G}_0\widetilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}_{\varphi_0}\tilde{\mathring{\theta}}_{\varphi_0}} \times \\ &\times(\widetilde{G}_0\widetilde{K}_0)_*^{-1}] - \Phi_{10*}\mathring{R}\Phi_0 S'_{\mathring{u}_0\mathring{\eta}_0}\}\delta\mathring{F}_{\mathring{v}_{0*}} + \delta\mathring{F}_{\mathring{v}_0}\{[S'_{\mathring{\eta}_0\mathring{\eta}_0}\mathring{F}_{\mathring{v}_{0*}}(\Phi_{10*}\mathring{R}\Phi_{10} + \mathring{C}) + \\ &+[S'_{\mathring{\eta}_0\mathring{u}_0}(\widetilde{G}_0\widetilde{K}_0)^{-1}S'_{\tilde{\mathring{\theta}}_{\varphi_0}\tilde{\mathring{\theta}}_{\varphi_0}}(\widetilde{G}_0\widetilde{K}_0)_*^{-1}]\,\Phi_{10*}\mathring{R}\Phi_{10} - S'_{\mathring{\eta}_0\mathring{u}_0}\Phi_{0*}\mathring{R}\Phi_{10}\}))\,ds.\end{aligned} \tag{26}$$

The following designations are introduced into the variation (26):

$$\overset{\circ}{\Gamma}_{*}\overset{\circ}{\Gamma}=\Phi_{10*}\overset{\circ}{R}\Phi_{10}+\overset{\circ}{C};\qquad \overset{\circ}{D}\overset{\circ}{D}_{*}=S'_{\overset{\circ}{\eta}_0\overset{\circ}{\eta}_0};$$
$$\overset{\circ}{\mathrm{T}}=\overset{\circ}{\mathrm{T}}_0+\overset{\circ}{\mathrm{T}}_{+}+\overset{\circ}{\mathrm{T}}_{-}=(\overset{\circ}{\Gamma}_{*})^{-1}\Phi_{10*}\overset{\circ}{R}\{\Phi_{10}[S'_{\overset{\circ}{u}_0\eta_0}+(\widetilde{G}_0\widetilde{K}_0)^{-1}S'_{\overset{\circ}{\theta}_{\varphi_0}\tilde{\theta}_{\varphi_0}}(\widetilde{G}_0\widetilde{K}_0)_{*}^{-1}]-\Phi_0 S'_{\overset{\circ}{u}_0\eta_0}\}(\overset{\circ}{D}_{*})^{-1}. \tag{27}$$

Using the designations (27), the variation (26) will take the following form:

$$\delta\overset{\circ}{e}=\frac{1}{j}\int_{-j\infty}^{j\infty}tr[\overset{\circ}{\Gamma}_{*}(\overset{\circ}{\Gamma}\overset{\circ}{F}_{\overset{\circ}{v}_0}\overset{\circ}{D}+\overset{\circ}{\mathrm{T}})\overset{\circ}{D}_{*}\,\delta\overset{\circ}{F}_{\overset{\circ}{v}_{0*}}+\delta\overset{\circ}{F}_{\overset{\circ}{v}_0}\overset{\circ}{D}(\overset{\circ}{D}_{*}\overset{\circ}{F}_{\overset{\circ}{v}_{0*}}\overset{\circ}{\Gamma}_{*}+\overset{\circ}{\mathrm{T}}_{*})\overset{\circ}{\Gamma}]\,ds,$$

and the condition of equality of the variation (26) to zero will be as follows:

$$\overset{\circ}{\Gamma}\overset{\circ}{F}_{\overset{\circ}{v}_0}\overset{\circ}{D}=-(\overset{\circ}{\mathrm{T}}_0+\overset{\circ}{\mathrm{T}}_{+}).$$

The synthesis algorithm for the optimum structure of matrix $\hat{\overset{\circ}{F}}_{\overset{\circ}{v}_0}$ should be written as follows:

$$\hat{\overset{\circ}{F}}_{\overset{\circ}{v}_0}=-\overset{\circ}{\Gamma}^{-1}(\overset{\circ}{\mathrm{T}}_0+\overset{\circ}{\mathrm{T}}_{+})\overset{\circ}{D}^{-1}.$$

The equation of constraints of matrices $\overset{\circ}{F}_{\overset{\circ}{v}_0}$ and $\overset{\circ}{F}_{\overset{\circ}{x}_0}$ has the following form:

$$\overset{\circ}{F}_{\overset{\circ}{x}_0}=\Phi_{10}\overset{\circ}{F}_{\overset{\circ}{v}_0}+P_0^{-1}.$$

The optimum structure of the regulator in the equivalent stabilization system of the stand simulator motion is defined by the following expression:

$$\overset{\circ}{W}_0=\hat{\overset{\circ}{F}}_{\overset{\circ}{v}_0}(\hat{\overset{\circ}{F}}_{\tilde{x}_0})^{-1}=\hat{\overset{\circ}{F}}_{\overset{\circ}{v}_0}(\Phi_{10}\hat{\overset{\circ}{F}}_{\overset{\circ}{v}_0}+P_0^{-1})^{-1}. \tag{28}$$

Hereby, the problem of synthesis of the optimum structure of the regulator $\hat{\overset{\circ}{W}}_0$ in the equivalent stabilization system of the stand simulator motion is solved.

Conclusion. This paper provides a practically effective methodology of analytic design of the optimum stochastic control system of five-degree-of-freedom stand simulator of the spacecraft motion. The stand simulator is required for preflight training of the spacecraft crew.

This paper addresses and strictly solves the issues of synthesis of the optimum structures of regulators in control systems under determinate and random stationary actions on examined objects.

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