

Ministry of Education and Science of Ukraine
National Aviation University

**CONTINUING AIRCRAFT
AIRWORTHINESS
(ICAO Doc 9760)**

SELF-STUDY METHOD GUIDE

Part II

**Application of the Multi-Optional
Functions Entropy Doctrine to Assess
the Aircraft Maintenance Process
Improvements for the Students of the
Field of Study 27 "Transport",
Specialty 272 "Aviation Transport",
Specialization 01 "Aircraft and Aero-Engines
Maintenance and Repair"**

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Містять декілька рекомендацій для самостійної роботи щодо застосування ентропійної доктрини багатоопційних функцій для оцінки удосконалення процесу технічного обслуговування повітряного судна в рамках елементів наукового дослідження.

Для студентів 1-го курсу галузі знань 27 «Транспорт», спеціальності 272 «Авіаційний транспорт», спеціалізації 01 «Технічне обслуговування та ремонт повітряних суден і авіадвигунів».

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The METHOD GUIDE contains a few recommendations on the Self-Study in regards with the Multi-Optional Functions Entropy Doctrine application to assess the aircraft maintenance process improvements in the framework of the scientific research elements.

Designed for the 1st year students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Maintenance and Repair of Aircraft and Aircraft Engines".

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INTRODUCTION

This **METHOD GUIDE ON THE SELF-STUDY (SS)** is contemplated in response to the needs of our students in more detailed elaborations concerning problems stated, set or given for the students' independent work, possibly used in their course projecting, further graduation papers or even Ph.D. studies. The whole material is split into portions. Each portion is intended to cover a fraction of probable applications aimed at **CONTINUING AIRCRAFT AIRWORTHINESS** or its retaining measures.

The presented in the, second, **PART II** of the **METHOD GUIDE ON THE SS** problems are dedicated, and a special attention is drawn here, to the scientific component of the SS work. Specifically, the objectives of the **PART II** are to help students cope with the challenging problems relating to the **AIRCRAFT (A/C)** technical operation in regards with the aeronautical engineering **MAINTENANCE (M/T)** optimal periodicities.

The set of the considered problems is based upon the **RECOMMENDED LITERATURE SOURCES** (the list is presented, but not limited to it). The **LIST OF LITERATURE** at the end of the **METHOD GUIDE** is basic (major) and compiled in the alphabetic order with respect to the matter of supposed (assumed) importance.

The **REFERENCES LIST** is selected, set in the order [1-144], does not pretend for completeness, but instead it is aimed at developing the students' abilities of thinking and to analyze, contemplate in the specified directory rather than their abilities to know and memorize. However, these are very significant too. Actually, in the contemporary informative boom world, the needed or required data can easily be retrieved from the internet, found in multiple references, guidance materials [1, 69, 71, 74, 111, 112, 134, 135, 137], studies [2, 66-68, 75-88, 113-116, 119-133, 136, 139-144], dictionaries [70, 110], comprehensive books [3, 72] or monographs [64, 65, 86-88, 113, 114, 117, 118, 123] etc. The **METHOD GUIDE** is designed for the 1st year students of the Field of Study: 27 "Transport", Specialty: 272 "Aviation Transport", Specialization: 01 "Maintenance and Repair of Aircraft and Aircraft Engines". It includes detailed solutions for obtaining reliability objective measures allowing assessing the improvements of the A/C functional system M/T process considered in reference [112].

AERONAUTICAL ENGINEERING MAINTENANCE OPTIMAL PERIODICITY DETERMINATION

The principal theoretical provisions can be found out in references [136, 141-143, 3, 72, 76, 79, 82, 84, 87, 110, 114, 116, 122, 124-133, 139, 140].

1. Determination of the periodicity via a graph construction and with the use (help) of the Laplace transformations

The presented case study centers the idea of the aeronautical engineering reliability probabilistic characteristics determination based on the related graph of the A/C given functional system functioning and the system's from state to state transitioning construction. The initial modeling is suggested (proposed) for the consideration with the use (help) of the graph represented in Fig. 1, as in the *Theory of Mass Service*, [136].

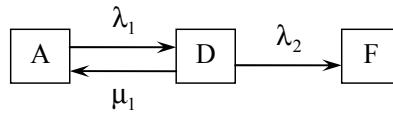


Fig. 1. Graph of three states of an aircraft functional system with a possible transition from the normal into damaged and backwards as well as from the damaged into failure state

Here, in Fig. 1, “A” designates the up state of the system; “D” – damage; “F” – failure. Accepting, for instance, the law of the damages appearance times distribution, as well as of the damages restoration times to the up state and the damages development times to the failure happening, as the exponential ones with the corresponding rates of λ_1 , μ_1 , and λ_2 (see Fig. 1), [143, p. 171], that is

$$f_1(t) = \lambda_1 e^{-\lambda_1 t}, \quad f_2(t) = \lambda_2 e^{-\lambda_2 t}, \quad f_3(t) = \mu_1 e^{-\mu_1 t}, \quad (1)$$

where t – time, there is a possibility to write down the corresponding, to the graph of Fig. 1, system of the differential equations by *Erlang* in the following view of the three ordinary linear equations of the first order of (2):

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D, \end{aligned} \right\} \quad (2)$$

where P_A , P_D , and P_F – probabilities of the corresponding states (Fig. 1).

The *Theory of Mass Service* [136, 82] provisions imply that the flow of events occurring in the system, which graph of states and transitions is described with the graph portrayed in Fig. 1, obeys the *Poisson Law*.

It means that for the *Poisson Flow* the number of the events getting into any time interval is distributed in accordance with the *Poisson Law*: [136, § 1.4, pp. 17, 18, (1.4.2)].

And for the *Simplest (Stationary Poisson) Flow* the time interval between any two neighboring events is distributed with the exponential law in the kind of (1) with the corresponding parameter of the type of λ or μ : [136, § 1.4, p. 19, (1.4.7)].

It results in a *Markovian Random Process* with the discrete states and continuous time going on in the corresponding *Poisson System* (see the graph of such system shown in Fig. 1): [136, § 2.2, pp. 51-53].

Such kinds of processes and systems are pretty (quite, relatively, fairly, rather) often (or at least not seldom) encountered in practical.

One of the methods of the system of Eq. (2) solution is presented with the *Laplace transformations* in the *operational calculus* [140, Chapter XIX, pp. 400-432].

The system of Eq. (2) is transformed with [140, Chapter XIX, § 1, p. 401, (4)]:

$$F(p) = \int_0^{+\infty} e^{-pt} f(t) dt, \quad (3)$$

where p – complex parameter (variable).

The function $F(p)$ is called the *Laplace transformant (image)* of the function $f(t)$, which is called the *initial function, or original*. The indication is [140, Chapter XIX, § 1, pp. 401, 402, (7)]:

$$L\{f(t)\} = F(p). \quad (4)$$

In accordance with the theorem for transformants of derivatives [140, Chapter XIX, § 8, p. 409, (27)], the system of Eq. (2) will have the corresponding algebraic system

$$\left. \begin{aligned} & \left(pF_A(p) - [P_A|_{t_0=0} = 1] = L\left\{\frac{dP_A}{dt}\right\} = \right. \\ & = -\lambda_1[F_A(p) = L\{P_A\}] + \mu_1[F_D(p) = L\{P_D\}]; \\ & \left. \left(pF_D(p) - [P_D|_{t_0=0} = 0] = L\left\{\frac{dP_D}{dt}\right\} = \right. \right. \\ & = \lambda_1[F_A(p) = L\{P_A\}] - (\lambda_2 + \mu_1)[F_D(p) = L\{P_D\}]; \\ & \left. \left(pF_F(p) - [P_F|_{t_0=0} = 0] = L\left\{\frac{dP_F}{dt}\right\} = \right. \right. \\ & = \lambda_2[F_D(p) = L\{P_D\}]. \end{aligned} \right\}$$

$$\left. \begin{aligned} pF_A(p) - 1 &= -\lambda_1 F_A(p) + \mu_1 F_D(p); \\ pF_D(p) - 0 &= \lambda_1 F_A(p) - (\lambda_2 + \mu_1) F_D(p); \\ pF_F(p) - 0 &= \lambda_2 F_D(p). \end{aligned} \right\}. \quad (5)$$

The obtained algebraic equations system, Eq. (5), solving is possible in different ways.

One of them is a matrix-vector.

Let us rewrite the system of Eq. (5) in the following style

$$\left. \begin{aligned} (p + \lambda_1)F_A & - \mu_1 F_D & + 0 & = 1; \\ -\lambda_1 F_A & + (p + \lambda_2 + \mu_1)F_D & + 0 & = 0; \\ 0 & - \lambda_2 F_D & + pF_F & = 0. \end{aligned} \right\} \quad (6)$$

The matrix for the transformation of the system of Eq. (6) will be [140, Chapter XXI, § 1, p. 510, (5)]:

$$\mathbf{M} = \begin{vmatrix} p + \lambda_1 & -\mu_1 & 0 \\ -\lambda_1 & p + \lambda_2 + \mu_1 & 0 \\ 0 & -\lambda_2 & p \end{vmatrix}. \quad (7)$$

The needed (unknown/wanted/sought) vector-column of transformants is

$$\mathbf{F} = \begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix}. \quad (8)$$

Then the transformation of the system of Eq. (6) is [140, Chapter XXI, § 8, p. 522, (5)]:

$$\begin{vmatrix} p + \lambda_1 & -\mu_1 & 0 \\ -\lambda_1 & p + \lambda_2 + \mu_1 & 0 \\ 0 & -\lambda_2 & p \end{vmatrix} \cdot \begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}. \quad (9)$$

Or, to make it shorter it is [140, Chapter XXI, § 8, p. 523, (6)]:

$$\mathbf{M} \cdot \mathbf{F} = \mathbf{B}. \quad (10)$$

where \mathbf{B} – vector-column of the free members of the system of Eq. (6):

$$\mathbf{B} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}. \quad (11)$$

The required solution of Eq. (8) will be [140, Chapter XXI, § 9, p. 523, (2)] found with the use of the inverse matrix \mathbf{M}^{-1} :

$$\mathbf{F} = \mathbf{M}^{-1} \cdot \mathbf{B}. \quad (12)$$

The last equation (12) with taking into account [140, Chapter XXI, § 7, p. 521, (5)]:

$$\mathbf{M}^{-1} = \frac{1}{\Delta(\mathbf{M})} \cdot \tilde{\mathbf{M}}, \quad (13)$$

where, $\Delta(\mathbf{M})$ – determinant of matrix \mathbf{M} , Eq. (7), [140, Chapter XXI, § 7, p. 520, (2)]; $\tilde{\mathbf{M}}$ – adjacent matrix to matrix \mathbf{M} , [140, Chapter XXI, § 7, p. 521, (4)]; can be written as [140, Chapter XXI, § 9, p. 523, (3)]:

$$\mathbf{F} = \frac{1}{\Delta(\mathbf{M})} \cdot \tilde{\mathbf{M}} \cdot \mathbf{B}, \quad (14)$$

or, in the developed view [140, Chapter XXI, § 9, p. 523, (4)]:

$$\begin{pmatrix} F_A \\ F_D \\ F_F \end{pmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{pmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (15)$$

where M_{ij} – algebraic addition of the element of m_{ij} , [140, Chapter XXI, § 2, p. 512], of the initial matrix Eq. (7).

Fulfilling multiplying of the matrixes in the right hand part of Eq. (15) we will obtain [140, Chapter XXI, § 9, p. 523, (5)]:

$$\begin{pmatrix} F_A \\ F_D \\ F_F \end{pmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{pmatrix} M_{11} \cdot 1 + M_{21} \cdot 0 + M_{31} \cdot 0 \\ M_{12} \cdot 1 + M_{22} \cdot 0 + M_{32} \cdot 0 \\ M_{13} \cdot 1 + M_{23} \cdot 0 + M_{33} \cdot 0 \end{pmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{pmatrix} M_{11} \\ M_{12} \\ M_{13} \end{pmatrix}. \quad (16)$$

In accordance with the initial matrix Eq. (7)

$$M_{11} = (-1)^{+1} \cdot [(p + \lambda_2 + \mu_1)p + \lambda_2 \cdot 0] = (p + \lambda_2 + \mu_1)p. \quad (17)$$

$$M_{12} = (-1)^{+2} \cdot [(-\lambda_1 p) - 0 \cdot 0] = \lambda_1 p. \quad (18)$$

$$M_{13} = (-1)^{+3} \cdot [\lambda_1 \lambda_2 - 0 \cdot (p + \lambda_2 + \mu_1)] = \lambda_1 \lambda_2. \quad (19)$$

$$\begin{aligned} \Delta = \Delta(\mathbf{M}) &= (p + \lambda_1)(p + \lambda_2 + \mu_1)p + (-\lambda_1)(-\lambda_2) \cdot 0 + (-\mu_1) \cdot 0 \cdot 0 - \\ &- (p + \lambda_2 + \mu_1) \cdot 0 \cdot 0 - (-\mu_1)(-\lambda_1)p - (p + \lambda_1)(-\lambda_2) \cdot 0. \end{aligned} \quad (20)$$

Finally

$$\Delta = (p + \lambda_1)(p + \lambda_2 + \mu_1)p - \lambda_1 \mu_1 p = [(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1 \mu_1]p. \quad (21)$$

It is comparable with the determinant of the characteristic equation adequate to the determinant of equation (7), in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (2) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)].

Now, applying the matrix-vector approach of Eq. (6)-(21) it yields for transformants

$$F_A = \frac{(p + \lambda_2 + \mu_1)p}{[(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1 \mu_1]p} = \frac{p + \lambda_2 + \mu_1}{(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1 \mu_1}. \quad (22)$$

$$F_A = \frac{p + \lambda_2 + \mu_1}{p^2 + \lambda_2 p + \mu_1 p + \lambda_1 p + \lambda_1 \lambda_2 + \lambda_1 \mu_1 - \lambda_1 \mu_1}. \quad (23)$$

$$F_A = \frac{p + \lambda_2 + \mu_1}{p^2 + (\lambda_1 + \lambda_2 + \mu_1)p + \lambda_1\lambda_2} = \frac{p + \lambda_2 + \mu_1}{(p - k_1)(p - k_2)}, \quad (24)$$

where roots k_1 and k_2 of the denominator of Eq. (24) (compare with the determinant of the characteristic equation adequate to the determinant of equation (7), in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (2) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)] are

$$k_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \quad k_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \quad (25)$$

where $a = 1$, $b = \lambda_1 + \lambda_2 + \mu_1$, $c = \lambda_1\lambda_2$ – corresponding coefficients of the denominator of Eq. (24).

Other transformant

$$F_D = \frac{\lambda_1 p}{[(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1\mu_1]p} = \frac{\lambda_1}{(p - k_1)(p - k_2)}. \quad (26)$$

The third transformant

$$F_F = \frac{\lambda_1\lambda_2}{[(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1\mu_1]p} = \frac{\lambda_1\lambda_2}{(p - k_1)(p - k_2)p}. \quad (27)$$

The other way of getting solutions for transformants from the system of algebraic Eq. (5) is as follows.

From the second equation of the system of Eq. (6) we have

$$-\lambda_1 F_A + (p + \lambda_2 + \mu_1)F_D = 0; \quad F_A = \frac{p + \lambda_2 + \mu_1}{\lambda_1} F_D. \quad (28)$$

Substituting Eq. (28) for its value into the first Eq. (6) we get

$$(p + \lambda_1)F_A - \mu_1 F_D = 1; \quad (p + \lambda_1)\frac{p + \lambda_2 + \mu_1}{\lambda_1} F_D - \mu_1 F_D = 1. \quad (29)$$

Solving Eq. (29) for F_D we obtain

$$F_D \left[\frac{(p + \lambda_1)(p + \lambda_2 + \mu_1)}{\lambda_1} - \mu_1 \right] = 1;$$

$$F_D \frac{(p + \lambda_1)(p + \lambda_2 + \mu_1) - \lambda_1\mu_1}{\lambda_1} = 1. \quad (30)$$

Using the expressions for the denominators of Eq. (22)-(27) the result yields

$$F_D \frac{(p-k_1)(p-k_2)}{\lambda_1} = 1; \quad F_D = \frac{\lambda_1}{(p-k_1)(p-k_2)}. \quad (31)$$

Thus, we have got the same to Eq. (26) result. Substituting it, Eq. (31), into Eq. (28) gives also the found Eq. (24):

$$F_A = \frac{p + \lambda_2 + \mu_1}{\lambda_1} \cdot \frac{\lambda_1}{(p-k_1)(p-k_2)}; \quad F_A = \frac{p + \lambda_2 + \mu_1}{(p-k_1)(p-k_2)}. \quad (32)$$

At last, applying the third equation of the system of Eq. (6) and expression (31) we are having

$$-\lambda_2 F_D + p F_F = 0; \quad -\frac{\lambda_1 \lambda_2}{(p-k_1)(p-k_2)} + p F_F = 0;$$

$$F_F = \frac{\lambda_1 \lambda_2}{p(p-k_1)(p-k_2)}. \quad (33)$$

Eq. (33) is also the same as Eq. (27) result. This means that we have checked all the previous derivations for their correctness (accuracy).

Now, we may look for original functions of the sought probabilities at the reference books and materials [79, p. 439], [122, pp. 234-242]. From the literature sources tables [79, p. 439] we choose

$$\frac{1}{(p+a)(p+b)} = L \left\{ \frac{1}{b-a} (e^{-at} - e^{-bt}) \right\}, \quad (34)$$

here a and b – another designation of the corresponding roots of the transformants fractions (24)-(27), (31)-(33); at the reference book [79].

Or from [122, p. 235, # 12]

$$\frac{1}{(s-a)(s-b)} = L \left\{ \frac{1}{a-b} (e^{at} - e^{bt}) \right\}, \quad (35)$$

here s – another designation of the complex parameter (variable) of the Laplace transformations in the operational calculus at the reference book [122]; a and b – also another designation of the corresponding roots of the transformants fractions (24)-(27), (31)-(33); at the reference book [122].

Thus, for the probability of the “normal” state, we have that value from Eq. (24) or (32), [79, p. 439] or [122, p. 235, # 13] together with Eq. (34) or (35)

$$\frac{p}{(p+a)(p+b)} = L\left\{\frac{1}{a-b}(ae^{-at} - be^{-bt})\right\};$$

or

$$\frac{s}{(s-a)(s-b)} = L\left\{\frac{1}{a-b}(ae^{at} - be^{bt})\right\}. \quad (36)$$

For the “damage” – Eq. (26) or (31), we apply Eq. (34) or (35). And for “failure” – Eq. (27) or (33), we use [79, p. 439]:

$$\frac{1}{p(p+a)(p+b)} = L\left\{\frac{1}{ab(a-b)}[(a-b) + be^{-at} - ae^{-bt}]\right\}; \quad (37)$$

or [122, p. 235, # 14]:

$$\frac{1}{(s-a)(s-b)(s-c)} = L\left\{-\frac{(b-c)e^{at} + (c-a)e^{bt} + (a-b)e^{ct}}{(a-b)(b-c)(c-a)}\right\}, \quad (38)$$

for the given case of the stated problem setting $c = 0$; or [122, p. 242, # 1.7]:

$$\frac{1}{s(s-a)(s-b)} = L\{Ae^{at} + Be^{bt} + k\}; \quad (39)$$

where

$$A = \frac{1}{a(a-b)}; \quad B = \frac{1}{b(b-a)}; \quad k = \frac{1}{ab}. \quad (40)$$

Computer simulation results are represented in Fig. 2. In Fig. 2 y_0 , y_1 , and y_2 represent numerical integration of the differential equations system (2); $\lambda_1 = 5 \cdot 10^{-3}$ h⁻¹; $\lambda_2 = 1 \cdot 10^{-3}$ h⁻¹; $\mu_1 = 2 \cdot 10^{-3}$ h⁻¹; at the initial conditions: $t_0 = 0$; $P_A|_{t=t_0} = 1$; $P_D|_{t=t_0} = 0$; $P_F|_{t=t_0} = 0$.

Symbols “0”, “1”, “2” depict the system states of “A”, “D”, and “F” respectively.

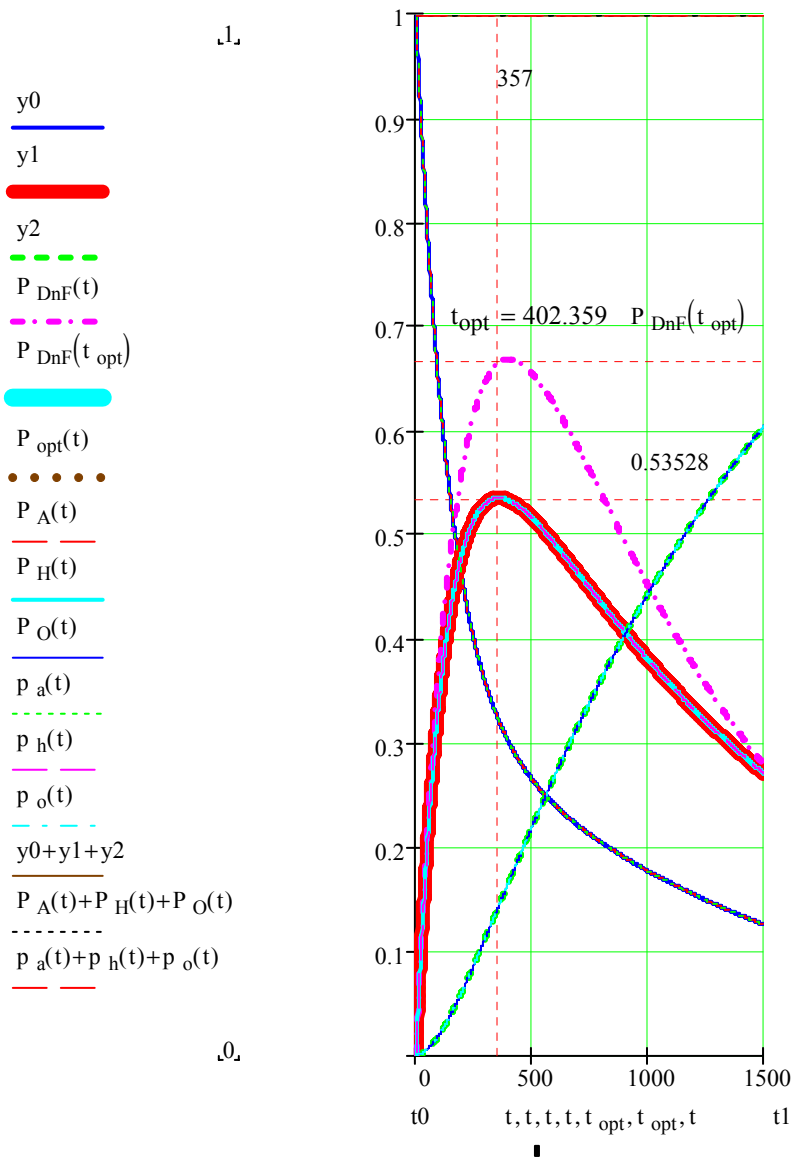


Fig. 2. Results of computer calculation experiments on the MathCad platform

The curve y_1 plotted for the damage state “D” (see Fig. 2) coincides with $P_H(t)$ for $P_D(t)$, calculated from matrix-vector solutions {see the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760): SELF-STUDY METHOD GUIDE. PART I**, Eq. (101)-(133), i.e. (105) and (108)}:

$$a = -1, \quad b = -(\lambda_1 + \lambda_2 + \mu_1), \quad c = -\lambda_1 \lambda_2, \\ k_1 = 0, \quad k_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad (41)$$

{(117), (119), (120) and (127)}:

$$\alpha_2^{(1)} = 0, \quad \alpha_1^{(1)} = 0, \quad \alpha_3^{(1)} = 1; \\ \alpha_2^{(2,3)} = 1; \quad \alpha_1^{(2,3)} = \frac{\mu_1}{\lambda_1 + k_{2,3}}; \quad \alpha_3^{(2,3)} = \frac{\lambda_2}{k_{2,3}}; \quad (42)$$

{(131)-(133)}:

$$C_3 = \frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}, \quad C_2 = -\frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}, \quad C_1 = -C_2 \alpha_3^{(2)} - C_3 \alpha_3^{(3)}; \quad (43)$$

{(114)}:

$$\left. \begin{aligned} P_A(t) &= C_1 \alpha_1^{(1)} e^{k_1 t} + C_2 \alpha_1^{(2)} e^{k_2 t} + C_3 \alpha_1^{(3)} e^{k_3 t}; \\ P_D(t) &= C_1 \alpha_2^{(1)} e^{k_1 t} + C_2 \alpha_2^{(2)} e^{k_2 t} + C_3 \alpha_2^{(3)} e^{k_3 t}; \\ P_F(t) &= C_1 \alpha_3^{(1)} e^{k_1 t} + C_2 \alpha_3^{(2)} e^{k_2 t} + C_3 \alpha_3^{(3)} e^{k_3 t}. \end{aligned} \right\} \quad (44)$$

Also (see Fig. 2) y_1 and $P_H(t)$ coincide with $p_h(t)$ for $P_D(t)$, calculated from solutions with Laplace transformations, Eq. (3)-(39), i.e. (25):

$$a = 1, \quad b = \lambda_1 + \lambda_2 + \mu_1, \quad c = \lambda_1 \lambda_2, \quad k_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad (45)$$

the roots for the Laplace transformants (images) and initial functions (originals) and their parameters (34)-(40):

$$a = k_1, \quad b = k_2, \quad A = \frac{1}{a(a-b)}, \quad B = \frac{1}{b(b-a)}, \quad k = \frac{1}{ab}; \quad (46)$$

$$\left. \begin{aligned} P_A(t) &= \frac{ae^{at} - be^{bt}}{a-b} + (\lambda_2 + \mu_1) \frac{e^{at} - e^{bt}}{a-b}; \\ P_F(t) &= \lambda_1 \lambda_2 (Ae^{at} + Be^{bt} + k); \\ P_D(t) &= \lambda_1 \frac{e^{at} - e^{bt}}{a-b}. \end{aligned} \right\} \quad (47)$$

The other curves coincide as well, i.e. y_0 , $P_A(t)$, and $p_a(t)$ for $P_A(t)$; y_2 , $P_O(t)$, and $p_o(t)$ for $P_F(t)$; being also calculated by the described above methods respectively (see Fig. 2).

The *MathCad* computer calculation platform gives a powerful tool for a variety of problem settings with the help (use) of graphs in the framework of the *Mass Service Theory* [136]. The next period of this manual is dedicated to some (a few, several) more complicated problem versions on the improvement of the A/C given functional system M/T process modeling.

The presented methods can be used for scheduling (appointing) the M/T periodicity of aviation engineering products, the reliability of which is predetermined by the failures that develop slowly (gradually) not quickly (sharply) [143, p. 172].

2. Determination of the maintenance optimal periodicity with taking into account (consideration) economical indexes (indicators, measures, criteria)

The given criterion ensures the maximal reliability of a unit (product) work at a between-scheduled period $P(t_p)$ at the minimal value of a labor-spending for its M/T T_{TO} (for execution of scheduled works and troubleshooting (elimination/removal of failures)), [143, p. 172]. The periodicity of M/T t_p in the considered case is determined on condition of achieving the maximal value of the ratio of [143, p. 172]:

$$\Pi_{t_p} = \max \left\{ \frac{P(t_p)}{T_{TO}} \right\}. \quad (48)$$

In some cases the maximal value of this ratio of (48) can be determined at subjecting (imposing, enforcing, putting into effect, implementing, applying) some constraints (limitations): minimal labor-consumption (labor

capacitance) T_{TO} at the predetermined (given, specified, required) reliability $P(t_p)$ level – $P(t_p)_{req}$; maximal reliability $P(t_p)_{max}$ at the predetermined (given, specified, required) labor-consumption (labor capacitance) T_{TO} level – $(T_{TO})_{req}$. Usually, the diagrams (graphics) of $P(t_p)$, T_{TO} , and Π_{t_p} dependencies changes on time t are plotted in the process of calculations, [143, p. 172, Fig. 15.3], and the M/T optimal periodicity is determined by them.

All the presented methods of the M/T periodicity determination are based upon the statistical data (material) of aeronautical engineering failures and damages, as well as of their runs times, accumulated at the process of operation. Besides (moreover, in addition, furthermore), one should have the data of the labor-spending for malfunctioning (troubleshooting) detection and removal, as well as for the data on the M/T carrying out (execution). Cards-orders for M/T, orders for defects detection and classification (determination), aeronautical engineering reliability level control checks, technological instructions and prescriptions for scheduled works carrying out (execution), results of expert inspections of M/T techniques (technological) processes can be used as the initial information, [143, p. 172].

The M/T periodicity of A/C certain units (products) calculated by any method is reckoned (considered, thought, supposed, deemed, guessed) to be a desired (preferred, most wanted) periodicity, since it (the determined, defined periodicity) may change at the grouping of fixed (separate, specific, exact, definite, particular) works into the scheduled forms for the A/C as an entirety, [143, p. 173].

3. Grouping works into the scheduled maintenance optimal forms

The task of multiple (numerous, many, several, various) separate (fixed, specific, exact, definite, particular) works grouping with their own carrying out (executions) periodicities into the optimal scheduled M/T forms is to determine the number of the M/T forms and point out their basic periodicities for the A/C as an entirety, [143, pp. 162-174, Chapter 15, especially Sub-Chapter 15.5, pp. 173, 174].

The works grouping by the labor-consuming forms of scheduled M/T can be fulfilled (carried out, realized, completed, performed) in different ways, [143, Sub-Chapter 15.5, pp. 173, 174].

1. The periodicity of the scheduled M/T forms is reckoned (considered, thought, supposed, deemed, guessed) to be known (it may be predetermined in technical requirements for the given A/C type). In this case, each work, completion of which influences flight safety, belongs (pertains, is relevant, relates) to the scheduled M/T form, the periodicity of which is less than the desired periodicity of that work completion and is closer to that periodicity than the all other works periodicities. The periodicity of the rest of the works is formed with taking into account (consideration) economical criteria [143, Sub-Chapter 15.5, p. 173].

2. The periodicities of the different scheduled M/T forms are not predetermined (given, prescribed) in advance. In this case, the number of the M/T forms, scope and periodicity of works for each form are prescribed (appointed, assigned, selected, chosen) on condition of the minimal labor-spending for the M/T per one flight hour assurance (providing, guaranteeing, insurance), on the condition that the given safety indexes (measures, criteria) having been ensured as well [143, Sub-Chapter 15.5, p. 173].

The essence of the method is as the following. All works in regards with the M/T process of the aeronautical engineering products are divided into two groups [143, Sub-Chapter 15.5, p. 173].

The works relating with the dangerous failures of the products prevention are included into the first group (M_1). For such works, it is considered to be known the assigned periodicity $t_{1,i}$ of each of which completion [143, Sub-Chapter 15.5, p. 173].

The works which optimal periodicity of completion $t_{2,i}$ can be assigned by the criterion of the minimum specific expenses for the A/C M/T are included into the second group (M_2) [143, Sub-Chapter 15.5, p. 173].

In calculations, each work of the groups of M_1 and M_2 is marked with the two indicators (indexes, subscripts): the first characterizing the number of the group of the works (1 or 2), the second – the ordinal number of the work ($i = 1, 2, \dots, n_1$ or $i = 1, 2, \dots, n_2$) by value of its periodicity completion growth: $t_{1,1} < t_{1,2} < \dots < t_{1,n_1}$. The value of i_1 takes all (every of the, each of the) values of the work group of M_1 , i.e. (that is) $i_1 \in M_1$; and $i_2 \in M_2$. It is supposed to be known the direct costs $y_{1,i}$ for the one time completion of each of the group works M_1 ; and for each of the second group M_2 works – the specified (given) dependencies of the specific M/T costs upon the

periodicities of the M/T: $y_{2,i} = f(t_{2,i})$. In the process of the works formation there may happen to be a few (several) groups m of the works having the same (identical) localized periodicity t_i . For each of those groups of the works it is necessary to know the mean (average) duration B_i of the group the works completion [143, Sub-Chapter 15.5, p. 173].

Having known the periodicity of the each work carrying out as well as labor expenses for the works onetime performance (or the dependence of the specific M/T costs upon the periodicity of the M/T), the each works group completion duration, it is possible to obtain (get, find) the M/T forms for the scheduled M/T as the entire integrity. The task of the works grouping into the optimal forms of the scheduled M/T is that for changing the values of the works periodicities of $t_{1,i}$ and $t_{2,i}$ in order to find such appropriate series (sequence, chain) of the basis M/T periodicity for the A/C as the entire integrity, at which the value of the annual (yearly) M/T and operational repair (restoration, overhaul) expenditures C_{TO} , with taking into account the losses due to the M/T standing is minimized [143, Sub-Chapter 15.5, p. 174]:

$$C_{TO} = \sum_{\substack{i=1 \\ i \in M_1}}^{n_1} y_{1,i} \left(\frac{T_r}{t_{1,i}} - 1 \right) + \sum_{\substack{i=1 \\ i \in M_2}}^{n_2} y_{2,i} T_r + \sum_{\substack{i=1 \\ i \in M}}^m B_i \left(\frac{T_r}{t_i} - 1 \right), \quad (49)$$

where T_r – yearly (annual) flight hours per an A/C, m – number of the groups of the works having the identical (same) localized periodicity t_i , B_i – average (mean) duration of the each group of works carrying out at the A/C [143, Sub-Chapter 15.5, p. 174].

In the cases when the presented above dependencies of $y_{1,i}$, $y_{2,i}$, and B_i are unknown, there are being used some more elaborated (complex, complicated) methods of the scheduled M/T optimization, for example, the linear programming method [143, Sub-Chapter 15.5, p. 174].

The detailed solutions, in the view of Eq. (1)-(49), including those for obtaining the reliability objective measures, allow successfully assessing the improvements of the A/C functional system M/T process considered in reference [112] on the basis of [136, 140].

It may be concluded. The obtained solutions results described with the mathematical expressions of (1)-(133) create a possibility for the students to model the situations relating with the reliability objective measures allowing successfully assessing the improvements of the A/C functional system M/T

process considered in reference [112] on the basis of the methods discussed in [136, 140] in application to [1-144].

Further **PARTS** of the **METHOD GUIDE ON THE SS** problems are going to be intended for several other scientific components of the SS work on the academic subject. Background readings of the new theories [4-60, 64, 65, 89-109, 117, 118, 138] are welcome here in respect with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144].

It is very attractive to use the calculus of variations theories [64, 65, 80, 85, 88, 95, 96, 117, 118] emphasizing the uncertainties of different kinds [4-60, 64, 65, 89-109, 117, 118, 138] for optimization of the problems stated in all references of [1-144].

MODELLING ON THE BASIS OF THE MULTI-OPTIONAL UNCERTAINTY CONDITIONAL OPTIMIZATION DOCTRINE

The principal theoretical provisions can be found out in references [61-65, 117, 118, 9, 12, 13, 31, 38, 44-46, 79, 122, 136, 139, 140, 143].

A modelling in regards to aeronautical engineering maintenance optimal periodicity determination can be performed with the use of the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine*, [9, 12, 13, 31, 38, 44-46].

The optimal values of aeronautical engineering maintenance periodicities illustrated in Fig. 2 can be obtained not only in the entire probabilistic way, but also in a hybrid optional way, [9, 12, 13, 31, 38, 44-46].

The essence of the method is to consider the process developing in the system from the position of some hybrid optional functions distribution optimality with making allowance for the hybrid multi-optional functions distribution uncertainty measured by the entropy of the hybrid multi-optional effectiveness functions, [9, 12, 13, 31, 38, 44-46].

1. The simplest case of the optimal maintenance periodicity determination

First, let us consider a graph which corresponds to the simplest problem. It is represented in Fig. 3.

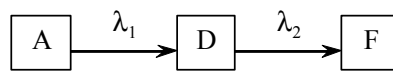


Fig. 3. Graph of three states of an aircraft functional system

Here, in Fig. 3, there are used the designations coinciding with those ones of the Fig. 1.

Starting with the simplest case, mathematically described with the expressions similar to the procedures of Eq. (1)-(47), as well as discussed above and represented by the graph for the three states of an aircraft functional system shown in Fig. 1, we might consider the corresponding intensities of λ_1 and λ_2 , for the given problem setting (see Fig. 3), as some

certain parameters of the multi-“optionality” (options). Therefore, we may use the apparatus of the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine* [9, 12, 13, 31, 38, 44-46], which is a kind of an evolution of the *Subjective Preferences Functions (Subjective Analysis, Subjective Entropy Paradigm, Subjective Entropy Maximum Principle) Theory* [64, 65, 117, 118] developed for solving the problems of the multi-alternativeness of the individual human being choice in different situations with a given set of attainable (achievable, reachable) alternatives; the *Theory* [64, 65, 117, 118] emerged (come out, appeared, materialized, originated) from the well-known in theoretical statistical physics *Jaynes’ Entropy Maximum Principle* [61-63]. According with that *Doctrine* [9, 12, 13, 31, 38, 44-46] the system states options are “A” – up state of the system, “D” – damaged state, and “F” – failure.

Suppose there are certain functions expressing some kind of the system functioning effectiveness and related to the options. The objectively existing optimality has to be reflected somehow in the connected functions distribution, thus, having attributes of the optimality on their own. In order to find optimal functions, the methods of *Calculus of Variations* [80, 85, 88] are widely used and it is generally accepted.

Let us try an attempt to find the extremals for the presented research. The considered options are logically to be chosen as the three possible states, that is the number of the options equals three. However, there is a state without an exit, which means the problem becomes a two-optional one, i.e. $i = 2$. The optional effectiveness functions F_i pertaining with the options are $F_i(\lambda_i)$ – relevant to the intensities of the transitions λ_1 and λ_2 . The hybrid functions $h_i[F_i(\lambda_i)]$ are related with effectiveness functions of the options, depending upon an uncertainty of the situation.

Assuming the uncertainty is measured with the entropy of the optional hybrid functions, we synthesize (construct, compile) the purpose (objective) functional for the three hypothetically possible options in the postulated in the *Subjective Analysis Theory* [64, 65, 117, 118] way [9, 12, 13, 31, 38, 44-46]:

$$\Phi_h = -\sum_{i=1}^3 h_i[F_i(\cdot)] \ln h_i[F_i(\cdot)] + \beta \sum_{i=1}^3 h_i[F_i(\cdot)] F_i(\lambda_i) + \gamma \left\{ \sum_{i=1}^3 h_i[F_i(\cdot)] - 1 \right\}, \quad (50)$$

where

$$-\sum_{i=1}^3 h_i[F_i(\cdot)] \ln h_i[F_i(\cdot)] = H_h \quad (51)$$

– entropy of the optional (options) hybrid functions; β – system's optimization internal parameter; γ – normalizing function (coefficient);

$$\left\{ \sum_{i=1}^3 h_i[F_i(\cdot)] - 1 \right\} \quad (52)$$

– normalizing condition.

The sign “+” in front of $\beta > 0$ means that higher values are prescribed to the hybrid optional functions connected with the higher values of the options' effectiveness functions. The sign “-” in front of $\beta > 0$ means vice versa that higher values are attached to the hybrid optional functions related with the lower values of the options' effectiveness functions.

Once again, in the case pictured in Fig. 3 the synthesized view functional of Eq. (50) drops out one of the three options' i.e. the third optional effectiveness function F_3 , pertaining with the option: $F_3(\lambda_3)$, as it is not relevant to the intensities of the transitions λ_1 and λ_2 . The hybrid functions $h_{1,2}[F_{1,2}(\lambda_{1,2})]$ are related with effectiveness functions of the options $F_{1,2}(\lambda_{1,2})$, depending upon an uncertainty of the situation.

The options of the situation may be considered as related to, connected with, the “A” and “D” states except for the “F” state since it has no exit. However, for some other considerations the other two states of the three might be essential. Anyway, in the stated problem setting, there are just the two optional parameters: the intensities of the transitions λ_1 and λ_2 , the two optional effectiveness function F_i pertaining with the options numbered, $i \in \overline{1,2}$: $F_{1,2}(\lambda_{1,2})$, and the two hybrid functions $h_{1,2}[F_{1,2}(\lambda_{1,2})]$ related with effectiveness functions of the options $F_{1,2}(\lambda_{1,2})$, depending upon an uncertainty of the situation.

Thus, for the system with no opportunity of going out of, let us say, the third state, we must synthesize the two option functional:

$$\Phi_h = -\sum_{i=1}^2 h_i[F_i(\cdot)] \ln h_i[F_i(\cdot)] + \beta \sum_{i=1}^2 h_i[F_i(\cdot)] F_i(\lambda_i) + \gamma \left\{ \sum_{i=1}^2 h_i[F_i(\cdot)] - 1 \right\}. \quad (53)$$

Now, for the functional of expression (53) to undergo an extremum with respect to the hybrid functions $h_i[F_i(\lambda_i)]$ the necessary conditions are as follows:

$$\frac{\partial \Phi_h}{\partial h_i} = 0. \quad (54)$$

In case that there must be an optimal time between M/T

$$t_p^* = \beta \quad (55)$$

– the periodicity of the scheduled M/T works performance, it gives

$$\Phi_h = -\sum_{i=1}^2 h_i(\cdot) \ln h_i(\cdot) + t_p^* \sum_{i=1}^2 h_i(\cdot) F_i(\cdot) + \gamma \left\{ \sum_{i=1}^2 h_i(\cdot) - 1 \right\}. \quad (56)$$

And

$$\frac{\partial \Phi_h}{\partial h_i} = -\ln h_i(\cdot) - 1 + t_p^* F_i(\cdot) + \gamma = 0, \quad \forall i \in \overline{1,2}. \quad (57)$$

Then

$$\ln h_1(\cdot) - t_p^* F_1(\cdot) = \gamma - 1 = \ln h_2(\cdot) - t_p^* F_2(\cdot). \quad (58)$$

From where

$$\ln h_1(\cdot) - t_p^* F_1(\cdot) = \ln h_2(\cdot) - t_p^* F_2(\cdot). \quad (59)$$

After that we have got the *Law of Subjective Conservatism* [55] on one hand and on the other hand

$$\ln h_1(\cdot) - \ln h_2(\cdot) = t_p^* [F_1(\cdot) - F_2(\cdot)]. \quad (60)$$

At last

$$t_p^* = \frac{\ln h_1(\cdot) - \ln h_2(\cdot)}{F_1(\cdot) - F_2(\cdot)}. \quad (61)$$

In case the options related effectiveness function is expressed explicitly with respect to λ_i in the way of

$$F_i(\lambda_i) = \lambda_i, \quad \text{and} \quad h_i[F_i(\lambda_i)] = h_i(\lambda_i) = x\lambda_i, \quad (62)$$

where x – unknown, uncertain multiplier in type of the Lagrange one, we obtain with the help of the procedure considered through (50)-(62) the needed optimal periodicity in the sense of the probability $P_D(t)$ maximum [143, p. 171].

Indeed substituting Eq. (62) into Eq. (61)

$$t_p^* = \frac{\ln \frac{x\lambda_1}{x\lambda_2}}{\lambda_1 - \lambda_2} = \frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}. \quad (63)$$

The sense of the uncertain multiplier x becomes obvious with the use of the normalizing condition (likewise Eq. (52)) of the initial functional Eq. (56). That is

$$\sum_{i=1}^2 h_i[F_i(\lambda_i)] = 1. \quad (64)$$

With respect to Eq. (62)

$$x\lambda_1 + x\lambda_2 = 1 \Rightarrow x = \frac{1}{\lambda_1 + \lambda_2}. \quad (65)$$

On objective functional, formulae (53), (55), and (56), extremum existence conditions of Eq. (54), (57)

$$\ln h_i(\cdot) = \gamma - 1 + t_p^* F_i(\cdot), \quad \forall i \in \overline{1, 2}. \quad (66)$$

Hence

$$h_i(\cdot) = \exp[\gamma - 1 + t_p^* F_i(\cdot)] = \exp[\gamma - 1] \exp[t_p^* F_i(\cdot)]. \quad (67)$$

Because of the normalizing conditions likewise equations of (52) or (64)

$$\sum_{j=1}^2 h_j[F_j(\lambda_j)] = 1 = \sum_{j=1}^2 \exp[\gamma - 1] \exp[t_p^* F_j(\cdot)] = \exp[\gamma - 1] \sum_{j=1}^2 \exp[t_p^* F_j(\cdot)]. \quad (68)$$

Therefore

$$\exp[\gamma - 1] = \frac{1}{\sum_{i=1}^2 \exp[t_p^* F_i(\cdot)]}. \quad (69)$$

This yields for the optimal distribution of the hybrid-optional functions

$$h_i^{(\text{opt})}[F_i(\lambda_i)] = \frac{\exp[t_p^* F_i(\cdot)]}{\sum_{j=1}^2 \exp[t_p^* F_j(\cdot)]}. \quad (70)$$

Thus, finally we obtained the known in Subjective Analysis the Canonical Distribution of the Subjective Preferences, [64, 65, 117, 118].

However in this work, we interpret it, Eq. (70), as the optional hybrid functions distribution [9, 12, 13, 31, 38, 44-46] since we do not consider any active elements or subjects (persons, individuals, or human beings) in the system. Instead we deal with the objectively existing optimal quality of the

system, corresponding with the system intrinsic nature, rather than subjectively preferred (although might be also essential, indispensable) matter.

Because of the assumptions expressed with the equations of (50), (53), (55), (56), and (62), as well as due to the derivations presented with the procedures following Eq. (50)-(65)

$$h_1[\cdot] = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \quad h_2[\cdot] = \frac{\lambda_2}{\lambda_1 + \lambda_2}. \quad (71)$$

Because of the same assumptions, as well as due to the derivations presented with the procedures following Eq. (50)-(57) and (66)-(70)

$$h_1^{(\text{opt})}[\cdot] = \frac{\exp[t\lambda_1]}{\exp[t\lambda_1] + \exp[t\lambda_2]}, \quad h_2^{(\text{opt})}[\cdot] = \frac{\exp[t\lambda_2]}{\exp[t\lambda_1] + \exp[t\lambda_2]}. \quad (72)$$

Furthermore, accordingly with [64, 65, 117, 118], for functional of Eq. (56), t – time is the independent variable for now, although.

And in any case whether t_p^* Eq. (63) is obtained on conditions of Eq. (56)-(62) or from the related probability $P_D(t)$ extremization

$$h_1^{(\text{opt})}(t_p^*) = \frac{\exp[t_p^*\lambda_1]}{\exp[t_p^*\lambda_1] + \exp[t_p^*\lambda_2]}, \quad h_2^{(\text{opt})}(t_p^*) = \frac{\exp[t_p^*\lambda_2]}{\exp[t_p^*\lambda_1] + \exp[t_p^*\lambda_2]}. \quad (73)$$

Now, there is a necessity to prove the identities of Eq. (71) to Eq. (73) for the considered problem setting.

Indeed.

Substituting the Eq. (63) value into Eq. (73)

$$h_1^{(\text{opt})}(t_p^*) = \frac{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_1\right]}{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_1\right] + \exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_2\right]},$$

$$h_2^{(\text{opt})}(t_p^*) = \frac{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_2\right]}{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_1\right] + \exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2} \lambda_2\right]}. \quad (74)$$

Then, for the nominator of the first Eq. (74)

$$\begin{aligned} \exp\left[\frac{\ln\lambda_1 - \ln\lambda_2}{\lambda_1 - \lambda_2} \lambda_1\right] &= \exp\left[\ln\frac{\lambda_1}{\lambda_2} \frac{\lambda_1}{\lambda_1 - \lambda_2}\right] = \left(\exp\left[\ln\frac{\lambda_1}{\lambda_2}\right]\right)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} = \\ &= \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}. \end{aligned} \quad (75)$$

The first Eq. (74) turns out

$$\begin{aligned} &\frac{\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}{\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} + \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}} = \frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} \left[\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} + \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}} \right]} = \\ &= \frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} \left[\left(\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}\right) + \left(\frac{(\lambda_1)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}\right) \right]} = \\ &= \frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} \left[\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}} + (\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_1)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}} \right]} = \\ &= \frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}} + (\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_1)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}} = \\ &= \frac{1}{1 + \frac{(\lambda_2)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_1)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}{(\lambda_1)^{\frac{\lambda_1}{\lambda_1 - \lambda_2}} (\lambda_2)^{\frac{\lambda_2}{\lambda_1 - \lambda_2}}}} = \frac{1}{1 + \frac{(\lambda_2)^{\frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2}}}{(\lambda_1)^{\frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2}}}} = \frac{1}{1 + \frac{(\lambda_2)^1}{(\lambda_1)^1}} = \frac{\lambda_1}{\lambda_1 + \lambda_2}. \end{aligned} \quad (76)$$

Thus, we have got the second Eq. (62) on condition of Eq. (65) for the first hybrid effectiveness multi-optional function:

$$h_1^{(\text{opt})}(t_p^*) = \frac{\exp[t_p^* \lambda_1]}{\exp[t_p^* \lambda_1] + \exp[t_p^* \lambda_2]} = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \quad (77)$$

as we presupposed above.

The obtained result expressed with Eq. (77) is obvious because the optimal interval for the M/T periodicity t_p^* has been found on the basis of the previously postulated optimality with taking into account entropy paradigm for the hybrid optional functions $h_i(\cdot)$; in their turn the hybrid optional functions have been modified (hybridized) with the unknown (uncertain) multiplier x (likewise implanted extra chromosome). Therefore optimal, with taking into consideration the uncertainty in the view of their entropy, hybrid optional functions distribution returns the assumed optimal hybrid optional functions values in accordance with the previously supposed view and accepted effectiveness (see Eq. (56)-(62)).

2. Probabilistic approach solution as a proof for the proposed Doctrine possible applications

The probabilistic approach solution in the stated problem setting pertaining with the graph shown in Fig. 3 is a partial case solution for the process described in the first section (see and compare Fig. 1 and Fig. 3 also analyze Eq. (1)-(47)).

For such partial solution obtaining, there is just a need of the $\mu_1 = 0$ acceptance.

This returns the following modifications in the Eq. (1)-(47), we distinguish just the necessary (indispensable, essential, important) ones.

Distribution densities come up without that for the restoration transition (see Eq. (1)):

$$f_1(t) = \lambda_1 e^{-\lambda_1 t}, \quad f_2(t) = \lambda_2 e^{-\lambda_2 t}. \quad (78)$$

The system of the differential equations by *Erlang* is modified in the way of simplification (see Eq. (2)):

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - \lambda_2 P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned} \right\} \quad (79)$$

Next up is the method different from the *Laplace transformations in the operational calculus* [140, Chapter XIX, pp. 400-432].

From the first equation of the system (79) we find

$$\frac{dP_A}{dt} = -\lambda_1 P_A; \quad \frac{dP_A}{P_A} = -\lambda_1 dt; \quad \int \frac{dP_A}{P_A} = -\int \lambda_1 dt + C, \quad (80)$$

where C – constant of integration, it will be found from the initial conditions:

$$t_0 = 0; \quad P_A|_{t=t_0} = 1; \quad P_D|_{t=t_0} = 0; \quad P_F|_{t=t_0} = 0, \quad (81)$$

Eq. (80) yields

$$\ln P_A = -\lambda_1 t + C, \quad (82)$$

from where

$$\ln 1 = -\lambda_1 \cdot 0 + C, \quad 0 = -0 + C, \quad C = 0. \quad (83)$$

Thus,

$$P_A(t) = e^{-\lambda_1 t}. \quad (84)$$

The same result as [143, pp. 170-172], which is correct because state “A” means not transition into the state of damage “D”, the probability is $P_A(t) = P_{\bar{D}}(t)$, the designation for such event is in the subscript of the probability of $P_{\bar{D}}(t)$ with the negation mark: \bar{D} ; and transition from “A” straight to the state of failure “F” is impossible: $P_{\bar{D}F}(t) = 0$, as well as any of the backward transitions. Therefore

$$P_A(t) = P_{\bar{D}}(t) = P_{\bar{D}F}(t) + P_{\bar{D}\bar{F}}(t) = 0 + e^{-\lambda_1 t} = e^{-\lambda_1 t}. \quad (85)$$

Substituting the result of Eq. (84) or (85) for the corresponding value of $P_A(t)$ in the second equation of the system (79) we get

$$\frac{dP_D}{dt} = \lambda_1 e^{-\lambda_1 t} - \lambda_2 P_D. \quad (86)$$

Thus, we have got a linear not uniform (homogeneous) differential equation of the first order. It can easily be solved with the method represented in [139, 140]. Exactly, accordingly to [140, Chapter XIII, especially § 7, pp. 30-33] the Eq. (86) has the view of [140, Chapter XIII, § 7, p. 30, (1)]

$$\frac{dy}{dx} + P(x)y = Q(x). \quad (87)$$

where $P(x)$ and $Q(x)$ – given continuous functions of x (or constants).

The solution of the linear equation (87). We will be finding the solution of the equation (87) in the view of a product of two functions of x [140, Chapter XIII, § 7, p. 30, (2)]:

$$y = u(x)v(x). \quad (88)$$

One of these functions can be taken arbitrary, the other one will be determined on the basis of Eq. (87).

Differentiating both parts of Eq. (88), we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}. \quad (89)$$

Substituting the obtained expression of the derivative of $\frac{dy}{dx}$ into Eq. (87), we will have, [140, Chapter XIII, § 7, p. 31],

$$u \frac{dv}{dx} + v \frac{du}{dx} + Puv = Q, \quad (90)$$

or, [140, Chapter XIII, § 7, p. 31, (3)],

$$u \left(\frac{dv}{dx} + Pv \right) + v \frac{du}{dx} = Q. \quad (91)$$

Let us choose function v as such as, [140, Chapter XIII, § 7, p. 31, (4)],

$$\frac{dv}{dx} + Pv = 0. \quad (92)$$

Dividing variables in this differential equation with respect to function v , we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dv}{v} = -Pdx. \quad (93)$$

Integrating, we get, [140, Chapter XIII, § 7, p. 31],

$$-\ln|C_1| + \ln|v| = -\int P dx, \quad (94)$$

where C_1 – constant of integration; or, [140, Chapter XIII, § 7, p. 31],

$$v = C_1 e^{-\int P dx}. \quad (95)$$

Since it is enough to have some different from zero solution of Eq. (92), then we will take [140, Chapter XIII, § 7, p. 31, (5)] as the function of $v(x)$, assuming and accepting $C_1 = 1$:

$$v(x) = e^{-\int P dx}, \quad (96)$$

where $\int P dx$ – some antiderivative (counterderivative). It is obviously that $v(x) \neq 0$.

Substituting the found value of $v(x)$ into Eq. (91), [140, Chapter XIII, § 7, p. 31, (3)], we will get (taking into account that $\frac{dv}{dx} + Pv = 0$), [140, Chapter XIII, § 7, p. 31],

$$v(x) \frac{du}{dx} = Q(x), \quad (97)$$

or, [140, Chapter XIII, § 7, p. 31],

$$\frac{du}{dx} = \frac{Q(x)}{v(x)}, \quad (98)$$

from where, [140, Chapter XIII, § 7, p. 31],

$$u = \int \frac{Q(x)}{v(x)} dx + C, \quad (99)$$

where C – constant of integration.

Substituting u and v into the formula of Eq. (88), [140, Chapter XIII, § 7, p. 30, (2)], we finally get

$$y = v(x) \left[\int \frac{Q(x)}{v(x)} dx + C \right], \quad (100)$$

or, [140, Chapter XIII, § 7, p. 32, (6)],

$$y = v(x) \int \frac{Q(x)}{v(x)} dx + Cv(x). \quad (101)$$

In the considered problem setting (see, compare, and substitute the corresponding functions, expressions, variables, and values of Eq. (86), (87), (101))

$$\frac{dP_D}{dt} + \lambda_2 P_D = \lambda_1 e^{-\lambda_1 t}, \quad \frac{dy}{dx} + P(x)y = Q(x),$$

$$y = P_D, \quad x = t, \quad P(x) = \lambda_2, \quad Q(x) = \lambda_1 e^{-\lambda_1 t}, \quad v(x) = e^{-\int \lambda_2 dt} = e^{-\lambda_2 t}.$$
(102)

Thus

$$P_D = e^{-\lambda_2 t} \int \frac{\lambda_1 e^{-\lambda_1 t}}{e^{-\lambda_2 t}} dt + C e^{-\lambda_2 t}.$$
(103)

Then

$$P_D = e^{-\lambda_2 t} \lambda_1 \int e^{(\lambda_2 - \lambda_1)t} dt + C e^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} [e^{(\lambda_2 - \lambda_1)t}] + C e^{-\lambda_2 t}.$$
(104)

From the initial conditions, expressions (81) $t_0 = 0$; $P_D|_{t=0} = 0$. It yields

$$0 = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 \cdot 0} [e^{(\lambda_2 - \lambda_1) \cdot 0}] + C e^{-\lambda_2 \cdot 0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \cdot 1 \cdot 1 + C \cdot 1.$$
(105)

Therefore

$$C = -\frac{\lambda_1}{\lambda_2 - \lambda_1}.$$
(106)

Then

$$P_D = \frac{\lambda_1}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t}] - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$
(107)

Final result Eq. (107) is the same as [143, p. 171, (15.1)]. It is the probability of the damage state “D” of the system (see Fig. 3).

The probability of the failure state “F” will be obtained as the solution from the third equation of the system (79), i.e.

$$\frac{dP_F}{dt} = \lambda_2 P_D = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$
(108)

Integration yields

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} [\int e^{-\lambda_1 t} dt - \int e^{-\lambda_2 t} dt] + C,$$
(109)

where C – constant of integration.

Then

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[\left(-\frac{1}{\lambda_1} \right) e^{-\lambda_1 t} + \frac{1}{\lambda_2} e^{-\lambda_2 t} \right] + C. \quad (110)$$

From the initial conditions, expressions (81) $t_0 = 0$; $P_F|_{t=0} = 0$. It yields

$$\begin{aligned} 0 &= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[\left(-\frac{1}{\lambda_1} \right) e^{-\lambda_1 \cdot 0} + \frac{1}{\lambda_2} e^{-\lambda_2 \cdot 0} \right] + C = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) + C = \\ &= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left(\frac{\lambda_1 - \lambda_2}{\lambda_2 \lambda_1} \right) + C = -1 + C \Rightarrow C = 1. \end{aligned} \quad (111)$$

Therefore

$$\begin{aligned} P_F &= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[\frac{e^{-\lambda_2 t}}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_1} \right] + 1 = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[\frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 \lambda_1} \right] + 1 = \\ &= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + 1 = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + 1 + e^{-\lambda_1 t} - e^{-\lambda_1 t} = \\ &= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + e^{-\lambda_1 t} = \\ &= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}. \end{aligned}$$

Finally

$$P_F = 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} = 1 - e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_2 t} - e^{-\lambda_1 t}). \quad (112)$$

The result (112) is the same as [143, pp. 170-172].

Now we will return to the *Laplace transformations* in the *operational calculus* [140, Chapter XIX, pp. 400-432].

The system (2), now (79), after the *Laplace transformations* in accordance with the theorem for transformants of derivatives [140, Chapter XIX, § 8, p. 409, (27)], will get the view of the corresponding algebraic equations system (see Eq. (5) and on):

$$\left. \begin{aligned} pF_A(p) - 1 &= -\lambda_1 F_A(p); \\ pF_D(p) - 0 &= \lambda_1 F_A(p) - \lambda_2 F_D(p); \\ pF_F(p) - 0 &= \lambda_2 F_D(p). \end{aligned} \right\} \quad (113)$$

$$\left. \begin{array}{l} (p+\lambda_1)F_A \quad +0 \quad +0=1; \\ -\lambda_1 F_A \quad +(p+\lambda_2)F_D \quad +0=0; \\ 0 \quad -\lambda_2 F_D \quad +pF_F=0. \end{array} \right\} \quad (114)$$

$$\mathbf{M} = \begin{vmatrix} p+\lambda_1 & 0 & 0 \\ -\lambda_1 & p+\lambda_2 & 0 \\ 0 & -\lambda_2 & p \end{vmatrix}, \quad \mathbf{F} = \begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix}, \quad \begin{vmatrix} p+\lambda_1 & 0 & 0 \\ -\lambda_1 & p+\lambda_2 & 0 \\ 0 & -\lambda_2 & p \end{vmatrix} \cdot \begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}. \quad (115)$$

$$\mathbf{M} \cdot \mathbf{F} = \mathbf{B}, \quad \mathbf{B} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}, \quad \mathbf{F} = \mathbf{M}^{-1} \cdot \mathbf{B}, \quad \mathbf{M}^{-1} = \frac{1}{\Delta(\mathbf{M})} \cdot \tilde{\mathbf{M}}, \quad \mathbf{F} = \frac{1}{\Delta(\mathbf{M})} \cdot \tilde{\mathbf{M}} \cdot \mathbf{B}. \quad (116)$$

$$\begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{vmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}. \quad (117)$$

$$\begin{vmatrix} F_A \\ F_D \\ F_F \end{vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{vmatrix} M_{11} \cdot 1 + M_{21} \cdot 0 + M_{31} \cdot 0 \\ M_{12} \cdot 1 + M_{22} \cdot 0 + M_{32} \cdot 0 \\ M_{13} \cdot 1 + M_{23} \cdot 0 + M_{33} \cdot 0 \end{vmatrix} = \frac{1}{\Delta(\mathbf{M})} \cdot \begin{vmatrix} M_{11} \\ M_{12} \\ M_{13} \end{vmatrix}. \quad (118)$$

$$M_{11} = (-1)^{1+1} \cdot [(p+\lambda_2)p + \lambda_2 \cdot 0] = (p+\lambda_2)p. \quad (119)$$

$$M_{12} = (-1)^{1+2} \cdot [(-\lambda_1 p) - 0 \cdot 0] = \lambda_1 p. \quad (120)$$

$$M_{13} = (-1)^{1+3} \cdot [\lambda_1 \lambda_2 - 0 \cdot (p+\lambda_2)] = \lambda_1 \lambda_2. \quad (121)$$

$$\Delta = (p+\lambda_1)(p+\lambda_2)p. \quad (122)$$

$$k_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a=1, \quad b=\lambda_1 + \lambda_2, \quad c=\lambda_1 \lambda_2. \quad (123)$$

$$k_{1,2} = \frac{-(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 + \lambda_2)^2 - 4\lambda_1 \lambda_2}}{2}. \quad (124)$$

$$k_{1,2} = \frac{-(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1)^2 + (\lambda_2)^2 - 2\lambda_1 \lambda_2}}{2}. \quad (125)$$

$$k_{1,2} = \frac{-(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2}}{2}, \quad k_{1,2} = \frac{-(\lambda_1 + \lambda_2) \pm (\lambda_1 - \lambda_2)}{2}.$$

(126)

$$k_1 = -\lambda_2, \quad k_2 = -\lambda_1. \quad (127)$$

$$F_D = \frac{\lambda_1 p}{[(p + \lambda_1)(p + \lambda_2)]p} = \frac{\lambda_1}{(p - k_1)(p - k_2)}. \quad (128)$$

$$\frac{1}{(s-a)(s-b)} = L \left\{ \frac{1}{a-b} (e^{at} - e^{bt}) \right\}, \quad a = k_1 = -\lambda_2, \quad b = k_2 = -\lambda_1. \quad (129)$$

$$P_D = \frac{\lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t}). \quad (130)$$

The result of Eq. (130) is the same as of Eq. (107).

And for the $P_D(t)$ Eq. (130) extremum existence

$$\frac{dP_D(t)}{dt} = \frac{\lambda_1}{\lambda_1 - \lambda_2} (\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_2 t}) = 0. \quad (131)$$

$$\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_2 t} = 0, \quad \lambda_1 e^{-\lambda_1 t} = \lambda_2 e^{-\lambda_2 t}, \quad \frac{\lambda_1}{\lambda_2} = e^{(\lambda_1 - \lambda_2)t}. \quad (132)$$

$$\ln \frac{\lambda_1}{\lambda_2} = (\lambda_1 - \lambda_2)t, \quad t_{\text{opt}} = \frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}. \quad (133)$$

The result of Eq. (133) is the same as of Eq. (63). However Eq. (63) is obtained in the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine* [9, 12, 13, 31, 38, 44-46] way Eq. (50)-(63) rather than in the demonstrated herewith this sub-chapter entirely probabilistic method.

Thus, the identity is proven and the optimum is visible in Fig. 2.

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Частина II

ЗАСТОСУВАННЯ ЕНТРОПІЙНОЇ ДОКТРИНИ
БАГАТООПЦІЙНИХ ФУНКЦІЙ ДЛЯ ОЦІНКИ
УДОСКОНАЛЕННЯ ПРОЦЕСУ ТЕХНІЧНОГО
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