MINISTRY OF SCIENCE AND EDUCATION OF UKRAINE
National Aviation University

Qualifying scientific work
on the rights of the manuscript

UDC 629.735.083.02/06(043.3)

DISSERTATION

MATHEMATICAL MAINTENANCE MODELS OF VEHICLES' EQUIPMENT

05.22.20 – maintenance and repair of vehicles

A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy

The thesis contains the results of own research. The use of ideas, results, and texts of other authors have references to the appropriate sources.

A. Raza

Scientific Advisor: Ulansky Volodymyr Vasylovych,
Doctor of Technical Sciences, Professor

Kyiv – 2018

ABSTRACT

Raza A. Mathematical maintenance models of vehicles' equipment. – Qualifying scientific work on the rights of the manuscript.

Dissertation for obtaining a scientific degree of Doctor of Philosophy within the specialty 05.22.20 «Maintenance and repair of vehicles». – National Aviation University, Kyiv, 2018.

The thesis addresses the critical scientific problem of creating the appropriate maintenance models for digital avionics systems and degrading equipment of vehicles, which increases the operational effectiveness of such systems significantly.

The thesis research includes the analysis of the current state and models of digital avionics maintenance. The study describes the necessity for developing the mathematical maintenance models for redundant digital avionics systems, considering the discontinuous nature of their operation, continuous nature of in-flight testing, possibility of both permanent and intermittent failures and organization of several maintenance levels using various diagnostic tools for detecting both failure types.

Another focus of the thesis is the analysis of modern trends and mathematical models of condition-based maintenance (CBM) of vehicles' equipment. The necessity of developing new CBM mathematical models for degrading equipment of vehicles, considering the probabilities of correct and incorrect decisions when checking system suitability for use in the upcoming operation interval, and the possibility of joint determination of the optimum inspection schedule and replacement thresholds for systems that affect and do not affect safety have been substantiated.

The scientific novelty of the primary results obtained in the course of the thesis research is as follows:

1. For the first time, mathematical models to evaluate the operational reliability indicators of continuously monitored line replaceable units/line replaceable modules (LRUs/LRMs) and redundant avionics systems over both finite and infinite time interval, which, unlike known models, consider the characteristics of both permanent and intermittent

failures, have been developed. These models allow evaluating the impact of intermittent failures on the availability and mean time between unscheduled removals (MTBUR) of LRU/LRM.

2. For the first time, generalized expressions to calculate the average maintenance costs of redundant avionics systems, considering the impact of permanent and intermittent failures, have been developed for alternative maintenance options that differ by the number of maintenance levels (one, two or three), which allows choosing the optimal maintenance option during warranty and post-warranty periods.

3. For the first time, a mathematical model of CBM, based on condition monitoring at scheduled times has been developed, which, unlike the known models, considers the probabilities of correct and incorrect decisions made when checking system suitability. This model allows formulating the criteria of determining the optimal replacement threshold for each inspection time and substantially reduce the likelihood of system failure in the forthcoming interval of operation.

4. For the first time, generalized mathematical expressions to calculate the effectiveness indicators of CBM over a finite time interval, as well as the criteria of joint optimization of the inspection schedule and replacement thresholds for systems that affect or do not affect the safety, have been developed. These results allow significantly improve the availability, reduce average maintenance costs and reduce the number of inspections.

The practical value of the results obtained in the thesis is as follows:

1. The techniques to calculate probabilistic and time-related indicators of maintenance effectiveness for digital avionics LRUs/LRMs over finite and infinite operating intervals have been developed. The proposed procedures allow to estimate the availability, operational reliability function (ORF), and mean time between unscheduled removals (MTBUR) of LRUs/LRMs during warranty and post-warranty maintenance periods for both federated avionics (FA) and integrated modular avionics (IMA) architectures;

2. A technique for minimizing the warranty maintenance cost of the redundant digital avionics systems has been developed, demonstrating (through the example of the ADIRS system of the Airbus A380 aircraft) that in the case of the optimal option of warranty maintenance, the average maintenance cost per aircraft decreases by 28 %;

3. A technique for minimizing the post-warranty maintenance cost of the redundant digital avionics systems has been developed. It demonstrates (through the example of the ADIRS system of the Airbus A380 aircraft) that a three-level maintenance option with an intermittent fault detector (IFD) at I and D levels, is optimal as it reduces the total expected maintenance costs by 11 times compared to a one-level option, and by over 8.5 times compared to a two-level option without IFD;

4. A technique for determining the optimal replacement thresholds when monitoring the condition of the degrading system at scheduled times has been developed, which allows to significantly reduce the system failure probability in the forthcoming interval of operation.

5. A technique for joint determination of the optimal replacement threshold and periodicity of suitability checking when monitoring the system condition has been developed, which allows to substantially increase the availability of systems while significantly reducing the number of inspections.

The results of the thesis research may be used in the development and maintenance of FA and IMA systems, as well as degrading equipment of vehicles.

Keywords: digital avionics systems, federated avionics, integrated modular avionics, degrading systems, maintenance, condition-based maintenance, redundancy, mathematical model, regenerative process, probabilities of correct and incorrect decisions, availability, operational reliability function, average system operating costs, warranty maintenance, post-warranty maintenance, built-in test equipment, intermittent fault detector, automated test equipment, permanent failure, intermittent failure.

List of the PhD student publications:

1. Раза, А. Математическая модель обслуживания цифровых систем авионики с учетом перемежающихся отказов/ А. Раза// Математические машины и системы. – 2018. - № 1. – С. 138-147.

2. Raza, A. Mathematical model of corrective maintenance based on operability checks for safety critical systems/ A. Raza// American journal of applied mathematics. – 2018. – V. 6(1). – P. 8-15.

3. Raza, A. Maintenance model of digital avionics/ A. Raza// Aerospace. – 2018. – V. 5(2). – P. 1-16.

4. Raza, A. Cost model for assessing losses to avionics suppliers during warranty period/ A. Raza, V. Ulansky// Advances in through-life engineering services. Decision Engineering: collective monograph. – Springer, 2017. – P. 291-307.

5. Raza, A. Optimal policies of condition-based maintenance under multiple imperfect inspections/ A. Raza, V. Ulansky// Transactions on engineering technologies: collective monograph. – Springer, 2016. – P. 285-299.

6. Raza, A. Modelling of predictive maintenance for a periodically inspected system/ A. Raza, V. Ulansky// Procedia CIRP. – 2017. – V. 59. – P. 95-101.

7. Raza, A. Minimizing total lifecycle expected costs of digital avionics' maintenance/ A. Raza, V. Ulansky// Procedia CIRP. – 2015. – V. 38. - P. 118-123.

8. Ulansky, V. Determination of the optimal maintenance threshold and periodicity of condition monitoring/ V. Ulansky, A. Raza// First world congress on condition monitoring (WCCM), 13-16 June 2017, London, UK. – WCCM proceedings, 2017. – P. 1-12.

9. Raza, A. Generalized cost functions of avionics breakdown maintenance strategy/ A. Raza, V. Ulansky, K. Augustynek, K. Warwas// 2017 IEEE Aerospace conference, 4-11 March, 2017, Big Sky, Montana, USA. – Conference proceedings, 2017. - P. 1-15.

10. Raza, A. Modelling condition monitoring inspection intervals/ A. Raza, V. Ulansky// Electronics and electrical engineering: Proceedings of the 2014 Asia-Pacific Electronics and Electrical Engineering Conference (EEEC 2014), December 27-28, 2014, Shanghai, China. — London: CRC Press, Taylor & Francis Group, 2015. - P. 45-51.

11. Raza, A. A Probabilistic model of periodic condition monitoring with imperfect inspections/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering 2015, WCE 2015, 1-3 July, 2015, London, UK. – V. II. – P. 999-1005.

12. Raza, A. Optimal thresholds for stochastically deteriorating systems/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering and computer science 2015, WCECS 2015, 21-23 October, 2015, San Francisco, USA. – V. II. – P. 934-939.

13. Ulansky, V. Modelling of condition monitoring with imperfect inspections/ V. Ulansky, A. Raza// Proceedings of the 19th World conf. on nondestructive testing 2016, WCNDT 2016, 13-17 June, 2016, Munich, Germany. – P. 1-9.

14. Raza, A. Assessing the impact of intermittent failures on the cost of digital avionics' maintenance/ A. Raza, V. Ulansky// 2016 IEEE Aerospace conference, 5-12 March, 2016, Big Sky, Montana, USA. – Conference proceedings, 2016. – P. 1-16.

15. Raza, A. Modelling of operational reliability and maintenance cost for avionics systems with permanent and intermittent failures/ A. Raza, V. Ulansky// Proceedings of the 9th IMA international conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. – P. 186-192.

16. Raza, A. Modeling of discrete condition monitoring for radar equipment/ A. Raza, V. Ulansky// 2014 IEEE Microwaves, radar and remote sensing symp., 23-25 September 2014, Kyiv, Ukraine. – MRRS proceedings, 2014. – P. 88-91.

CONTENTS

ABBREVIATIONS

ADC	Air data computer
ADCN	Avionics data communication network
ADIRS	Air data inertial reference system
ADIRU	Air data inertial reference unit
AFDX	Avionics full duplex switched Ethernet
ARINC	Aeronautical radio incorporation
ATE	Automated test equipment
BITE	Built-in test equipment
CAN	Controller area network
CBM	Condition-based maintenance
CPIOM	Core processing input/output module
CRDC	Common remote data concentrator
DME	Distance measuring equipment
FA	Federated avionics
FAA	Federal Aviation Association
FMECA	Failure mode effect and criticality analysis
FMSA	Failure modes symptoms analysis
ICP	Integrated control panels
IFD	Intermittent fault detector
IFDIS	Intermittent fault detection & isolation system
IMA	Integrated modular avionics
IOM	Input/output module
LRM	Line replaceable module
LRU	Line replaceable unit
IVHM	Integrated vehicle health management
MMR	Multi-mode receiver
MSG	Maintenance steering group

MTBF	Mean time between failures
MTBUR	Mean time between unscheduled removals
NBAA	National Business Aviation Association
NDE	Non-destructive equipment
NFF	No fault found
ORF	Operational reliability function
OSA-CBM	Open system architecture for condition-based maintenance
PDF	Probability density function
PWMO	Post-warranty maintenance option
PWTEC	Post-warranty total expected costs
RCM	reliability centred maintenance
SRU	Shop replaceable unit
UGF	Universal generating function
VIFD	Voyager Intermittent Fault Detector
WTEC	Warranty total expected costs
WMO	Warranty maintenance option

INTRODUCTION

The relevance of the thesis topic. The relevance of the dissertation topic is due to the tendency of continuous growth of the functional complexity of vehicles' equipment and, in particular, avionics systems of aircraft and as a consequence, a significant increase in maintenance costs. The cost of avionics maintenance is very high for modern aircraft. It can be as high as 30% of the aircraft maintenance cost. Therefore, the extremely crucial task is to reduce the cost of maintenance of vehicles' equipment while ensuring a high level of operational reliability. Currently, the most frequently used maintenance strategies of vehicles equipment are run-to-failure and condition-based maintenance. For most modern avionics systems, the run-to-failure maintenance strategy is dominant. With this maintenance strategy, a high level of flight safety is ensured by using redundant avionics systems, while the flight regularity is provided by a sufficient number of spare LRUs in the airline warehouse. However, the run-to-failure maintenance strategy proves to be ineffective in the case of high rate of unconfirmed failures or so-called No Fault Found (NFF) events. International aviation data suggest that over 400,000 NFF events occur each year [1]. As shown in [2], the estimated NFF rates for avionics systems range from 20 % to 50 %. The study [3] indicates that the avionics system component failures account for 80.4 % of all NFF cases, which resulted in an additional 26.6 % of unscheduled removals of avionics LRUs. The main reason for NFF cases related to electronic LRUs is the occurrence of intermittent failures in flight [4]. The general-purpose automated test equipment (ATE) is not adapted for detecting intermittent failures, and they may repeatedly occur during next flights. The negative impact of NFF on airlines includes increased service times, disruption of flight regularity, and increased quantity of spare LRUs in the exchange fund, which ultimately leads to an increase in the avionics system lifecycle costs.

Therefore, the modern aircraft operation practices confirm the **relevance** of assessing the impact of intermittent failures on the avionics system lifecycle costs and selection of a specific maintenance option to minimize the adverse effect of these failures.

CBM is currently considered as a promising approach for improving operational reliability and reducing operating costs of various vehicles. Economic assessment of CBM usage for different vehicles equipment has been discussed by many authors [5-15]. For instance, the economic effects from adopting CBM of 6TL electric batteries in the United States Department of Defence (DoD) amounted to USD 1,017 million over 25 years [5]. As indicated in [16], the use of CBM can prevent 80 % of all failures of A340-600 air-conditioning system. At the same time, the airline's financial losses can be reduced by 90 % due to a reduction of unplanned flight delays. CBM is a maintenance strategy that utilizes monitoring of the actual system condition to decide on performing the specific maintenance. Typically, this type of maintenance is necessary to perform when one or more parameters of system condition show signs of performance degradation or imminent failure. The primary activity of this maintenance type is condition monitoring, which can be continuous or periodic. In some cases, continuous condition monitoring is not feasible due to higher operating costs or physical impossibility to install the state sensors inside the system. In such cases, periodic condition monitoring is deemed more appropriate. For example, wet arc propagation resistance tests of the aircraft electric wiring interconnect systems are possible only with periodic condition monitoring [17, 18]. CBM is a preferred strategy in cases, where system degradation can be measured or when a system fails as a particular system state parameter exceeds the functional failure threshold.

The most critical controlled variables for CBM optimization are inspection times and replacement thresholds, which correspond to the pre-failure condition of the system [19]. Therefore, the issues associated with determining the optimal replacement thresholds and inspection schedule of deteriorating equipment of vehicles are currently most essential. The most significant effect from using CBM is expected to achieve by combined optimization of inspection schedule and replacement thresholds.

Aim and objectives of the study. The purpose of the dissertation work is the development of mathematical models of run-to-failure maintenance and CBM, which are intended to increase the maintenance effectiveness of digital avionics systems and degrading equipment of vehicles.

To achieve this aim, the following tasks must be solved:

- To develop mathematical models for evaluating the operational reliability of continuously monitored LRUs/LRMs of avionics systems over a finite and an infinite time interval that would consider the impact of both permanent and intermittent failures.

- To develop generalized relationships for calculation of average operating costs during warranty and post-warranty periods of operation of redundant avionics systems for alternative maintenance options that differ by the number of maintenance levels (one, two or three).

- To develop a new decision rule for system condition monitoring at discrete time points and a corresponding mathematical model of maintenance that would significantly reduce the probability of system failure during operation between time points of condition monitoring by rejecting potentially unreliable systems.

- To develop a new mathematical model of CBM for a finite time interval of system operation, considering the probabilities of correct and incorrect decisions when checking system suitability for use in the forthcoming period of operation.

- To develop CBM effectiveness indicators based on suitability checking, as well as criteria to determine the optimal inspection schedule and replacement thresholds for systems that affect or do not affect the safety.

- To develop techniques that allow using the proposed mathematical models in solving problems of maintenance optimization of vehicles' equipment.

The object of the research shall be the processes of maintenance of digital avionics systems and deteriorating equipment of vehicles.

The subject of the research shall be the mathematical models of maintenance of vehicles’ equipment.

Methods of the research. The methods of mathematical reliability theory, probability theory, and statistics, the theory of regenerative stochastic processes, as well as methods of numerical analysis and simulation modelling are used to address the objectives stated in the thesis.

Scientific novelty of the obtained results.

1. For the first time, mathematical models have been developed to evaluate the operational reliability indicators of continuously monitored LRUs/LRMs and redundant avionics systems over both a finite and an infinite time interval, which, unlike known models, consider the characteristics of both permanent and intermittent failures, allowing to assess the influence of intermittent failures on the maintenance effectiveness.

2. For the first time, generalized maintenance cost functions during warranty and post-warranty periods of operation of redundant avionics systems, considering the characteristics of permanent and intermittent failures, have been developed for alternative maintenance options that differ by the number of maintenance levels (one, two or three), allowing to choose the optimal maintenance option for each maintenance period.

3. For the first time, a mathematical model of CBM, based on condition monitoring at scheduled time points has been developed, which, unlike the known models, takes into consideration the unconditional probabilities of correct and incorrect decisions made when checking system suitability. The model allows formulating the criteria for determining the optimal replacement threshold for each inspection time and substantially reduce the likelihood of system failure in the forthcoming interval of operation.

4. For the first time, generalized mathematical expressions to calculate the effectiveness indicators of CBM over a finite time interval, as well as the criteria of joint optimization of the inspection schedule and replacement thresholds for systems that affect or do not affect safety, have been developed. These results allow significantly improve the availability, reduce average maintenance costs and number of inspections.

Validity and trustworthiness of the obtained research results have been confirmed by the sufficient and proper application of the mathematical apparatus of probability theory and reliability theory, consistency of the obtained theoretical results with operational data, as well as the results of simulation modelling.

The practical significance of the obtained results. The thesis creates a scientific and technical basis for further improvement of maintenance of digital avionics and degrading equipment of vehicles. The following practical results of the thesis research have been achieved:

- The techniques to calculate the availability, ORF and MTBUR of avionics LRUs/LRMs during warranty and post-warranty period for both FA and IMA architectures have been developed, allowing to evaluate the impact of both permanent and intermittent failures.
- A technique to minimize the cost of warranty maintenance for redundant digital avionics systems, considering the impact of both permanent and intermittent failures, has been developed. On the example of the ADIRS system of the Airbus A380, it has been shown that in the case of the optimal arrangement of the warranty maintenance, the average maintenance costs per aircraft may reduce by 28%.
- A technique to minimize the costs of post-warranty maintenance for redundant digital avionics systems has been developed. It has demonstrated (through the example of the ADIRS system mounted in the Airbus A380) that a three-level maintenance option, with IFD at I- and D-level of maintenance, is optimal as it reduces the total expected maintenance costs by 11 times compared to a one-level maintenance option and by over 8.5 times compared to a two-level maintenance option without IFD.
- A technique of determining the optimal replacement thresholds for the case of a known inspection schedule of the deteriorating system has been developed, which allows reducing the system failure probability between inspections significantly.
- A technique for joint optimization of the replacement threshold and periodicity of suitability checking, when monitoring the system condition, has been developed. It allows to significantly increase the availability of systems that do not affect safety, as well as to reduce the maintenance costs for systems that affect safety while reducing the number of inspections significantly.

Personal contribution of the candidate. The main results of the thesis research were obtained by the author independently. Studies [20-22] were conducted independently by the author. The candidate has made the following contributions in the articles published in co-authorship: in [32, 35] — analytical expressions to calculate the probabilities of correct and incorrect decisions made when checking system suitability in the case of monotonically decreasing and increasing stochastic processes of degradation. In [30] — development of a new decision rule for one-time system condition monitoring based on the evaluation of the system remaining operation-to-failure time, as well as analytical expressions to calculate the probabilities of correct and incorrect decisions. In [24, 31] — development of mathematical models to determine the optimal replacement thresholds in the case of multiple suitability checking. In [29] — a mathematical model to determine the optimal moments of condition monitoring. In [25] — analytical expressions to calculate the mean times spent by the system in various states in the case of CBM. In [27] — analytical expressions to calculate CBM effectiveness indicators for safety-critical systems. In [26] — a mathematical model to calculate the operational reliability of continuously monitored LRUs and redundant avionics systems over an infinite time interval. In [33] — mathematical expressions to evaluate the reliability and maintenance costs in the case of arbitrary distribution of operating time to intermittent or permanent failure over a finite time interval. In [34] — mathematical formulas to calculate the ORF, MTBUR and average cost of unscheduled repairs of LRUs over an infinite time interval of operation. In [23] — a mathematical model of the LRU operational reliability in the case of arbitrary distribution of operating time to permanent and intermittent failure over a finite time interval. In [28] — development of analytical expressions to calculate the average maintenance costs of redundant avionics systems during warranty and post-warranty periods for alternative maintenance options that differ by the number of maintenance levels (one, two or three), considering the impact of permanent and intermittent failures.

Approbation of the results of the dissertation work. The research results have been discussed at 14 international congresses, symposiums and conferences: 1) 2014 IEEE Microwaves, Radar and Remote Sensing Symposium (Kyiv, 2014); 2) 2014 Asia-Pacific Conference on Electronics and Electrical Engineering (Shanghai, China, 2014); 3) World Conference on Control, Electronics and Electrical Engineering (Shanghai, China, 2015); 4) World Congress on Engineering (London, UK, 2015); 5) World Congress on Engineering and Computer Science (San Francisco, USA, 2015); 6) 4th International Conference on Through-life Engineering Services (Cranfield, UK, 2015); 7) 2016 IEEE Aerospace Conference (Big Sky, USA, 2016); 8) International Symposium on No Fault Found (Cranfield, UK, 2016); 9) 9th IMA International Conference on Modelling in Industrial Maintenance and Reliability (London, UK, 2016); 10) 19th World Conference on Non-

Destructive Testing (Munich, Germany, 2016); 11) 5th International Conference on Through-life Engineering Services (Cranfield, UK, 2016); 12) 2017 IEEE Aerospace Conference (Big Sky, USA, 2017); 13) First World Congress on Condition Monitoring — WCCM 2017 (London, UK, 2017); 14) 2018 IEEE Aerospace Conference (Big Sky, USA, 2018).

Publications. The main contents of the thesis have been published in 16 printed publications, including three periodicals, two collective monographs, 11 proceedings of international congresses, symposiums, and conferences. All publications indexed in international scientometric databases, including 10 in the Scopus database and 4 in the Web of Science database.

Structure and content of the thesis. The thesis consists of an introduction, four chapters, conclusions, list of used references represented after each chapter, and three appendices. The total number of pages is 196. In the thesis, there are 36 figures (including 14 figures on seven separate pages), 17 tables (including seven tables on six separate pages), 204 references on 28 pages and seven pages of appendices.

REFERENCES

1. Roadmap to self-serving assets in civil aerospace/A. M. Brintrup, D. C. Ranasinghe, S. Kwan *et al.*//1st CIRP Industrial product-service systems (IPS2) conference, 1-2 April 2009, Cranfield, UK. – Conference proceedings, 2009. - P. 323-331.

2. Soderholm, P. A system view of the no-fault-found (nff) phenomenon/P. Soderholm//Reliability engineering and system safety. - 2007. – V. 92, No. 1. – P. 1-14.

3. Hockley, C. The impact of no fault found on through-life engineering services/C. Hockley, P. Phillips//Journal of quality in maintenance engineering. – 2012. – V. 18, No. 2. – P. 141–153.

4. No Fault Found events in maintenance engineering Part 1: Current trends, implications and organizational practices/S. Khan, P. Phillips, I. Jennions, C. Hockley//Reliability engineering and system safety. – 2014. – V. 123. – P. 183-195.

5. How engineers can conduct a cost-benefit analysis for PHM systems/J. Banks, K. Reichard, E. Crow, K. Nickell//2005 IEEE Aerospace conference, 5-12 March 2005, Big Sky, MT, USA. – Aerospace conference proceedings, 2005. – P. 1-10.

6. Feldman, K. A methodology for determining the return on investment associated with prognostics and health management/K. Feldman, T. Jazouli, P. Sandborn//IEEE Transactions on reliability. - 2009. - V. 58, No. 2. - P. 305-316.

7. Hölzell, N. B. Cost-benefit analysis of prognostics and condition-based maintenance concepts for commercial aircraft considering prognostic errors/N. B. Hölzell, V. Gollnick//2015 Annual conference of the prognostics and health management society, 19-24 October, 2015, San Diego, California. – Conference proceedings, 2015. – P. 1-16.

8. Cost-benefit analysis methodology for PHM applied to legacy commercial aircraft/B. P. Leao, K. T. Fitzgibbon, L. C. Puttini, G. P. B. Melo//2008 IEEE Aerospace conference, 1-8 March 2008, Big Sky, MT, USA. – Aerospace conference proceedings, 2008. – P. 1-13.

9. Sandborn, P. A. A maintenance planning and business case development model for the application of prognostics and health management (PHM) to electronic systems/P. A. Sandborn, C. Wilkinson//Microelectronics reliability. – 2007. - V. 47, No. 12. - P. 1889-1901.

10. Scanff, E. Lifecycle cost impact using prognostic health management (PHM) for helicopter avionics/E. Scanff, K. L. Feldman, S. Ghelam *et al.*//Microelectronics reliability. – 2007. - V. 47. - P. 1857-1864.

11. Feldman, K. The analysis of return on investment for PHM applied to electronic systems/K. Feldman, P. Sandborn, T. Jazouli//2008 IEEE International conference on prognostics and health management (PHM), 6-9 October, 2008, Denver, CO, USA. - Conference proceedings, 2008. – P. 1-9.

12. Kahlert, A. Cost-benefit analysis and specification of component-level PHM systems in aircraft/A. Kahlert, S. Giljohann, U. Klingauf//Universal journal of mechanical engineering. 2016. – V. 4, No. 4. – P. 88-98.

13. Hölzell, N B. An aircraft lifecycle approach for the cost-benefit analysis of prognostics and condition-based maintenance based on discrete-event

simulation/N. B. Hölzell, T. Schilling, V. Gollnick//2014 Annual conference of the prognostics and health management society, September 27 – 3 October 2014, Fort Worth, Texas, USA. – Conference proceedings, 2014. – P. 1-16.

14. Lasch, R. Condition-based maintenance planning within the aviation industry/R. Lasch, R. Fritzsche//Logistics Management. Lecture Notes in Logistics: col. monogr. – Cham: Springer, 2016. – P.

15. Li, J. Condition based maintenance optimization of an aircraft assembly process considering multiple objectives/J. Li, T. Sreenuch, A. Tsourdos//ISRN Aerospace engineering. – 2014. – V. 2014. – P. 1-13.

16. Gerdes, M. Effects of condition-based maintenance on costs caused by unscheduled maintenance of aircraft/M. Gerdes, D. Galar, D. Scholz//Journal of quality in maintenance engineering. - 2016. - V. 22, No. 4. - P. 394-417.

17. Bui, H. T. Evaluation of deterioration of the aircraft electrical wiring system due to undetected arcing events: thesis for a Master's of Engineering degree/ Hau T. Bui; Rensselaer Polytechnic Institute. - Hartford, USA, 2006. – 45 p.

18. Aging aircraft wiring fault detection survey/K. R. Wheeler, D. A. Timucin, I. X. Twombly *et al.*//NASA Ames Research Center, Moffett Field, CA. – Available at: http://ti.arc.nasa.gov/m/pub-archive/1342h/1342%20%28Wheeler%29.pdf/ -1.06.2006.

19. Grall, A. A condition-based maintenance policy for stochastically deteriorating systems/A. Grall, C. B´erenguer, L. Dieulle//Reliability engineering and system safety. – 2002. - V. 76, No. 2. – P. 167–180.

20. Раза, А. Математическая модель обслуживания цифровых систем авионики с учетом перемежающихся отказов/ А. Раза// Математические машины и системы. – 2018. - № 1. – С. 138-147.

21. Raza, A. Mathematical model of corrective maintenance based on operability checks for safety critical systems/ A. Raza// American journal of applied mathematics. – 2018. – V. 6(1). – P. 8-15.

22. Raza, A. Maintenance model of digital avionics/ A. Raza// Aerospace. – 2018. – V. 5(2). – P. 1-16.

23. Raza, A. Cost model for assessing losses to avionics suppliers during warranty period/ A. Raza, V. Ulansky// Advances in through-life engineering services. Decision Engineering: a collective monograph. – Springer, 2017. – P. 291-307.

24. Raza, A. Optimal policies of condition-based maintenance under multiple imperfect inspections/ A. Raza, V. Ulansky// Transactions on engineering technologies: a collective monograph. – Springer, 2016. – P. 285-299.

25. Raza, A. Modelling of predictive maintenance for a periodically inspected system/ A. Raza, V. Ulansky// Procedia CIRP. – 2017. – V. 59. – P. 95-101.

26. Raza, A. Minimizing total lifecycle expected costs of digital avionics' maintenance/ A. Raza, V. Ulansky// Procedia CIRP. – 2015. – V. 38. - P. 118-123.

27. Ulansky, V. Determination of the optimal maintenance threshold and periodicity of condition monitoring/ V. Ulansky, A. Raza// First world congress on condition monitoring (WCCM), 13-16 June 2017, London, UK. – WCCM proceedings, 2017. – P. 1-12.

28. Raza, A. Generalized cost functions of avionics breakdown maintenance strategy/ A. Raza, V. Ulansky, K. Augustynek, K. Warwas// 2017 IEEE Aerospace conference, 4-11 March, 2017, Big Sky, Montana, USA. – Conference proceedings, 2017. - P. 1-15.

29. Raza, A. Modelling condition monitoring inspection intervals/ A. Raza, V. Ulansky// Electronics and electrical engineering: Proceedings of the 2014 Asia-Pacific Electronics and Electrical Engineering Conference (EEEC 2014), December 27-28, 2014, Shanghai, China. — London: CRC Press, Taylor & Francis Group, 2015. - P. 45-51.

30. Raza, A. A Probabilistic model of periodic condition monitoring with imperfect inspections/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering 2015, WCE 2015, 1-3 July, 2015, London, UK. – V. II. – P. 999-1005.

31. Raza, A. Optimal thresholds for stochastically deteriorating systems/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering and computer science 2015, WCECS 2015, 21-23 October, 2015, San Francisco, USA. – V. II. – P. 934-939.

32. Ulansky, V. Modelling of condition monitoring with imperfect inspections/ V. Ulansky, A. Raza// Proceedings of the 19th World conf. on nondestructive testing 2016, WCNDT 2016, 13-17 June, 2016, Munich, Germany. – P. 1-9.

33. Raza, A. Assessing the impact of intermittent failures on the cost of digital avionics’ maintenance/ A. Raza, V. Ulansky// 2016 IEEE Aerospace conference, 5-12 March, 2016, Big Sky, Montana, USA. – Conference proceedings, 2016. – P. 1-16.

34. Raza, A. Modelling of operational reliability and maintenance cost for avionics systems with permanent and intermittent failures/ A. Raza, V. Ulansky// Proceedings of the 9th IMA international conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. – P. 186-192.

35. Raza, A. Modeling of discrete condition monitoring for radar equipment/ A. Raza, V. Ulansky// 2014 IEEE Microwaves, radar and remote sensing symp., 23-25 September 2014, Kyiv, Ukraine. – MRRS proceedings, 2014. – P. 88-91.

CHAPTER 1:
ANALYSIS OF BREAKDOWN AND CONDITION-BASED MAINTENANCE STRATEGIES FOR MODERN AVIATION EQUIPMENT AND STATEMENT OF THE RESEARCH OBJECTIVES

1.1. Analysis of the current state of digital avionics maintenance

Until the late 1990s, digital avionics of most civilian aircraft, including Boeing (B757, B767) and Airbus (A320, A330, and A340) series, was based on FA architecture with each avionics system consisting of several LRUs. Each LRU has built-in test equipment (BITE), which continuously monitors LRU operation in flight. Dedicated applications perform most of the functions of conventional LRUs of IMA architecture used in B777 and A380. Generic IMA modules called Core Processing Input/Output Modules (CPIOMs) run these dedicated applications. The IMA structure also includes I/O modules (IOMs). CPIOMs and IOMs are LRMs, which are linked via the avionics data communication network (ADCN). Avionics of A380 includes 30 LRMs, 50 LRUs with an Avionics Full Duplex Switched Ethernet (AFDX) interface and approximately 30 LRUs with an ARINC 429 interface. LRUs/LRMs operate until failure (permanent or intermittent), which is registered in flight or after landing. RUs/LRMs, rejected in flight, are removed from the aircraft and may be repaired either by the manufacturer or at the airport maintenance facility.

Modern digital avionics systems are designed as modular units with high requirements to testability and maintainability. A non-redundant avionics system usually consists of one or several LRUs/LRMs. In turn, an LRU/LRM comprises a set of shop replaceable units (SRUs), each being a printed circuit board (PCB) assembly with non-repairable electronic components. According to the structural classification of avionics systems, the following three levels of maintenance are usually considered: organizational maintenance (O-level), intermediate maintenance (I-level) and depot maintenance (D-level). O-level maintenance is performed at an aircraft parking and is used for the rapid turnover of LRUs/LRMs where these LRUs/LRMs can be removed and replaced with spare LRUs/LRMs from the warehouse within a short period if they are rejected by BITE during the pre-flight test or in-flight monitoring. LRUs/LRMs represent the spare parts at this maintenance level. Specialized airport workshops, located usually at the base airports, perform I-level maintenance. The I-level maintenance is more skilled and allows testing and repairing the LRUs/LRMs removed from the aircraft at the O-level maintenance. Ground ATE allow automating most test procedures at this maintenance level. Inoperable LRUs/LRMs are repaired by replacing the faulty SRUs. Therefore, SRUs are the spare parts at the I-level maintenance. The depot maintenance is more an opportunity than a location. D-level maintenance supports O-level and I-level maintenance by providing engineering and technical assistance that exceeds the scope of O- and I-levels. D-level maintenance is usually performed in specialized repair centres or at original equipment manufacturing facilities. The standard spare parts used at D-level within a three-level maintenance system are non-repairable electronic components. The maintenance system may consist of two levels if the only O- and D-levels are applicable. In this case, the spare parts at the D-level maintenance include SRUs and non-repairable electronic components.

Many airlines currently dismiss the third level and sometimes the second maintenance level to outsource work to specialized companies, which are called "repair stations" [1]. Major U.S. air carriers now use outsourced repair stations for up to 47 % of their maintenance costs [2]. However, one of the largest U.S. airlines, American Airlines, conducts most of its maintenance operations in-house [3]. Thus, there are many options for avionics maintenance using one-, two- or three-level maintenance systems.

Unconfirmed failures, commonly called NFF events, significantly impact aviation equipment operating costs [4]. By ARINC 672 [5], an NFF is the result of testing when a unit removed as inoperable at a certain maintenance level is judged as operable by the results of checking at the lower maintenance level. According to [6], the average financial losses due to NFF per aircraft in U.S. civil aviation amounted to approximately $ 200,000 in 2013. Similar losses exist in military aviation [7, 8]. For instance, according to the U.S. Department of Defence, 3 out of 4 (75 %) weapon systems are subject to NFF-type failures [8]. The U.S. DoD loses from $ 2 billion to $ 10 billion annually due to NFF events [9]. The study [10] provides statistics on NFF events of avionics of F-16 aircraft of the Turkish Air Force for 2013-2014. According to this study, NFF events amount to 45 % of the total number of failures. It should be specially noted that the problems of avionics account for up to 75 % of all cases of NFF in the aviation industry [11].

Some of the terms used to describe NFF phenomenon include “trouble not identified” (TNI), “no trouble found” (NTF), “cannot duplicate” (CND), and “re-test OK” (RTOK) [12, 13]. NFF is a chain of events, which usually starts with symptoms of a fault registered in-flight, and later, after ground diagnostics and repair, the same symptoms are repeatedly recorded in one of the subsequent flights. Financial losses due to NFF include additional maintenance costs, costs for extra spare parts, losses due to the irregularity of flights, costs for more technical diagnostics, etc.

The study [14] outlines the main NFF causes in civil and military aviation:

1) Intermittent failures.

2) Experience of technicians in failure diagnostics.

3) Experience of technicians in intermittent failure diagnostics.

4) Experience of technicians related to NFF.

5) Fault diagnostics guide.

6) Training of technicians in NFF diagnostics.

7) Environment factors (heat, vibration, etc.).

8) Component integrity (wiring).

9) Component integrity (connectors).

10) Experience of technicians with diagnostics equipment.

As seen from this list, intermittent failures (item 1) are the leading NFF cause, and they are also present in the “experience of technicians in intermittent failure diagnostics” cause factor (item 3). An intermittent failure usually implies a multiple-occurrence of self-eliminating failure of the same nature. Intermittent failures in avionics systems may be software-induced or hardware-induced. Intermittent failures of hardware, such as electronic assemblies, are divided into four categories: printed circuit boards; connectors; components; interconnections [15]. Intermittent failures are mechanical by nature. They occur due to faults of electrical wiring, solder joints, screening braid, connectors, metal lines of integrated circuits, etc. Modern electronic components are reliable, so the intermittent discontinuity between printed circuit board components is becoming the main cost driver [9].

Conventional ATEs cannot detect the locations of intermittent failures that cause the appearance of NFF, for the following reasons: [9, 16]:

1) The parameter of the monitoring object is usually tested only once for a short period, which generally does not coincide with the time of occurrence of the intermittent failure;

2) Digital averaging, scanning and sampling of the test signal miss out intermittent failures.

3) The removed LRUs/LRMs are tested in the laboratory rather than in the operating environment where failures occur; the electrical wiring interconnect system is also tested in a static environment.

4) Conventional ATE are designed to detect functional failures, faulty components, as well as short circuits and breaks in electrical circuits and, is not capable of intermittent failure diagnostics.

5) Intermittent failures that cause NFF do not correspond to any particular failure pattern.

The second reason for ATE failing to detect intermittent failures requires some clarification. Under the operating principle, digital multimeters take sample readings and average them over a specified period before presenting to the operator. Most digital multimeters take samples at millisecond intervals. Therefore, even if a high ohmic resistance is detected in one or more readings, they would be entirely averaged out over a thousand readings. Thus, digital multimeters are incapable of detecting intermittent ohmic faults of Category 3 or shorter. According to the standard [17], Category 3 includes durations of intermittent failures ranging from 501 microseconds to 5 milliseconds. Finally, let us consider the fourth reason. Conventional ATE may not be used to test intermittent failures for the following reasons [18]:

- ATE does not check all circuits or functional paths of the LRU/LRM, including all connection paths to SRUs, simultaneously.

- Conventional ATE does not test the LRUs/LRMs in the appropriate operating environment.

- Conventional ATE is incapable of detecting short-duration intermittent failures that cause NFF.

Given the conventional ATE disadvantages, as stated above, specific specialized tools for intermittent failure diagnostics are currently in development. For instance, Universal Synaptics Corporation (USA) produces a Voyager Intermittent Fault Detector (VIFD) [19] and an Intermittent Fault Detection and Isolation System (IFDS) [20].

VIFD is capable of connecting up to 256 or 512 electrical test points. VIFD is a diagnostic tool that simultaneously and continuously monitors all devices tested in the object, at the same time detecting intermittent faults with the duration as short as 50 ns.

IFDIS presents a modern intermittent fault diagnosis system that simultaneously and continuously monitors each electrical path in the chassis, at the same time subjecting the diagnosed object to a simulated operating environment. The hardware circuit of the neural network detects and isolates intermittent faults of up to 50 ns occurring in any circuit of the object during the test. The graphical test results indicate the exact location of the intermittent fault for rapid repairs.

The U.S. Air Force used the IFDIS to diagnose one of its least reliable LRUs — modular low-power radio frequency system (MLPRF) of F-16 AN/APG-68 [18]. 400 LRUs were tested to detect intermittent faults, with the results showing that over 60 % of LRUs had such faults. These LRUs had been tested earlier using ground ATE and were found to be operable. All LRUs were returned to operation after repairs. Furthermore, MTBUR of the repaired LRUs increased more than three-fold, and the economic effect amounted to approximately $ 60 million. It should be noted that the cost of a single IFDS system is $ 2.2 million, which limits its use by airlines. Therefore, the adoption of IFDIS or VIFD by airlines may occur only after a feasibility study on the applicability of these systems.

1.2. Analysis of mathematical maintenance models of digital avionics systems

Avionics system performance indicators can be roughly divided into probabilistic and cost-related indicators [21]. Probabilistic indicators include the availability, the operational readiness coefficient, the coefficient of technical use, and the operational reliability function. The cost-related performance indicators include the average maintenance cost and the average maintenance cost per unit time. Definitions of these indicators of maintenance effectiveness are given in many studies, for example [22-27].

One of the most important indicators of the operational reliability of aviation equipment, which is used by most airlines for inventory management, is the MTBUR [28]. Statistically, MTBUR is calculated as follows [28, 29]:

$$MTBUR = mt_{FH} \big/ N_R \, , \tag{1.1}$$

where t_{FH} is the number of flying hours for a fleet of aircraft over a period; m is the number of identical-type LRUs in the aircraft fleet; N_R is the number of unscheduled removals of LRUs over the same period. Calculation of MTBUR by formula (1.1) is possible only after the accumulation of a significant amount of statistical information.

The following analytical expression was proposed in [21, 30-32] to calculate MTBUR of digitally monitored avionics systems over an infinite time interval:

$$MTBUR = \frac{1 - e^{-\lambda\tau}}{\lambda\left[1 - (1-\alpha)e^{-\lambda\tau}\right]}, \tag{1.2}$$

where λ is the LRU unrevealed failure rate; τ is the periodicity of LRU checking; α is the conditional probability of a “false alarm” when checking the operability of LRU.

A similar expression was proposed in [33] for the case of a finite time interval:

$$\begin{gathered} MTBUR = \frac{\tau}{\alpha}\left[1 - (1-\alpha)^N e^{-(N+1)\lambda\tau}\right] + \left[(1 - e^{-\lambda\tau})\cdot\left(\frac{1}{\lambda} - \frac{\tau}{\alpha}\right) - \tau\cdot e^{-\lambda\tau}\right] \times \\ \times \frac{1 - (1-\alpha)^{N+1} e^{-(N+1)\lambda\tau}}{1 - (1-\alpha)e^{-\lambda\tau}} + \tau(1-\alpha)^N e^{-(N+1)\lambda\tau}, \end{gathered} \tag{1.3}$$

where N is the number of LRU operability checks over the finite time interval T.

Expressions (1.2) and (1.3) may not be used to evaluate the MTBUR of digital avionics LRUs/LRMs, since, firstly, modern avionics systems are monitored continuously in flight, rather than periodically and secondly, these expressions do not consider the rate of LRU intermittent failures.

A decision model of avionics system maintenance strategy is considered in [34]. The ranking of maintenance strategies is based on the cognitive uncertainty information processing. The example given in the study shows that a reliability-centred maintenance strategy is the most suitable for avionics systems. However, it should be noted that the study does not consider the differentiation of avionics system failures into permanent and intermittent ones. Also, the number of maintenance levels and ATE cost at different levels, is not considered at all.

A mathematical model is proposed in [35] to calculate the maintenance cost per LRU socket in the aircraft.

$$C_{socket,i} = fC_{LRU,i} + (1-f)C_{LRU\,repair,i} + fT_{replace,i}V + (1-f)T_{repair,i}V\,, \tag{1.4}$$

where f is the probability of replacing the LRU with a new one as a result of maintenance; $C_{LRU,i}$ is the cost of the i-th type LRU; $C_{LRUrepair,i}$ is the i-th type LRU repair cost; $T_{replace,i}$ is the i-th type LRU replacement time; V is the unit cost of losses due to non-use of the LRU; $T_{repair,i}$ is the i-th type LRU repair time. Stochastic simulation modelling is used to determine the parameters of expression (1.4). A multifunctional display of Boeing 737 was used as a tested object. This model disregards intermittent failures.

The study [36] uses a simulation model to compare the lifecycle cost of helicopter avionics systems concerning various maintenance strategies, including unscheduled maintenance (after failure registration), fixed-interval scheduled maintenance and condition-based maintenance. The simulation model disregards intermittent failures.

In [37] a hierarchical simulation model was developed to assess the aircraft reliability. The model considers system specifics and flight mission approaches. The proposed model consists of submodules that correspond to components involved in carrying out a specified flight task. The model is used to estimate the operational probability of mission execution in cases with various combinations of system failures. The effect of intermittent failures on the likelihood of performing a flight task is disregarded.

A simple mathematical model was proposed in [38] to evaluate the cost of maintenance of an air data computer, which is a part of aircraft avionics:

$$TMC_{ac} = (AMC_u)m/1000\,, \tag{1.5}$$

where TMC_{ac} is the total cost of airborne computer maintenance; m is the number of years in operation; AMC_u is the annual maintenance cost of the airborne computer. Expression (1.5) disregards the reliability characteristics of the airborne computer.

The study [39] analyses the FMECA method (Failure Mode Effect and Criticality Analysis) which specifies the maintenance scope and procedure for each of the analysed aircraft systems. Reliability modelling is conducted under the function performed by each part of the system, and then the entire system is divided into many subsystems, and each of the subsystems is further divided into separate parts and components. The described method does not divide failures into permanent and intermittent.

In [40], a two-level and three-level maintenance system of the F-117A stealth fighter avionics are compared. The following formulas are used to determine the required number of spare LRUs:

$$S_r^{2LM} = \frac{TT + RCT}{MTBUR/DFH}, \tag{1.6}$$

$$S_r^{3LM} = \frac{TT + RCT}{MTBF/DFH}, \tag{1.7}$$

where S_r^{2LM} and S_r^{3LM} is the number of spare LRUs for a two-level and three-level maintenance system, respectively; TT is the expected time of transportation of the dismantled LRU to the repair site and back; DFH is the daily flying hours; $MTBF$ is the mean time between LRU failures. Formulas (1.6) and (1.7) can only be used to compare two-level and three-level maintenance systems regarding the number of spare LRUs.

The study [41] also compares a two-level and three-level maintenance systems of dismantled avionics LRUs. Cost savings granted by the transition from a three-level to a two-level maintenance system are calculated using the following formula:

Total cost savings = (*labour savings*) - (*increase in labour costs at the D-level maintenance*) - (*increase in transportation costs*) + *or* - (*increase or decrease in spare parts cost*) + *or* - (*change in the LRU cost by the manufacturer*).

Simulation modelling determines each of the cost components. It should be noted that the study does not differentiate between permanent and intermittent failures.

The study [42] considers a mathematical maintenance model for a continuously tested single-unit digital electronic system that is subject to the revealed, unrevealed and intermittent failures. In the case of an exponential distribution of time to failure, the availability is determined by the following formula:

$$A = \frac{\mu + \theta}{(\mu + \lambda + \theta)(1 + \mu t_{CR}) + \theta\left[(\mu + \theta)t_{FR} + \lambda t_{CR}\right]}, \quad (1.8)$$

where μ is the revealed system failure rate; λ and θ are, respectively, the unrevealed and intermittent failure rate. It should be noted that formula (1.8) is valid only for systems with a continuous mode of operation. Avionics systems have an intermittent operation due to the alternation of flights and landings. Therefore, expression (1.8) may not be used to evaluate the avionics system availability.

The following generalized expression for operational costs determination was proposed in [43]:

$$TLOC = m\left(\frac{T_0}{MTBUR}\right)\left\{\sum_{t=1}^{T_K} N_{A,t} ECC_t (1+\varepsilon)^{1-t}\right\} + \sum_{t=1}^{T_K} K_t (1+\varepsilon),^{1-t} \quad (1.9)$$

where *m* is the number of single-type LRUs installed on board an aircraft; T_0 is the average aircraft flying hours per year; T_K is the total LRU maintenance time, expressed in the number of years in operation; ECC_t is the expected airline costs per one LRU repair in year *t*; K_t is the airline capital investments over the year *t*; ε is the time discount rate, expressed in fractions or percent per year; $N_{A,t}$ is the number of aircraft in operation over the year *t*. The formula (1.9) is of a generic nature. The parameters included in (1.9) shall be determined for each of the maintenance options. For example, MTBUR is calculated in [43] using (1.3), which does not consider the possibility of intermittent failures in flight.

The study [44] proposes an empirical model to estimate the maintenance unit cost per hour depending on the total flight hours (*flhours*), aircraft size (*assize*), load factor (*lf*), average flight duration (*avst*), exchange rate (*ers*), number of passengers per airline (*airl_rpax*) and gross domestic production (*gdp*):

$$U_{cmainh} = \beta_0 + \beta_1 flhours + \beta_2 asize + \beta_3 lf + \beta_4 avst + \beta_5 ers2 + \beta_6 airl\_rpax + \beta_7 gdp, \quad (1.10)$$

where $\overline{\beta_0, \beta_7}$ are empirical coefficients. Expression (1.10) can be used for purely economic estimations, but not for the selection of an optimal avionics system maintenance option.

The study [45] considers several NFF-related maintenance strategies. The authors give statistics on NFF events, according to which NFF events make approximately 70 % of all avionics failures in military aviation. The following three maintenance strategies are considered:

1) It is assumed that after rechecking by ground ATE all dismantled LRUs with NFF are mounted back on board the aircraft.

2) It is assumed that all dismantled LRUs with NFF are defective, so such LRUs are re-tested in conditions close to flight conditions to identify and remove all defects that cause the appearance of NFF in flight.

3) Only those LRUs with NFF are repaired, which have been selected by the technicians based on their experience.

The average maintenance cost for the first strategy is:

$$C_{NFF,G} = C_T + 55.125\ \% C_G + 14.875\ \% C_B, \tag{1.11}$$

where $C_{NFF,G}$ is the cost of testing LRU with NFF using ground ATE; C_G is the cost of handling and administering each LRU in an operable state; C_B is the cost of handling and administering each LRU with NFF.

The average maintenance cost for the second strategy is determined as follows:

$$C_{NFF,\mathrm{E}} = 174.4625\ \% C_T + C_{CE} + 4.4625\ \% C_{R2} + 55.26\ \% C_G + 12.57\ \% C_B, \tag{1.12}$$

where C_{CE} is the cost of testing the LRU with NFF in conditions close to flight; C_{R2} is the cost of repairing the LRU with NFF.

The average cost of maintenance when using the third strategy is

$$C_{NFF,T} = C_T + 17.5\ \% C_{R1} + 59.281\ \% C_G + 14.9\ \% C_B, \tag{1.13}$$

where C_{R1} is the average cost of repairing the LRU dismantled from the aircraft (4,375% of repairs correspond to the LRU with NFF and 13,125% of repairs correspond to operable LRUs). It should be noted that expressions (1.11)–(1.13) are empirical and can only be used with the specified percentage distributions of confirmed and unconfirmed failures.

The following mathematical formula is proposed in [21, 27] to calculate the ORF $P_э(kt_\Pi, t)$ of a periodically checked avionics system:

$$P_э\left(kt_П, t\right) = \sum_{j=0}^{k=1} P_c\left(jt_П\right) e^{-\lambda(t - jt_П)} \cdot \left(1 - \alpha\right)^{k-j} + P_c\left(kt_П\right) e^{-\lambda(t - kt_П)}, \tag{1.14}$$

where $kt_\Pi < t \leq (k+1)t_\Pi$; $P_c(jt_\Pi)$ is the probability of removing the LRU from the aircraft board by the results of the *j*-th pre-flight check of the LRU with the help of the BITE; t_Π is the aircraft flight time; λ is the LRU unrevealed failure rate; α is the conditional probability of the BITE “false alarm”. Expression (1.14) applies to the case of a periodically checked LRU and absence of intermittent failures.

The following expression is proposed in [46] to evaluate the costs of aircraft equipment maintenance:

$$CMT = C_s + C_m + C_p + C_{tl} + C_f + C_d \,, \tag{1.15A}$$

where C_s is the cost of spare parts; C_m is the cost of necessary materials; C_p is the maintenance personnel costs; C_{te} is the cost of tools and supporting equipment; C_f is the cost of renting premises; C_d is the cost of technical data.

The study [47] considers a logical model of the primary cost drivers of the total cost of NFF consequences. The analysis shows how to choose the most suitable drivers to represent the total losses due to NFF. The generic framework for NFF cost estimation demonstrates how both qualitative and quantitative information can be used to achieve the maintenance goals. This article outlines the relevance of NFF consequences cost estimation problem, but no mathematical or other models are proposed to solve it.

The following references are not related to the models of avionics maintenance which are the subject of this study. However, these references are essential for understanding the proposed mathematical model of operational reliability assessment of avionics LRUs.

The study [48] analyses an intermittent fault detection strategy which incorporates testing at regular intervals. The exponential distribution of time to permanent and intermittent failures is assumed. The optimal testing periodicity is determined, which maximizes the probability of detecting an intermittent failure upon its first appearance:

$$\tau_{opt} = \frac{\log\theta - \log\lambda}{\theta - \lambda} \,, \tag{1.15}$$

where θ and λ are, respectively, the intermittent and permanent failure rate.

It should be noted that this model is not suitable for assessing the operational reliability of avionics systems for two reasons. Firstly, avionics systems have continuous in-flight monitoring rather than a periodical. Secondly, the duration of intermittent failures is tens of nanoseconds to hundreds of microseconds, and the testing periodicity determined by formula (1.15) may be hundreds of hours.

The study [49] considers a telecommunication system with intermittent faults. The time to a fault has an exponential distribution. Faults become permanent when the unrevealed state duration exceeds the upper time limit. This model is not appropriate for assessing the reliability of avionics systems because the operating mode of telecommunication systems is continuous, while avionics systems operate in a discontinuous mode.

The study [50] presents a Markov model of general-purpose reliability with three states for systems with both permanent and intermittent faults. A comparison is made between the reliability of duplex and redundant systems. As in [49], the proposed model can be used for systems of continuous operation only.

The study [51] considers a reliability model to determine the optimal intermittent fault-testing periodicity for pipelined embedded processors regarding the minimum testing cost. The continuous-time two-state Markov model is used for probabilistic modelling of intermittent faults. The cost function has the following form:

$$C(T) = (1-q)\times N\times D + q\times E\{R\}, \tag{1.16}$$

where T is the testing periodicity; q is the probability that the existing fault becomes active (any number of times) during the time interval [0, B] required to complete one task; $N = B/T$ is the number of tests within $[0, B]$ time interval; D is the single test duration; $E\{R\}$ is the mathematical expectation of the time required to detect the intermittent fault in the processor. As in [49, 50], this model can only be used for continuously-operated systems with periodic condition monitoring.

A model for assessing the reliability of digital systems that are subject to both permanent and intermittent faults is considered in [52]. The digital system reliability assessment is based on a three-state Markov model. The following formula determines the probability of failure-free operation:

$$R(t) = \frac{\mu}{\mu+\upsilon}e^{-\lambda t} + \frac{\upsilon}{\mu+\upsilon}e^{-(\mu+\upsilon+\lambda)t}, \tag{1.17}$$

where υ is the transition rate from an operable state to an intermittent failure state; μ is the transition rate from an intermittent failure state to an operable state; λ is the permanent failure rate. This model is intended for continuously-operated digital systems, therefore may not be used to assess the operational reliability of avionics systems.

A Bayesian network based on the method for diagnosing transient and intermittent faults was proposed in [53]. This Markov model may not be used to model avionics system faults since transient faults usually occur in electrical power systems.

The study [54] assesses the impact of various scenarios of restoring the processor after occurring intermittent faults on its performance. The operation of a fault-tolerant multi-core processor is simulated in the presence of intermittent faults, subjected to exponential and Weibull distribution. The study shows that 40 % of processor faults are intermittent and 60 % are permanent. The results of the study may be used to select the law of time distribution until an intermittent fault occurrence in digital modules of avionics systems.

In the study [55], a strategy of imperfect checks is investigated to detect intermittent faults in a computer system. The system is tested periodically, and its failure is detected with a certain probability at the next test time. The expected cost of detection of an intermittent fault is determined. In the case of exponential distribution of time to failure, the optimal testing interval T^* is determined by solving the following equation:

$$\frac{C_p(T^*)}{c_D/\lambda} = \left(e^{-\lambda T^*} - 1\right)\left[\frac{p^2}{q^2}\left(1 - e^{-\lambda T^*}\right) + \frac{p+1}{q}\right], \tag{1.18}$$

where p and q = 1 - p are, respectively, the probability of detection and non-detection of an intermittent fault; λ is the intermittent fault rate; c_D is the expenses per unit time due to the faulty state of the computer; C_p is the total expected costs for the computer operation. This model can be used only for systems with periodic testing and continuous operation.

1.3. Analysis of current trends in the condition-based maintenance of vehicles' equipment

The US military standard ADS-79D-HDBK [56] describes the CBM system in the US Army and defines the general guidance needed to achieve the CBM goals of the Air Force and unmanned aerial vehicles. The standard contains specific proven methods to achieve the CBM functional objectives while stating that these methods should not be considered as the only means for achieving the set goals. The ADS-79D-HDBK standard gives the following definition to the CBM: CBM is a set of maintenance processes and capabilities derived primarily from real-time system condition assessment obtained from embedded sensors and external measurements using portable equipment. CBM depends on the collection of data from sensors and the processing, analysis, and correlation of these data with the material conditions that require maintenance actions. The system condition assessment is conducted through continuous or periodic condition monitoring. Condition monitoring provides data for the equipment "health" assessment and maintenance is performed only when necessary.

As explained in ADS-79D-HDBK, the goals of the CBM system are to minimize the volume of heavy forms of maintenance, increase aircraft availability, improve flight safety and reduce maintenance costs.

The following objectives shall be addressed to achieve these goals:

- Determine the time intervals after which specific maintenance or replacement of equipment components is required;
- Accumulate the statistical data on the use of each piece of equipment or aircraft.
- Determine the equipment component degradation rate and estimate the remaining useful life.
- Utilize data to support a balanced approach in establishing repair limits.
- Accumulate data required for effective risk management of the aircraft fleet.

The ADS-79D-HDBK standard pays particular attention to setting the task of determining the optimal inspection schedule of equipment components that are subject to wear and degradation. At a qualitative level, the problems of determining the inspection times and the threshold for replacement of equipment components are considered to prevent its failure. Figure 1.1 illustrates the concept of a widely used inspection planning methodology for detecting damage and defects in equipment during operation. To include the failure mitigation effect, the replacement threshold a_{NDE} is considered. In general, the a_{NDE} threshold provides a high probability of failure mitigation.

If there is a probabilistic defect detection model, the replacement threshold a_{NDE} is selected from the condition that a 90% probability of defect detection is provided with 95% certainty. By analogy with the concept of the P-F interval [57-59], the remaining useful life presents the period during which the value of the degradation parameter changes from the a_{NDE} to the failure threshold of the equipment component, as shown in Fig. 1.1.

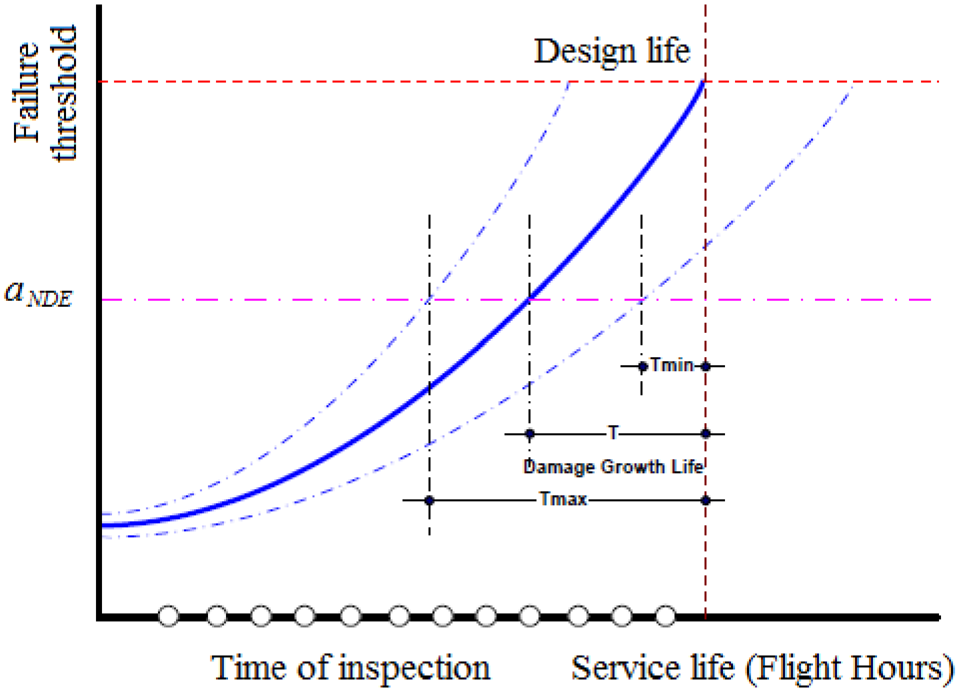


Fig. 1.1. The current approach to determine inspection interval [56]

Due to the random nature of the degradation process, the remaining useful life is also a random variable. To eliminate the effect of this randomness, the evaluated average residual lifetime is further divided by the tolerance coefficient of the inspection interval to damage, *SF*. The *SF* value is usually in the range of 3 ~ 4. In this manner, the following formula determines the inspection interval value for $t > T$:

$$T_{inspection} = T/SF. \tag{1.19}$$

Theoretically, the first inspection shall be carried out at the time which corresponds to the intersection of the degradation parameter curve with the threshold a_{NDE}. In practice, the first inspection is carried out after the equipment has been in operation for 1/4 or 1/2 of the mean time of failure-free operation.

From the above description of the existing methodology to determine the inspection schedule of the equipment degrading components, it can be seen that in fact, this technique is heuristic, and not mathematically justified. The standard ADS-79D-HDBK specifies the relevance and necessity of solving the task of achieving the most effective inspection schedule within the limits of the target reliability constraints, which should ensure a minimum number of inspections, while the risk of failure should not exceed the maximum allowable level.

At present, CBM is widely introduced into the operating practice of ground and on-board equipment of vehicles. For instance, Boeing and GE Aviation have jointly developed a standard for introducing CBM in civil aviation and other industries [60, 61]. The standard defines the Open System Architecture for CBM (OSA-CBM). The OSA-CBM standard enables and presents ways to reduce maintenance costs, improve communication between different departments and incorporate changes in system design and further cooperation in the field of aircraft systems CBM.

ISO standards for condition monitoring and diagnostics of machines offer a trustworthy guide for establishing a typical CBM procedure. For example, ISO 17359 [62] presents an overview of a generic method recommended for use when a condition monitoring programme is being implemented and provides further detail on the critical steps to follow. The standard presents the concept of activity by condition monitoring to identify the root causes of failure and describes a general approach to the definition of alarm criteria associated with the forthcoming of failure, carrying out diagnosis and prognostics, as well as increasing the trustworthiness of diagnostic and prediction results that are further developed in other international standards.

ISO 13379-1 [63] presents the main aspects of the data interpretation and diagnostics methods. This standard considers the Failure Modes Symptoms Analysis (FMSA) method in detail. The task of the FMSA method is to select a technology and a condition monitoring strategy that maximizes the trustworthiness of the diagnosis and the prediction of this type of failure. The FMSA method is a further modification of the FMECA method [64-67], aimed at selecting diagnostic parameters by which it is possible to determine the type of defect or failure and to formulate the strategies of condition monitoring.

Another standard that serves as a guide for the maintenance systems development is Maintenance Steering Group-3 (MSG-3) [68]. The title of this document is "Operator/Manufacturer Scheduled Maintenance Development". MSG-3 was developed by the maintenance steering group of the U.S. Aviation Transport Association (ATA). The

MSG-3 standard is used for the development of maintenance plans for aircraft engines and systems before the aircraft goes into service. MSG-3 is a top-down approach to determine the consequences (safety, timeliness, and economy) of failure, from the system level and down to the component level. The failure effects are divided into five categories, and if the effects fail to be mitigated, then the system must be redesigned. For example, the use of MSG-3 led to necessary design changes for control and lightning protection systems of Boeing 787-8. Besides, the MSG-3 methodology has helped to increase safety while reducing maintenance costs by up to 30 percent [69]. MSG-3 is currently the only methodology for commercial aircraft manufacturers. According to Advisory Circular AC-121-22C [70], the Federal Aviation Association (FAA) policy states that the latest MSG-3 analysis procedures should be used to develop routine scheduled maintenance tasks for all new aircraft. This is the only methodology adopted by the U.S. airworthiness authorities. Most major manufacturers of business aircraft also adopted the MSG-3 standard with the support of the National Business Aviation Association (NBAA).

In 2012, the U.S. DoD approved Instruction 4151.22 "Condition Based Maintenance Plus (CBM^+) for Materiel Maintenance" [71]. CBM^+ is the application and integration of appropriate processes, technologies, and knowledge-based capabilities to achieve the targeted availability, reliability, operational and support costs of the DoD systems and components throughout their life cycle. At its core, CBM^+ is the maintenance performed by identifying the most efficient approach to maintenance, integrating technologies and CBM capabilities with other maintenance strategies that enhance the availability and efficiency of the systems and components of the US DoD.

The International Association of Aerospace, Automotive and Commercial Engineers (SAE International) has issued the standard JA6268 "Design & Run-Time Information Exchange for Health-Ready Components" [72]. The purpose of this document is to clearly define the best methods by which the IVHM functionality relating to components and subsystems should be integrated into applications at the vehicle or platform level.

1.4. Analysis of CBM mathematical models of vehicles' equipment

A growing interest in CBM is manifested by a large number of studies devoted to various mathematical models and methods of optimization. The majority of the existing CBM models with scheduled inspections can be conditionally divided into two groups: CBM models with perfect inspections and CBM models with imperfect inspections. First, let us consider CBM models with perfect checks.

An optimal replacement strategy for CBM with optimal inspection intervals is considered in [73] for the case when degradation corresponds to an inverse Gaussian process with random effects. According to the proposed model, inspections are carried out at regular intervals $\{0, \delta, 2\delta, \dots, k\delta, \dots\}$, where δ is the inspection periodicity. The following formula calculates the total cost of inspections:

$$S(\delta) = \frac{c_i}{1 - \exp(-r\delta)}, \tag{1.20}$$

where c_i is the cost of a single system inspection; r is the discount factor.

The corrective and preventive maintenance costs as well as the cost of idle time, are added to the total cost of inspections to get a target optimization function for determining the optimal inspection periodicity.

The studies [74, 75] present a model of optimal periodic inspections based on the class of increasing Markov processes:

$$X(t) = X_0 + \sum_{s \le t} (X_s - X_{s-1}), \tag{1.21}$$

where X_0 is the initial process value; $X_s - X_{s-1}$ is the process increment at the s-th step of its development. A stochastic gamma process, which belongs to this class of Markov processes, is used as a degradation model. The inspection periodicity and preventive maintenance threshold are considered as controlled variables. Failure can be detected only through inspection. The system is renewed when the check indicates that the preventive maintenance threshold or failure threshold has been exceeded. The optimal values of the inspection periodicity and preventive maintenance threshold are determined by minimizing the expected average maintenance cost per unit time.

The study [76] discusses the problem of functional checks in reliability centered maintenance (RCM). A general cost model is developed under the assumption of a nondecreasing system degradation process. Joint optimization of the potential failure threshold and inspection intervals is considered to minimize the expected operating cost per unit time. A gamma process describes a stochastic process of degradation.

The study [77] considered a system that is subjected to stochastic degradation and monitored using perfect inspections. When the system condition exceeds the failure threshold L, the system goes into a failed state, and a corrective replacement is immediately performed, after which the system becomes as good as new. When the system condition upon inspection is found to exceed the given critical threshold λ, the system is still functioning but considered as "worn-out," and a preventive replacement is carried out. N ($N > 1$) thresholds $\overline{\xi_1, \xi_N}$ are set in the range of system states ($\xi_0 = 0$) in order to distinguish the N intervals of possible decisions ($0 < \xi_1 < ... < \xi_N < L$). The last threshold ξ_N denotes the critical threshold λ. The measurement error of the monitored parameter is considered negligible. The system operating costs per unit are minimized. The following formula presents the cost function per unit time:

$$\mathbf{E}C_{\infty}\left(\overline{\xi_1, \xi_N}\right) = \frac{1}{\Delta t}\left(c_i \int_0^{\xi_N} g_1(x)dx + c_p \int_{\xi_N}^{L} g_1(x)dx + c_c \int_L^{+\infty} g(x)dx + c_0 \int_0^{\xi_N} g(x)dx\right), \quad (1.22)$$

where c_i, c_p, and c_c are the cost of an inspection, a preventive replacement cost and a corrective replacement cost, respectively; c_o is the income per unit time when the system is in operating state; $g_1(x)$ is the density function of the system stationary state equal to x with an inspection scheduled; $g_2(x)$ is the density function of the system stationary state equal to x with no inspection scheduled; $g(x) = g_1(x) + g_2(x)$. The selection of the inspection schedule and the critical threshold λ affect the cost-effectiveness of the maintenance strategy.

The study [78] proposes a mathematical model to investigate the joint influence of the preventive replacement threshold and inspection schedule on the total costs of the system maintenance. The gamma process describes the stochastic process of degradation of the system, and the inspection schedule can be any positive and decreasing function that must be optimized. The global cumulative cost function over the interval [0, t] is determined as:

$$C(t) = C_i N_i(t) + C_p N_p(t) + C_c N_c(t) + C_d d(t), \quad (1.23)$$

where C_i, C_p, C_c are, respectively, the costs for inspection, preventive repair and corrective repair; C_d is the cost of idle per unit time; $d(t)$ is the system downtime over interval [0, t].

The study [79] considered a decision-making model that allows determining the schedule of inspections, depending on the failure characteristics of the equipment to be inspected. The model considers the maintainability of the system and the preferences of the decision-maker related to costs and idle time.

A model proposed in [80], considered preventive and corrective maintenance cost, as well as inspection cost to determine the optimal inspection and replacement policies, to minimize the total average cost of replacements and inspections.

The study [81] considered an approach to the construction and optimization of CBM policy for an accumulative degradation system. It is assumed that the system condition inspections are perfect. The optimization target function is the total cost of various inspections, replacements, and idle time, which is determined by the following formula:

$$\overline{C}_{\infty}\left(\tau_0,\tau_1,\tau_2,\xi\right)=\frac{c_{ix}\mathrm{E}_{\pi}\left(N_{ix}\left(T_1\right)\right)+c_p+\left(c_c-c_p\right)\mathrm{P}_{\pi}\left(X_{T_1}>L\right)+c_u\mathrm{E}_{\pi}\left(D_u\left(T_1\right)\right)}{\mathrm{E}_{\pi}\left(T_1\right)}, \qquad (1.24)$$

where τ_0, τ_1, and τ_2 are inspection periodicities, which correspond to different durations of the system in a stressed state; ξ is the preventive replacement threshold; c_p and c_c are, respectively, the preventive and corrective replacement cost; $N_{ix}(T_1)$ is the number of scheduled inspections until time T_1; $D_u(T_1)$ is the system idle time in the interval $(0, T_1)$; $T_1$is the duration of the semi-renewal cycle ; E_{π} is the mathematical expectation symbol; P_{π} is the event probability symbol.

The study [82] presented a CBM structure for a gradually degrading single-unit system. The proposed decision-making model is used to determine the optimal inspection schedule and if necessary, the replacement times as well, to balance the system failure costs and the losses due to the idle time over an infinite time interval. The controlled variables are the replacement threshold and inspection schedule. Inspections are assumed to be perfect.

The study [83] considered CBM of a single-unit system subject to dependent failures due to deterioration and traumatic shock events. A periodic inspection (replacement) strategy is proposed based on the results of condition monitoring, which minimizes the following cost function:

$$C_{ins}(T,M)=\frac{C_p P_p(T,M)+C_c\left(1-P_p(T,M)\right)+C_d E\left[W_r\right]+C_i E\left[N_i\right]}{E\left[\tau_r\right]}, \qquad (1.25)$$

where $E[\tau_r]$, $P_p(T,M)$, $E[W_r]$, and $E[N_i]$ are, respectively, expected time to replace the system, the probability of preventive replacement during a regeneration cycle, the expected system idle time and the expected number of inspections during the regeneration cycle; T is the system inspection periodicity; M is the preventive replacement threshold.

The study [84] introduced the maintenance scheduling threshold for organizing the maintenance resources according to the system state. The maintenance scheduling threshold is used as a controlled variable in combination with the preventive maintenance threshold and failure threshold. The formula determines the expected maintenance cost for one regeneration cycle

$$CR\left(X_S,X_M\right)=C_0+\frac{\sum_{j=1}^{\infty}\left[P_1(j)C_1+P_2(j)C_2+P_3(j)C_3+E\left(T_{WS}(j)\right)C_{WS}+E\left(T_{WC}(j)\right)C_{WC}\right]}{\sum_{j=1}^{\infty}\left\{\left[P_1(j)+P_2(j)+P_3(j)\right]\left(t+T_L\right)+E\left(T_{WS}(j)\right)-E\left(T_{WC}(j)\right)\right\}}, \qquad (1.26)$$

where X_S is the maintenance scheduling threshold; X_M is the preventive maintenance threshold; j is the time index for the period from $(j-1)\Delta t$ to $j\Delta t$; $P_1(j)$, $P_2(j)$, and $P_3(j)$ are the probabilities that at time $j\Delta t$ the system state parameter is less than the preventive maintenance threshold, greater than the preventive maintenance threshold and less than the failure threshold, and greater than the failure threshold, respectively; C_1, C_2, and C_3 are cost values, which correspond to the three system states; T_{WS} is the delay in the supply of spare parts; T_{WC} is the operator delay; C_{WS} is the spare parts supplier delay's cost per unit time; C_{WC} is the cost of waiting for the operator per unit time; E is the mathematical expectation symbol; T_L is the lead time of the order.

The study [85] considered a redundant dual-unit configuration typically used in avionics systems. Simulation of the system operation is carried out by a Markov process with a finite number of states and continuous time. The study does not consider the replacement threshold and the probability of incorrect decisions made during a periodic inspection of the second unit.

The study [86] considers a CBM strategy with three possible actions: periodic inspection, preventive replacement, and corrective replacement, respectively. A preventive or corrective replacement renews the system state. It is assumed that the system's periodical inspections are perfect. The study considers two maintenance policies about the change point of the degradation process T_c: maintenance policy for $t < T_c$ and maintenance policy for $t \geq T_c$. Each policy has its preventive maintenance threshold, where $A_2 < A_1$. The adaptive maintenance optimization task comes down to finding values Δt, A_1 and A_2, which minimize the expected long-term costs $E[C(T)]$:

The study [86] considers a CBM strategy with three possible actions: periodic inspection, preventive replacement, and corrective replacement, respectively. A preventive or corrective repair renews the system state. It is assumed that the system's periodical inspections are perfect. The study considers two maintenance policies concerning the change point of the degradation process T_c: maintenance policy for $t < T_c$ and maintenance policy for $t \geq T_c$. Each policy has its preventive maintenance threshold, where $A_2 < A_1$. The adaptive maintenance optimization task comes down to finding values Δt, A_1 and A_2, which minimize the expected long-term costs $E[C(T)$:

$$E\left[C(T)\right]=C_I E\left[N_I(T)\right]+C_P P_P\left(\Delta t,A_1,A_2\right)+C_C P_C\left(\Delta t,A_1,A_2\right)+C_D E\left[W(T)\right], \quad (1.27)$$

where T is the regeneration cycle duration; C_I is the cost per inspection; $E[N_I(T)]$ is the expected number of inspections during the regeneration cycle; $P_P(\Delta t, A_1, A_2)$ is the probability that the regeneration cycle will end due to preventive maintenance; $P_C(\Delta t, A_1, A_2)$ is the probability that the regeneration cycle will end due to corrective maintenance; C_P and C_C are the preventive and corrective maintenance costs, respectively; $E[W(T)]$ is the average system idle time during the regeneration cycle; C_D is the system idle time cost per unit time.

The study [87] proposes a maintenance policy for degrading systems with state-dependent operating costs. The system is replaced when the level of its degradation exceeds the replacement threshold. By denoting the minimum total maintenance cost as $V(k, X_k)$ over an infinite time interval with the initial state (k, X_k), the optimality equation satisfies the Bellman equation, expressed as

$$V\left(k,X_k\right)=\begin{cases} c_c+V(0,0), \text{ if } X_k>l; \\ \min\left[c_p+V(0,0), e^{-r\tau}U\left(k,X_k\right)+e^{-r\tau}W\left(k,X_k\right)\right], \text{ if } X_k\leq l, \end{cases} \quad (1.28)$$

where τ is the inspection periodicity; X_k is the system state during the k-th inspection; c_p is the preventive maintenance cost; l is the system functional failure threshold; r is the discount factor; c_c is the corrective maintenance cost; $U(k, X_k)$ is the expected cost of transition to (k+1)-th inspection.

The study [88] presents a CBM optimization method for series-parallel systems aimed at minimizing the maintenance costs. The target function in this study represents a sum of average maintenance costs per unit time and system installation costs. The constraint in the optimization task is imposed on the availability. The theory of semi-regenerating processes and the universal generating function [89] are used to evaluate the target cost function and availability. The formulation of the optimization problem is as follows:

$$\begin{cases} \text{Target: } \min C_S = C_{IN} + C_M, \\ \text{Limit: } \mathrm{A}_S \geq A_0, \end{cases} \tag{1.29}$$

where C_S is the target cost function; C_{IN} is the system installation cost; C_M is the system maintenance cost; A_S is the system availability; A_0 is the required value of system availability. The controlled variables in (1.29) are the reliability structure of the system and periodicity of condition monitoring.

The tasks of joint optimization of the preventive maintenance threshold and schedule of perfect inspections when using CBM strategy were also considered in [90-101].

Let us now analyse CBM models with imperfect inspections. The study [102] examined a maintenance model with periodic inspections. Imperfect periodic inspections occur with a periodicity t within the selected time interval $[0, (n+1)t]$. When inspecting the system its failure is detected with the probability $p \in (0.1)$. After failure detection, a corrective repair of the system is performed. If no failure occurred over $[0, (n+1)t]$ or it was not detected, then the system is replaced with a new one at the time point $(n+1)t$. The goal is to determine the optimal frequency of imperfect inspections between preventive maintenance, which minimizes the cost of maintenance. The following formula determines the expected maintenance cost:

$$\mathrm{E}(C) = c_2\left[\sum_{i=0}^{n}\gamma_i\left(n - i + \sum_{j=0}^{i-1}(1-p)^j\right) + n\overline{F_X}\left((n+1)t\right)\right] + \sum_{i=0}^{n}\gamma_i \mathrm{E}\left[\int_0^{D_i} c_3(u)du\right] + \zeta\sum_{i=0}^{n}\gamma_i + c_1, \tag{1.30}$$

where c_1 is the cost of system repair; c_2 is the system inspection cost; ζ is the instantaneous cost of unrevealed failure; γ_i is the probability of unrevealed failure, which occurs after preventive maintenance when i inspections remain until the next scheduled preventive maintenance; D_i is the time until unrevealed failure detection when an unrevealed failure occurs between $(n-i)$ and $(n-i+1)$ inspections; $c_3(u)$ are the losses due to an unrevealed failure when the time duration in the unrevealed failure state is equal to u; $F_X(x)$ is the distribution function of the time to unrevealed failure.

The remarks concerning this paper are as follows:

1) Only two probabilities associated with imperfect inspections are considered, namely the conditional probability of true negative (p) and the conditional probability of missed detection $(1-p)$. The conditional probability of false alarm and the conditional probability of true positive are disregarded.

2) A preventive maintenance threshold for system inspection is disregarded.

3) The conditional probability of true negative (p) is constant at each check and does not depend on the system degradation process parameters.

It should be noted that the majority of maintenance models with imperfect inspections typically consider two types of errors: "false alarms" with probability α and "missed detection" with probability β and, accordingly, correct decisions with probabilities 1 – α and 1 – β. Examples of such maintenance models are presented in [103-107]. As an example, we will analyse the maintenance model proposed in [107]. A periodically inspected system can be in one of the following states: good, defective, and failed. The system in a defective state enters a wear state but still functions.

Failure of the system is detected immediately since the system ceases to function. However, the inspection only can identify the defective state. In the case the system failure has significant consequences, then it is desirable to replace the system if the defective state is detected during the inspection, rather than to allow operation until failure. The system becomes as good as new after replacement. The following formula determines the expected cost of system maintenance during one regeneration cycle:

$$\mathrm{E}\left[C(\tau)\right]=c_0\mathrm{E}(K)+c_pP(c_p)+c_f\left[1-P(c_p)\right], \qquad (1.31)$$

where c_0 and τ are, respectively, the cost and periodicity of inspection; c_f is the cost incurred due to failure; c_p is the cost of preventive maintenance; $\mathrm{E}(K)$ is the mathematical expectation of the number of inspections during one regeneration cycle; $P(c_p)$ is the probability that the regeneration cycle will end with preventive maintenance.

As in [102], the threshold of preventive maintenance is not considered when checking the system and the conditional probabilities of incorrect decisions α and β are constant and do not depend on the parameters of the system degradation process. Strictly speaking, the maintenance models proposed in [102-107] are not CBM models, since in reality the error probabilities when checking the system condition are not constant coefficients, but depend on the time and parameters of the degradation process. Moreover, as shown in [108, 109], such models also depend on the results of the previous checks. Therefore, in the further analysis, we will consider only those studies in which the observed process at the condition monitoring includes either a measurement error or the probabilities of correct and incorrect decisions that are functions of the system degradation model.

The study [110] considered a decision-making model for the case of periodic inspections of the system condition, which minimizes the expected average cost per year. After repair, the system becomes as good as new. The observed stochastic process $Y(t)$ includes the original process $X(t)$ and a normally distributed measurement error ε:

$$Y(t) = X(t) + \varepsilon,\ \varepsilon \to N(0, \sigma_\varepsilon), \tag{1.32}$$

The convolution determines the likelihood function of measurement y at a given system parameter degradation rate μ

$$l(y\,|\,\mu) = f_{Y(t)}(y) = \int_{-\infty}^{\infty} f_{X(t)}(\mathrm{y}-\varepsilon) f_{\varepsilon}(\varepsilon)\,d\varepsilon, \tag{1.33}$$

where $f_{X(t)}(y - \varepsilon) = Ga(y - \varepsilon|at, b)$ is the likelihood function of increment $X(t)$; a and b are the parameters of a stochastic process with a gamma distribution. The average cost per unit time is described by the relation of the expected cost per regeneration cycle to the expected regeneration cycle duration:

$$C(p, m, \Delta k) = \mathbf{E}(C_1)/\mathbf{E}(I), \tag{1.34}$$

where Δk is the interval between inspections. The expected cost per regeneration cycle is

$$\mathbf{E}(C_1)=\sum_{j=1}^{\infty}\left(jc_1+c_p\right)\Pr\left\{X\left((j-1)\Delta k\right)\le pm, pm<X\left(j\Delta k\right)\le m\right\}+ \\ \left[(j-1)c_1+c_F\right]\Pr\left\{X\left((j-1)\Delta k\right)\le pm, X\left(j\Delta k\right)>m\right\}, \tag{1.35}$$

where c_1, c_p, and c_F are the costs for inspection, preventive and corrective replacement due to a failure, respectively.

The study [111] considered a CBM model, in which the measurement result included the initial process of system degradation along with a normally distributed measurement error. Probabilities of incorrect decisions made during inspections are disregarded.

A CBM model, which utilizes a stochastic Wiener process $X(t)$ to model degradation with measurement errors ε, is studied in [112]:

$$Y(t)=X(t)+\varepsilon, \tag{1.36}$$

$$X(t)=\beta\Lambda(t)+\sigma\mathrm{B}\left[\Lambda(t)\right], \tag{1.37}$$

where $Y(t)$ is the measurement result of the system state parameter; β is the drift rate parameter; $\Lambda(t)$ is the drift function; σ is the volatility parameter; $\mathrm{B}(t)$ is the standard Brownian motion. Within this model, the distribution of the remaining useful life is calculated, which is used to make decisions about restoring or using the system.

A novel approach to the evaluation of the remaining useful lifetime of lithium-ion batteries is proposed in [113] based on the Wiener process with measurement error:

$$X(t)=\beta\Lambda(t)+\sigma\mathrm{B}\left[\Lambda(t)\right], \tag{1.38}$$

where λ is the drift parameter; σ_B is the diffusion parameter. The remaining notations have the same meaning as in (1.37). The inspection schedule is assumed to be known.

The studies [114-123] considered the degradation process models related to CBM for objects of different physical nature.

1.5. Statement of research objectives

The analysis of the current state of digital avionics systems maintenance shows that three maintenance levels (O, I and D) are practiced in civil and military aviation, which differ by the place of maintenance performing and depth of fault detection by the available diagnostic equipment. Besides, many potential maintenance options of avionics systems are possible that differ by the number of maintenance levels and the use of outsourcing. No

Fault Found events have a significant impact on the maintenance cost of removed LRUs/LRMs, with their main cause being intermittent failures that occur in flight. Conventional ATE cannot diagnose intermittent failures in removed LRUs/LRMs for various reasons. Therefore, specialized types of diagnostics equipment are used to detect the intermittent failures, which are capable of simulating flight conditions with simultaneous and continuous monitoring of all tested devices of the diagnosed object. The cost of such diagnostic equipment is high; therefore, before its use in multilevel maintenance systems, it is necessary to conduct a feasibility study.

The analysis of the mathematical models of digital avionics maintenance shows that currently no mathematical models exist to calculate MTBUR, availability, ORF and average maintenance costs of continuously monitored redundant avionics systems that differ by the number of maintenance levels (O-, I- and D-levels), and consider maintainability and fault-tolerance to permanent and intermittent failures. Moreover, no mathematical models exist to evaluate the applicability of IFDs at the D-level maintenance of avionics LRUs/LRMs.

The analysis of the latest international standards, instructions, and guidelines for vehicles' equipment maintenance shows that CBM is based on the condition monitoring, which determines the equipment health to provide maintenance only when it is needed. The goals of CBM are to minimize a heavy volume maintenance checks, increase aircraft availability, improve flight safety, and reduce maintenance costs. To achieve the set goals, it is necessary to address a number of tasks among which the most relevant are the tasks related to determining optimal time intervals at the end of which specific actions are required to maintain or replace equipment components subjected to wear and degradation, determination of the equipment component degradation rate and evaluation of the remaining useful life. CBM effectiveness largely depends on solving the problem of determining the optimal schedule of condition monitoring within the limits of the target reliability thresholds, which must ensure a minimum number of inspections, while maintaining the failure risk below the maximum allowable level.

The analysis of CBM mathematical models shows that a large number of research papers published in top-tier scientific journals and proceedings of prestigious scientific conferences are devoted to mathematical modelling of various CBM problems. The published studies pay particular attention to the determination of the optimal preventive replacement threshold, optimal inspection schedule, the trustworthiness of inspections, optimization criteria, as well as degradation process models. The majority of published CBM mathematical models consider perfect inspections, in which the system condition is determined accurately. The mathematical models of maintenance with imperfect inspections are based on the decision rule, aimed at rejecting only systems that are inoperable at the time of condition monitoring. The drawback of this decision rule is the impossibility of discarding the systems that may fail within the period before the next time point of inspection. Also, these mathematical models assume that the probabilities of incorrect decisions are constants and do not depend on the time and degradation process parameters, which does not reflect the real conditions. The reviewed studies do not include any statement of joint optimization tasks of the inspection schedule and preventive replacement threshold for each time of inspection. The publications pay little attention to the development of mathematical models and criteria for optimizing CBM of systems that affect safety.

Thus, the purpose of the thesis is to develop mathematical models of run-to-failure maintenance of digital avionics systems, considering permanent and intermittent failures, as well as the CBM models of vehicles' degrading equipment based on imperfect inspections.

From the conducted analysis it follows that to achieve this goal it is necessary to solve the following tasks:

- To develop mathematical models for evaluating the operational reliability of continuously monitored avionics LRUs/LRMs over finite and infinite time intervals that would consider the impact of both permanent and intermittent failures.
- To develop generalized relationships for calculation of average maintenance costs during warranty and post-warranty periods of redundant avionics systems operation for alternative maintenance options that differ by the number of maintenance levels (1, 2 or 3).
- To develop a new decision rule for system condition monitoring at discrete times and a corresponding mathematical model of CBM. As well as to formulate the tasks of determining the optimal replacement threshold for each inspection based on the cost and

probabilistic optimization criteria, that would significantly reduce the probability of system failure during the forthcoming interval of operation.

- To develop generalized mathematical expressions to calculate CBM performance indicators that consider the probabilities of correct and incorrect decisions made when checking the system suitability in the forthcoming interval of operation, as well as the criteria for joint optimization of inspection time-moments and replacement thresholds for systems that affect and do not affect safety.
- To develop techniques and programs that make it possible to use the proposed mathematical models in solving tasks of optimizing maintenance of vehicles' equipment.

1.6. Conclusions

1. The analysis of the current state of digital avionics maintenance allows us to conclude that:

- Modern avionics systems are redundant systems, each including several LRUs/LRMs that are continuously monitored in flight by BITE and have a modular design that provides easy access to PCBs for ground diagnostics and maintenance.
- Three maintenance levels (O, I and D) are practiced in civil and military aviation, differing in the place of maintenance performing and depth of fault detection in the dismantled LRUs / LRMs by available diagnostic equipment, what is more, the third and sometimes the second level of maintenance often outsourced to repair stations;
- NFF events have a significant impact on the maintenance cost of dismantled LRUs/LRMs, with their main cause being intermittent failures that occur in flight.
- Conventional ATE is incapable of diagnosing intermittent failures in dismantled LRUs/LRMs for a variety of reasons, therefore specialized diagnostic equipment is developed that is capable of simulating flight conditions with simultaneous and continuous monitoring of all tested devices of the diagnosed object to detect intermittent failures with the duration as short as 50 ns.
- The cost of specialized equipment for diagnosing intermittent failures is quite high, so they have not found broad application in practice because of the lack of economic justification for the expediency of their use.

2. Based on the analysis of mathematical maintenance models of digital avionics systems the following is shown:

- No mathematical models exist to calculate the MBTUR, availability and average operating costs over finite and infinite intervals of operation of continuously monitored redundant avionics systems for alternative maintenance options that differ by the number of maintenance levels (O-, I- and D-levels), which would consider the characteristics of maintainability and failure-free operation to permanent and intermittent failures.

- No mathematical models exist to evaluate the application expediency of intermittent failure detectors at D-level maintenance of avionics LRUs/LRMs.

3. The following may be concluded based on the analysis of the latest international standards, instructions, and guidelines for the CBM of vehicles' equipment:

- The goals of the CBM are to minimize a heavy volume of maintenance checks, increase aircraft availability, improve flight safety and reduce maintenance cost.

- To achieve the set goals, a number of tasks must be addressed, among which the most relevant are the tasks of determining the optimal time intervals after which specific actions are required to maintain or replace equipment components that are subject to wear and degradation, determination of the equipment component degradation rate and evaluation of the remaining useful life.

- CBM effectiveness largely depends on solving the problem of determining the optimal schedule of condition monitoring within the limits of the target reliability thresholds, which must ensure a minimum number of inspections, while maintaining the failure risk below the maximum allowable level.

4. The analysis of CBM mathematical models shows that:

- A significant number of publications are devoted to various problems of CBM mathematical modelling, and particular attention is paid to the determination of the optimal preventive replacement threshold, optimal inspection schedule, inspection trustworthiness (perfect, imperfect), optimization criteria as well as degradation process models.

- The majority of CBM mathematical models consider perfect inspections, in which the system condition is assumed to be determined error-free.

- The CBM mathematical models with imperfect inspections incorporate a decision rule that is incapable of rejecting at least part of the systems that may fail before the next inspection time. Also, these mathematical models assume that the probabilities of incorrect decisions are constants and do not depend on the time and parameters of the degradation process, which does not reflect the real conditions.
- No mathematical models of CBM with imperfect inspections and an arbitrary degradation process exist that consider the probabilities of correct and incorrect decisions as functions of time and parameters of the degradation process.
- There are no statements of tasks for joint optimization of inspection schedule and preventive replacement thresholds, as well as CBM mathematical models and optimization criteria for systems that affect safety.

REFERENCES

1. Air carriers' outsourcing of aircraft maintenance. FAA Report Number: AV-2008-090, Sept. 30, 2008.
2. Review of air carriers' use of aircraft repair stations. FAA Report Number: AV-2003-047, July 8, 2003.
3. Hurst, N. Congress wary of outsourcing aircraft maintenance. - Available: http://www.rollcall.com/news/congress_wary_of_outsourcing_aircraft_maintenance-222830-1.html – 4.03.2013.
4. No fault found: The search for the root cause: monograph/ I. K. Jennions, S. Khan, C. Hockley, P. Phillips. – London: SAE International, 2015. – 202 p.
5. ARINC Report 672. Guidelines for the reduction of no fault found (NFF). – 2008.
6. NFF – no fault found. American Institute of Aeronautics and Astronautics. – Available:https://info.aiaa.org/tac/AASG/PSTC/Lists/Calendar/Attachments/1/NFF%20Subcommittee.pdf - 22.04.2013.
7. Hockley, C. A research study of no fault found (NFF) in the Royal Air Force/ C. Hockley, L. Laceya//Procedia CIRP. – 2017. - V. 59. – P. 263-267.
8. Hockley, C. J. Report for air command on the impact of no fault found (NFF) on a selection of RAF aircraft fleets/C. J. Hockley, L. Lacey, J. G. Pelham//Technical report

TES 02-01-2015. – 2015. - EPSRC through-life engineering services centre, Cranfield University, UK.

9. Anderson, K. Intermittent fault detection and isolation reduces NFF and enables cost effective readiness/K. Anderson//USC. – Available at: http://www.ncms.org/wp-content/uploads/3-Steadman-CTMA-Briefing-2017. pdf – 4.04.2017.

10. Ilarslan, M. Mitigating the impact of false alarms and no fault found events in military systems/M. Ilarslan, L. Y. Ungar//IEEE Instrumentation & measurement magazine. – 2016. – V. 19, No. 4. – P. 16-22.

11. Burchell, B. No fault found/B. Burchell//Aviation week. – Available at: http://aviationweek.com/awin/no-fault-found - 1.02.2007.

12. No fault found events in maintenance engineering Part 1: Current trends, implications and organizational practices/S. Khan, P. Phillips, I. Jennions, C. Hockley//Reliability engineering and system safety. – 2014. – V. 123. – P. 183-195.

13. Thomas, D. A. The trouble not-identified in automotive electronics/ D. A. Thomas, K. Ayers, M. Pecht// Microelectron. reliab. – 2002. – V. 42. – P. 641–651.

14. No fault found. Aerospace survey results//Copernicus technology Ltd. - Available:https://connect.innovateuk.org/documents/3329929/3676148/Aerospace+Survey+on+No+Fault+Found+Impacts+-+Copernicus+Technologies+Ltd.pdf/a20b7a42-9430-410c-a6f9 edc022f4388b – 1.03.2012.

15. Qi, H. No-fault-found and intermittent failures in electronic products/H. Qi, S. Ganesan, M. Pecht//Microelectronics reliability. – 2008. – V. 48. – P. 663–674.

16. Steadman, B. Intermittent fault detection and isolation system/B. Steadman, F. Berghout, N. Olsen//2008 IEEE AUTOTESCON, 8-11 September, 2008, Salt Lake City, UT, USA. – AUTOTESCON proceedings, 2008. – P. 37-40.

17. Performance specification. Electronic test equipment, intermittent fault detection and isolation for chassis and backplane conductive paths: MIL-PRF-32516. – [Put into operation 23.03.2015]. - Washington: USA, 2015. – 26 p. - (Military standard of USA).

18. Anderson, K. Intermittent fault detection & isolation system (IFDIS)/ K. Anderson//Aerospace & defense. – Available at: https://contest.techbriefs.com/2014/entries/aerospace-and-defense/4014 - 5.03.2014.

19. Voyager intermittent fault detector - VIFD/Universal Synaptics Corporation. – Available at: http://www.usynaptics.com/index.php/products/ncompass voyager - 1.1.2017

20. Intermittent fault detection & isolation system - IFDS/Universal Synaptics Corporation. – Available at: http://www.usynaptics.com/ index.php/products/ ifdis - 01.01.2017.

21. Уланский, В. В. Организация системы технического обслуживания и ремонта радиоэлектронного комплекса Ту-204: учебное пособие/В. В. Уланский, Г. Ф. Конахович, И. А. Мачалин. – К.: КИИГА, 1992. – 110 с.

22. Надійність техніки. Терміни та визначення: ДСТУ 2860-94. - [Введ. в дію 01.01.1996]. – К.: Держстандарт України, 1994. – 26 с. – (Національний стандарт України).

23. Military handbook. Electronic reliability design handbook: MIL-HDBK-338B. – [Put into operation 01.10.1998]. – Washington: USA, 1998. – 1046 p. – (Military standard of USA).

24. Rausand, M. System reliability theory. Models and statistical methods: a monograph/M. Rausand, A. Hsyland. – New York: John Wiley & Sons, 2009. – 536 p.

25. Nakagawa, T. Maintenance theory of reliability: monograph/T. Nakagawa. – London: Springer, 2005. – 274 p.

26. Уланский, В. В. Оценка апостериорной надежности дискретно контролируемых технических систем/В. В. Уланский//Проблемы повышения эффективности эксплуатации авиационного и радиоэлектронного оборудования воздушных судов гражданской авиации: Сб. науч. тр. – К.: КИИГА, 1987. – С. 19-31.

27. Уланский, В. В. Оценка эксплуатационной надежности периодически контролируемой одноблочной системы авионики при наличии явных и скрытых отказов/В. В. Уланский, И. А. Мачалин//Авиационно-космическая техника и технология. – 2007. - № 6(42). – С. 87-93.

28. IATA. Guidance material and best practices for inventory management. – Montreal, 2015. – 155 p.

29. Galisanskis, A. Statistical evaluation and analysis of mutual interactions of components/A. Galisanskis, V. Giniotis//Aviation. – 2006. V. X, No. 3. – P. 9-15.

30. Уланский, В. В. Диагностическое обеспечение эксплуатации радиоэлектронных систем воздушных судов: дис. докт. техн. наук: 05.22.14/ В. В. Уланский. — К.: КИИГА. — 1989. — 351 с.

31. Ulansky, V. V. The choice of optimum variant of warranty maintenance management of modern avionics products/V. V. Ulansky, I. A. Machalin//Матеріали VI Міжнар. НТК “Авіа–2004”. – К.: НАУ, 2004. – Т. 2. – С. 22.31–22.34.

32. Ulansky, V. V. Optimization of post-warranty maintenance of avionics systems/V. V. Ulansky, I. A. Machalin//International conference on aeronautical science and air transportation (ICASAT2007), 23-25 April, 2007, Tripoli, Libya. — Conference proceedings, 2007. — P. 619-628.

33. Уланский, В. В. Уточненная модель обслуживания одноблочной системы авионики/В. В. Уланский, И. А. Мачалин//Электронное моделирование. – 2008. – Т. 30, № 2. – С. 55–67.

34. Maintenance strategy decision for avionics system based on cognitive uncertainty information processing/J. Tu, C. Sun, X. Zhang *et al.*//Maintenance and reliability. - 2015. - V. 17, 2. - P. 297–305.

35. Feldman, K. A methodology for determining the return on investment associated with prognostics and health management/K. Feldman, T. Jazouli, P. Sandborn//IEEE Transactions on reliability. - 2009. - V. 58, No. 2. - P. 305-316.

36. Scanff, E. Life cycle cost impact using prognostic health management (PHM) for helicopter avionics/E. Scanff, K. L. Feldman, S. Ghelam *et al.*//Microelectronics reliability. – 2007. - V. 47. - P. 1857-1864.

37. Modeling aircraft operational reliability/ K. Tiassou, K. Kanoun, M. Ka *et al.*//Lecture notes in computer science: coll. monogr. – Berlin: Springer, 2011. – V. 6894. - P. 157-170.

38. Dhillon, B. S. Maintainability, maintenance, and reliability for engineers: monograph/B. S. Dhillon. – London: CRC Press, 2006. – 278 p.

39. Jun, L. Reliability analysis of aircraft equipment based on FMECA method/L. Jun, X. Huibin//Physics procedia. – 2012. – V. 25. – P. 1816–1822.

40. Mason, R. L. Stealth fighter avionics: 2LM versus 3LM/R. L. Mason//Air force journal of logistics. – 1998. – V. 22, No. 3. – P. 31-34.

41. Wang, Y. Manpower management benefits predictor method for aircraft two-level maintenance concept/Y. Wang, B. Song//Modern applied science. - 2008. - V. 2, No. 4. - P. 33-37.

42. Ulansky, V. Availability modelling of a digital electronic system with intermittent failures and continuous testing/V. Ulansky, I. Terentyeva//Engineering letters. - 2017. - V. 25, No. 2. - P. 104-111.

43. Уланский, В. В. Обобщенные функции стоимости обслуживания до безопасного отказа легкозаменяемых блоков систем авионики/ В. В. Уланский, И. А. Мачалин //Електроніка та системи управління. – 2008. – № 1 (15). – С. 86–97.

44. Vega, D. J. G. Assessing the influence of the scale of operations on maintenance costs in the airline industry/D. J. G. Vega, D. A. Pamplona, A. V. M. Oliveira//Journal of transport literature. – 2016. – V. 10, No. 3. – P. 10-14.

45. Ilarslan, M. An economic analysis of false alarms and no fault found events in air vehicles/M. Ilarslan, L. Y. Ungar, K. Ilarslan//2016 IEEE AUTOTESTCON, 12-15 September, 2016, Anaheim, CA, USA. - AUTOTESTCON proceedings, 2016. - P. 1-7.

46. Saltoglu, R. Aircraft scheduled airframe maintenance and downtime integrated cost model/R. Saltoglu, N. Humaira, G. Inalhan//Advances in operations research. – 2016. - V. 2016. – P. 1-12.

47. A framework to estimate the cost of no-fault-found events/J. A. Erkoyuncu, S. Khan, S. M. F. Hussain, R. Roy//International journal of production economics. - 2016. - V. 173. - P. 207-222.

48. Nakagawa, T. Maintenance theory of reliability: monograph/ T. Nakagawa. - London: Springer, 2005. – 269 p.

49. Nakagawa, T. Advanced reliability models and maintenance policies: monograph/T. Nakagawa. - London: Springer, 2008. – 234 p.

50. Hsu, Y. T. Novel model of intermittent faults for reliability and safety measures in long-life computer systems/Y. T. Hsu, C. F. Hsu//International journal of electronics. – 1991. – V. 71, No. 6. – P. 917-937.

51. Optimal periodic testing of intermittent faults in embedded pipeline processor applications/N. Kranitis, A. Merentitis, N. Laoutaris *et al.*//Design, automation and test in Europe conference, 6-10 March, 2006, Munich, Germany. – Conference proceedings, 2006. - P. 65-70.

52. Prasad, V. B. Digital systems with intermittent faults and Markovian models/ V. B. Prasad//1992 IEEE 35th Midwest symposium on circuits and systems, 9-12 August, 1992, Washington, USA. – Symposium proceedings, 1992. - V. 1. – P. 195-198.

53. Cai, B. A dynamic-Bayesian-network-based fault diagnosis methodology considering transient and intermittent faults/B. Cai, Y. Liu, M. Xie//IEEE Transactions on automation science and engineering. - 2017. - V. 14, No. 1. - P. 276-285.

54. Rashid, L. Intermittent hardware errors recovery: modelling and evaluation/L. Rashid, K. Pattabiraman, S. Gopalakrishnan//2012 Ninth international conference on quantitative evaluation of systems (QEST), 17-20 September, 2012, London, UK. - QEST proceedings, 2012. - P. 1-10.

55. Zhao, X. Optimal time and random inspection policies for computer systems/X. Zhao, M. Chen, T. Nakagawa//Applied mathematics & information sciences. - 2014. - V. 8, No. 1L. - P. 413-417.

56. Aeronautical design standard handbook. Condition based maintenance system for US army aircraft: ADS-79D-HDBK. – [Put into operation 07.03.2013]. – Huntsville: USA, 2013. – 284 p. – (Military standard of USA).

57. Blan, D. R. Maximizing the P-F interval through condition-based maintenance/D. R. Blan//Maintworld. Maintenance & asset management. – Available at: http://www.maintworld.com/Applications/Maximizing-the-P-F-Interval-Through-Condition-Based-Maintenance - 07.10.2013.

58. Moubray, J. Reliability-centered maintenance: monograph/J. Moubray. – New York: Industrial press, inc., 1997. – 448 p.

59. On-condition maintenance using P-F interval or failure detection threshold (FDT)/Reliability hot wire. – Available at: http://www.weibull.com/hotwire/issue76/hottopics76.htm - 1.06.2007.

60. GE, Boeing implement condition-based maintenance standard/ Reliable plant. – Available at: http://www.reliableplant.com/Read/18448/ge%2C-boeing-implement-condition-based-maintenance-stard – 29.06.2010.

61. Open system architecture for condition-based maintenance/MIMOSA OSA-CBM. – Available at: http://www.mimosa.org/mimosa-osa-cbm - 29.06.2010.

62. Condition monitoring and diagnostics of machines — General guidelines: ISO 17359. – [Put into operation 15.04.2011]. – Geneva: Switzerland, 2011. – 26 p.

63. Condition monitoring and diagnostics of machines — Data interpretation and diagnostics techniques — Part 1: General guidelines: ISO 13379-1. – [Put into operation 01.05.2012]. - Geneva: Switzerland, 2012. – 33 p.

64. Cetin, E. N. FMECA applications and lessons learnt/E. N. Çetin//2015 Annual reliability and maintainability symposium (RAMS), 26-29 January, 2015, Palm Harbor, FL, USA. – RAMS proceedings, 2015. – P. 1-5.

65. Improved multi-faults diagnosis for CNC machine tools/B. Sheng, C. Deng, Y. Wang, S. Xie//2016 12th IEEE/ASME International conference on mechatronic and embedded systems and applications (MESA), 29-31 August, 2016, Auckland, New Zealand. – MESA proceedings, 2016. – P. 1-6.

66. Failure mode and effect analysis under uncertainty: an integrated multiple criteria decision-making approach/H. C. Liu, J. X. You, P. Li, Q. Su//IEEE Transactions on reliability. – 2016. – V. 65, No. 3. – P. 1380-1392.

67. Stamatis, D. H. Failure mode and effect analysis: FMEA from theory to execution: monograph/D. H. Stamatis. - New York: ASQC Press, 2003. – 300 p.

68. Operator/manufacturer scheduled maintenance development. Fixed wing aircraft – V. 1/Air transport association of America: MSG 3, V. 1, version 2015.1. – [Put into operation 01.01.2015] – New York: USA, 2015. – 128 p.

69. Vogl, G. W. Standards for prognostics and health management (PHM) techniques within manufacturing operations/G. W. Vogl, B. A. Weiss, M. A. Donmez//2014 Annual conference of the prognostics and health management society, 29 September-02 October, 2014, Fort Worth, TX, USA. - Conference proceedings, 2014. - P. 576-588.

70. Maintenance review boards, maintenance type boards, and OEM/TCH recommended/Advisory circular 121-22C. U.S. Department of transportation. Federal aviation administration. – [Put into operation 27.08.2012]. – USA, 2012. – 81 p.

71. Condition based maintenance plus (CBM+) for materiel maintenance/DoD Instruction (DoDI) 4151.22. - [Put in operation 16.10.2012]. – USA Department of defense, 2012. – 8 p.

72. Design & run-time information exchange for health-ready components/ SAE standard JA6268. [Put in operation 04.02.2018] - SAE International, 2018. - Available: https://doi.org/10.4271/JA6268_201804.

73. Condition-based maintenance using the inverse Gaussian degradation model/N. Chen, Z. S. Ye, Y. Xiang, L. Zhang//European journal of operational research. - 2015. - V. 243, No. 1. - P. 190-199.

74. Abdel-Hameed, M. Inspection and maintenance policies of devices subject to deterioration/M. Abdel-Hameed//Advances in applied probability. – 1987. – V. 19. – P. 917-931.

75. Abdel-Hameed, M. Correction to: "Inspection and maintenance policies of devices subject to deterioration"/M. Abdel-Hameed//Advances in applied probability. – 1995. – V. 27. – P. 584.

76. Jia, X. A prototype cost model of functional check decisions in reliability-centered maintenance/X. Jia, A. H. Christer//Journal of the operational research society. – 2002. – V. 53, No. 12. – P. 1380-1384.

77. Grall, A. A condition-based maintenance policy for stochastically deteriorating systems/A. Grall, C. Berenguer, L. Dieulle//Reliability engineering and system safety. – 2002. – V. 76. – P. 167-180.

78. Sequential condition-based maintenance scheduling for a deteriorating system/L. Dieulle, C. Bérenguer, A. Grall, M. Roussignol//European journal of operational research. – 2003. – V. 150, No. 2. – P. 451–461.

79. Ferreira, R. A multi-criteria decision model to determine inspection intervals of condition monitoring based on delay time analysis/R. Ferreira, A. de Almeida, C. Cavalcante//Reliab. engineering and system safety. – 2009. – V. 94, No. 5. – P. 905-912.

80. Golmakani, H. Age-based inspection scheme for condition-based maintenance/H. Golmakani, F. Fattahipour//Journal of quality in maintenance engineering. – 2011. – V. 17, No. 1. – P. 93-110.

81. Deloux, E. An adaptive condition-based maintenance policy with environmental factors/E. Deloux, B. Castanier, C. Bérenguer//Risk and decision analysis in maintenance optimization and flood management: coll. monogr. – Amsterdam: IOS Press, 2009. – P. 137-148.

82. Continuous-time predictive-maintenance scheduling for a deteriorating system/A. Grall, L. Dieulle, C. Bérenguer, M. Roussignol//IEEE Transactions on reliability. - 2002. - V. 51, No. 2. - P. 141-150.

83. A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events/K. T. Huynh, A. Barros, C. Bérenguer, I. Castro//Reliability engineering and system safety. - 2011. - V. 96, No. 4. - P. 497-508.

84. Condition-based maintenance with scheduling threshold and maintenance threshold/H. K. Wang, H. Z. Huang, Y. F. Li, Y. J. Yang//IEEE Transactions on reliability. - 2016. - V. 65, No. 2. - P. 513-524.

85. Catelani, M. Condition-based maintenance and Markov modelling for avionics devices/M. Catelani, L. Ciani, M. Venzi//2017 IEEE International workshop on metrology for aerospace (MetroAeroSpace), 21-23 June 2017, Padua, Italy. - MetroAeroSpace proceedings, 2017. - P. 1-5.

86. Guo, C. Maintenance optimization for systems with non-stationary degradation and random shocks/C. Guo, Y. Bai, Y. Jia//9th IMA International conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. - Conference proceedings, 2016. - P. 77-83.

87. Liu, B. Condition-based maintenance for degrading systems with state-dependent operating cost/B. Liu, M. Xie, W. Kuo//9th IMA International conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. - Conference proceedings, 2016. - P. 121-126.

88. Inspection and maintenance optimization of a multi-state series-parallel system/M. A. Yin, Z. L. Sun, J. Wang, Y. Guo//9th IMA International conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. - Conference proceedings, 2016. - P. 235-241.

89. Levitin, G. The universal generating function in reliability analysis and optimization: monograph/G. Levitin. - 2005. - London: Springer. – 442 p.

90. Safety constraints applied to an adaptive Bayesian condition-based maintenance optimization model/R. Flage, D. W. Coit, J. T. Luxhoj, T. Aven//Reliability engineering & system safety. - 2012. - V. 102. - P. 16-26.

91. Fouladirad, M. On the use of on-line detection for maintenance of gradually deteriorating systems/M. Fouladirad, A. Grall, L. Dieulle//Reliability engineering & system safety. - 2008. - V. 93, No. 12. - P. 1814-1820.

92. Fouladirad, M. On-line change detection and condition-based maintenance for systems with unknown deterioration parameters/M. Fouladirad, A. Grall//IMA journal of management mathematics. - 2012. - V. 25. - P. 139-155.

93. Wang, W. A model to determine the optimal critical level and the monitoring intervals in condition-based maintenance/W. Wang//International journal of production research. - 2000. - V. 38, No. 6. - P. 1425-1436.

94. Fouladirad, M. Monitoring and condition-based maintenance with abrupt change in a system's deterioration rate/M. Fouladirad, A. Grall//International journal of systems science. - 2015. - V. 46, No. 12. - P. 2183-2194.

95. van der Weide, J. Stochastic analysis of shock process and modeling of condition-based maintenance/J. van der Weide, M. D. Pandey//Reliability engineering & system safety. - 2011. - V. 96, No. 6. - P. 619-626.

96. Amari, S. V. Optimal design of a condition-based maintenance model/S. V. Amari, L. McLaughlin//Annual symposium on reliability and maintainability (RAMS), 26-29 January, 2004, Los Angeles, CA, USA. – RAMS proceedings, 2004. - P. 528-533.

97. Li, W. An inspection-maintenance model for systems with multiple competing processes/W. Li, H. Pham//IEEE Transactions on reliability. - 2005. - V. 54, No. 2. - P. 318-327.

98. Condition-based inspection/replacement policies for non-monotone deteriorating systems with environmental covariates/X. Zhao, M. Fouladirad, C. Bérenguer, L. Bordes//Reliability engineering & system safety. – 2010. – V. 95, No. 8. – P. 921-934.

99. Optimal maintenance policy and residual life estimation for a slowly degrading system subject to condition monitoring/D. Tang, V. Makis, L. Jafari, J. Yu// Reliability engineering & system safety. – 2015. – V. 134. – P. 198-207.

100. Golmakani, H. R. Optimal replacement policy and inspection interval for condition-based maintenance/H. R. Golmakani, F. Fattahipour//International journal of production research. - 2011. - V. 49, No. 17. - P. 5153-5167.

101. Golmakani, H. R. Condition-based inspection scheme for condition-based maintenance/H. R. Golmakani//International journal of production research. - 2012. - V. 50, No. 14. - P. 3920-3935.

102. He, K. Scheduling preventive maintenance as a function of an imperfect inspection interval/K. He, L. M. Maillart, O. A. Prokopyev//IEEE Transactions on reliability. - 2015. - V. 64, No. 3. - P. 983-997.

103. Berrade, M. Maintenance scheduling of a protection system subject to imperfect inspection and replacement/M. Berrade, A. Cavalcante, P. Scarf//European journal of operational research. – 2012. – V. 218. – P. 716-725.

104. Zequeira, R. I. Optimal scheduling of non-perfect inspections/R. I. Zequeira, C. Bérenguer//IMA J. of management mathematics. - 2006. - V. 17, No. 2. - P. 187-207.

105. Badıa, F. Optimal inspection and preventive maintenance of units with revealed and unrevealed failures/F. Badıa, M. D. Berrade, C. A. Campos//Reliability engineering and system safety. – 2002. – V. 78. – P. 157-163.

106. Lam, Y. An inspection-repair-replacement model for a deteriorating system with unobservable state/Y. Lam//Journal of applied probability. – 2003. – V. 40, No. 4. – P. 1031–1042.

107. Imperfect inspection and replacement of a system with a defective state. A cost and reliability analysis/M. D. Berrade, P. A. Scarf, C. A. V. Cavalcante, R. A. Dwight//Reliability engineering and system safety. – 2013. – V. 120. – P. 80–87.

108. Уланский, В. В. Оптимальные стратегии обслуживания электронных систем на основе диагностирования/В. В. Уланский//Вопросы технической диагностики. - 1987. - С. 137-143.

109. Уланский, В. В. Достоверность многоразового контроля работоспособности невосстанавливаемых электронных систем/ В. В. Уланский//Ресурсосберегающие технологии обслуживания радиоэлектронного оборудования воздушных судов гражданской авиации. - 1992. - С. 14-25.

110. Kallen, M. Optimal maintenance decisions under imperfect inspection/ M. Kallen, J. Noortwijk//Reliability engineering and system safety. – 2005. –V. 90, No. (2-3). – P. 177-185.

111. Newby, M. Optimal inspection policies in the presence of covariates/ M. Newby, R. Dagg//European safety and reliability conference (ESREL'02), 19-21 March, 2002, Lyon, France. – Conference proceedings, 2002. – P. 131-138.

112. Ye, Z. A Bayesian approach to condition monitoring with imperfect inspections/Z. Ye, N. Chen, K. L. Tsui//Quality and reliability engineering international. – 2015. – V. 31, No. 3. – P. 513–522.

113. Remaining useful life prediction of lithium-ion batteries based on the Wiener process with measurement error/S. Tang, C. Yu, X. Wang *et al.*//Energies. – 2014. – V. 7, No. 2. – P. 520–547.

114. Whitmore, G. A. Estimating degradation by a wiener diffusion process subject to measurement error/G. A. Whitmore//Lifetime data analysis. – 1995. - V. 1(3). - P. 307-319.

115. Prognosis of structural damage growth via integration of physical model prediction and Bayesian estimation/Y. Liu, Q. Shuai, S. Zhou, J. Tang//IEEE Transactions on reliability. - 2017. - V. PP, No. 99. - P. 1-12.

116. Wen, Y. Multiple-phase modeling of degradation signal for condition monitoring and remaining useful life prediction/Y. Wen, J. Wu, Y. Yuan// IEEE Transactions on reliability. - 2017. - V. 66, No. 3. - P. 924-938.

117. Remaining useful life prediction based on a general expression of stochastic process models/N. Li, Y. Lei, L. Guo *et al.*//IEEE Transactions on industrial electronics. - 2017. - V. 64, No. 17. - P. 5709-5718.

118. Liu, Y. Dynamic reliability assessment for nonrepairable multistate systems by aggregating multilevel imperfect inspection data/Y. Liu, C. J. Chen//IEEE Transactions on reliability. - 2017. - V. 66, No. 2. - P. 281-297.

119. Zhai, Q. RUL prediction of deteriorating products using an adaptive wiener process model/Q. Zhai, Z. S. Ye//IEEE Transactions on industrial informatics. - 2017. - V. PP, No. 99. - P. 1-10.

120. Kwon, D. Remaining-life prediction of solder joints using RF impedance analysis and Gaussian process regression/D. Kwon, M. H. Azarian, M. Pecht//IEEE Trans. on components, packaging, and manuf. technology. - 2015. - V. 5, No. 11. - P. 1602-1609.

121. Sikorska, J. Prognostic modelling options for remaining useful life estimation by industry/J. Sikorska, M. Hodkiewicz, L. Ma//Mechanical systems and signal processing. – 2011. - V. 25, No. 5. - P. 1803-1836.

122. Zhai, Q. Robust degradation analysis with non-Gaussian measurement errors/Q. Zhai, Z. S. Ye//IEEE Trans. on instrumentation and measurement. - 2017. – No. 99. - P. 1-10.

123. Optimal two-variable accelerated degradation test plan for Gamma degradation processes/T. R. Tsai, W. Y. Sung, Y. L. Lio *et al.*//IEEE transactions on reliability. - 2016. - V. 65, No. 1. - P. 459-468.

CHAPTER 2:
MAINTENANCE MODELS OF AVIONICS SYSTEMS CONSIDERING PERMANENT AND INTERMITTENT FAILURES

2.1. Statement of tasks

As shown in Chapter 1, modern avionics systems are redundant and comprise a set of easily replaceable LRUs/LRMs. Usually, avionics systems use permanent redundancy in combination with the active mode of operation. Each LRU/LRM includes several SRUs and has BITE, which provides continuous in-flight testing of the LRU/LRM. The modular design of avionics systems provides easy access to PCBs and components for maintenance and diagnostics. Each LRU/LRM operates until the event of a safe failure (permanent or intermittent), which is registered in flight or after landing.

The analysis of maintenance mathematical models of digital avionics provided in Chapter 1 showed that no mathematical models exist to calculate the operational reliability indicators of redundant avionics systems, which consider the alternating mode of avionics systems operation, periodic nature of pre-flight maintenance, continuous character of in-flight monitoring and possibility of occurrence of both permanent and intermittent failures in flight. Therefore, subsections 2.2-2.4 describe mathematical models of LRU/LRM and redundant avionics system operation over finite and infinite operating intervals, considering all of the factors as mentioned above.

The analysis conducted in Chapter 1 shows that three maintenance levels (O, I and D) are practiced in civil and military aviation, differing in the place of maintenance performing and depth of fault detection in the dismantled LRUs/LRMs. Therefore, many potential avionics system maintenance options are possible, which differ in the number of maintenance levels and the use of outsourcing. It should also be noted that NFF events have a significant impact on the cost of maintenance of dismantled LRUs/LRMs. As noted in Chapter 1, intermittent failures are the leading cause of NFF occurrence in avionics systems. Furthermore, conventional ATE designed to detect functional failures, faulty components, short circuits and breaks in electrical circuits, is not capable of intermittent fault diagnostics. Therefore, specific intermittent failure diagnostics equipment such as VIFD and IFDS, is being developed, capable of simulating flight conditions, with simultaneous and continuous monitoring of all tested devices of the diagnosed object, to detect intermittent failures with the duration as short as 50 ns. However, the specialized intermittent failure diagnostics equipment is not widely used because of the high price and the lack of economic justification of its application. The analysis conducted in Chapter 1 shows that no mathematical models exist to calculate the average operating costs over finite and infinite intervals of operation of continuously monitored redundant avionics systems that differ by the number of maintenance levels (O, I and D levels), which consider the maintainability characteristics and the possibility of diagnosing both permanent and intermittent failures. Therefore, in subsections 2.5-2.7, mathematical models are developed to calculate the average operating costs considering all the above factors.

Both probabilistic and cost-related indicators we will use to assess maintenance effectiveness. The availability and ORF we further use as probabilistic maintenance effectiveness indicators. MTBUR is an essential indicator of the operational reliability of avionics LRUs/LRMs. A high rate of intermittent failures significantly reduces MTBUR. As indicated in [2], MTBUR of avionics systems is about 50 % of the MTBF, which results in a 40–50 % increase in direct operating costs [3]. Further, we use total expected maintenance costs for a given time interval as a generalized cost indicator. Maintenance effectiveness indicators will be determined individually for the warranty and post-warranty period of avionics system operation.

2.2. A mathematical model of LRU/LRM operation and maintenance over a finite time interval

2.2.1. Possible LRU/LRM states during operation and maintenance. When developing the mathematical model of LRU/LRM operation and maintenance, we assume that the operation interval is finite. Let us denote this interval as T. It may be associated, for example, with the warranty maintenance period duration. The BITE continuously monitors the LRU/LRM condition in a flight of duration τ, where τ is the average time between aircraft landings at the base airport. In case of permanent failure, the LRU/LRM is disabled, or the on-board computer ignores information from this unit. In case of intermittent failure, the LRU/LRM is usually not disabled during flight. However, the on-board computer records information about intermittent failures that occurred during the flight.

The procedure for repair operations is as follows: if a permanent or intermittent failure occurs in flight, then the LRU/LRM is dismantled after landing at the base airport and sent for repair. After any repair, the LRU/LRM is assumed to be as good as new. We describe the LRU/LRM behaviour in the interval (0, T) by a finite-state stochastic process $S(t)$. The process $S(t)$ changes with jumps. Each jump of $S(t)$ is due to the LRU/LRM transition to one of the possible states. We assume that $S(t)$ is a regenerative stochastic process and permanent failure of LRU/LRM occurs at time ξ, where $k\tau < \xi \leq (k+1)\tau$. Then, at an arbitrary time t, the LRU/LRM can be in one of the following states [4, 5]: S_1, if at time t the LRU/LRM is operable; S_2, if at time t the LRU/LRM is inoperable due to the detected permanent failure that occurs in flight; S_3, if at time t the LRU/LRM is in O-level maintenance (dismantling or mounting the LRU/LRM on aircraft board); S_4, if at time t the LRU/LRM is in the state of waiting for a spare LRU/LRM from the airline warehouse; S_5, if at time t the LRU/LRM is shipped to repair or from repair; S_6, if at time t the LRU/LRM is repaired due to an intermittent failure; S_7, if at time t the LRU/LRM is repaired due to a permanent failure.

If BITE has rejected the LRU/LRM, then it must be replaced with operable LRU/LRM from the warehouse. The LRU/LRM replacement time should be short enough to avoid any disruptions to the flight regularity. Any aircraft departure delays will result in financial losses. Therefore, the warehouse must have a sufficient number of spare LRUs/LRMs. On the other hand, the surplus of spare LRUs/LRMs in the warehouse will be associated with financial losses as well, as the cost of avionics is exceptionally high.

Let us denote the time spent by the LRU/LRM in the state S_i ($i = \overline{1,7}$) as TS_i., which is a random variable with the expected time $E[TS_i]$. Let Ξ and Σ be, respectively, the random time to a permanent and intermittent failure with the probability density function (PDF) $\omega(\xi)$ and $f(\varepsilon)$, and cumulative distribution function (CDF) $\Omega(\xi)$ and $F(\varepsilon)$. The mean time of the LRU/LRM regeneration cycle is determined using the addition theorem of mathematical expectations:

$$E\left[TS_0\right] = \sum_{i=1}^{7} E\left[TS_i\right] \tag{2.1}$$

2.2.2. Mean time of staying the LRU/LRM in different states. Let us determine the mean times $\overline{E[TS_1], E[TS_7]}$. Assume that the LRU/LRM works being operable till time ξ, where $k\tau < \xi \le (k+1)\tau$, $k = (\overline{0, N})$, $(N+1)\tau = T$. The conditional mathematical expectation of the time spent by the LRU in the state S_1 provided that $\Xi = \xi$ is [4-6]:

$$E\left[TS_1 \middle| \xi\right] = \begin{cases} \sum\limits_{\nu=1}^{k} \nu\tau \int\limits_{(\nu-1)\tau}^{\nu\tau} f(z)dz + \xi\left[1 - F(k\tau)\right], \text{ if } k\tau < \xi \le (k+1)\tau,\ k = \overline{0, N}, \\ \sum\limits_{\nu=1}^{N} \nu\tau \int\limits_{(\nu-1)\tau}^{\nu\tau} f(z)dz + T\left[1 - F(T)\right], \text{ if } \xi > T. \end{cases} \tag{2.2}$$

To determine the mathematical expectation of the time spent by the LRU/LRM in the state S_i $(i = \overline{1,7})$, we will use the modified formula for the total mathematical expectation of a continuous random variable [4-6]:

$$E[TS_i] = \sum_{k=0}^{N} \int\limits_{k\tau}^{(k+1)\tau} E\left[T_i \middle| k\tau < \xi \le (k+1)\tau\right]\omega(\xi)d\xi + \int\limits_{T}^{\infty} E\left[T_i \middle| \xi > T\right]\omega(\xi)d\xi,\ i = \overline{1,6}\ . \tag{2.3}$$

Applying formula (2.3) to expression (2.2), we obtain [4, 6]:

$$\begin{gathered} E[TS_1] = \sum_{k=0}^{N} \int\limits_{k\tau}^{(k+1)\tau} \left\{ \sum_{\nu=1}^{k} \nu\tau \int\limits_{(\nu-1)\tau}^{\nu\tau} f(z)dz + \upsilon\left[1 - F(k\tau)\right] \right\} \omega(\upsilon)d\upsilon + \\ \int\limits_{T}^{\infty} \left\{ \sum_{\nu=1}^{N} \nu\tau \int\limits_{(\nu-1)\tau}^{\nu\tau} f(z)dz + T\left[1 - F(T)\right] \right\} \omega(\upsilon)d\upsilon. \end{gathered} \tag{2.4}$$

The conditional mathematical expectation of the time spent by the LRU/LRM in the state S_2 provided that $\Xi = \xi$ is equal to [4-6]:

$$E\left[TS_2 \middle| \xi\right] = \begin{cases} \left[(k+1)\tau - \xi\right] \int\limits_{k\tau}^{\infty} f(z)dz, \text{if } k\tau < \xi \le (k+1)\tau, \\ 0, \text{ if } \xi > T. \end{cases} \tag{2.5}$$

By substituting (2.5) into (2.3), we obtain the mean time spent by the LRU/LRM in the state S_2 [4-6]:

$$\begin{gathered} E\left[TS_2\right] = \sum_{k=0}^{N} \int\limits_{k\tau}^{(k+1)\tau} \left\{ \int\limits_{k\tau}^{\infty} f(z)dz\left[(k+1)\tau - \upsilon\right] \right\} \omega(\upsilon)d\upsilon = \\ \sum_{k=0}^{N} \left[1 - F(k\tau)\right] \left\{ (k+1)\tau\left[\Omega((k+1)\tau) - \Omega(k\tau)\right] - \int\limits_{k\tau}^{(k+1)\tau} \upsilon\omega(\upsilon)d\upsilon \right\}. \end{gathered} \tag{2.6}$$

If the duration of the maintenance operations is constant over the interval (0, T), the mean times $\overline{E[TS_3], E[TS_5]}$ are determined by the following formulas [4]:

$$E\left[TS_3\right] = t_M^{O-level}, \tag{2.7}$$

$$E\left[TS_4\right]=\left(t_{spare}+t_M^{O-level}-t_{stop}\right), \tag{2.8}$$

$$E\left[TS_5\right]=t_{shipping}, \tag{2.9}$$

where $t_M^{O-level}$ is the average duration of maintenance operations at O-level; t_{spare} is the average time of waiting for spare LRU/LRM from the warehouse in the situation "aircraft on ground"; t_{stop} is the average scheduled stop time of the aircraft at the base airport; and $t_{shipping}$ is the average time of shipping the failed LRU/LRM to the repair and back.

If an intermittent failure occurs during the ν-th flight and until this moment there is no permanent failure, the conditional mathematical expectation of the time spent by the LRU/LRM in the state S_6 provided that $\Xi = \xi$ is equal to [4, 5]:

$$E\left[TS_6|\xi\right]=t_{IFR}\begin{cases}\sum\limits_{\nu=1}^{k}\int\limits_{(\nu\text{-}1)\tau}^{\nu\tau}f(z)dz, \text{ if } k\tau<\xi\le(k+1)\tau,\ k=\overline{0,N},\\ \sum\limits_{\nu=1}^{N+1}\int\limits_{(\nu\text{-}1)\tau}^{\nu\tau}f(z)dz, \text{ if } \xi>T,\end{cases} \tag{2.10}$$

where t_{IFR} is the average time of the LRU/LRM repair due to intermittent failure.

By substituting (2.10) into (2.3), we obtain the mean time spent by the LRU/LRM in the state S_6 [4]:

$$\begin{gathered}E\left[TS_6\right]=t_{IFR}\left\{\sum_{k=0}^{N}\int\limits_{k\tau}^{(k+1)\tau}\left[\sum_{\nu=1}^{k}\int\limits_{(\nu\text{-}1)\tau}^{\nu\tau}f(z)dz\right]\omega(\upsilon)d\upsilon+\int\limits_{T}^{\infty}\left[\sum_{\nu=1}^{N+1}\int\limits_{(\nu\text{-}1)\tau}^{\nu\tau}f(z)dz\right]\omega(\upsilon)d\upsilon\right\}=\\ t_{IFR}\left\{\sum_{k=0}^{N}F(k\tau)\left[\Omega((k+1)\tau)-\Omega(k\tau)\right]+F(T)\left[1-\Omega(T)\right]\right\}.\end{gathered} \tag{2.11}$$

If a permanent failure occurs during the k-th flight and until moment $k\tau$ there is no intermittent failure, the conditional mathematical expectation of the time spent by the LRU/LRM in the state S_7 provided that $\Xi = \xi$ is equal to [4]:

$$E\left[TS_7|\xi\right]=t_{PFR}\begin{cases}\int\limits_{k\tau}^{\infty}f(z)dz\int\limits_{k\tau}^{(k+1)\tau}\omega(\upsilon)d\upsilon, \text{ if } k\tau<\xi\le(k+1)\tau,\\ \left[1-F(T)\right], \text{ if } \xi>T,\end{cases} \tag{2.12}$$

where t_{PFR} is the average time of the LRU/LRM repair due to permanent failure.

By substituting (2.12) into (2.3), we obtain the mean time spent by the LRU/LRM in the state S_7 [4]:

$$E\left[TS_7\right]=t_{PFR}\left\{\sum_{k=0}^{N}\int_{k\tau}^{(k+1)\tau}\left[\int_{k\tau}^{\infty}f(z)dz\right]\omega(\upsilon)d\upsilon+\left[1-F(T)\right]\int_{T}^{\infty}\omega(\upsilon)d\upsilon\right\}=$$
$$t_{PFR}\left\{\sum_{k=0}^{N}\left[1-F(k\tau)\right]\left[\Omega((k+1)\tau)-\Omega(k\tau)\right]+\left[1-F(T)\right]\left[1-\Omega(T)\right]\right\}. \quad (2.13)$$

2.2.3. The case of an exponential failure distribution. As is well known [7], the exponential distribution is suitable to describe failure distribution of complex systems. On-board electronic LRUs/LRMs have a complex structure and include a considerable number of components. Extrinsic and intrinsic failure mechanisms of these components may cause the units to fail. These failure mechanisms can combine forming a constant failure rate, which is only possible with an exponential distribution of time to failure [8]. Therefore, the exponential distribution with the PDF

$$\omega(\xi)=\lambda e^{-\lambda\xi} \quad (2.14)$$

is the appropriate failure distribution for most electronic avionics LRUs, where λ is the LRU/LRM permanent failure rate.

Let us consider the case when intermittent failures are also subject to the exponential distribution with the PDF

$$f(\varepsilon)=\theta e^{-\theta\varepsilon}, \quad (2.15)$$

where θ is the LRU/LRM intermittent failure rate.

Substituting (2.14) and (2.15) into (2.4), (2.6), (2.11), and (2.13), we obtain [4-6]:

$$E\left[TS_1\right]=\frac{\tau}{1-e^{-\theta\tau}}\left[1-e^{-(\lambda+\theta)T}\right]+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}+\tau e^{-(\lambda+\theta)T}, \quad (2.16)$$

$$E\left[TS_2\right]=\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right], \quad (2.17)$$

$$E\left[TS_6\right]=t_{IFR}\left\{\left(1-e^{-\lambda\tau}\right)\left[\frac{e^{-\lambda\tau}-e^{-\lambda T}}{1-e^{-\lambda\tau}}-\frac{e^{-(\lambda+\theta)\tau}-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]+\left(1-e^{-\theta T}\right)e^{-\lambda T}\right\}, \quad (2.18)$$

$$E\left[TS_7\right]=t_{PFR}\left\{\left(1-e^{-\lambda\tau}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]+e^{-(\lambda+\theta)T}\right\}. \quad (2.19)$$

Expressions for mean times $\overline{E[TS_1],E[TS_7]}$ may be used to determine complex reliability indicators. For example, the LRU/LRM availability is determined as follows:

$$A=E\left[TS_1\right]/E\left[TS_0\right]. \quad (2.20)$$

Figure 2.1 shows the dependence of the LRU/LRM availability on the rate of intermittent failures in the case of an exponential distribution.

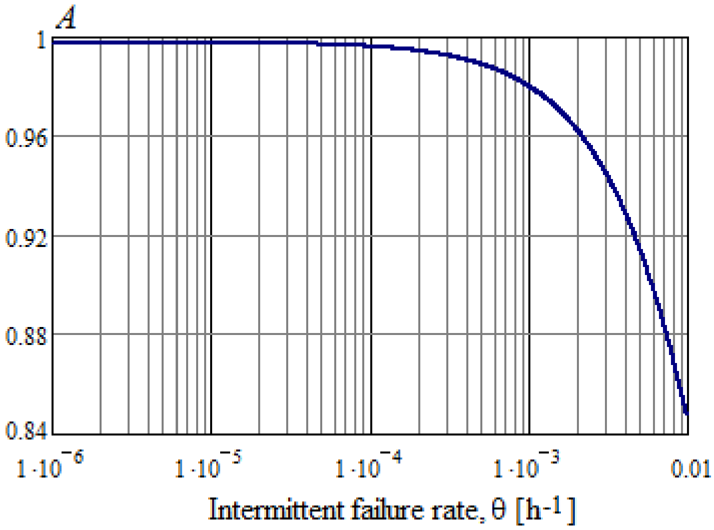


Fig. 2.1. Dependence of the LRU/LRM availability on the intermittent failure rate at T = 5,000 h, τ = 5 h and $\lambda = 1\times10^{-4}$ h^{-1}

As can be seen in Fig. 2.1, the availability begins to decrease significantly when $\theta > 1\times10^{-4}$ h^{-1}. For $\theta < 5\times10^{-5}$ h^{-1}, the availability does not depend on the intermittent failure rate.

2.2.4. Mean time between unscheduled removals. As previously noted, MTBUR is widely used by airlines as an LRU/LRM operational reliability indicator. Let us determine MTBUR considering permanent and intermittent failures. Assume as in paragraph 2.2.2 that the LRU/LRM permanent failure occurs at time ξ, where $k\tau < \xi \le (k+1)\tau\,, k = \overline{0,N}$. Then, the LRU/LRM will be removed from the aircraft board at the time $\nu\tau$, if an intermittent failure occurs during the ν-th flight, where $\nu = \overline{1,k}$. Also, the LRU/LRM will be removed from the aircraft at time $(k+1)\tau$, unless an intermittent failure occurs before the instant $k\tau$. Therefore, under the condition that $\Xi = \xi$, the mathematical expectation of the time to the LRM/LRU removal is equal to [4, 6]:

$$E\left[TBUR_T\left|\xi\right.\right]=\begin{cases}\sum\limits_{\nu=1}^{k}\nu\tau\int\limits_{(\nu-1)\tau}^{\nu\tau}f(z)dz+(k+1)\tau\left[1-F(k\tau)\right], \text{ if } k\tau<\xi\le(k+1)\tau,\ k=\overline{0,N},\\ \sum\limits_{\nu=1}^{N}\nu\tau\int\limits_{(\nu-1)\tau}^{\nu\tau}f(z)dz+T\left[1-F(T)\right], \text{ if } \xi>T,\end{cases} \tag{2.21}$$

where $TBUR_T$ is the random time between unscheduled removals.

Substituting (2.21) into (2.3), we obtain the following expression for MTBUR [4]:

$$E[TBUR_T]=\sum_{k=0}^{N}\int_{k\tau}^{(k+1)\tau}\left\{\sum_{\nu=1}^{k}\nu\tau\int_{(\nu-1)\tau}^{\nu\tau}f(z)\,dz+(k+1)\tau\left[1-F(k\tau)\right]\right\}\omega(\upsilon)\,d\upsilon+ \\ \int_{T}^{\infty}\left\{\sum_{\nu=1}^{N}\nu\tau\int_{(\nu-1)\tau}^{\nu\tau}f(z)\,dz+T\left[1-F(T)\right]\right\}\omega(\upsilon)\,d\upsilon. \tag{2.22}$$

Let us assume that both failure types have an exponential distribution. Then, after substituting (2.14) and (2.15) into (2.22) and performing the necessary mathematical transformations, we obtain [4]:

$$E[TBUR_T]=\frac{\tau}{1-e^{-\theta\tau}}\left[1-e^{-(\lambda+\theta)T}\right]+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}+\tau e^{-(\lambda+\theta)T}+ \\ \left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]. \tag{2.23}$$

If $\lambda\to 0$, (2.23) transforms into the form

$$E[TBUR_T]=\tau\left(1-e^{-\theta T}\right)/\left(1-e^{-\theta\tau}\right). \tag{2.24}$$

If $\theta\to 0$, using (2.23) yields

$$E[TBUR_T]=\frac{\tau\left(1-e^{-\lambda T}\right)}{1-e^{-\lambda\tau}}. \tag{2.25}$$

Using (2.24) and (2.25), we determine the upper estimate of MTBUR:

$$E[TBUR_T]\le\min\left\{\frac{\tau\left(1-e^{-\theta T}\right)}{1-e^{-\theta\tau}},\frac{\tau\left(1-e^{-\lambda T}\right)}{1-e^{-\lambda\tau}}\right\}. \tag{2.26}$$

Example 2.1. Calculation of MTBUR for avionics LRU when T = 5,000 h, τ = 4 h, $\theta = 5\times 10^{-4}$ h^{-1}, and $\lambda = 1\times10^{-4}$ h^{-1}.

Solution. Using (2.24)–(2.26), we obtain

$$E[TBUR_T]\le\min(1837,3935)=1837\,\text{h}.$$

Substitution of the initial data into (2.23) yields $E[TBUR_T]$ = 1,586 h. In this manner, the upper estimate of MTBUR is 16 % higher than its real value. Therefore, one may use inequality (2.26) only for approximate calculations of MTBUR.

Figure 2.2 illustrates the dependence of MTBUR on the intermittent failure rate for an exponential distribution of time to both failure types. As can be seen in Fig. 2.2, MTBUR rapidly decreases when the intermittent failure rate increases. It should also be noted that $E[TBUR_T]$ = 3,935 h for θ = 0.

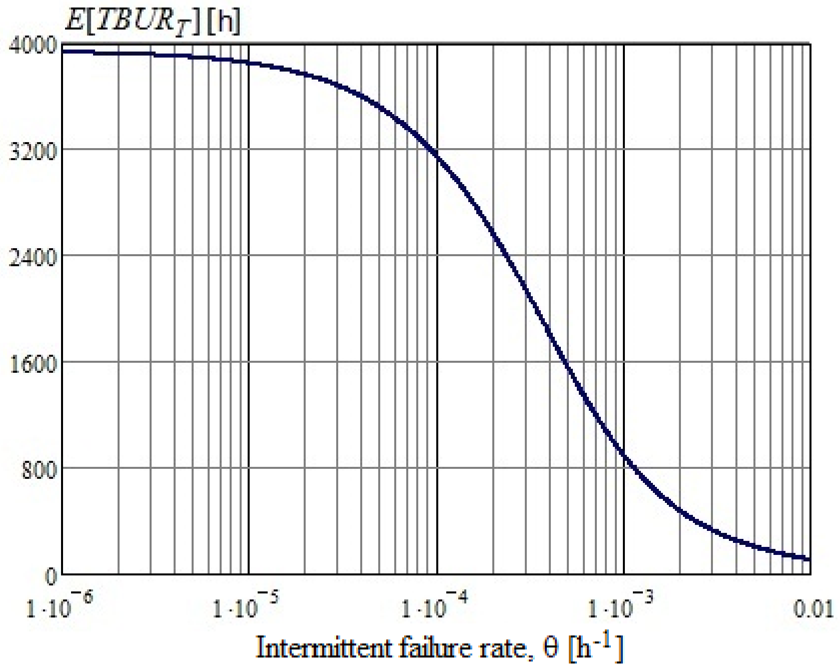


Fig. 2.2. Dependence of MTBUR on the intermittent failure rate at T = 5,000 h, τ = 4 h and $\lambda = 1\times10^{-4}$ h^{-1}

2.2.5. Operational reliability function. ORF is the leading indicator of maintenance effectiveness for systems that affect safety. Onward, we shall use the ORF to describe the LRU/LRM failure-free operation over the time interval ($k\tau$, t), $k\tau < t \leq (k+1)\tau$, considering that the LRU/LRM could undergo an unscheduled repair due to a permanent or intermittent failure at time-moments $\overline{\tau, k\tau}$.

To determine the ORF, we use the joint PDF of random variables $\Xi, \overline{\Theta_1, \Theta_k}$, which we denote as $\omega_0(\xi, \overline{\theta_1, \theta_k})$, where $\Theta_i = \Theta - (i-1)\tau$ is the remainder of the operating time to intermittent failure after i - 1 flights ($i = \overline{1, k}$). Using the multiplication theorem of the PDFs, we can write the following expression [5, 6]:

$$\omega_0\left(\xi, \overline{\theta_1, \theta_k}\right) = \omega(\xi)\, f\left(\overline{\theta_1, \theta_k} | \xi\right), \tag{2.27}$$

where $f(\overline{\Theta_1, \Theta_k} | \xi)$ is the conditional PDF of random variables $\overline{\Theta_1, \Theta_k}$ provided that $\Xi = \xi$.

To determine the ORF, we introduce some conditional probabilities associated with intermittent failures [5, 6]. The conditional probability of intermittent failure occurrence during the ν-th ($\nu = \overline{1,k}$) flight is described as follows:

$$P_{IF|\xi}\left(\nu\tau|\xi\right) = P\left\{\bigcap_{i=1}^{\nu-1}\Theta_i > \tau \bigcap \Theta_\nu < \tau|\xi\right\} \tag{2.28}$$

The conditional probability of intermittent failure non-occurrence during the k-th flight is described as follows:

$$P_{\overline{IF}|\xi}\left(k\tau|\xi\right) = P\left\{\bigcap_{i=1}^{k}\Theta_i > \tau|\xi\right\}. \tag{2.29}$$

The probabilities (2.28) and (2.29) are determined by integrating PDF $f(\overline{\Theta_1, \Theta_k}|\xi)$ between the corresponding limits [5, 6]:

$$P_{IF|\xi}\left(\nu\tau|\xi\right) = \int_\tau^\infty \cdot(\nu-1)\cdot \int_\tau^\infty \int_0^\tau f\left(\overline{u_1, u_\nu} \mid \xi\right)\overline{du_1 du_\nu} \tag{2.30}$$

$$P_{\overline{IF}|\xi}\left(k\tau|\xi\right) = \int_\tau^\infty \cdot(k)\cdot \int_\tau^\infty f\left(\overline{u_1, u_k}|\xi\right)\overline{du_1 du_k} \tag{2.31}$$

The ORF over the finite time interval T is determined as follows [9]:

$$R\left(k\tau, t\right) = \sum_{j=0}^{k} \frac{P_R\left(j\tau\right)}{\int_0^{T-j\tau} \omega\left(x\right)dx} \int_{t-j\tau}^{T} P_{\overline{IF}|\xi}\left(\left(k-j\right)\tau|\upsilon\right)\omega\left(\upsilon\right)d\upsilon,\ t \geq k\tau, \tag{2.32}$$

$$P_R\left(j\tau\right) = P_{IF}\left(j\tau\right) + P_{PF}\left(j\tau\right), \tag{2.33}$$

$$P_{IF}\left(j\tau\right) = \sum_{\nu=0}^{j-1} \frac{P_R(\nu\tau)}{\int_0^{T-\nu\tau} \omega(x)dx} \int_{(j-\nu)\tau}^{T} P_{IF|\xi}\left((j-\nu)\tau|\upsilon\right)\omega\left(\upsilon\right)d\upsilon, \tag{2.34}$$

$$P_{PF}\left(j\tau\right) = 1 - \sum_{\nu=0}^{j-1} \frac{P_R(\nu\tau)}{\int_0^{T-\nu\tau} \omega(x)dx} \int_{(j-\nu)\tau}^{T} P_{\overline{IF}|\xi}\left((j-\nu-1)\tau|\upsilon\right)\omega\left(\upsilon\right)d\upsilon, \tag{2.35}$$

where $P_R(\tau), P_{IF}(j\tau)$ и $P_{PF}(j\tau)$ are, respectively, the probabilities of total LRU repairing, the LRU/LRM repairing due to intermittent failure, and the LRU repairing due to permanent failure at the time $j\tau$.

Let us start the proof of relations (2.32)–(2.35) with expression (2.33). Let us introduce the following events: $\Pi(j\tau)$ is the event of LRU/LRM repairing at the time $j\tau$ after the j-th flight; $h_{IF}(j\tau)$ and $h_{PF}(j\tau)$ are the events of LRU/LRM repairing due to intermittent and permanent failure, respectively. The event h_{IF} $(j\tau)$ will appear in the case if the intermittent failure occurs in the j-th flight. It should be noted that some avionics LRUs/LRMs are dismantled after registering several intermittent failures. Similarly, the event $h_{PF}(j\tau)$ will appear in the case if the permanent failure occurs during the j-th flight. The LRU/LRM will be restored at the time $j\tau$ in case one of the events $h_{IF}(j\tau)$ or $h_{PF}(j\tau)$ occurs. Consequently,

$$\Pi(j\tau) = h_{IF}(j\tau) + h_{PF}(j\tau). \tag{2.36}$$

Assuming that $h_{IF}(j\tau)$ and $h_{PF}(j\tau)$ are mutually exclusive events, and applying the addition theorem of probability to (2.36), we obtain (2.33).

To prove (2.32), (2.34), and (2.35), let us write down the probabilistic definition of the probabilities $R(k\tau, t)$, $P_{IF}(j\tau)$ and $P_{PF}(j\tau)$.

Operational reliability function can be formulated as follows:

$$R(j\tau, t) = P\left\{\bigcup_{j=0}^{k}\left[\Pi(j\tau) \cap \Xi > t - j\tau \cap \left(\bigcap_{i=j+1}^{k} \Theta_i > \tau\right)\right]\right\}. \tag{2.37}$$

The formulation of the probability of LRU/LRM recovery due to intermittent or permanent failure is as follows:

$$P_{IF}(j\tau) = P\left\{\bigcup_{\nu=0}^{j-1}\left[\Pi(\nu\tau) \cap \left(E_1((j-\nu)\tau) \setminus E_2((j-\nu)\tau)\right)\right]\right\}, \tag{2.38}$$

$$P_{PF}(j\tau) = 1 - P\left\{\bigcup_{\nu=0}^{j-1}\left[\Pi(\nu\tau) \cap E_1((j-\nu)\tau)\right]\right\}, \tag{2.39}$$

where

$$E_1((j-\nu)\tau) = \Xi > (j-\nu)\tau \cap \left(\bigcap_{i=\nu+1}^{j-1} \Theta_i > \tau\right)$$

is the event consisting in the fact that in the LRU/LRM, which began to operate at time $\nu\tau$, no permanent failure will occur during the time $(j\text{-}\nu)\tau$, and no intermittent failure - during flights $\overline{\nu + 1, j - 1}$;

$$E_2((j-\nu)\tau) = \Xi > (j-\nu)\tau \cap \left(\bigcap_{i=\nu+1}^{j} \Theta_i > \tau\right)$$

is the event similar to $E_1((j - \nu)\tau)$, with the only difference that no intermittent failure occurs in the j-th flight as well;

'\' is a symbol of the difference between the two events.

Assuming that $P_R(0) = 1$ and applying the addition and multiplication theorems of probability to expressions (2.37)–(2.39), we obtain (2.32), (2.34) and (2.35).

Let us determine the probabilities (2.32)-(2.35) for an exponential distribution of permanent and intermittent failures. Due to memoryless property inherent to the exponential distribution, the probabilities (2.30) and (2.31) transform into [5]:

$$P_{IF|\xi}\left(\nu\tau|\xi\right) = e^{-(\nu-1)\theta\tau}\left(1-e^{-\theta\tau}\right), \tag{2.40}$$

$$P_{\overline{IF}|\xi}\left(k\tau|\xi\right) = e^{-k\theta\tau}. \tag{2.41}$$

Substituting (2.14), (2.15), (2.40), and (2.41) into (2.32), (2.34) and (2.35), we obtain

$$R\left(k\tau,t\right) = \sum_{j=0}^{k} \frac{P_R\left(j\tau\right)}{1-e^{-\lambda(T-j\tau)}} e^{-(k-j)\theta\tau}\left[e^{-\lambda(t-j\tau)} - e^{-\lambda T}\right], \tag{2.42}$$

$$P_{IF}\left(j\tau\right) = \left(1-e^{-\theta\tau}\right)\sum_{\nu=0}^{j-1} \frac{P_R\left(\nu\tau\right)}{1-e^{-\lambda(T-\nu\tau)}} e^{-(j-\nu-1)\theta\tau}\left[e^{-(j-\nu)\lambda\tau} - e^{-\lambda T}\right], \tag{2.43}$$

$$P_{PF}\left(j\tau\right) = 1 - \sum_{\nu=0}^{j-1} \frac{P_R\left(\nu\tau\right)}{1-e^{-\lambda(T-\nu\tau)}} e^{-(j-\nu-1)\theta\tau}\left[e^{-(j-\nu)\lambda\tau} - e^{-\lambda T}\right]. \tag{2.44}$$

It should be noted that beginning from the fourth or fifth flight, the probabilities (2.41) and (2.42) reach the steady-state values

$$P_{IF}^{*}\left(\tau\right) = \left(1-e^{-\theta\tau}\right)\left(e^{-\lambda\tau} - e^{-\lambda T}\right) \Big/ \left[\left(1-e^{-\lambda T}\right)\left(1-e^{-\theta T}\right)\right], \tag{2.45}$$

$$P_{PF}^{*}\left(\tau\right) = 1 - \left(e^{-\lambda\tau} - e^{-\lambda T}\right) \Big/ \left(1-e^{-\lambda T}\right). \tag{2.46}$$

2.3. A mathematical model of LRU/LRM operation and maintenance over an infinite time interval

An infinite interval of operation usually means a time interval that significantly exceeds the average time between LRU/LRM recoveries. To obtain expressions over an infinite time interval, it suffices to set $T = \infty$ in the previously derived formulas.

2.3.1. Mean time spent by the LRU/LRM in different states. By substituting $T = \infty$ into (2.9), (2.11), (2.15), and (2.17), we obtain

$$E[TS_1] = \sum_{k=0}^{\infty} \int_{k\tau}^{(k+1)\tau} \left\{ \sum_{\nu=1}^{k} \nu\tau \int_{(\nu-1)\tau}^{\nu\tau} f\left(z\right)dz + \upsilon\left[1-F\left(k\tau\right)\right] \right\} \omega\left(\upsilon\right)d\upsilon, \tag{2.47}$$

$$E[TS_2]=\sum_{k=0}^{\infty}\int_{k\tau}^{(k+1)\tau}\left\{\int_{k\tau}^{\infty}f(z)dz\left[(k+1)\tau-\upsilon\right]\right\}\omega(\upsilon)d\upsilon=$$
$$\sum_{k=0}^{\infty}\left[1-F(k\tau)\right]\left\{(k+1)\tau\left[\Omega((k+1)\tau)-\Omega(k\tau)\right]-\int_{k\tau}^{(k+1)\tau}\upsilon\omega(\upsilon)d\upsilon\right\}, \tag{2.48}$$

$$E[TS_6]=t_{IFR}\left\{\sum_{k=0}^{\infty}\int_{k\tau}^{(k+1)\tau}\left[\sum_{\nu=1}^{k}\int_{(\nu-1)\tau}^{\nu\tau}f(z)dz\right]\omega(\upsilon)d\upsilon\right\}=t_{IFR}\left\{\sum_{k=0}^{\infty}F(k\tau)\left[\Omega((k+1)\tau)-\Omega(k\tau)\right]\right\}, \tag{2.49}$$

$$E[TS_7]=t_{PFR}\left\{\sum_{k=0}^{\infty}\int_{k\tau}^{(k+1)\tau}\left[\int_{k\tau}^{\infty}f(z)dz\right]\omega(\upsilon)d\upsilon\right\}=t_{PFR}\left\{\sum_{k=0}^{\infty}\left[1-F(k\tau)\right]\left[\Omega((k+1)\tau)-\Omega(k\tau)\right]\right\}. \tag{2.50}$$

Substituting $T=\infty$ into (2.16)–(2.19), we obtain the mean times $E[TS_1], E[TS_2], E[TS_6]$ и $E[TS_7]$ in the case of exponential distribution of failures [5, 10]:

$$E[TS_1]=\frac{\tau}{1-e^{-\theta\tau}}+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1}{1-e^{-(\lambda+\theta)\tau}}, \tag{2.48}$$

$$E[TS_2]=\frac{1}{1-e^{-(\lambda+\theta)\tau}}\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right), \tag{2.49}$$

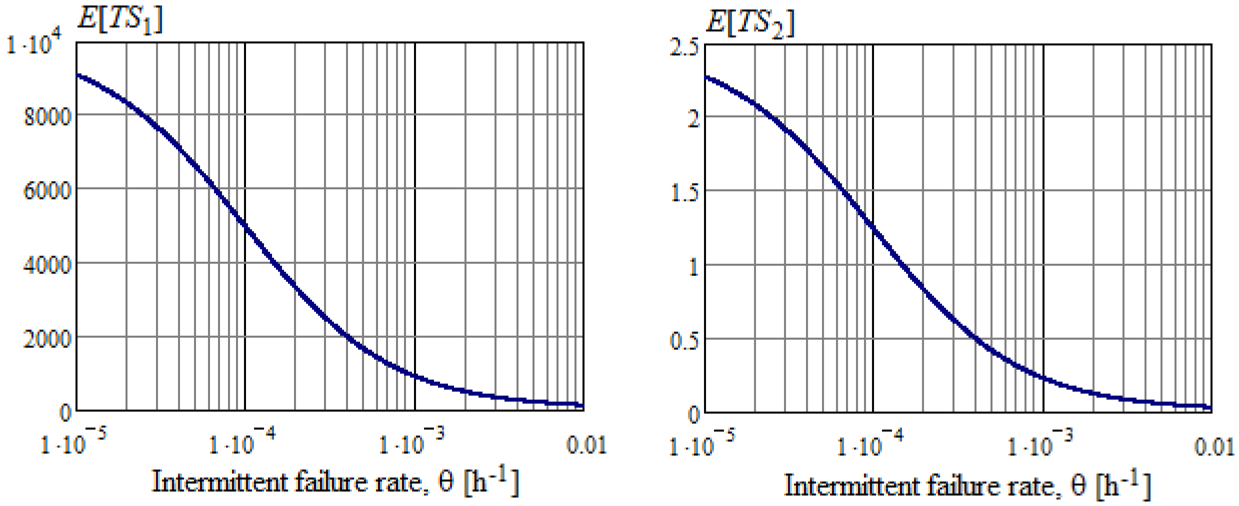
$$E[TS_6]=t_{IFR}\left(1-e^{-\lambda\tau}\right)\left[\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}-\frac{e^{-(\lambda+\theta)\tau}}{1-e^{-(\lambda+\theta)\tau}}\right], \tag{2.50}$$

$$E[TS_7]=t_{PFR}\left[\frac{1-e^{-\lambda\tau}}{1-e^{-(\lambda+\theta)\tau}}\right]. \tag{2.51}$$

Figure 2.3 shows the dependences of $E[TS_1]$ и $E[TS_2]$ on the intermittent failure rate. As can be seen in Fig. 2.3, both mean times decrease as θ increases.

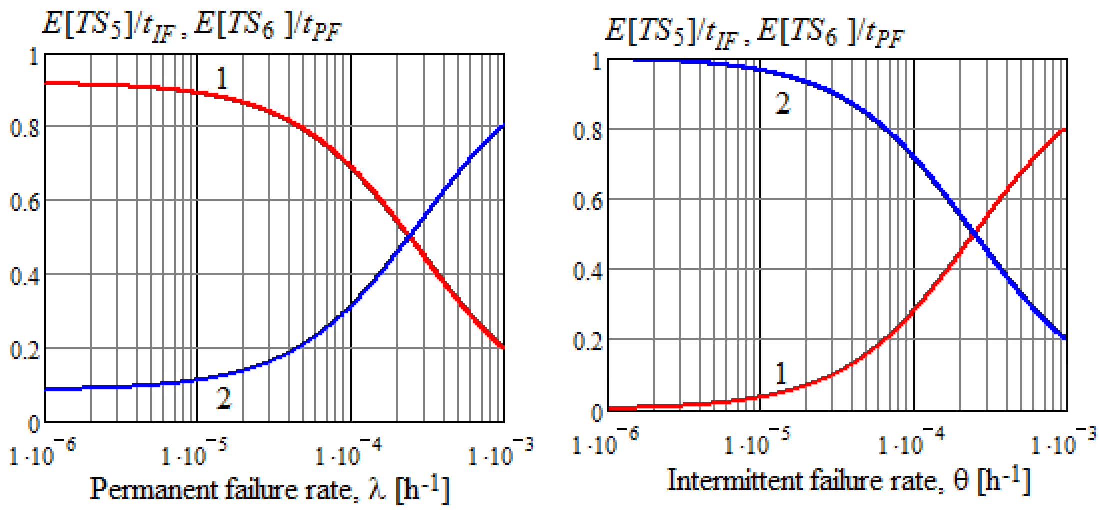
Figure 2.4 (a, b) illustrates the dependences of $E[TS_5]/t_{IFR}$ (curve 1) and $E[TS_6]\ /\ t_{PFR}$ (curve 2) on the permanent (a) and intermittent (b) failure rate. Figure 2.4 (a) clearly shows that $E[TS_5]/t_{IF}$ decreases, and $E[TS_6]\ /\ t_{PF}$ increases as λ increases. As can be seen in Fig. 2.4 (b), the dependence of $E[TS_5]/t_{IF}$ и $E[TS_6]\ /\ t_{PF}$ on θ is reversed.

It should be noted that the sum of $E[TS_5]/t_{IF}$ и $E[TS_6]\ /\ t_{PF}$ is equal to unity for any combination of θ and λ, because the relations $E[TS_5]/t_{IF}$ and $E[TS_6]/t_{PF}$ determine a posteriori probabilities P_{IF} and P_{PF} that the LRU/LRM was removed from the aircraft either due to intermittent or permanent failure.



(a) (b)

Fig. 2.3. Dependence of mean times $E[TS_1]$ (a) and $E[TS_2]$ (b) on the intermittent failure rate at τ = 5 h and λ = 1×10 $^{-4}$ h^{-1}



(a) (b)

Fig. 2.4. Dependence of $E[TS_5]/t_{IF}$ (curve 1) and $E[TS_6]/t_{PF}$ (curve 2) on the permanent (a) and intermittent (b) failure rate at τ = 5 h, $t_{IF} = t_{PF} = 1$ h, θ = 1 × 10^{-4} h^{-1} (a), and λ = 1 × 10^{-4} h^{-1} (b)

2.3.2. Mean time between unscheduled removals. By substituting $T = \infty$ into the expression (2.22), we obtain the general formula to calculate MTBUR [4]:

$$E[TBUR] = \sum_{k=0}^{\infty} \int_{k\tau}^{(k+1)\tau} \left\{ \sum_{\nu=1}^{k} \nu\tau \int_{(\nu-1)\tau}^{\nu\tau} f(z)\,dz + (k+1)\tau\left[1 - F(k\tau)\right] \right\} \omega(\upsilon)\,d\upsilon. \qquad (2.52)$$

At $T = \infty$ from (2.23), we get

$$E[TBUR]=\frac{\tau}{1-e^{-\theta\tau}}+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1}{1-e^{-(\lambda+\theta)\tau}}+\frac{1}{1-e^{-(\lambda+\theta)\tau}}\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right). \quad (2.53)$$

Figure 2.5 shows the dependence of MTBUR on the intermittent failure rate. By comparing Fig. 2.2 and 2.5 we can conclude, firstly, that MTBUR has a strong dependence on θ for both finite and infinite operation intervals, and secondly, MTBUR has a significant dependency on the length of the interval (0, *T*).

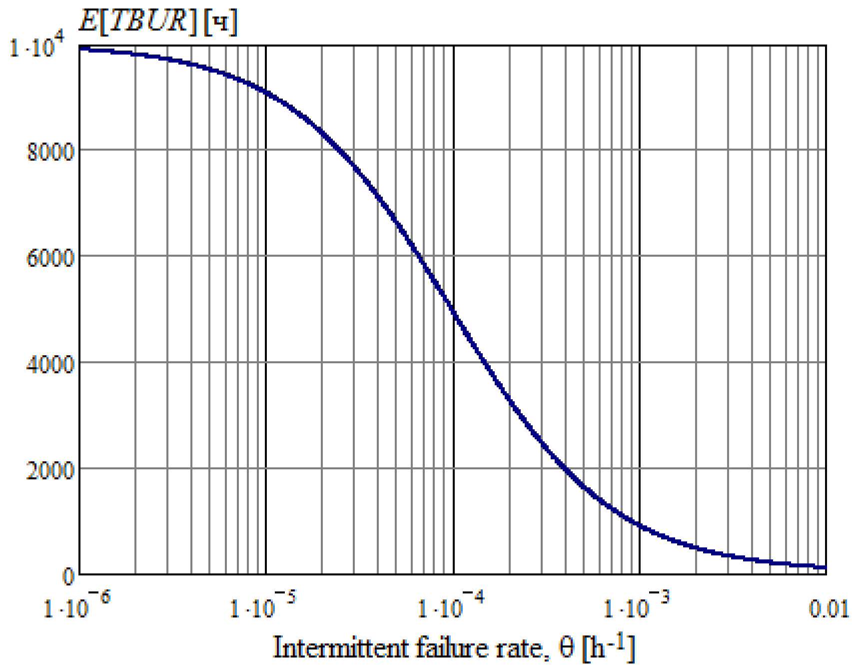


Fig. 2.5. Dependence of MTBUR on the intermittent failure rate for $T = \infty$, $\tau = 4$ h and $\lambda = 1\times10^{-4}$ h^{-1}

Indeed, $E[TBUR] = 5{,}000\ \text{h}$ for $T = \infty$ and $\theta = 10^{-4}\text{h}^{-1}$, and $E[TBUR_T] = 3{,}200\ \text{h}$ for $T = 5{,}000\ \text{h}$ and $\theta = 10^{-4}\text{h}^{-1}$.

2.3.3. Operational reliability function. Substituting $T = \infty$ into (2.32), (2.34), and (2.35), we obtain expressions for ORF and probabilities of LRU/LRM repairing due to intermittent and permanent failure at time $j\tau$ [11]:

$$R(k\tau,t)=\sum_{j=0}^{k}P_R(j\tau)\int_{t-j\tau}^{\infty}P_{\overline{IF}|\xi}\left((k-j)\tau|\upsilon\right)\omega(\upsilon)d\upsilon,\ t\geq k\tau, \quad (2.54)$$

$$P_{IF}(j\tau)=\sum_{\nu=0}^{j-1}P_R(\nu\tau)\int_{(j-\nu)\tau}^{\infty}P_{IF|\xi}\left((j-\nu)\tau|\upsilon\right)\omega(\upsilon)\,d\upsilon, \quad (2.55)$$

$$P_{PF}(j\tau)=1-\sum_{\nu=0}^{j-1}P_R(\nu\tau)\int_{(j-\nu)\tau}^{\infty}P_{\overline{IF}|O}\big((j-\nu-1)\tau|\upsilon\big)\omega(\upsilon)d\upsilon. \quad (2.56)$$

By substituting $T=\infty$ into (2.42)–(2.44) and performing the necessary mathematical transformations, we obtain the following formulas that are valid for the exponential failure distribution [11]:

$$R(k\tau,t)=\sum_{j=0}^{k}P_R(j\tau)\exp\{-[(k-j)\theta\tau+\lambda(t-j\tau)]\}, \quad (2.57)$$

$$P_{IF}(j\tau)=(1-e^{-\theta\tau})\sum_{\nu=0}^{j-1}P_R(\nu\tau)\exp\{-[(j-\nu-1)\theta+(j-\nu)\lambda]\tau\}, \quad (2.58)$$

$$P_{PF}(j\tau)=1-\sum_{\nu=0}^{j-1}P_R(\nu\tau)\exp\{-[(j-\nu-1)\theta+(j-\nu)\lambda]\tau\}. \quad (2.59)$$

Example 2.2. Calculate (2.57)–(2.59) assuming that $\lambda = 10^{-4}$ h^{-1}, $\theta = 2\times10^{-4}$ h^{-1}, τ = 5 h, and $t = (k+1)\tau$ in (2.57).

Table. 2.1 presents the calculation results.

Table 2.1. Calculated probabilities as a function of the flight number

k	$P_R(k\tau)$	$P_{IF}(k\tau)$	$P_{PF}(k\tau)$	$R[k\tau, (k+1)\tau]$
0	1.0	0	1.0	0.999500124979169
1	0.001498875562289	0.000999000541458	0.000499875020831	0.999498252229370
2	0.001498876161360	0.000999001140529	0.000499875020831	0.999500125577941
3	0.001498874964116	0.000999000542057	0.000499874422059	0.999500124979169
4	0.001498875562289	0.000999000541458	0.000499875020831	0.999500124979169
5	0.001498875562289	0.000999000541458	0.000499875020831	0.999500124979169

As can be seen in Table 2.1, all probabilities reach steady-state values beginning from the fourth flight. It should also be noted, that P_{IF} is twice larger than P_{PF}.

Since the stochastic process of changing the LRU/LRM states is regenerative, then by the limiting theorem the following limits must exist:

$$\Big\{R^*(\tau)=\lim_{k\to\infty}R[k\tau,(k+1)\tau],\ P_{IF}^*(\tau)=\lim_{j\to\infty}P_{IF}(j\tau), P_{PF}^*(\tau)=\lim_{j\to\infty}P_{PF}(j\tau),\ P_R^*(\tau)=\lim_{j\to\infty}P_R(j\tau). \quad (2.60)$$

In the case of exponential failure distributions, the limits (2.60) take the following form:

$$\{R^*(\tau)=e^{-\lambda\tau},\ P_{IF}^*(\tau)=(1-e^{-\theta\tau})e^{-\lambda\tau},\ P_{PF}^*(\tau)=1-e^{-\lambda\tau},\ P_R^*(\tau)=1-e^{-(\lambda+\theta)\tau}. \quad (2.61)$$

The proof of the limits (2.61) follows from (2.42), (2.45) and (2.46) when $T = \infty$. Indirect substantiation is the data given in Table 2.1.

The limits (2.60) and (2.61) allow us to calculate the steady-state values of a series of maintenance effectiveness indicators. The average number of removals due to intermittent failures over time *t* (flight hours) is determined by the following formula [11]:

$$N_{IF}(t) = tP_{IF}^{*}(\tau)/\tau \tag{2.62}$$

The average number of removals due to permanent failures over time *t* is [11]

$$N_{PF}(t) = tP_{PF}^{*}(\tau)/\tau \tag{2.63}$$

The average total number of removals due to permanent and intermittent failures over time *t* is [11]

$$N_{R}(t) = t\,P_{R}^{*}(\tau)/\tau = N_{IF}(t) + N_{PF}(t) \tag{2.64}$$

The proofs of the relations (2.62)–(2.64) are omitted because of their obviousness.

The average repair cost of a set of *m* LRUs over time *t* may be used as a cost-related indicator [11]:

$$C(t) = \frac{mt}{\tau}\left[C_{IF}P_{IF}^{*}(\tau) + C_{PF}P_{PF}^{*}(\tau)\right], \tag{2.65}$$

where C_{IF} and C_{PF} are, respectively, the average cost of LRU/LRM repair due to intermittent and permanent failure.

Example 2.3. Using (2.64) and (2.65), plot the dependencies of $N_R(t)$ and $C(t)$ as functions of the intermittent failure rate θ at $t = 40{,}000$ h, $\lambda = 10^{-4}$ h^{-1}, $\tau = 5$ h, C_{IF} = \$ 1,000 and C_{PF} = \$ 2,000.

Figure 2.6 shows the graphs of these dependencies. As can be seen in Fig. 2.6, the average number of replacements and the average repair cost begin to increase significantly when $\theta > 10^{-4}$ h^{-1}. Therefore, the following requirement related to the intermittent failure rate should be ensured when designing and operating the electronic avionics LRUs/LRMs: $\theta \leq 10^{-4}$ h^{-1}. Particular attention should be paid to the case of a high intermittent failure rate. So for $\theta = 10^{-2}$ h^{-1}, the cost of repairing the dismantled LRUs/LRMs reaches \$ 12×10^{6}, which is 50 times the repair cost for $\theta = 10^{-5}$ h^{-1}.

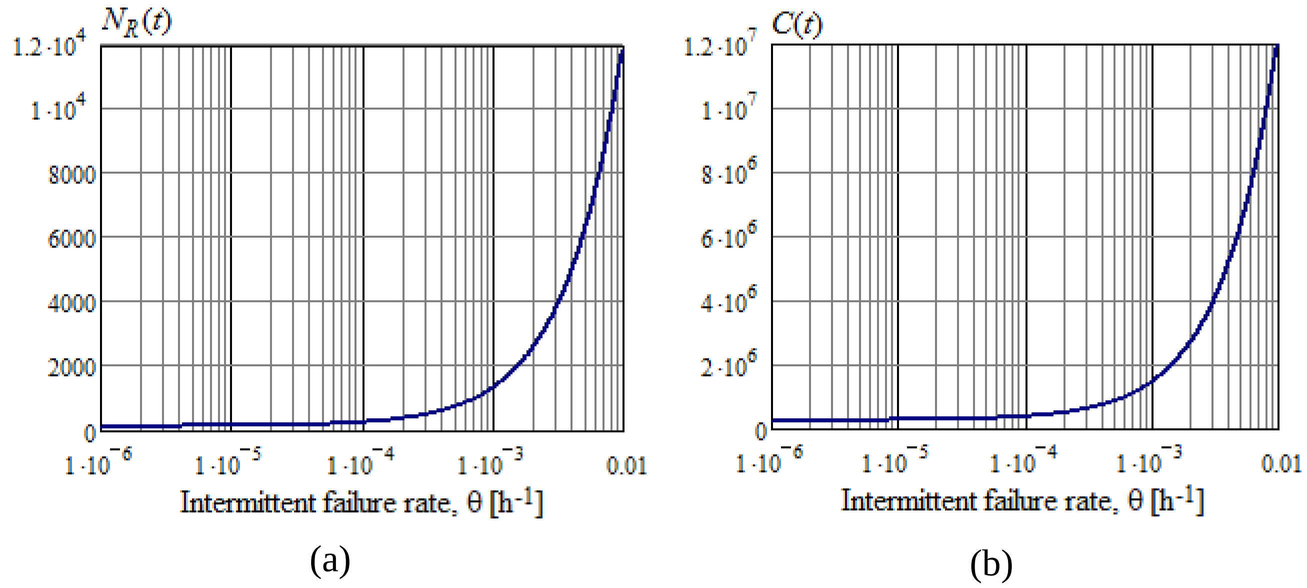


Fig. 2.6. Dependencies of the average total number of removals (a) and the average repair cost over time t (b) on the rate of intermittent failures

2.4. Maintenance effectiveness indicators of redundant avionics systems

Most avionics systems comprise several identical LRUs/LRMs, and the system reliability is a function of the reliability of individual LRUs/LRMs. We assume further that LRU/LRM failures are statistically independent. Let us consider an active redundancy case, in which all m identical LRUs/LRMs are operational from the start of system operation. Avionics systems operate in an intermittent mode due to the alternation of flights and landings in airports. Such LRU/LRM maintenance operations as dismantling, mounting, and pre-flight inspection may be carried out during stops at the base or transit airports. Consequently, such LRU/LRM states as S_3, S_5, S_6, and S_7 are related only to maintenance, repair, and shipping costs and do not affect the probabilistic indicators of redundant avionics systems. Therefore, we introduce a new time axis associated only with the LRU/LRM states S_1, S_2, and S_4, which assume LRUs/LRMs usage for their intended purpose. To determine the probabilistic effectiveness indicators, we are going to use the method of structure functions [12, 13], which is used to represent any probabilistic reliability indicator as a mathematical expectation of the corresponding structure function.

2.4.1. The case of a parallel redundancy structure. In the case of a parallel redundancy structure, the steady-state availability and steady-state ORF are determined by the following formulas [4, 10]:

$$A = 1 - \left\{1 - E(TS_1) / \left[E(TS_1) + E(TS_2) + E(TS_4)\right]\right\}^m, \qquad (2.66)$$

$$R_m^*(\tau) = 1 - \left[1 - R^*(\tau)\right]^m. \qquad (2.67)$$

The unavailability $\bar{A}$ and the probability of system failure in flight $Q_m^*(\tau)$ are determined as the complementary events probabilities, i. e.:

$$\bar{A} = \left\{1 - E(TS_1) / \left[E(TS_1) + E(TS_2) + E(TS_4)\right]\right\}^m, \qquad (2.68)$$

$$Q_m^*(\tau) = \left[1 - R^*(\tau)\right]^m. \qquad (2.69)$$

Example 2.4. Calculation of the unavailability of a redundant avionics system as a function of the intermittent failure rate with the following initial data: $m = 2$, $\tau = 5$ h, $E[TS_4] = 1$ h and $\lambda = 1 \times 10^{-4}$ h^{-1}.

Figure 2.7 illustrates the unavailability dependence on the intermittent failure rate. As can be seen in Fig. 2.7, the unavailability begins to increase substantially when $\theta > 10^{-4}$ h^{-1}.

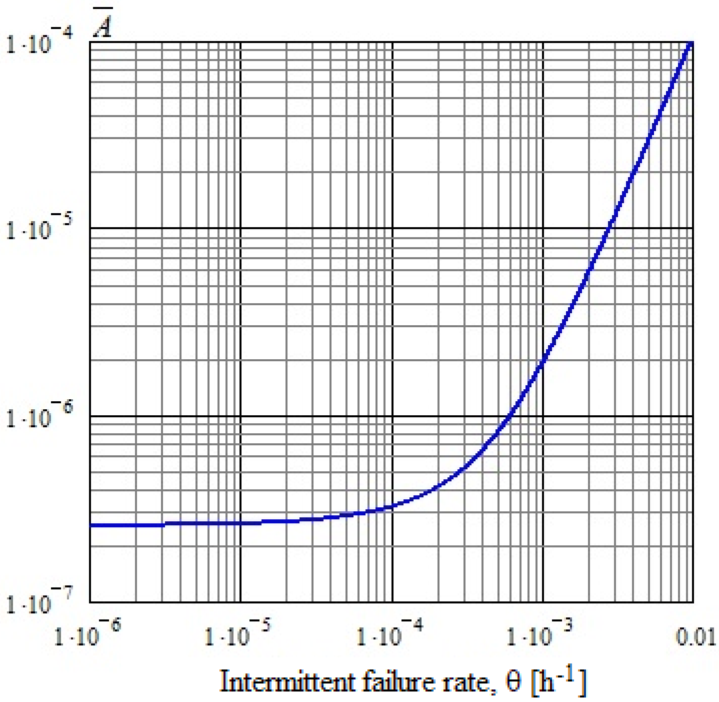


Fig. 2.7. Dependence of unavailability of the duplicated avionics system on the intermittent failure rate

2.4.2. The case of a majority vote redundant structure *k*-out-of-*m*. With the majority structure of redundancy, the avionics system is operable if ***k***-out-of-***m*** LRUs/LRMs are operable, where $k > m/2$.

Using the method of structure functions, we determine availability and ORF [4, 5]:

$$A=\sum_{i=k}^{m}\binom{m}{i}\left[\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^i\left[1-\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^{m-i}, \quad (2.70)$$

$$R_m^*(\tau)=\sum_{i=k}^{m}\binom{m}{i}\left[R^*(\tau)\right]^i\left[1-R^*(\tau)\right]^{m-i}. \quad (2.71)$$

The unavailability and the probability of system failure in flight are determined as the complementary events probabilities.

Parallel and majority redundancy structures of avionics systems are the most common. Among other structures of avionics systems, regarding reliability, parallel-series structure should also be specified.

2.4.3. The case of a parallel-series structure. In this case, a non-redundant system consists of l multi-type LRUs, which form a series structure regarding reliability. A redundant system is a parallel connection of m series structures. Using the structure functions method, we derive the following expressions for the system steady-state availability and ORF during flight:

$$A=1-\left\{1-\prod_{i=1}^{l}\frac{E(TS_{1,i})}{E(TS_{1,i})+E(TS_{2,i})+E(TS_{4,i})}\right\}^m, \quad (2.72)$$

$$R_m^*(\tau)=1-\left[1-\prod_{i=1}^{l}R_i^*(\tau)\right]^m. \quad (2.73)$$

In conclusion, it should be noted that formulas (2.67), (2.69), (2.71), and (2.73) have a known form for the probabilistic reliability indicators of redundant systems. At the same time, the formulas to calculate the ORF of LRUs/LRMs included in the indicated relations and calculated by (2.54), (2.57), (2.60), and (2.61) have been derived by the author and are new. Therefore, formulas (2.67), (2.69), (2.71), and (2.73) are given in this chapter only to show how to calculate the ORF of a redundant avionics system when the values of ORF for LRUs/LRMs are known.

2.5. Warranty maintenance model of redundant avionics systems

2.5.1. Analysis of warranty time-related indicators of avionics systems' suppliers. Efficient operation of an aircraft is primarily determined by the proper regulation of the relationship between an aircraft supplier (manufacturer) and aircraft operator (airline). The acutest problem faced in this relationship is during the warranty period, where the main cost for aircraft systems' repair is borne by the supplier, i.e., the supplier will remedy the defect in a system free of charge and in a reasonable time, by either repairing or replacing the defective system. However, the aircraft buyer must purchase a sufficient number of spare LRUs/LRMs to ensure regularity of flights. The number of spare LRUs/LRMs depends on the selected warranty maintenance option (WMO). The selection of the optimal WMO for avionics systems depends on the following factors: 1) supplier's warranty obligations; 2) operational reliability indicators; 3) whether the buyer has ground test equipment.

Manufacturers and suppliers of avionics systems typically use the following warranty time-related indicators: warranty period (T_W); guaranteed repair time (T_{RS}); guaranteed expedited delivery time of a spare LRU/LRM (T_{ED}). Warranty time-related indicators have specific units of measurement. The warranty period is expressed in a calendar period (years, months) from the beginning of the warranty. The guaranteed repair time is measured in a calendar duration (days) from the beginning of repairs. The guaranteed expedited delivery time of a spare LRU/LRM is expressed in a calendar duration (days) from the claim date. Values of T_W, T_{RS}, and T_{ED} are specified in any contract for the supply of avionics systems. For example, one of the avionics manufacturers uses the following values of warranty time-related indicators for its products: T_W = 3 years, T_{RS} = 15 days and T_{ED} = 5 days.

The initial time-moments of accounting for warranty obligations are described as follows: 1) for T_W, the date of official reception of the avionics system from the supplier (manufacturer); 2) for T_{RS}, the date when the manufacturer is notified about the avionics system failure; 3) for T_{ED}, the date when the manufacturer should deliver a spare LRU/LRM to the buyer.

The supplier's warranty time-related indicators may be presented in other units of measurement. For example, if the avionics system has a 3-year warranty and the aircraft has an annual flight time of 2,000 hours, if necessary, the warranty period can be measured in hours, e. g. $T_W = 3 \times 2{,}000 = 6{,}000$ hours. In some cases, the length of the warranty period is specified separately in the calendar duration and flight hours. In such cases, the indicator T_W is equal to the value that is achieved first.

2.5.2. Analysis of possible warranty maintenance options. During the warranty period, the buyer of the aircraft does not pay the failed LRUs/LRMs repair costs. However, he pays for spare LRUs/LRMs needed to ensure the regularity of flights. Therefore, the buyer must choose such WMO for which the total operating costs during the warranty period will be minimal. Possible WMOs may differ in the presence or absence of ground test equipment at the base airport and the values of T_{RS} and T_{ED}. Presence of ground test equipment at the base airport allows testing the dismantled LRUs/LRMs and shipping to the manufacturer only the ones with confirmed failures. LRUs/LRMs with unconfirmed failures are sent to the warehouse and then installed in the aircraft on demand. The use of ground test equipment reduces the mean repair time of dismantled LRUs/LRMs and, consequently, reduces the number of spare LRUs/LRMs.

The first WMO includes only O-level maintenance, where all dismantled LRUs/LRMs are shipped to the manufacturer for repair. The most significant cost measure for an aircraft buyer is the total expected cost of maintenance during the warranty period, which we will denote by WTEC (warranty total expected costs). WTEC is the total costs incurred by the buyer during the warranty period, which comprise the following two main components: the cost of dismantling and installing the LRUs/LRMs on board the aircraft during the warranty period and the cost of spare LRUs/LRMs recalculated per one aircraft. We denote the average costs associated with the first WMO as $WTEC_1$. Evident that the cost of spare LRUs/LRMs has a significant impact on $WTEC_1$. Since each LRU/LRM has a manufacturer's warranty, it will be repaired free of charge in case of a failure.

$WTEC_1$ is determined as follows [5]:

$$WTEC_1 = m \times LC \times t_M^{O-level} N_R(T_W) + (PS + US) \times C_{LRU} / N_W , \tag{2.74}$$

where LC is the operational maintenance labour cost per hour ($ /h); $N_R(T_W)$ is the average number of unscheduled removals of LRUs/LRMs due to permanent and intermittent failures for time T_W; PS is the planned number of spare LRUs/LRMs in the warehouse; US is the unplanned number of spare LRUs/LRMs that will need to be supplied from the manufacturer to ensure regularity of flights; C_{LRU} is the cost of a spare LRU/LRM; N_W is the number of aircraft under the supplier's warranty. In equation (2.74), the warranty period T_W is expressed in flight hours.

The average number of unscheduled removals of LRUs/LRMs due to permanent and intermittent failures during the period T_W is calculated by (2.63), or by the following formula:

$$N_R(T_W) = T_W / E\left[TBUR_{T_W}\right]. \tag{2.75}$$

The second WMO assumes that ground test equipment is used at the base airport at the I-level maintenance to recheck the dismantled LRUs/LRMs. Since conventional test equipment cannot practically detect the presence of intermittent failures in dismantled LRUs/LRMs, such LRUs/LRMs are sent to the warehouse after testing. We denote the average costs associated with the second WMO as $WTEC_2$. These costs include the following components: the cost of dismantling and mounting LRUs/LRMs on board aircraft during the warranty period; the cost of rechecking the dismantled LRUs/LRMs using ground test equipment; the cost of ground test equipment and the cost of spare LRUs/LRMs recalculated per one aircraft.

$WTEC_2$ is determined as follows [5]:

$$WTEC_2 = m \times LC\left(t_M^{O-level} + t_{TE}^{I-level}\right) N_R(T_W) + \frac{C_{TE}^{I-level}}{N_W \times F_{TE}^{I-level}} + (PS + US) C_{LRU} / N_W\,, \tag{2.76}$$

where $t_{TE}^{I-level}$ is the average LRU/LRM testing time using the ground test equipment at the I-level maintenance; $F_{TE}^{I-level}$ is the number of LRU/LRM types that can be tested by the ground test equipment at the I-level maintenance; $C_{TE}^{I-level}$ is the cost of the ground test equipment used at the I-level maintenance.

As seen from (2.74) and (2.76), $WTEC_1$ and $WTEC_2$ are the functions of the planned (PS) and unplanned (US) number of spare LRUs/LRMs in the warehouse at the base airport. The optimal number of spare LRUs/LRMs is determined by the following criterion:

$$E\left[TS_4\right] = \left[t_{spare}\left(PS, US\right) + t_M^{O-level} - t_{stop}\right] \to 0, \tag{2.77}$$

which is a modification of the criterion proposed in [13].

We can see from (2.76) that if $t_{stop} \geq t_{spare}(PS, US) + t_M^{O-level}$, then no disruption of the regularity of flights will occur. In contrast, if $t_{stop} < \Delta t_{spare}(PS, US) + t_M^{O-level}$, there will be a disruption of the flight regularity. Therefore, the optimal number of spare

LRUs/LRMs is the minimum possible number of LRUs/LRMs, which ensures that there are no delays in departures from the base airport.

The mathematical relationships to calculate the waiting time for a spare LRU/LRM at the base airport are given in Appendix 2.

Example 2.5. Calculation of the optimal number of spare LRUs/LRMs in the warehouse of the base airport with the following initial data: $T_W = 5{,}000$ h; $m = 3$; $t_M^{O-level} = 0.5$ h; $\lambda = 1\times10^{-4}$ h^{-1}; $T_{RS} = 103$ h; $T_{ED} = 34$ h; $t_{STOP} = 1$ h.

The dependence of the mean waiting time t_{spare} on the number of planned spare LRUs/LRMs (PS) for the first WMO at $N_W = 10$ is shown in Fig. 2.8 (a). Since the mean waiting time for a spare LRU/LRM does not exceed 0.5 h, 4 spare LRUs/LRMs are required for $\theta = 1\times10^{-4}$ h^{-1} and 6 spare LRUs/LRMs for $\theta = 1\times10^{-3}$ h^{-1}. As can be seen in Fig. 2.8 (a), the mean waiting time t_{spare} largely depends on the LRU/LRM intermittent failure rate.

Figure 2.8 (b) shows the dependence of the optimal number of spare LRUs/LRMs on the number of aircraft for the 1st WMO. The following conclusions may be drawn from the analysis of Fig. 2.8 (b): the function $PS(N_W)$ is an integer increasing function of N_W; an increase in the intermittent failure rate leads to a significant increase in the number of spare LRUs/LRMs (PS); the case when $\theta = 1\times10^{-3}$ h^{-1} is more sensitive to the increase in the number of aircraft than the case when $\theta = 1\times10^{-4}$ h^{-1}. The latter circumstance indicates that a high intermittent failure rate has a significant impact on the effectiveness of the 1st WMO.

Figure 2.9 (a) shows the dependence of the expected unplanned number of spare LRUs/LRMs (US) on the planned number of spare LRUs/LRMs (PS) for the 1st WMO. As seen, the US decreases rapidly as PS increases. Besides, the US is highly dependent on the expedited delivery time (T_{ED}). With a decrease of T_{ED}, US decreases as well.

Let us suppose that the second WMO is used. Using formulas (A.2.3), (A.2.7) and (A.2.8), we calculate that $E[T_R] = 12.8$ h, $P_{IF} = 0.905$ and $P_{PF} = 0.095$ for $\theta = 1\times10^{-3}$ h^{-1}, and $E[T_R] = 73.5$ h, $P_{IF} = 0.316$ and $P_{PF} = 0.684$ for $\theta = 1\times10^{-4}$ h^{-1}.

Figure 2.9 (b) shows the dependence of unavailability on the number of spare LRUs/LRMs of a triple redundant avionics system for the 2nd WMO when $m = 3$, $T_W = 5{,}000$ h, $N_W = 10$, $\tau = 4$ h, $\lambda = 1\times10^{-4}$h^{-1}, $T_{RS} = 103$ h, and $T_{ED} = 34.3$ h.

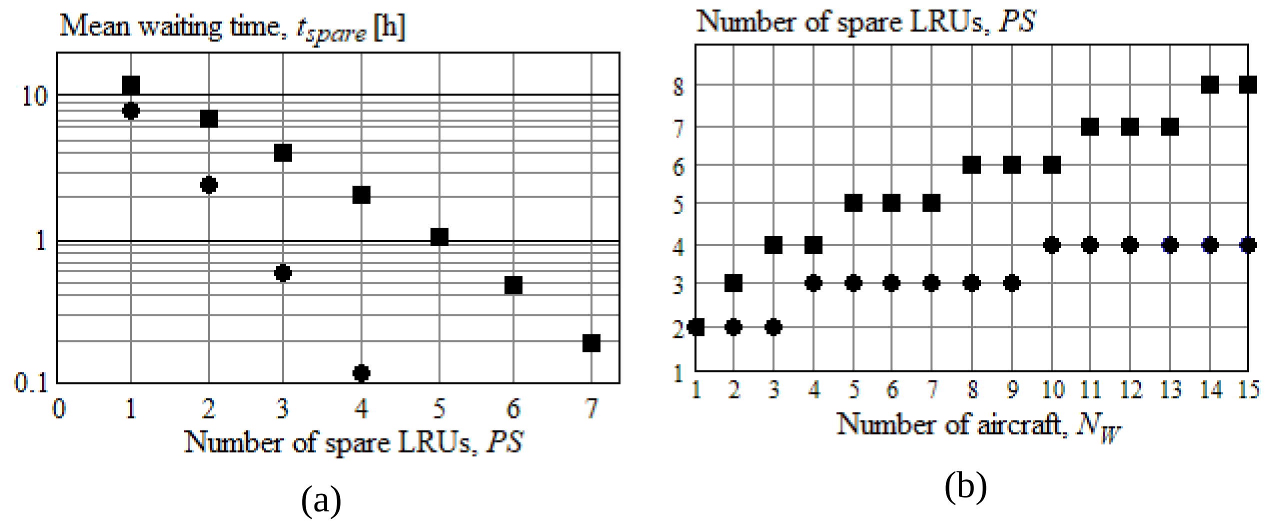


Fig. 2.8. Dependences of the mean waiting time on the number of planned spare LRUs/LRMs (PS) for the first WMO (a) and the number of spare LRUs/LRMs on the number of aircraft in the airline (b). Circles: $\theta = 1 \times 10^{-4}$ h^{-1}; squares: $\theta = 1{\times}10^{-3}$ h^{-1}

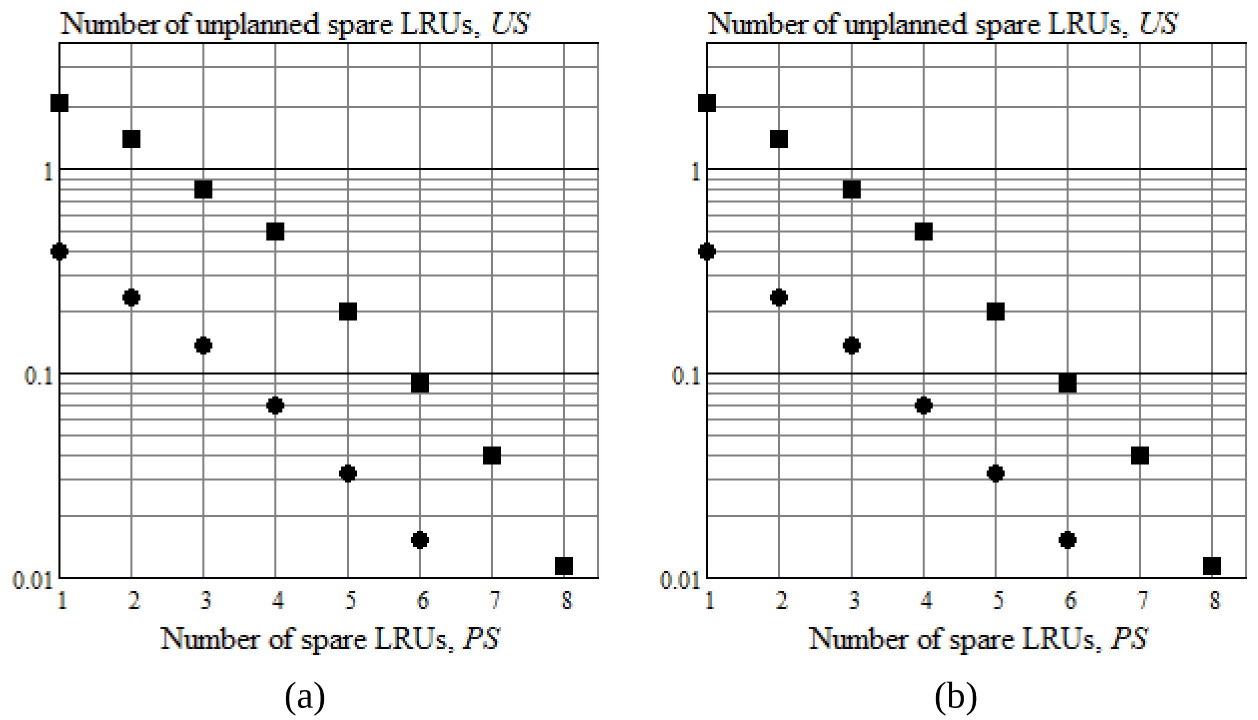


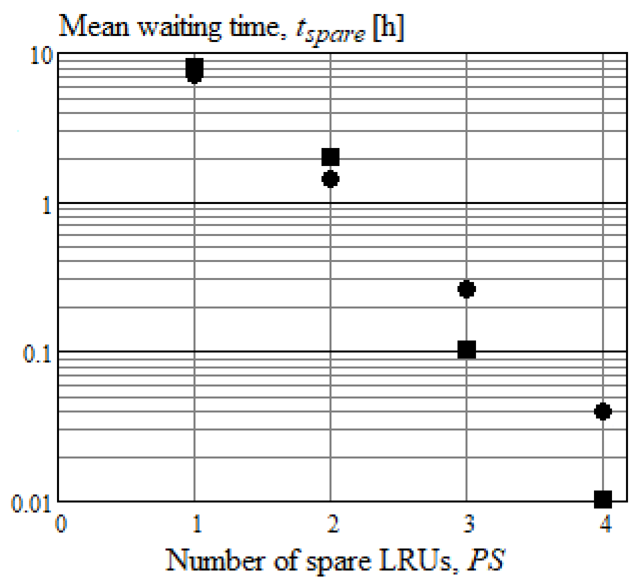
Fig. 2.9. (a) Dependence of the expected unplanned number of spare LRUs/LRMs (US) on the planned number of spare LRUs/LRMs (PS) for the first WMO. Squares: $T_{RS} = T_{ED} =$ 103 h; circles: $T_{RS} = 103$ h and $T_{ED} = 34.3$ h. (b) Dependence of unavailability on the number of spare LRUs/LRMs for a triple redundant avionics system. Circles: $\theta = 1{\times}10^{-4}$ h^{-1}; squares: $\theta = 1{\times}10^{-3}$ h^{-1}

As can be seen in Fig. 2.9 (b), with a high intermittent failure rate, more spare LRUs/LRMs are required to achieve the minimum possible unavailability.

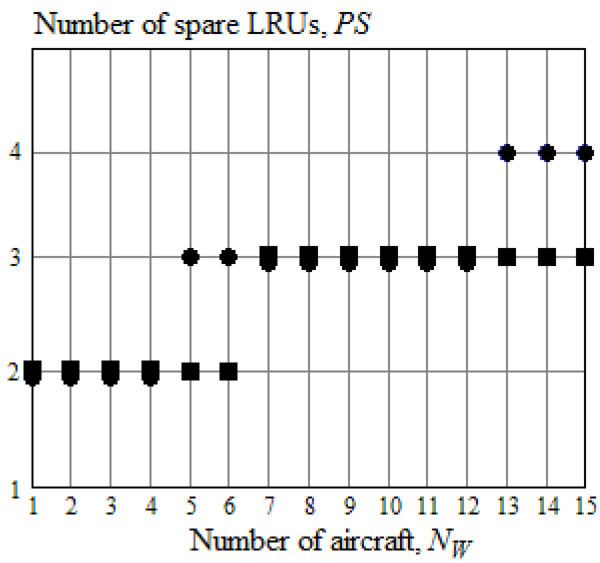
The dependence of the mean waiting time (t_{spare}) on the number of spare LRUs/LRMs (PS) for the second WMO when $N_W = 10$ is shown in Fig. 2.10 (a). As can be seen in Fig. 2.10 (a), t_{spare} has a much weaker dependence on the intermittent failure rate in comparison with the same dependence for the 1st WMO.

The dependence of the number of spare LRUs/LRMs on the number of aircraft operated by the airline for the 2nd WMO is shown in Fig. 2.10 (b). As can be seen in Fig. 2.10 (b), the optimal number of spare LRUs/LRMs is a little dependent on the intermittent failure rate.

Moreover, for $N_W = 1, 2, 3, 7, \ldots, 12$, the number of spare LRUs/LRMs does not depend on θ at all. In addition, the optimal number of spare LRUs/LRMs is significantly lower for the 2nd WMO compared to the 1st WMO, especially in the case of a high rate of θ. Thus, with the use of ground test equipment to recheck dismantled LRUs/LRMs, the number of spare LRUs/LRMs needed to ensure flight regularity reduces.



(a)



(b)

Fig. 2.10. (a) Dependence of the waiting time (t_{spare}) on the number of spare LRUs/LRMs (PS) for the 2nd WMO. Circles: $\theta = 1\times10^{-4}$ h^{-1}; squares: $\theta = 1\times10^{-3}$ h^{-1}. (b) Dependence of the number of spare LRUs/LRMs on the number of aircraft operated by the airline for the second WMO. Circles: $\theta = 1\times10^{-4}$ h^{-1}; squares: $\theta = 1\times10^{-3}$ h^{-1}

2.5.3. Choosing the optimal warranty maintenance option. Let us consider the problem of choosing the optimal WMO through the example of distance measuring equipment (DME). Optimal WMO should ensure the minimum value of the average buyer's costs during the warranty period, i. e.:

$$WTEC_{opt} = \min\left(WTEC_i,\, i = 1,2\right). \tag{2.78}$$

Modern wide-body aircraft, as a rule, have a duplicated DME system on board (m = 2). Suppose that the DME is used on board of a medium-range aircraft. The following parameter values are selected to compare the first and second WMO: $N_W = 10$; T_W = 5,000 h; C_{LRU} = \$ 13,000; LC = \$ 20; τ = 4 h; $t_M^{O-level} = 0.5$ h; $\lambda = 1\times10^{-4}$ h^{-1}; t_{stop} = 1 h; T_{RS} = 103 h; T_{ND} = 34 h; t_{TE} = 3 h; C_{TE} = \$ 23,000.

The results of calculations for the first and second WMO are given in Tables 2.2 and 2.3, respectively.

As can be seen in Table 2.2, when $\theta = 10^{-3}$ h^{-1}, $WTEC_1$ is 40.5 % higher than for $\theta = 10^{-4}$ h^{-1}. Therefore, $WTEC_1$ depends strongly on the intermittent failure rate θ.

Table 2.2. Results of calculations for the first WMO

Intermittent failure rate (h^{-1})	*MTBUR* (h)	The optimal number of spare LRUs (*PS*)	$WTEC_1$ (USD)	Unavailability ($\bar{A}$)
$\theta = 10^{-4}$	3,163	3	3,933	4×10^{-8}
$\theta = 10^{-3}$	907	5	6,612	4×10^{-8}

Table 2.3. Results of calculations for the second WMO

Intermittent failure rate (h^{-1})	*MTBUR* (h)	The optimal number of spare LRUs (*PS*)	$WTEC_2$ (USD)	Unavailability ($\bar{A}$)
$\theta = 10^{-4}$	3,163	3	5,272	4×10^{-8}
$\theta = 10^{-3}$	907	3	5,836	4×10^{-8}

On the contrary, as can be seen in Table 2.3, $WTEC_2$ is less dependent on the intermittent failure rate θ. Also, the number of spare LRUs is the same for both low and high intermittent failure rate θ. This result may be explained by analysing the formulas (A.2.1) and (A.2.3). Indeed, when the intermittent failure rate increases, the rate of unscheduled removal of LRU $\lambda_{i,i+1}$ increases as well. In this case, the mean repair time of the removed LRU ($M[T_R]$) reduces due to a decrease of the probability P_{PF}. As a result, when ground test equipment is used to recheck the dismantled LRUs, the optimal number of spare LRUs may almost not dependent on the intermittent failure rate.

2.6. Post-warranty maintenance model of redundant avionics systems

2.6.1. Analysis of possible post-warranty maintenance options. Small or low-cost airlines with a small number of aircraft may not have enough money to implement O-, I- and D-level maintenance. Therefore, such airlines can eliminate I- and D-level maintenance and use only O-level. Heavy maintenance checks and routines can be transferred to specialized companies, the so-called repair stations. As noted in [14], U.S. airlines outsourced 71 % of heavy maintenance in 2008. Therefore, only the organizational level maintenance (O-level) is considered for the first PWMO. The first PWMO is simple for airlines but may turn to be very expensive. All LRUs/LRMs that are rejected by BITE in flight are shipped to the manufacturer for repair. In this case, the airline must have a relatively large number of spare LRUs/LRMs to ensure flight regularity. This PWMO implies that LRUs/LRMs delivered to the manufacturer will include units with permanent and intermittent failures. The airline will have to pay for repairing the LRUs/LRMs with both failure types. As the warranty period is over, the airline is obliged to pay LRU/LRM repair costs, transportation costs, labour costs, and the cost of spare LRUs/LRMs throughout the post-warranty period. Therefore, the integral maintenance cost indicator for the airline is the total expected maintenance cost during the post-warranty service life of the avionics system, which will be denoted as PWTEC (post-warranty total expected cost). $PWTEC_1$ comprises the following components for the first PWMO: the cost of dismantling and mounting LRUs/LRMs in the aircraft during the post-warranty period; the cost of shipping the dismantled LRUs/LRMs to the manufacturer and back; the cost of LRU/LRM repairs at the manufacturer; the cost of spare LRUs/LRMs per aircraft. $PWTEC_1$ is determined as follows:

$$PWTEC_1 = m\left[LC \times t_M^{O-level} + C_{TR} + C_{IF}P_{IF} + C_{PF}P_{PF}\right]N_R(T_{PW}) + C_{LRU}\left(PS + US\right)/N_{PW}, \quad (2.79)$$

where C_{TR} is the average cost of LRU/LRM shipping to manufacturer and back; C_{IF} and C_{PF} are, respectively, the average cost of repairing the LRU/LRM with intermittent and permanent failure at the manufacturer; T_{PW} is the post-warranty period expressed in flight hours; N_{PW} is the number of aircraft operated in the airline without warranty.

Let's consider three different options of a two-level maintenance system, which includes O- and I-levels. For the first option, the airline uses ground test equipment at the I-level to recheck the dismantled LRUs/LRMs. After rechecking, the LRUs/LRMs with confirmed failures are shipped to the manufacturer or to repair station (in the case of outsourcing) for repair. LRUs/LRMs, whose failures have not been confirmed, are delivered to the warehouse of spare LRUs/LRMs.

For this option of a two-level maintenance system, $PWTEC_2$ includes the following cost components: the cost of O-level maintenance during the post-warranty period; the cost of re-checking the dismantled LRUs/LRMs using ground test equipment; the cost of shipping LRUs/LRMs with permanent failures to the manufacturer and back; the repair cost of failed LRUs/LRMs at the manufacturer; the cost of ground test equipment recalculated per one aircraft; the cost of spare LRUs/LRMs recalculated per one aircraft.

The indicator $PWTEC_2$ is determined as follows:

$$\begin{gathered} PWTEC_2 = m\left[\left(C_{TR} + C_{PF}\right)P_{PF} + LC\left(t_M^{O-level} + t_{TE}^{I-level}\right)\right]N_R(T_{PW}) + \\ C_{TE}^{I-level}/\left(N_{PW} \times F_{TE}^{I-level}\right) + C_{LRU}\left(PS + US\right)/N_{PW}. \end{gathered} \quad (2.80)$$

In the second option of the two-level maintenance system, the I-level maintenance uses a ground ATE to recheck the dismantled LRUs/LRMs and detect the failure location with depth up to the faulty SRU. As noted in Chapter 1, conventional ATE is not capable of detecting intermittent failures. Therefore, dismantled LRUs/LRMs with intermittent failures after rechecking by ATE will be delivered to the warehouse of spare LRUs/LRMs. Repair of LRUs/LRMs with permanent failures is carried out by replacing the faulty SRUs. After identifying the faulty SRUs, they are shipped to the manufacturer or the repair station (in the case of outsourcing) for repair. $PWTEC_3$ indicator represents the sum of the following cost components: the cost of O-level maintenance during post-warranty period; the cost of rechecking the dismantled LRUs/LRMs using ATE and detection of the faulty SRUs; the cost of shipping the faulty SRUs to the manufacturer, as well as the repaired SRUs back to the airline; the cost of SRU repairs at the manufacturer; the cost of ATE and spare LRUs/LRMs and SRUs recalculated per one aircraft.

$PWTEC_3$ is defined as follows:

$$PWTEC_3 = m\left[LC\left(t_M^{O-level} + t_{ATE}^{I-level}\right) + \left(C_{PF,R} + LC \times t_{PF,D}^{I-level}\right)P_{PF} + C_{TR,SRU}\right]N_R\left(T_{PW}\right) + C_{ATE}^{I-level} \Big/ \left(N_{PW} \times F_{ATE}^{I-level}\right) + \left[C_{LRU}\left(PS + US\right) + \sum_{j=1}^{n} C_j SRU_j\right] \Big/ N_{PW}, \tag{2.81}$$

where $t_{ATE}^{I-level}$ is the average time of rechecking the LRU/LRM using ATE at the I-level maintenance; $C_{PF,R}$ is the average cost of repairing the SRU with permanent failure; $t_{PF,D}^{I-level}$ is the average time to detect a permanent failure in the dismantled LRU/LRM with depth to SRU using ATE; $C_{TR,SRU}$ is the average cost of shipping the faulty SRU to the manufacturer and back; $C_{ATE}^{I-level}$ is the cost of ATE used at the I-level maintenance; $F_{ATE}^{I-level}$ is the number of LRUs that can be rechecked using ATE; C_j is the cost of the *j*-th SRU ($j = \overline{1,n}$); SRU_j is the number of spare SRUs of the *j*-th type; *n* is the number of SRU types in the examined LRU/LRM.

The number of spare SRUs can be calculated from the condition of guaranteed provision of all LRUs/LRMs (on-board and spare) with spare SRUs at a high probability (0.95–0.99) [13]. In the case of the simplest flow of failures, the optimal number of spare SRUs of the *j*-th type is determined by the Poisson formula as the minimal integer number SRU_j satisfying the following inequality [13]:

$$1 - P\left(\mathrm{H}_j\right) > \frac{\left(\mathrm{H}_j \lambda_j t_{RS,j}\right)^{(SRU_j + 1)}}{\left(SRU_j + 1\right)!} \exp\left(-\mathrm{H}_j \lambda_j t_{RS,j}\right), \tag{2.82}$$

where $\mathrm{H}_j = mN_{PW} + PS_j + US_j$ is the total number of SRUs of the *j*-th type installed on the on-board and spare LRUs/LRMs; $P(\mathrm{H}_j)$ is the probability that all LRUs/LRMs will be provided with SRUs of the *j*-th type; λ_j is the rate of permanent failures of *j*-th type SRUs; $t_{RS,j}$ is the average repair time of the *j*-th type SRU at the manufacturer.

In the third option of the two-level maintenance system, both ATE and IFD are used at the I-level to recheck the dismantled LRUs/LRMs and detect faulty SRUs. The combination of ATE and IFD allows not only check the dismantled LRUs/SRUs for permanent and intermittent failures but also detect faulty SRUs. Repair of LRUs/LRMs with permanent and intermittent failures is performed by replacing the identified faulty SRUs. Further, the airline will ship the SRUs with detected permanent and intermittent failures to the manufacturer for repair. It should be noted that IFD can also be used to detect intermittent failures in SRUs with the depth up to non-repairable elements.

The indicator $PWTEC_4$ comprises the same cost components as $PWTEC_3$, as well as the additional cost of IFD and operations to detect SRUs with intermittent failures, and is determined by the following formula [6]:

$$PWTEC_4 = m\left[LC\left(t_M^{O-level} + t_{ATE}^{I-level}\right) + \left(C_{PF,R} + LC \times t_{PF,D}^{I-level}\right)P_{PF} + \left(C_{IF,R} + LC \times t_{IF,D}^{I-level}\right)P_{IF} + C_{TR,SRU}\right]N_R\left(T_{PW}\right) + C_{ATE}^{I-level}\Big/\left(N_{PW} \times F_{ATE}^{I-level}\right) + C_{IFD}\Big/\left(N_{PW} \times F_{IFD}^{I-level}\right) + \left[C_{LRU}\left(PS + US\right) + \sum_{j=1}^{n} C_j SRU_j\right]\Big/ N_{PW}, \tag{2.83}$$

where $C_{IF,R}$ is the average cost of the SRU repair due to intermittent failure at the manufacturer; $t_{IF,D}^{I-level}$ is the average time to detect the location of an intermittent failure in the dismantled LRU with depth up to SRU using IFD; C_{IFD} is the IFD cost; $F_{IFD}^{I-level}$ is the number of LRU types that can be tested using IFD to detect intermittent failures.

By analogy with (2.82), the optimal number of spare SRUs of the j-th type is determined as the minimal integer number SRU_j satisfying the following inequality [6]:

$$1 - P\left(\mathrm{H}_j\right) > \frac{\left[\mathrm{H}_j\left(\lambda_j + \theta_j\right)t_{RS,j}\right]^{(SRU_j+1)}}{\left(SRU_j + 1\right)!}\exp\left[-\mathrm{H}_j\left(\lambda_j + \theta_j\right)t_{RS,j}\right], \tag{2.84}$$

where θ_j is the intermittent failure rate of the j-th type SRU.

Let's consider one of the possible options of realization of a three-level maintenance system, including O-, I- and D-level maintenance. As previously mentioned, D-level maintenance is carried out in specialized repair back-shops equipped with diagnostic tools capable of detecting faulty non-repairable electronic component or a group of elements in a printed circuit board (SRU) rejected at I-level. Maintenance at the D-level can function successfully if there is a sufficient number of spare non-repairable electronic components in the relevant warehouse. The indicator $PWTEC_5$ includes the following cost components: the cost of dismantling and mounting the LRUs/LRMs on-board the aircraft during the post-warranty period; the cost of rechecking the dismantled LRUs/LRMs using ATE and identification of faulty SRUs at the I-level maintenance; the cost of diagnosing and repairing SRUs at the D-level maintenance; the cost of ATE, IFD, spare LRUs/LRMs, SRUs, and non-repairable electronic components recalculated per one aircraft.

$PWTEC_5$ is calculated using the following formula:

$$PWTEC_5 = m\Big\{LC\left(t_M^{O-level} + t_{ATE}^{I-level}\right) + \left[LC\left(t_{PF,D}^{I-level} + t_{PF,D}^{D-level}\right) + C_{PF,R}^{D-level}\right]P_{PF} + \left[\left(LC\left(t_{IF,D}^{I-level} + t_{IF,D}^{D-level}\right) + C_{IF,R}^{D-level}\right)P_{IF}\right]\Big\}N_R\left(T_W\right) + C_{ATE}^{I-level}\Big/\left(N_{PW}\times F_{ATE}^{I-level}\right) + C_{IFD}^{I-level}\Big/\left(N_{PW}\times F_{IFD}^{I-level}\right) + C_{DRT}^{D-level}/Z_{DRT}^{D-level} + \left[C_{LRU}\left(PS+US\right) + \sum_{j=1}^{n} C_j SRU_j + \sum_{l=1}^{n}\sum_{q=1}^{SRU_l} C_{l,q}X_{l,q}\right]\Big/N_{PW}, \tag{2.85}$$

where $t_{PF,D}^{D-level}$ is the average time to detect a permanent failure in the SRU with depth to one or more non-repairable electronic components and replace them at the D-level maintenance; $t_{IF,D}^{D-level}$ is the average time to detect an intermittent failure in the SRU with depth to one or more non-repairable electronic components and replace them at the D-level maintenance; $C_{PF,R}^{D-level}$ is the average cost of replaced non-repairable components when repairing the SRU with a permanent failure at the D-level maintenance; $C_{IF,R}^{D-level}$ is the average cost of replaced non-repairable components when repairing the SRU with an intermittent failure at the D-level maintenance; $C_{DRT}^{D-level}$ is the cost of diagnostics and repair equipment used at the D-level maintenance; $Z_{DRT}^{D-level}$ is the number of SRU types repaired at the D-level maintenance; $C_{l,q}$ $(l=\overline{1,n}, q=\overline{1,SRU_l})$ is the cost of a spare electronic component of the q-th type in the l-th SRU; $X_{l,q}$ is the number of spare non-repairable components of the q-th type in the l-th SRU.

To determine the optimal number of spare LRUs/LRMs, we will use the warehouse management model of LRUs/LRMs as shown in Fig. A.2.1. For all PWMOs, we calculate the rates $\lambda_{i,i+1}$ and $\mu_{i+1,i}$ by formulas (A.2.1) and (A.2.2). However, the LRU/LRM average repair time ($E[T_R]$) is calculated using different formulas. So, for the 1st and 2nd PWMO, the indicator $E[T_R]$ is determined by (A.2.3), and for the 3rd PWMO - by the formula given below as it follows from (2.83):

$$E\left[T_R\right]=t_{ATE}^{I-level}+t_{PF,D}^{I-level}P_{PF}. \tag{2.86}$$

Further, for the 4th and 5th PWMO, as it follows from (2.83) and (2.85), the indicator $E[T_R]$ is determined by the following formula:

$$E\left[T_R\right]=t_{ATE}^{I-level}+t_{PF,D}^{I-level}P_{PF}+t_{IF,D}^{I-level}P_{IF}. \tag{2.87}$$

Since the 5th PWMO implies repairing the faulty SRUs at the D-level of maintenance, and not shipping them to the manufacturer for repair, the optimal number of spare SRUs is determined from the following inequality:

$$1-P\left(\mathrm{H}_j\right)>\frac{\left[\mathrm{H}_j\left(\lambda_j+\theta_j\right)\left(t_{PF,D}^{D-level}P_{PF}+t_{IF,D}^{D-level}P_{IF}\right)\right]^{(SRU_j+1)}}{\left(SRU_j+1\right)!}\exp\left[-\mathrm{H}_j\left(\lambda_j+\theta_j\right)\left(t_{PF,D}^{D-level}P_{PF}+t_{IF,D}^{D-level}P_{IF}\right)\right]. \tag{2.88}$$

Let us calculate the optimal number of spare LRUs for different PWMOs when $T_{PW}=40{,}000$ h, $N=10$, $m=3$, $t_M^{O-level}=0.5$ h, $t_{TE}^{I-level}=t_{ATE}^{I-level}=3$ h, $t_{PF,D}^{I-level}=t_{IF,D}^{I-level}=2$ h, $\lambda=1\times10^{-4}$ h^{-1}, $T_{RS}=103$ h, $T_{ND}=34$ h, and $t_{STOP}=1$ h.

Figures 2.11 and 2.12 show the dependence of the optimal number of planned spare LRUs/LRMs on the number of aircraft owned by the airline from the 1st to the 5th PWMO, respectively. From a comparison of the graphs in Figs. 2.11 and 2.12 follows that the optimal number of spare LRUs/LRMs is significantly less for the 2nd and 3rd than for the 1st PWMO. Also, the optimal number of spare LRUs/LRMs for the 3rd PWMO is practically independent of the rate of intermittent failures.

As can be seen in Fig. 2.12 (b), the number of spare LRUs/LRMs does not depend on the intermittent failure rate for $N=1,6\ldots15$. When $N=2\ldots5$, the number of spare LRUs/LRMs for $\theta=1\times10^{-3}$ h^{-1} is twice larger than that for $\theta=1\times10^{-4}$ h^{-1}. Regarding the dependences of $PS(N)$ presented in Fig. 2.12 (a, b), we can observe that the 3rd PWMO is less sensitive to changes in the intermittent failure rate than the 4th and 5th PWMO. Besides, $PS=1$ when $N=\overline{1,8}$ for both values of θ for the 3rd PWMO, and $PS=1$ only when $N=\overline{1,5}$ for the 4th and 5th PWMO. The latter is due to the fact that when $\theta>0$ the average repair time is longer for the 4th and 5th than for the 3rd PWMO. This conclusion can be drawn from a comparison of formulas (2.86) and (2.87).

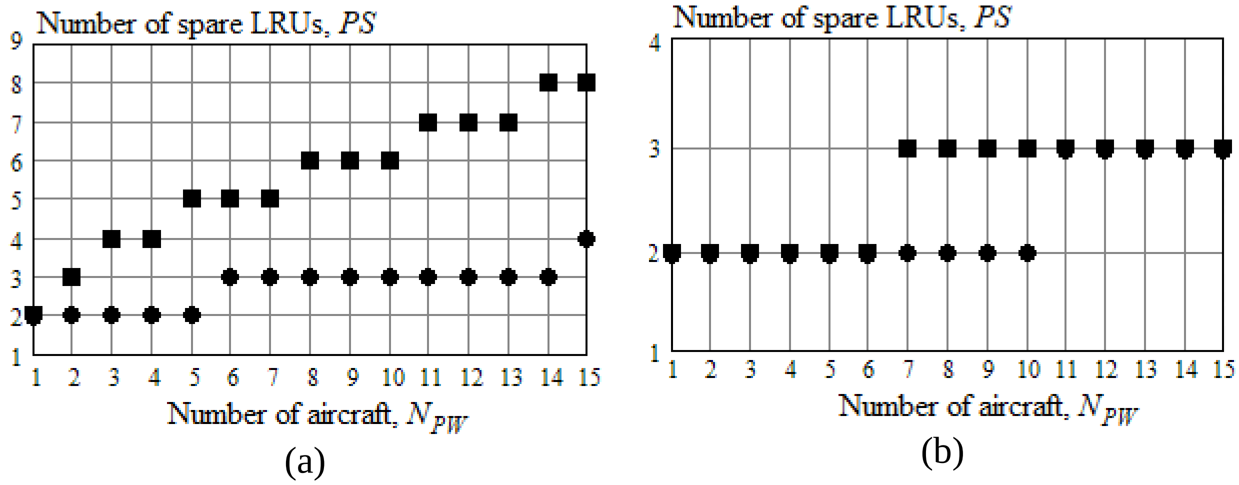


Fig. 2.11. Dependence of the number of spare LRUs/LRMs on the number of aircraft for the 1st (a) and 2nd (b) PWMO. Circles: θ = 1×10^{-4} h^{-1}; squares: θ = 1×10^{-3} h^{-1}

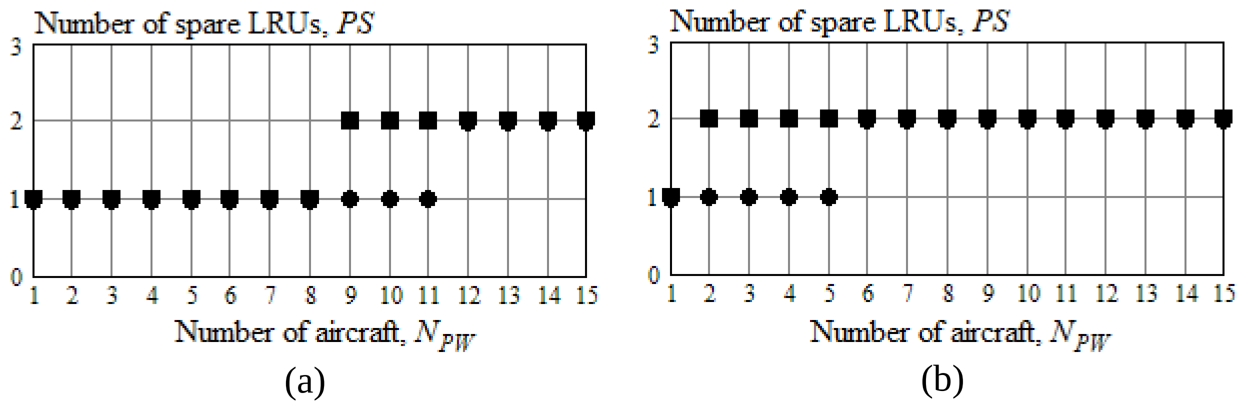


Fig. 2.12. Dependence of the number of spare LRUs/LRMs on the number of aircraft for the 3rd (a), 4th and 5th (b) PWMO. Circles: θ = 1×10^{-4} h^{-1}; squares: θ = 1×10^{-3} h^{-1}

2.6.2. Choosing the optimal option of the post-warranty maintenance. The optimal PWMO should satisfy the following criterion:

$$PWTEC_{opt} = \min\left(PWTEC_i,\ i = \overline{1,5}\right). \tag{2.89}$$

Let us consider an example of choosing the optimal PWMO for DME. Suppose, as before, that DME is used in a medium-range aircraft. The considered DME type has a modular design and comprises the following three SRUs: transmitter module (TM), receiver module (RM) and electronic power supply module (EPSM). The following parameter values are chosen to be the same for all compared PWMO: $N = 15$; $T_{PW} = 40{,}000$ h; τ = 4 h; $C_{TR} = \$\,120$; $C_{LRU} = \$\,13{,}000$; $LC = \$\,20$; $t_M^{O-level} = 0.5$ h; λ = 1 × 10^{-4} h^{-1}; $t_{stop} =$ 1 h; $T_{ED} = 34.3$ h. For the first PWMO, we assume that $T_{RS} = 103$ h, $C_{IF} = \$\,1{,}300$ and

$C_{PF} = \$\ 2{,}600$. For the 1st, 2nd and 3rd PWMO, we assume that θ = 1×10^{-4} h^{-1}. The use of IFD in the repair of on-board electronic LRUs/LRMs allows the identification and replacement of electronic components and connections that cause intermittent failures. As noted in Chapter 1, the intermittent failure rate of LRUs, which have been repaired using IFD, is significantly reduced. Therefore, in further calculations, we assume that the intermittent failure rate is less by a factor of 10, i. e. θ = 1×10^{-5} h^{-1}, for the 4th and 5th PWMO. For the 2nd PWMO, the parameter values related to ground ATE are the same as for the 2nd WMO, i. e. $F_{TE}^{I-level} = 2$, $t_{TE}^{I-level} = 3$ h and $C_{TE}^{I-level} = \$\ 23{,}000$. The following data were used for the 3rd and 4th PWMO: $F_{ATE}^{I-level} = 10$; $C_{ATE}^{I-level} = \$\ 200{,}000$; $F_{IFD}^{I-level} = 50$; $C_{IFD}^{I-level} = \$\ 150{,}000$; $t_{ATE}^{I-level} = 0.5$ h; $t_{PF,D}^{I-level} = 0.25$ h; $t_{IF,D}^{I-level} = 0.5$ h; $C_{PF,R} = C_{IF,R} = \$\ 400$; $C_{TR,SRU} = \$\ 70$.

Table 2.4 shows the initial data for SRUs. The symbols in Table 3.4 denote the following parameters of the j-th ($j = \overline{1,n}$) SRU: λ_j is the permanent failure rate; θ_j is the intermittent failure rate; C_j is the cost; $t_{RS,j}$ is the average repair time at the manufacturer. For the 3rd and 4th PWMO, $t_{RS,j} = 103$ h. The average repair time of SRUs for the 5th PWMO is included into (2.86).

Table 2.4. SRU initial data

SRU name	λ_j (h^{-1})	θ_j (h^{-1})	C_j ($)
TM (j = 1)	6.7 × 10^{-5}	0.7 × 10^{-5}	4700
RM (j = 2)	2.3 × 10^{-5}	0.2 × 10^{-5}	7000
EPSM (j = 3)	1 × 10^{-5}	0.1 × 10^{-5}	1300

The following initial data relating to the D-level maintenance are used for the 5th PWMO: $\sum_{l=1}^{n}\sum_{q=1}^{SRU,l} C_{l,q}X_{l,q} = \$\ 12{,}000$; $C_{PF,R}^{D-level} = C_{PF,R}^{D-level}$ =$ 150; $t_{PF,D}^{D-level} = t_{IF,D}^{D-level} = 1$ h; $C_{DRT}^{D-level}$= $ 200,000; $Z_{DRT}^{D-level} = 250$. The rest of the initial data have the same values as for the 3rd and 4th PWMO.

Table 2.5 presents the calculation results. The unplanned number of spare LRUs (US) is zero for all PWMO. The number of spare SRUs (SRU_j) was calculated for $P(\mathrm{H}_j) =$

0.99 ($j = 1,2,3$). Using the inequalities (2.82), (2.84) and (2.88), we obtain that $F_1 = 2$ and $F_2 = F_3 = 1$ for the 3rd and 4th PWMO, and $F_1 = F_2 = F_3 = 1$ for the 5th PWMO.

As follows from Table 2.5, the 5th PWMO has the smallest value of PWTEC. Therefore, the 5th PWMO is the best choice regarding the initial data used. Indeed, $PWTEC_5$ is 4.6 times less than $PWTEC_1$ and over three times less than $PWTEC_2$.

Table 2.5. Calculation results

PWMO (i)	$E[TBUR]$ (h)	The optimal number of spare LRUs (PS)	$PWTEC_i$ ($)	Unavailability ($\bar{A}$)
$i = 1$	5,000	3	35,880	4×10^{-8}
$i = 2$	5,000	2	25,380	4×10^{-8}
$i = 3$	5,000	1	10,400	4×10^{-8}
$i = 4$	8,982	1	10,200	4×10^{-8}
$i = 5$	8,982	1	7,778	4×10^{-8}

It should also be noted that the 3rd and 4th PWMO have almost the same PWTEC values, which are respectively 34 % and 31 % higher than $PWTEC_5$. The 5th PWMO requires the least number of spare parts, namely one LRU and one SRU of each type.

2.7. Minimizing the total expected costs of avionics systems' maintenance during the service life

Consider the task of minimizing the total expected costs of a redundant avionics system, including m LRUs/LRMs, over its service life. The service life of any technical device is defined as the period from the beginning of its operation to the point of discard. We denote the avionics system service life as T_{LT}. Obviously, the service life includes warranty and post-warranty periods. Therefore, T_{LT} can be defined in the following form:

$$T_{LT} = T_W + T_{PW} \,. \tag{2.90}$$

In subsections 2.5 and 2.6, we analysed two WMOs and five PWMOs. In this way, theoretically, ten different combinations of maintenance options can be obtained for the entire service life. The combination of WMO and PWMO must satisfy the following criterion:

$$TEC_{(i,j)opt} = \min\left\{WTEC_i + PWTEC_j; i = 1, 2; j = \overline{1,5}\right\}, \qquad 2.91)$$

where $TEC_{(i,j)opt}$ is the minimum value of the total expected costs.

Table 2.6 presents the calculated values of $TEC_{i,j}$ for various combinations of $WTEC_i$ and $PWTEC_j$.

Table 2.6. Calculated values $TEC_{i,j}$

$TEC_{i,j}$	$PWTEC_1$	$PWTEC_2$	$PWTEC_3$	$PWTEC_4$	$PWTEC_5$
$WTEC_1$	$TEC_{1,1}=\$39{,}813$	$TEC_{1,2}=\$29{,}313$	$TEC_{1,3}=\$14{,}333$	$TEC_{1,4}=\$14{,}133$	$\mathbf{TEC_{1,5}=\$11{,}711}$
$WTEC_2$	$TEC_{2,1}=\$41{,}152$	$TEC_{2,2}=\$30{,}652$	$TEC_{2,3}=\$15{,}672$	$TEC_{2,4}=\$15{,}472$	$TEC_{2,5}=\$13{,}050$

As can be seen in Table 2.6, the combination of the 1st WMO and the 5th PWMO is optimal. It should also be noted that the unavailability value is the same for all maintenance options. Therefore, option $(1, 5)$ is indeed optimal.

2.8. Conclusions

1. A new mathematical model has been developed to evaluate the operational reliability indicators of continuously monitored LRUs/LRMs over a finite and an infinite time interval, which, unlike the known models, considers the impact of both permanent and intermittent failures.

2. The generalized analytic expressions (2.4), (2.6), (2.11), (2.13), (2.22), (2.47)–(2.50), and (2.52) have been developed for mean times spent by the LRU/LRM in different operating states in the case of an arbitrary distribution of operating time to permanent and intermittent failures over a finite and an infinite time intervals, which allows calculation of availability and MTBUR.

3. The mathematical expressions (2.16)–(2.19), (2.23), (2.48)–(2.51) and (2.53) have been derived to calculate the mean times spent by the LRU/LRM in different states,

which are used to calculate the availability and MTBUR for cases of an exponential distribution of the operating time to permanent and intermittent failures over a finite and an infinite time interval. The numerical example shows that MTBUR decreases rapidly as the intermittent failure rate increases.

4. The relations (2.32)–(2.35) and (2.54)–(2.56) have been proved, which allow determining the ORF and the probability of unscheduled repair of LRU/LRM for an arbitrary distribution of the operating time to permanent and intermittent failures over a finite and an infinite time interval.

5. The relationships (2.42)–(2.44) and (2.57)–(2.59) have been derived to calculate the ORF and the probability of unscheduled LRU/LRM repair for an exponential distribution of the operating time to permanent and intermittent failures over a finite and an infinite time interval.

6. The relationships (2.62)–(2.65) have been developed to calculate the average number of unscheduled removals due to permanent and intermittent failures and the average cost of LRU repairs during a specified period; the numerical examples show that the average number of unscheduled removals and the average cost of LRU/LRM repairs sharply increase when the intermittent failure rate exceeds 10^{-4} h^{-1}.

7. The relations (2.66), (2.68), (2.70), and (2.72) have been developed to calculate the availability of redundant avionics systems with a parallel, majority, and parallel-series redundancy structure, and an illustrative numerical example shows that the unavailability of a duplicated avionics system begins to increase sharply when the intermittent failure rate exceeds 10^{-4} h^{-1}.

8. The equations (2.74) and (2.76) have been developed to calculate the total operating costs during the warranty period for two WMOs that are distinguished by the presence or absence of ground test equipment at the I-level maintenance. A numerical example shows that the use of the test equipment can almost eliminate the harmful effect of intermittent failures on the number of spare LRUs/LRMs and maintenance costs.

9. The generalized relations (2.79) - (2.81), (2.83) and (2.85) have been developed to calculate the average operating costs during the post-warranty period for five alternative maintenance options that differ by the existence of I- and D-level maintenance. A numerical

example shows that a three-level post-warranty maintenance system is optimal for the selected source data since it results in a minimum number of spare LRUs and SRUs, as well as minimum operating costs that are 4.6 times less than for a single-level maintenance system and over 30 % less than for any two-level maintenance system.

10. The criterion (2.91) to minimize the total expected cost of avionics systems' maintenance during the service life has been formulated, which consists in a comparative analysis of all possible combinations of warranty and post-warranty maintenance options and the selection of such combination that provides the lowest expected maintenance cost. The numerical example shows that the combination of single-level warranty maintenance (O-level) and three-level post-warranty maintenance (O-, I- and D-levels) is optimal for the selected initial data.

REFERENCES

1. M. Kayton, M. Avionics navigation systems: monograph / M. Kayton, W. R. Friend. 2nd ed. – New York: John Wiley&Sons Inc, 1997. – 800 p.
2. Hess, R. Electromagnetic environment/ R. Hess// Digital avionics handbook. – 3rd ed.: coll. monogr. – London: CRC Press, 2015. – P. 87–102.
3. ARINC Report 672. Guidelines for the reduction of no fault found (NFF). – 2008.
4. Raza, A. Maintenance model of digital avionics/ A. Raza// Aerospace. – 2018. – V. 5(2). – P. 1–16.
5. Raza, A. Minimizing total lifecycle expected costs of digital avionics' maintenance/ A. Raza, V. Ulansky// Procedia CIRP. – 2015. – V. 38. – P. 118–123.
6. Raza, A. Assessing the impact of intermittent failures on the cost of digital avionics' maintenance/ A. Raza, V. Ulansky// 2016 IEEE Aerospace conference, 5–12 March, 2016, Big Sky, Montana, USA. – Conference proceedings, 2016. – P. 1–16.
7. Drenick, R. F. The failure law of complex equipment/ R. F. Drenick// Journal of the society for industrial & applied mathematics. – 1960. - V. 8(4). – P. 680–690.
8. Reliability analysis in the commercial aerospace industry/ J. Qin, B. Huang, J. Walter, and et al.// The Journal of the Reliability Analysis Center. – 2005. – V. 13, № 1. – P. 1-5.

9. Raza, A. Cost model for assessing losses to avionics suppliers during warranty period/A. Raza, V. Ulansky//Advances in through-life engineering services. Decision engineering: coll. monogr. - Switzerland: Springer, 2017. – P. 291-307.
10. Раза, А. Математическая модель обслуживания цифровых систем авионики с учетом перемежающихся отказов/ А. Раза// Математические машины и системы. - № 1, 2018. – С.137-148.
11. Raza, A. Modelling of operational reliability and maintenance cost for avionics systems with permanent and intermittent failures/ A. Raza, V. Ulansky// Proceedings of the 9th IMA international conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. – P. 186-192.
12. Barlow, R. Statistical theory of reliability and life testing: probability models/ R. Barlow, F. Proshan. – Canada: Holt, Rinehart & Winston, 1975. – 435 p.
13. Уланский, В. В. Организация системы технического обслуживания и ремонта радиоэлектронного комплекса Ту-204: учебное пособие/В. В. Уланский, Г. Ф. Конахович, И. А. Мачалин. – К.: КИИГА, 1992. – 110 с.
14. McFadden, M. Global outsourcing of aircraft maintenance/ M. McFadden, D. S. Worrells// Journal of aviation technology and engineering. – 2012. – V. 1, № 2. – P. 63–73.

CHAPTER 3:
CONDITION-BASED MAINTENANCE MODELS OF DEGRADING SYSTEMS

3.1. Statement of tasks

As noted in Chapter 1, the goals of the CBM are to minimize the volume of heavy maintenance, increase aircraft availability, improve safety and reduce the cost of maintenance. To achieve these objectives, some tasks must be solved, among which the problems of determining the optimal time intervals for maintenance or replacement of equipment components subject to wear and degradation are the most relevant.

The analysis of CBM models presented in Chapter 1 shows that the published maintenance models with imperfect inspections incorporate a decision rule, which is aimed to reject the systems that are inoperable at the time of condition monitoring. The drawback of this decision rule is the impossibility of rejecting the systems that may fail in the period before the next inspection time. Therefore, subsection 3.2 develops a mathematical model of CBM, which makes it possible to substantially reduce the probability of system failure in the interval between inspections due to the rejection of potentially unreliable systems. To achieve this effect, in addition to the functional failure threshold, a replacement threshold is introduced for each inspection time. A new decision rule is proposed for checking the system condition, which is based on comparing the time of the check with the time estimate to the pre-failure state. Following this decision rule, general expressions are derived to calculate the probabilities of correct and incorrect decisions when checking system suitability, considering the results of previous inspections. The optimality criteria such as maximum net income, minimum Bayesian risk, maximum a posteriori probability, and minimum total error probability are used to determine optimal replacement thresholds.

In subsection 3.3, a mathematical model is developed that allows simultaneously to determine the optimal inspection times and replacement thresholds for the case of imperfect inspections. The model is based on the properties of the regenerative stochastic process of changing the system states. When checking the system suitability, the same decision rule is used as in subsection 3.2. Conditional probabilities of correct and incorrect decisions are determined when checking the system suitability for failure-free operation in the forthcoming time interval, which is necessary to calculate the mean time spent by the system in different states. We consider the system operation and maintenance over a finite time interval. The developed mathematical model supposes the use of both a sequential and periodic inspection schedule. The proposed criteria for optimizing the inspection schedule include maximum availability, minimum average costs when operating the system, and minimum average maintenance costs with a restriction on ORF.

3.2. A mathematical model of CBM to determine the optimal replacement thresholds on an infinite interval of system operation

3.2.1. The decision rule when checking the system suitability for use in the forthcoming interval of operation. This subsection considers a degrading system subjected to random failures. We assume that the system state parameter $L(t)$, which is a non-stationary stochastic process with continuous time, completely identifies the system condition. The system is monitored at successive times t_k (k = 1, 2, ...), where $t_0 = 0$. If the system state parameter value exceeds the functional failure threshold FF, the system goes into a failed state. If there is a measurement error (or noise) $Y(t_k)$, the measurement result $Z(t_k)$ is related to the true value $L(t_k)$ as follows:

$$Z(t_k) = L(t_k) + Y(t_k)\,. \tag{3.1}$$

Figure. 3.1 shows a typical realization of the stochastic process $L(t_k)$, measured at time points t_k $(k = \overline{1, j})$.

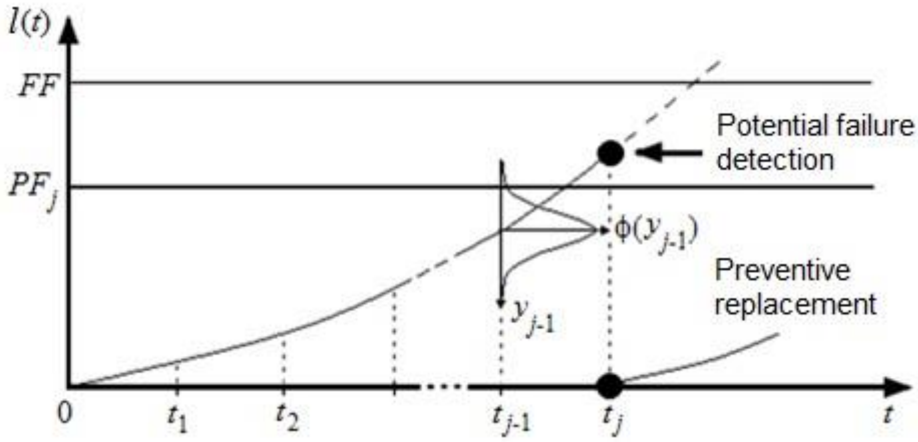


Fig. 3.1. A realization of the stochastic process $L(t)$ measured at times t_k (k = 1, 2, ..., j) with an error y_k having a PDF φ (y_k)

We introduce the following decision rule when checking the system condition at time t_k. If $z(t_k) < PF_k$, the system is judged as suitable for operation over the time interval (t_k, t_{k+1}); if $z(t_k) \geq PF_k$ the system is judged as unsuitable and excluded from operation over the time interval (t_k, t_{k+1}), where PF_k ($PF_k < FF$) is the replacement threshold at time t_k. As $PF_k < FF$, this decision rule is aimed to reject the systems that can fail over the time interval between inspections.

With the introduced decision rule, two system repair or replacement strategies are possible. If $PF_k \leq Z(t_k) < FF$, then a preventive repair or replacement of the system is carried out at time t_k. If $Z(t_k) \geq FF$, then a corrective repair or replacement of the system is performed at time t_k. We assume that any repair or replacement leads to a complete renewal of the system, i. e. the system becomes as good as new.

3.2.2. The space of events. Regarding the system suitability for operation over the time interval (t_k, t_{k+1}), when checking the parameter $L(t)$ at time t_k, one of the following mutually exclusive events might occur [1]:

$$\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L(t_{k+1})<FF\bigcap\left[\bigcap_{i=1}^{k}Z\left(t_i\right)<PF_i\right]\right\}, \tag{3.2}$$

$$\Gamma_2\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L(t_{k+1})<FF\bigcap Z(t_k)\geq PF_k\bigcap\left[\bigcap_{i=1}^{k-1}Z\left(t_i\right)<PF_i\right]\right\}, \tag{3.3}$$

$$\Gamma_3\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L(t_k)<FF\bigcap X\left(t_{k+1}\right)\geq FF\bigcap\left[\bigcap_{i=1}^{k}Z(t_i)<PF_i\right]\right\}, \tag{3.4}$$

$$\Gamma_4\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L\left(t_k\right)<FF\bigcap L\left(t_{k+1}\right)\geq FF\bigcap Z\left(t_k\right)\geq PF_k\bigcap\left[\bigcap_{i=1}^{k-1}Z(t_i)<PF_i\right]\right\}, \tag{3.5}$$

$$\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L(t_k)\geq FF\bigcap\left[\bigcap_{i=1}^{k}Z(t_i)<PF_i\right]\right\}, \tag{3.6}$$

$$\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{L(t_k)\geq FF\bigcap Z(t_k)\geq PF_k\bigcap\left[\bigcap_{i=1}^{k-1}Z\left(t_i\right)<PF_i\right]\right\}, \tag{3.7}$$

where $\Gamma_1(\overline{t_1,t_k};t_{k+1})$ is the joint occurrence of the following events: the system is suitable for operation over the time interval $(t_{k,}t_{k+1})$ and is judged as suitable at inspection times $\overline{t_1,t_k}$; $\Gamma_2(\overline{t_1,t_k};t_{k+1})$ is the joint occurrence of the following events: the system is suitable for operation over the time interval $(t_{k,}t_{k+1})$, is judged as suitable at inspection times $(t_{1,}t_{k-1})$, and is judged as unsuitable at inspection time t_k; $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ is the joint occurrence of the following events: the system is operable at inspection time t_k, fails within interval (t_k, t_{k+1}), and is judged as suitable at inspection times $\overline{t_1, t_k}$; $\Gamma_4(\overline{t_1, t_k}; t_{k+1})$ is the joint occurrence of the following events: the system is operable at inspection time t_k, fails during interval (t_k, t_{k+1}), is judged as suitable at inspection times $\overline{t_1, t_{k-1}}$, and is judged as unsuitable at inspection time t_k; $\Gamma_5(\overline{t_1, t_k}; t_{k+1})$ is the joint occurrence of the following events: the system has failed until inspection time t_k and has been judged suitable at inspection times $\overline{t_1, t_k}$; $\Gamma_6(\overline{t_1, t_k}; t_{k+1})$ is the joint occurrence of the following events: the system has failed until inspection time t_k, judged to be suitable at inspection times $\overline{t_1, t_{k-1}}$ and unsuitable at time t_k.

Figure 3.2 shows the graph of decision making when checking system suitability [2]. As can be seen in Fig. 3.2, the system can a priori be in one of the three states when checking the system suitability at time t_k: suitable with probability $P(t_{k+1})$; operable but unsuitable with probability $P(t_k) - P(t_{k+1})$; inoperable with probability $1 - P(t_k)$, where $P(t)$ is the reliability function of the system.

Let us determine the probabilities of the events (3.2)–(3.7). Assume that the random variable Η (Η ≥ 0) denotes the time to failure of the system with a failure PDF ω(η).

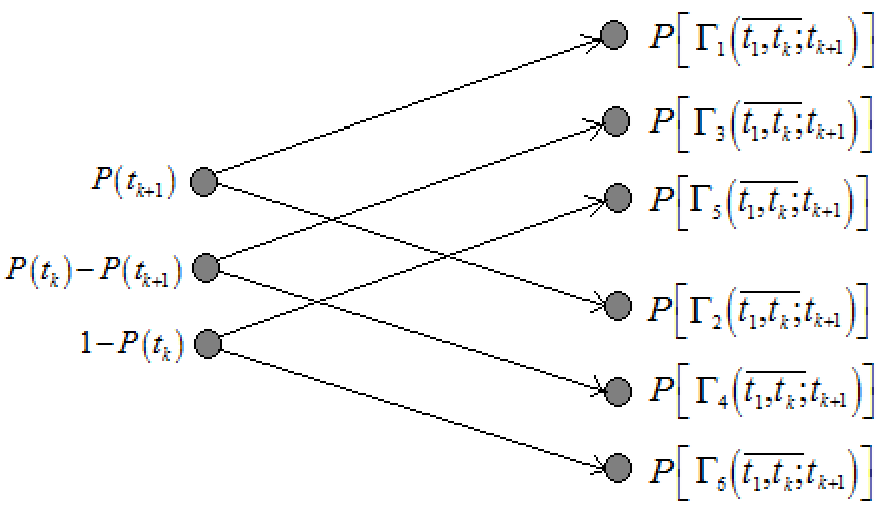


Fig. 3.2. The graph of decision making when checking system suitability at time t_k

We introduce two new random variables associated with the replacement threshold PF_k. Let $\mathrm{H}_{0,k}$ denote a random time of the system operation until it exceeds the replacement threshold PF_k by the parameter $X(t)$, and let H_k denote a random assessment of $\mathrm{H}_{0,k}$ based on the results of inspection at time t_k. Random variables H, $\mathrm{H}_{0,k}$, and H_k are defined as the smallest roots of the following stochastic equations [3]:

$$L(t) - FF = 0, \tag{3.8}$$

$$L(t) - PF_k = 0, \tag{3.9}$$

$$Z(t_k) - PF_k = 0. \tag{3.10}$$

The following implies from the definition of the random variable H_k

$$\mathrm{H}_k = \begin{Bmatrix} t_k, \text{ if } Z(t_k) \geq PF_k \, (k = 1, 2, \ldots) \\ > t_k, \text{ if } Z(t_k) < PF_k \end{Bmatrix}. \tag{3.11}$$

Based on equation (3.11), the previously introduced decision rule can be converted to the following form: the system is judged to be suitable at the time point t_k if $\eta_k > t_k$; otherwise (i.e. if $\eta_k \leq t_k$), the system is judged to be unsuitable, where η_k is the realization of H_k for the system under inspection.

Equations (3.1) and (3.10) imply that H_k is a function of random variables $L(t_k)$, $Y(t_k)$, and the replacement threshold PF_k. Presence of $Y(t_k)$ in (3.10) leads to appearing a random measurement error of the time to failure at inspection time t_k, which is defined as follows:

$$\Delta_k = \mathrm{H}_k - \mathrm{H}, \; k = 1, 2, \ldots. \tag{3.12}$$

Random variables H ($0 < \mathrm{H} < \infty$) and Δ_k ($-\infty < \Delta_k < \infty$) have an additive relationship. Therefore, the random variable H_k is defined in a continuous range of values from -∞ to +∞. Mismatch between the solutions of equations (3.8) and (3.10) results in the appearance of one of the following mutually exclusive events when checking suitability at time t_k [3]:

$$\Gamma_1\left(\overline{t_1, t_k}; t_{k+1}\right) = \left\{\mathrm{H} > t_{k+1} \cap \left(\bigcap_{i=1}^{k} \mathrm{H}_i > t_i\right)\right\}, \tag{3.13}$$

$$\Gamma_2\left(\overline{t_1, t_k}; t_{k+1}\right) = \left\{\mathrm{H} > t_{k+1} \cap \mathrm{H}_k \leq t_k \cap \left(\bigcap_{i=1}^{k-1} \mathrm{H}_i > t_i\right)\right\}, \tag{3.14}$$

$$\Gamma_3\left(\overline{t_1, t_k}; t_{k+1}\right) = \left\{t_k < \mathrm{H} \leq t_{k+1} \cap \left(\bigcap_{i=1}^{k} \mathrm{H}_i > t_i\right)\right\}, \tag{3.15}$$

$$\Gamma_4\left(\overline{t_1, t_k}; t_{k+1}\right) = \left\{t_k < \mathrm{H} \leq t_{k+1} \cap \mathrm{H}_k \leq t_k \cap \left(\bigcap_{i=1}^{k-1} \mathrm{H}_i > t_i\right)\right\}, \tag{3.16}$$

$$\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{\mathrm{H}\le t_k\bigcap\left(\bigcap_{i=1}^{k}\mathrm{H}_i>t_i\right)\right\}, \tag{3.17}$$

$$\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{\mathrm{H}\le t_k\bigcap\mathrm{H}_k\le t_k\bigcap\left(\bigcap_{i=1}^{k-1}\mathrm{H}_i>t_i\right)\right\}. \tag{3.18}$$

The events (3.13)–(3.18) are equivalent to the events (3.2)–(3.7). The events (3.2)–(3.7) and (3.13)–(3.18) differ by the fact that the former are formulated on the spatial axis (vertical axis), and the latter — on the time axis (horizontal axis). Since reliability indicators are usually formulated regarding the events that occur on the time axis, then we will use (3.13)–(3.18) further when evaluating the operational reliability indicators.

Equations (3.15) and (3.16) show that with regards to the system suitability over the time interval $(t_{k,}t_{k+1})$, the event $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ corresponds to an incorrect decision, while the event $\Gamma_4(\overline{t_1,t_k};t_{k+1})$ — to the correct decision. When the event $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ occurs, the unsuitable system is mistakenly allowed to use over the time interval$(t_{k,}t_{k+1})$. It should be noted that in the case of the operability checking at time t_k, the events $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ and $\Gamma_4(\overline{t_1,t_k};t_{k+1})$ are, respectively, match the correct and incorrect decision. This is the fundamental difference between the suitability and operability checking. So, the operability checking does not allow to reject potentially unreliable systems.

Further, the event $\Gamma_2(\overline{t_1,t_k};t_{k+1})$ is called a “false alarm” (“false positive”), and events $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ and $\Gamma_5(\overline{t_1,t_k};t_{k+1})$ are called “missed detection 1” and “missed detection 2”, respectively. The events $\Gamma_1(\overline{t_1,t_k};t_{k+1})$, $\Gamma_4(\overline{t_1,t_k};t_{k+1})$, and $\Gamma_6(\overline{t_1,t_k};t_{k+1})$ correspond to the correct decisions on the system suitability and unsuitability.

It should be specially noted that even for $Y(t_k)=0$ $(k=1,2,\dots)$, incorrect decisions are possible when checking system suitability. Indeed, if $Y(t_k)=0$, then equations (3.13)–(3.18) are converted to the following form:

$$\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{\mathrm{H}>t_{k+1}\bigcap\left(\bigcap_{i=1}^{k}\mathrm{H}_{0,i}>t_i\right)\right\}, \tag{3.19}$$

$$\Gamma_2\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{\mathrm{H}>t_{k+1}\bigcap\mathrm{H}_{0,k}\le t_k\bigcap\left(\bigcap_{i=1}^{k-1}\mathrm{H}_{0,i}>t_i\right)\right\}, \tag{3.20}$$

$$\Gamma_3\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{t_k<\mathrm{H}\le t_{k+1}\bigcap\left(\bigcap_{i=1}^{k}\mathrm{H}_{0,i}>t_i\right)\right\}, \tag{3.21}$$

$$\Gamma_4\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{t_k<\mathrm{H}\le t_{k+1}\bigcap \mathrm{H}_{0,k}\le t_k\bigcap\left(\bigcap_{i=1}^{k-1}\mathrm{H}_{0,i}>t_i\right)\right\},\tag{3.22}$$

$$\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)=\varnothing,\tag{3.23}$$

$$\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)=\left\{\Gamma\le t_k\bigcap \Gamma_{0,k}\le t_k\bigcap\left(\bigcap_{i=1}^{k-1}\Gamma_{0,i}>t_i\right)\right\},\tag{3.24}$$

where Ø denotes the impossible event.

The errors arising at $Y(t_k) = 0$ are methodological in nature and non-removable with the decision rule used herein.

3.2.3. The probabilities of correct and incorrect decisions when checking system suitability. Determination of the probabilities of the events (3.13) - (3.18) reduces to the calculation of the probability that the random point $\{\mathrm{H},\overline{\mathrm{H}_1,\mathrm{H}_k}\}$ falls into the $(k+1)$-dimensional region formed by the limits of the variation of each random variable, and is equal to $(k+1)$ - fold integral over this region.

We denote the joint PDF of the random variables $\{\mathrm{H},\overline{\mathrm{H}_1,\mathrm{H}_k}\}$ as $\omega_0(\eta,\overline{\eta_1,\eta_k})$. The event $\Gamma_1(\overline{t_1,t_k};t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the following limits: $t_{k+1}\le \mathrm{H}<\infty$ and $t_i<H_i<\infty, i=\overline{1,k}$. By integrating the PDF $\omega_0(\eta,\overline{\eta_1,\eta_k})$ within the specified region, we determine the probability of the event $\Gamma_1(\overline{t_1,t_k};t_{k+1})$ [3]

$$P\left[\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_{k+1}}^{\infty}\int_{t_k}^{\infty}\ldots\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}\,d\vartheta.\tag{3.25}$$

The event $\Gamma_2(\overline{t_1,t_k};t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the limits: $t_{k+1}\le \mathrm{H}<\infty$, $-\infty<\mathrm{H}_k\le t_k$, and $t_i<H_i<\infty, i=\overline{1,k-1}$. Integrating the PDF $\omega_0(\eta,\overline{\eta_1,\eta_k})$ within the limits, we obtain the probability of the event $\Gamma_2(\overline{t_1,t_k};t_{k+1})$ [3]

$$P\left[\Gamma_2\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_{k+1}}^{\infty}\int_{-\infty}^{t_k}\int_{t_{k-1}}^{\infty}\ldots\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}\,d\vartheta.\tag{3.26}$$

The event $\Gamma_3(\overline{t_1,t_k};t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the following limits: $t_k\le \mathrm{H}<t_{k+1}$ and $t_i<H_i<\infty, i=\overline{1,k}$. By integrating the PDF $\omega_0(\eta,\overline{\eta_1,\eta_k})$ within the indicated limits, we obtain [3]

$$P\left[\Gamma_3\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_k}^{t_{k+1}}\int_{t_k}^{\infty}\ldots\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}\,d\vartheta.\tag{3.27}$$

The event $\Gamma_4(\overline{t_1, t_k}; t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the following limits: $t_k \le \mathrm{H} < t_{k+1}$, $-\infty < \mathrm{H}_k \le t_k$, and $t_i < H_i < \infty, i = \overline{1, k-1}$. By integrating the PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$ within the given limits, we obtain [3]

$$P\left[\Gamma_4\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_k}^{t_{k+1}}\int_{-\infty}^{t_k}\int_{t_{k-1}}^{\infty}...\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}d\vartheta. \quad (3.28)$$

The event $\Gamma_5(\overline{t_1, t_k}; t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the following limits: $0 < \mathrm{H} \le t_k$ and $t_i < H_i < \infty, i = \overline{1, k}$. By integrating the PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$ within the corresponding limits, we obtain [3]

$$P\left[\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{0}^{t_k}\int_{t_k}^{\infty}...\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}d\vartheta. \quad (3.29)$$

The event $\Gamma_6(\overline{t_1, t_k}; t_{k+1})$ corresponds to a $(k+1)$-dimensional region with the following limits: $0 < \mathrm{H} \le t_k$, $-\infty < \mathrm{H}_k \le t_k$, and $t_i < H_i < \infty, i = \overline{1, k-1}$. By integrating the PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$ within the specified limits, we obtain [3]

$$P\left[\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{0}^{t_k}\int_{-\infty}^{t_k}\int_{t_{k-1}}^{\infty}...\int_{t_1}^{\infty}\omega_0\left(\vartheta,\overline{u_1,u_k}\right)\overline{du_1du_k}d\vartheta. \quad (3.30)$$

As seen from (3.25)–(3.30), to find the probabilities of correct and incorrect decisions made when checking system suitability, the joint PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$ of random variables $\mathrm{H}, \overline{\mathrm{H}_1, \mathrm{H}_k}$ must be known. We denote the conditional PDF of random variables $\overline{\Delta_1, \Delta_k}$ as $\psi_0(\overline{\delta_1, \delta_k}|\eta)$ provided that $\mathrm{H} = \eta$. Following the multiplication theorem of PDFs, we represent the PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$ in the following form [4]:

$$\omega_0\left(\eta,\overline{\eta_1,\eta_k}\right)=\omega(\eta)\varphi\left(\overline{\eta_1,\eta_k}|\eta\right), \quad (3.31)$$

where $\varphi(\overline{\eta_1, \eta_k}|\eta)$ is the conditional PDF of random variables $\overline{\mathrm{H}_1, \mathrm{H}_k}$ provided that $\mathrm{H} = \eta$. In the case of $\mathrm{H} = \eta$, the random variables $\overline{\mathrm{H}_1, \mathrm{H}_k}$ are defined as $\overline{\mathrm{H}_1 = \eta + \Delta_1, \mathrm{H}_k = \eta + \Delta_k}$.

By virtue of the additive relationship between random variables H and Δ_i $(i = \overline{1, k})$ the following equality holds:

$$\varphi\left(\overline{\eta_1,\eta_k}|\eta\right)=\psi_0\left(\overline{\eta_1-\eta,\eta_k-\eta}\middle|\eta\right). \quad (3.32)$$

By substitution of (3.32) into (3.31), we obtain the following expression for the multidimensional PDF $\omega_0(\eta, \overline{\eta_1, \eta_k})$:

$$\omega_0\left(\eta,\overline{\eta_1,\eta_k}\right)=\omega(\eta)\psi_0\left(\overline{\eta_1-\eta,\eta_k-\eta}\middle|\eta\right). \tag{3.33}$$

The relation (3.33) makes it possible to simplify (3.25)–(3.30). Substituting (3.33) into (3.25) gives [3]

$$P\left[\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_{k+1}}^{\infty}\omega(\vartheta)\int_{t_k}^{\infty}...\int_{t_1}^{\infty}\psi_0\left(\overline{u_1-\vartheta,u_k-\vartheta}\middle|\vartheta\right)\overline{du_1du_k}\,d\vartheta. \tag{3.34}$$

Assuming that $g_i = u_i - \vartheta$ $(i = \overline{1,k})$ in (3.34), we obtain:

$$P\left[\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_{k+1}}^{\infty}\omega(\vartheta)\int_{t_k-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta. \tag{3.35}$$

By performing analogous changes of variables in (3.26)–(3.30), we obtain [3]:

$$P\left[\Gamma_2\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_{k+1}}^{\infty}\omega(\vartheta)\int_{-\infty}^{t_k-\vartheta}\int_{t_{k-1}-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta, \tag{3.36}$$

$$P\left[\Gamma_3\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_k}^{t_{k+1}}\omega(\vartheta)\int_{t_k-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta, \tag{3.37}$$

$$P\left[\Gamma_4\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{t_k}^{t_{k+1}}\omega(\vartheta)\int_{-\infty}^{t_k-\vartheta}\int_{t_{k-1}-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta, \tag{3.38}$$

$$P\left[\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{0}^{t_k}\omega(\vartheta)\int_{t_k-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta, \tag{3.39}$$

$$P\left[\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_{0}^{t_k}\omega(\vartheta)\int_{-\infty}^{t_k-\vartheta}\int_{t_{k-1}-\vartheta}^{\infty}...\int_{t_1-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}\,|\vartheta\right)\overline{dg_1dg_k}\,d\vartheta. \tag{3.40}$$

As seen from (3.35)–(3.40), to calculate the probabilities of correct and incorrect decisions made when checking system suitability, the PDF $\omega(\eta)$ and ψ_0 $(\overline{\delta_1,\delta_k}\ |\eta)$ must be known. It should also be noted that formulas (3.35)–(3.40) are generalized, i. e. they can be used for any stochastic process $L(t)$.

3.2.4. Determination of the optimal replacement threshold. The problem of determining the optimal replacement threshold PF_k^{opt} at inspection time $t_k (k = 1, 2, ...)$ depends on the chosen optimization criterion. Let us consider some optimization criteria.

The maximum net income criterion is given by [3]

$$PF_k^{opt} \Rightarrow \min_{PF_k}\Big\{C_{profit}(t_{k+1}-t_k)P\Big[\Gamma_1\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]-C_{pr}P\Big[\Gamma_4\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]-$$
$$\Big(C_{pr}+C_{sp}\Big)P\Big[\Gamma_2\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]-C_{cr}P\Big[\Gamma_6\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]- \tag{3.41}$$
$$C_{uf}\Big\{P\Big[\Gamma_3\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_5\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]\Big\}\Big\},$$

where C_{profit} is the average profit per unit time of system operation; C_{pr} is the average cost of preventive replacement (repair) of the system; C_{sp} is the average cost of additional spare parts due to untimely preventive replacement (repair) of the system; C_{cr} is the average cost of corrective replacement (repair) of the system; C_{uf} is the average loss due to missed failure detection when checking the system suitability.

The minimum Bayes risk criterion can be formulated as follows [3]:

$$PF_k^{opt} \Rightarrow \min_{PF_k}\Big\{\Big(C_{pr}+C_{sp}\Big)P\Big[\Gamma_2\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+C_{uf}\Big\{P\Big[\Gamma_3\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_5\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]\Big\}\Big\}. \tag{3.42}$$

The criterion of minimum total error probability is represented as [5]

$$PF_k^{opt} \Rightarrow \min_{PF_k}\Big\{P\Big[\Gamma_2\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_3\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_5\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]\Big\}. \tag{3.43}$$

The criterion of a given a posteriori probability of failure-free operation of the system in the forthcoming interval of time is represented in the following form [5]:

$$PF_k^{opt} \Rightarrow P\{\mathrm{H} > t_{k+1} | \mathrm{H}_k > t_k\} = P_A, \tag{3.44}$$

where $P\{\mathrm{H} > t_{k+1}|\mathrm{H}_k > t_k\}$ is the a posteriori probability of the system failure-free operation in the interval $(t_{k,}t_{k+1})$, under condition that at time t_k the system was judged as suitable; P_A is the minimum allowable value of the a posteriori probability of the system failure-free operation.

The probability $P\{\mathrm{H} > t_{k+1}|\mathrm{H}_k > t_k\}$ is defined as the ratio of the probability $P\{\Gamma_1(\overline{t_1,t_k};t_{k+1})\}$ to the sum of probabilities $P\{\Gamma_1(\overline{t_1,t_k};t_{k+1})\}$, $P\{\Gamma_3(\overline{t_1,t_k};t_{k+1})\}$ and $P\{\Gamma_5(\overline{t_1,t_k};t_{k+1})\}$, i.e.

$$P\{\mathrm{H} > t_{k+1}|\mathrm{H}_k > t_k\} = \frac{P\Big[\Gamma_1\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]}{P\Big[\Gamma_1\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_3\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]+P\Big[\Gamma_5\Big(\overline{t_1,t_k};t_{k+1}\Big)\Big]}. \tag{3.45}$$

3.2.5. Model of the degradation process. Let us assume that a monotone stochastic function describes the process of degradation of a system

$$L(t) = A_0 + A_1 t, \tag{3.46}$$

where A_0 is the initial random value of the system state parameter $L(t)$, defined in the range from 0 to FF; A_1 is the random degradation rate of the system state parameter, defined in the interval from 0 to ∞. It should be noted that a linear model of the stochastic degradation process has been used in many other studies to describe the real physical deterioration processes. For example, a linear regression model is presented in [6] to describe the changes in the output voltage of the radar transmitter power supply over time. Also, a linear model was used to describe the corrosion strength function in [7].

Let us determine the conditional PDF $\psi_0(\overline{\delta_1,\delta_k}|\eta)$ of the stochastic process described by equation (3.46). Let us prove that if the measurement errors $\overline{Y(t_1),Y(t_k)}$ are independent random variables, then [8]

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=\int_0^{FF} f(a_0)\omega\left(\eta|a_0\right)\left(\frac{FF-a_0}{\eta}\right)^k\times\prod_{i=1}^{k}\Omega\left[\left(\frac{a_0-FF}{\eta}+PF_k-FF\right)\delta_i\right]\frac{da_0}{\omega(\eta)}, \quad (3.47)$$

where $f(a_0)$ is the PDF of the random variable A_0; $\Omega(y_i)$ is the PDF of the random variable $Y(t_i)$; $\omega(\eta|a_0)$ is the conditional PDF of the random variable H provided that $A_0 = a_0$.

Since the random variables $\overline{Y(t_1),Y(t_k)}$ are independent, for the stochastic process (3.46) we can write:

$$f\left(\overline{\delta_1,\delta_k}\,|\,\eta,a_0\right)=\prod_{i=1}^{k} f\left(\delta_i\,|\,\eta,a_0\right), \quad (3.48)$$

where $f(\overline{\delta_1,\delta_k}|\eta,a_0)$ is the conditional PDF of random variables $\overline{\Delta_1,\Delta_k}$ provided that $\mathrm{H} = \eta$ and $A_0 = a_0$.

Denoting $Y_i = Y(t_i)$ $(i = 1, 2, ...)$, we solve the stochastic equations

$$A_0 + A_1\mathrm{H} = FF\,, \quad (3.49)$$

$$A_0 + A_1\mathrm{H}_i + Y_i = PF_i \quad (3.50)$$

concerning variables H and H_i. As a result, we obtain

$$\mathrm{H}=\frac{FF-A_0}{A_1}, \quad (3.51)$$

$$\mathrm{H}_i=\frac{PF_i-A_0-Y_i}{A_1}. \quad (3.52)$$

By substituting (3.51) and (3.52) into (3.12), we obtain

$$\Delta_i = \frac{PF_i - Y_i - FF}{A_1}. \tag{3.53}$$

By solving equation (3.49) concerning A_1, we obtain

$$A_1 = \frac{FF - A_0}{\mathrm{H}}. \tag{3.54}$$

Substitution of (3.54) into (3.53) gives

$$\Delta_i = \frac{\mathrm{H}\left(PF_i - Y_i - FF\right)}{FF - A_0}. \tag{3.55}$$

For any values, $Y_i = y_i$, $A_0 = a_0$, and $\mathrm{H} = \eta$, the random variable Δ_i with the probability of unity has only one value. Therefore, the conditional PDF of random variables $\overline{\Delta_1, \Delta_k}$ relative to $\overline{Y_1, Y_k}$, A_0, and H is the Dirac delta function (δ_f):

$$f\left(\overline{\lambda_1, \lambda_k} \middle| \overline{y_1, y_k}, \eta, a_0\right) = \prod_{i=1}^{k} \delta_f \left[\delta_i - \frac{\eta\left(PF_i - y_i - FF\right)}{FF - a_0}\right] \tag{3.56}$$

Using the chain rule for PDFs, we find the joint PDF of random variables $\overline{\Delta_1, \Delta_k}$, $\overline{Y_1, Y_k}$, H, and A_0

$$f\left(\overline{\lambda_1, \lambda_k}, \overline{y_1, y_k}, \eta, a_0\right) = f\left(\overline{y_1, y_k}, \eta, a_0\right) f\left(\overline{\delta_1, \delta_k} \middle| \overline{y_1, y_k}, \eta, a_0\right) \tag{3.57}$$

Considering (3.56), expression (3.57) transforms into the form

$$f\left(\overline{\delta_1, \delta_k}, \overline{y_1, y_k}, \eta, a_0\right) = f\left(\overline{y_1, y_k}, \eta, a_0\right) \prod_{i=1}^{k} \delta_f \left[\delta_i - \frac{\eta\left(PF_i - y_i - FF\right)}{FF - a_0}\right]. \tag{3.58}$$

Since random variables $\overline{Y_1, Y_k}$ are assumed to be independent and do not depend on H and A_0, then

$$f\left(\overline{y_1, y_k}, \eta, a_0\right) = \omega\left(\eta, a_0\right) \prod_{i=1}^{k} \Omega\left(y_i\right). \tag{3.59}$$

By substitution of (3.59) into (3.58), we obtain

$$f\left(\overline{\delta_1, \delta_k}, \overline{y_1, y_k}, \eta, a_0\right) = \omega\left(\eta, a_0\right) \prod_{i=1}^{k} \Omega\left(y_i\right) \delta_f \left[\delta_i - \frac{\eta\left(PF_i - y_i - FF\right)}{FF - a_0}\right]. \tag{3.60}$$

By integrating (3.60) over the independent variables $\overline{Y_1, Y_k}$ gives

$$f\left(\overline{\delta_1,\delta_k},\eta,a_0\right)=\omega(\eta,a_0)\prod_{i=1}^{k}\int_{-\infty}^{\infty}\Omega\left(u_i\right)\delta_f\left[\delta_i-\frac{\eta\left(PF_i-u_i-FF\right)}{FF-a_0}\right]du_i\,. \tag{3.61}$$

Using the chain rule for PDFs, we define the joint PDF of random variables H and A_0 as follows:

$$\omega(\eta,a_0)=f(a_0)\omega(\eta|a_0)\,. \tag{3.62}$$

Considering (3.62), expression (3.61) takes the form

$$f\left(\overline{\delta_1,\delta_k},\eta,a_0\right)=f(a_0)\omega(\eta|a_0)\prod_{i=1}^{k}\int_{-\infty}^{\infty}\Omega\left(u_i\right)\delta_f\left[\delta_i-\frac{\eta\left(PF_i-u_i-FF\right)}{FF-a_0}\right]du_i\,. \tag{3.63}$$

Let us analyse the integral under the product sign in (3.63). Considering the delta function properties, we can obtain the following equality:

$$\int_{-\infty}^{\infty}\Omega\left(u_i\right)\delta_f\left[\delta_i-\frac{\eta\left(PF_i-u_i-FF\right)}{FF-a_0}\right]du_i=\left(\frac{FF-a_0}{\eta}\right)\Omega\left[\left(\frac{a_0-FF}{\eta}\right)\delta_i+PF_i-FF\right]. \tag{3.64}$$

Substituting (3.64) into (3.63) results in

$$f\left(\overline{\delta_1,\delta_k},\eta,a_0\right)=f(a_0)\omega(\eta|a_0)\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\Omega\left[\left(\frac{a_0-FF}{\eta}\right)\delta_i+PF_i-FF\right]. \tag{3.65}$$

By integrating the PDF (3.65) over the variable a_0, we obtain

$$f\left(\overline{\delta_1,\delta_k},\eta\right)=\int_{0}^{FF}f(a_0)\omega(\eta|a_0)\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\Omega\left[\left(\frac{a_0-FF}{\eta}\right)\delta_i+PF_i-FF\right]da_0\,. \tag{3.66}$$

Using the PDF chain rule, we have

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=f\left(\overline{\delta_1,\delta_k},\eta\right)\Big/\omega(\eta)\,. \tag{3.67}$$

Finally, by substituting (3.66) in (3.67), we obtain (3.47). This completes the proof.

In the absence of a replacement threshold, i. e. for $PF_i=FF$ $(i=\overline{1,k})$, expression (3.47) is converted into the form [8]:

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=\int_{0}^{FF}f(a_0)\omega(\eta|a_0)\left(\frac{FF-a_0}{\eta}\right)^k\times\prod_{i=1}^{k}\Omega\left[\left(\frac{a_0-FF}{\eta}\right)\delta_i\right]\frac{da_0}{\omega(\eta)}\,.$$

If the initial value of the system state parameter is considered almost identical for different systems, i. e. $A_0 = a_0 = constant$, then (3.47) is converted into the form [3]:

$$\psi_0\left(\overline{\delta_1,\delta_k}\,|\eta\right)=\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\Omega\left[\frac{(a_0-FF)\delta_i}{\eta}+PF_i-FF\right]. \tag{3.68}$$

For the case of normal distribution of the measurement error Y_i $(i=\overline{1,k})$ from (3.47) and (3.68), we obtain

$$\psi_0\left(\overline{\delta_1,\delta_k}\,|\eta\right)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^k\int_0^{FF} f\left(a_0\right)\omega\left(\eta|a_0\right)\left(\frac{FF-a_0}{\eta}\right)^k\times$$
$$\prod_{i=1}^{k}\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{(a_0-FF)\delta_i}{\eta}+PF_i-FF\right]^2\right\}\frac{da_0}{\omega\left(\eta\right)}, \tag{3.69}$$

$$\psi_0\left(\overline{\delta_1,\delta_k}\,|\eta\right)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^k\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{(a_0-FF)\delta_i}{\eta}+PF_i-FF\right]^2\right\}. \tag{3.70}$$

Example 3.1. Assume the system state parameter is the output voltage of the radar transmitter power supply [6]. Let us assume that the random measurement error of the system state parameter has a normal distribution with zero mathematical expectation and standard deviation $\sigma_y = 1$ kV . Let $a_0 = 20$ kV, $FF = 25$ kV, and $\eta = 450$ h. It is required to plot a conditional PDF of the error in evaluating the operating time to failure.

For $k = 1$, from expression (3.70), we obtain

$$\psi_0(\delta|\eta)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)\left(\frac{FF-a_0}{\eta}\right)\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{(a_0-FF)\delta}{\eta}+PF-FF\right]^2\right\}. \tag{3.71}$$

Figure 3.3 (a) shows a plot for the conditional PDF of the measurement error of the operating time to failure at $PF = FF = 25$ kV. As can be seen in Fig. 3.3 (a), the conditional PDF of the random variable Δ has a symmetric Gaussian distribution when $PF = FF$.

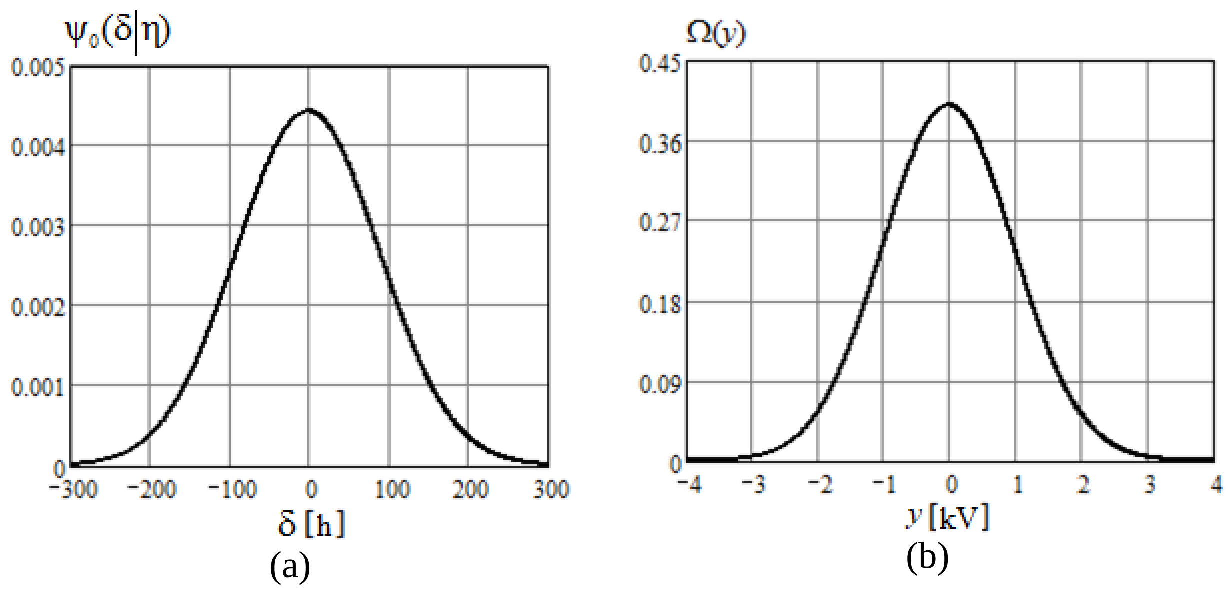


Fig. 3.3. Plots of the conditional PDF of the error in evaluating the operating time to failure (a) and the PDF of the measurement error of the system state parameter (b)

For comparison, Fig. 3.3 (b) shows a plot for the PDF of the measurement error of the system state parameter. By comparing the plots presented in Fig. 3.3 (a) and (b), we can see that both PDFs are of the same form, though the abscissa axis denotes the error in evaluating the time to failure for the PDF $\psi_0(\delta|\eta)$ and the measurement error of the system state parameter for the PDF $\Omega(y)$. Therefore, expressions (3.47), (3.68), (3.69) and (3.70) are used to convert the PDF of the measurement error of the system state parameter into the conditional PDF of the error in evaluating the operating time to failure.

Figure 3.4 shows a 3D presentation of the conditional PDF $\psi_0(\delta|\eta)$ when *PF* = *FF*, rendered in 3D Surface Plotter. As can be seen in Fig. 3.4, the conditional PDF $\psi_0(\delta|\eta)$ flattens with an increase in the failure time η, which indicates an increase in the variance of the error in evaluating the operating time to failure.

Figure 3.5 shows a 3D presentation of the conditional PDF $\psi_0(\delta|\eta)$ when *PF* = 24 kV and *FF* = 25 kV. As can be seen in Fig. 3.5, the introduction of even a small anticipatory tolerance leads to a shift in the mathematical expectation of the PDF $\psi_0(\delta|\eta)$ to the left, in the direction of negative errors in measuring the operating time to failure. It should also be noted that the greater the failure time η, the greater the shift to the left of the mathematical expectation of the conditional PRB $\psi_0(\delta|\eta)$.

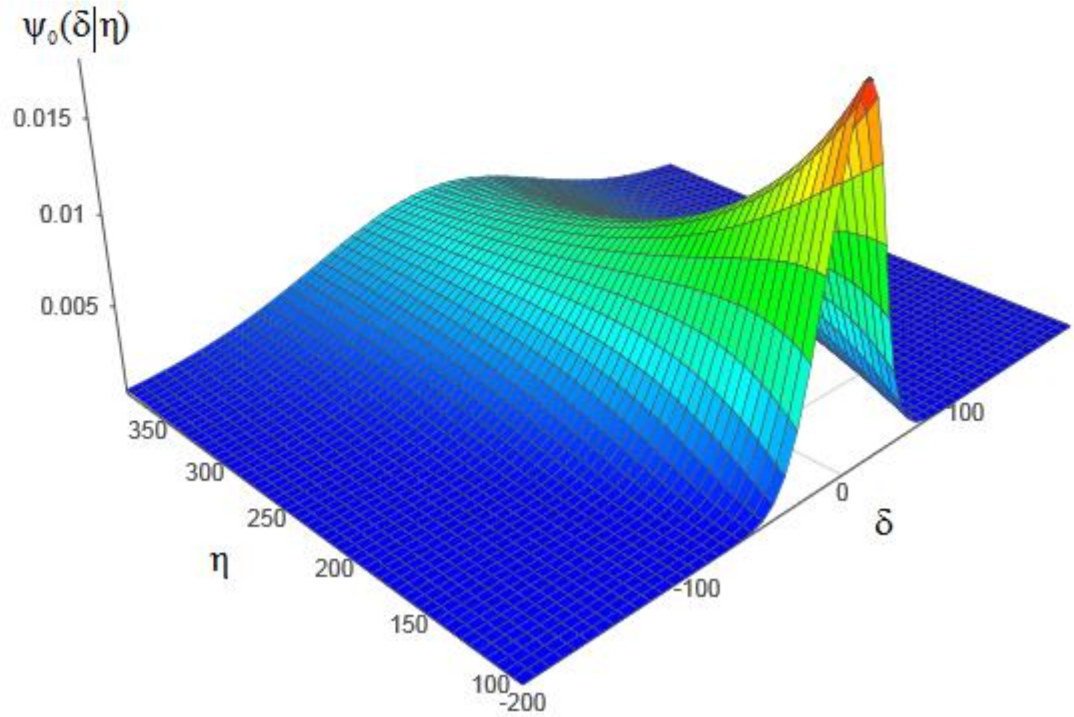


Fig. 3.4. A 3D presentation of the conditional PDF of the error in evaluating the operating time to failure as a function of arguments δ and η at *PF* = *FF*

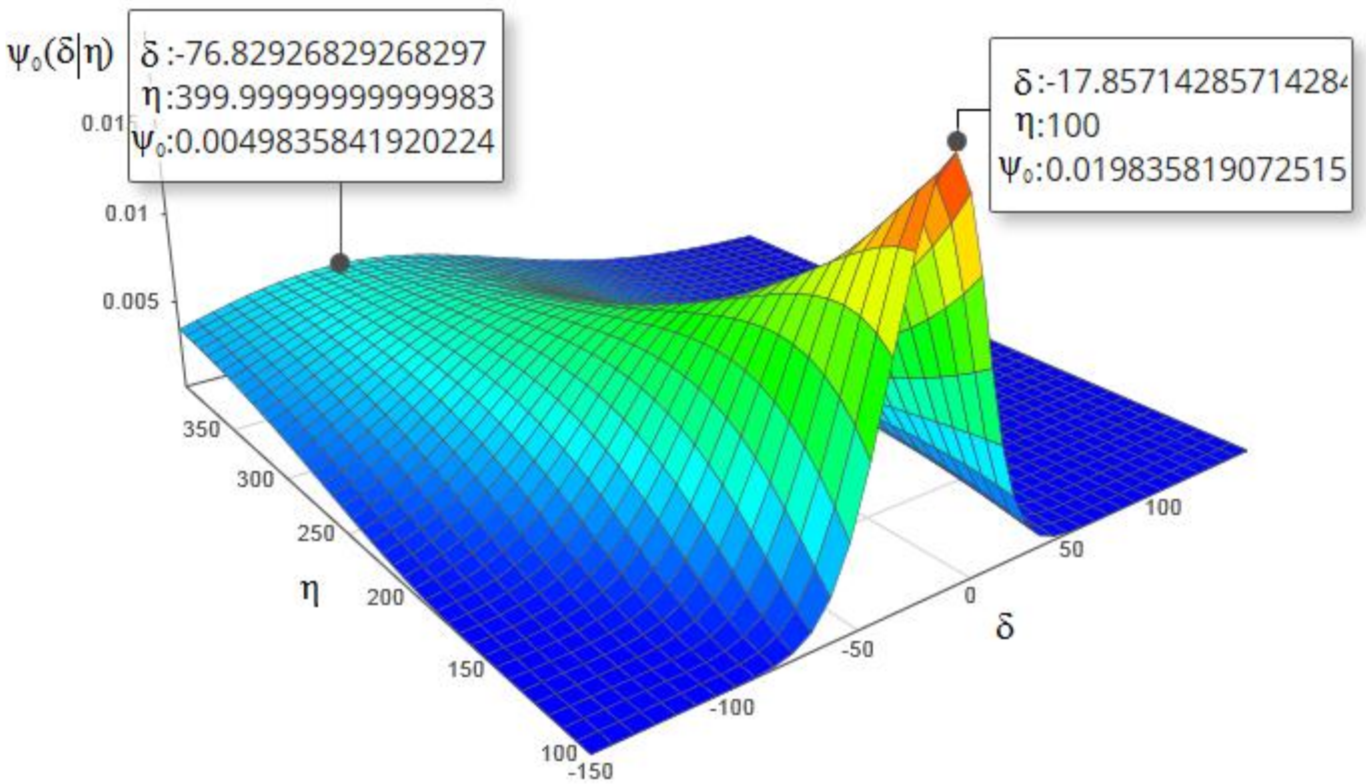


Fig. 3.5. A 3D presentation of the conditional PDF of the error in evaluating the time to failure as a function of arguments δ and η at *PF* = 24 kV and *FF* = 25 kV

So, for $\eta = 100$ h, the shift to the left equals 17.86 h, and for $\eta = 400$ h this shift is 76.83 h, i. e. it increases by more than four times. The dominance of negative measurement errors increases the probability of a false alarm and reduces the probability of a missed detection when checking the system suitability.

Figure 3.6 shows a 3D presentation of the conditional PDF of the error in evaluating the operating time to failure as a function of arguments σ_y and η at $\delta = 50$ h, *PF* = 24 kV and *FF* = 25 kV. The blue colour in Fig. 3.6 shows the region where, for an error $\delta = 50$ h, the value of the PDF $\psi_0(\delta|\eta)$ is practically zero. The boundary of this region is marked by a black line in Fig. 3.6. The ends of the black line in Fig. 3.6 correspond to the following coordinates: $\sigma_y = 1$ kV and $\eta = 88$ h; $\sigma_y = 0.4$ kV and $\eta = 500$ h.

Analysis of the 3D images of the conditional PDF $\psi_0(\delta|\eta)$ allows determining the requirements for the measurement accuracy of the system state parameter.

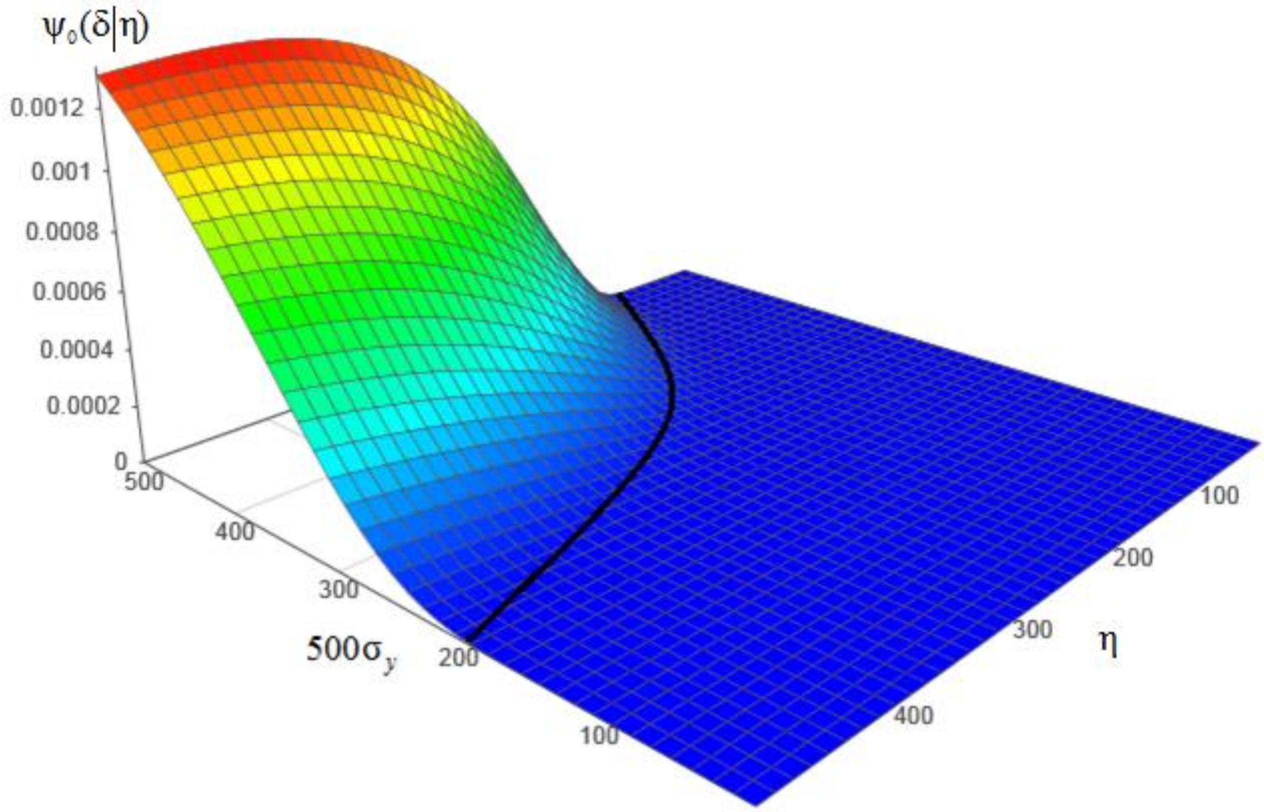


Fig. 3.6. A 3D presentation of the conditional PDF of the error in evaluating the time to failure as a function of arguments σ_y and η for $\delta = 50$ h, *PF* = 24 kV and *FF* = 25 kV

Therefore, considering the graph in Fig. 3.6, it can be said that the standard deviation of the measurement error should be at least 0.4 kV, i. e. $\sigma_y \geq 0.4$ kV.

Let us determine the probabilities (3.35)–(3.40) when $A_0 = a_0$ in the stochastic process model (3.46). By substituting the PDF (3.68) into (3.35), we obtain

$$P\left[\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \prod_{i=1}^{k} \int_{t_i-\vartheta}^{\infty} \left(\frac{FF-a_0}{\vartheta}\right) \Omega\left[\frac{(a_0-FF)g_i}{\vartheta} + PF_i - FF\right] dg_i d\vartheta . \quad (3.72)$$

We introduce a new variable

$$x_i = \frac{(a_0-FF)g_i}{\vartheta} + PF_i - FF . \quad (3.73)$$

Note that the new variable varies from

$$\frac{(a_0-FF)(t_k-\vartheta)}{\vartheta} + PF_i - FF$$

to $-\infty$. By substituting (3.73) into (3.72) and considering that

$$dg_i = \vartheta dx_i \big/ (a_0 - FF)$$

after simple transformations, we obtain [3]:

$$P\left[\Gamma_1\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \left[\prod_{i=1}^{k} \int_{-\infty}^{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF} \Omega(x_i) dx_i\right] d\vartheta . \quad (3.74)$$

Performing similar transformations over the formulas (3.36) - (3.40), we find [3]

$$P\left[\Gamma_2\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{t_{k+1}}^{\infty} \omega(\vartheta) \left[\prod_{i=1}^{k} \int_{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF}^{\infty} \Omega(x_i) dx_i\right] d\vartheta , \quad (3.75)$$

$$P\left[\Gamma_3\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{t_k}^{t_{k+1}} \omega(\vartheta) \left[\prod_{i=1}^{k} \int_{-\infty}^{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF} \Omega(x_i) dx_i\right] d\vartheta , \quad (3.76)$$

$$P\left[\Gamma_4\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{t_k}^{t_{k+1}} \omega(\vartheta) \left[\prod_{i=1}^{k} \int_{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF}^{\infty} \Omega(x_i) dx_i\right] d\vartheta , \quad (3.77)$$

$$P\left[\Gamma_5\left(\overline{t_1,t_k};t_{k+1}\right)\right] = \int_{0}^{t_k} \omega(\vartheta) \left[\prod_{i=1}^{k} \int_{-\infty}^{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF} \Omega(x_i) dx_i\right] d\vartheta , \quad (3.78)$$

$$P\left[\Gamma_6\left(\overline{t_1,t_k};t_{k+1}\right)\right]=\int_0^{t_k}\omega(\vartheta)\left[\prod_{i=1}^{k}\int_{\frac{(a_0-FF)(t_i-\vartheta)}{\vartheta}+PF_i-FF}^{\infty}\Omega(x_i)dx_i\right]d\vartheta. \tag{3.79}$$

It is easy to comprehend that the sum of (3.74)–(3.79) is equal to unity.

Example 3.2. Suppose that a linear stochastic process describes the output voltage of the radar transmitter power supply (3.46), while $A_0 = a_0$ and A_1 is a normal random variable. In this case, the PDF of the operating time to failure is given by [9]:

$$\omega(t)=\frac{m_1\sigma_1^2t^2+\sigma_1^2t\left(FF-a_0-m_1t\right)}{\sqrt{2\pi}\sigma_1^3t^3}\exp\left\{-\frac{\left(FF-a_0-m_1t\right)^2}{2\sigma_1^2t^2}\right\}, \tag{3.80}$$

where m_1 and σ_1 are the mathematical expectation and standard deviation of the random variable A_1. Assume that $a_0 = 19.645$ kV, $m_1 = 0.025$ kV/h, $\sigma_1 = 0.012$ kV/h, $\sigma_y = 0.1$ kV, and $m_y = 0$, where m_y is the mathematical expectation of $Y(t)$.

Let us determine the optimal thresholds PF_k $(k = 1, 2, \ldots)$ when checking the power supply suitability at time points $t_k = k\tau$ by the criterion of minimum total error probability (4.43), where $\tau = 100$ h. Figure 3.7 (a) shows the dependence of the total error probability on the threshold PF_1 for $k = 1$, $t_1 = 100$ h, and $t_2 = 200$ h.

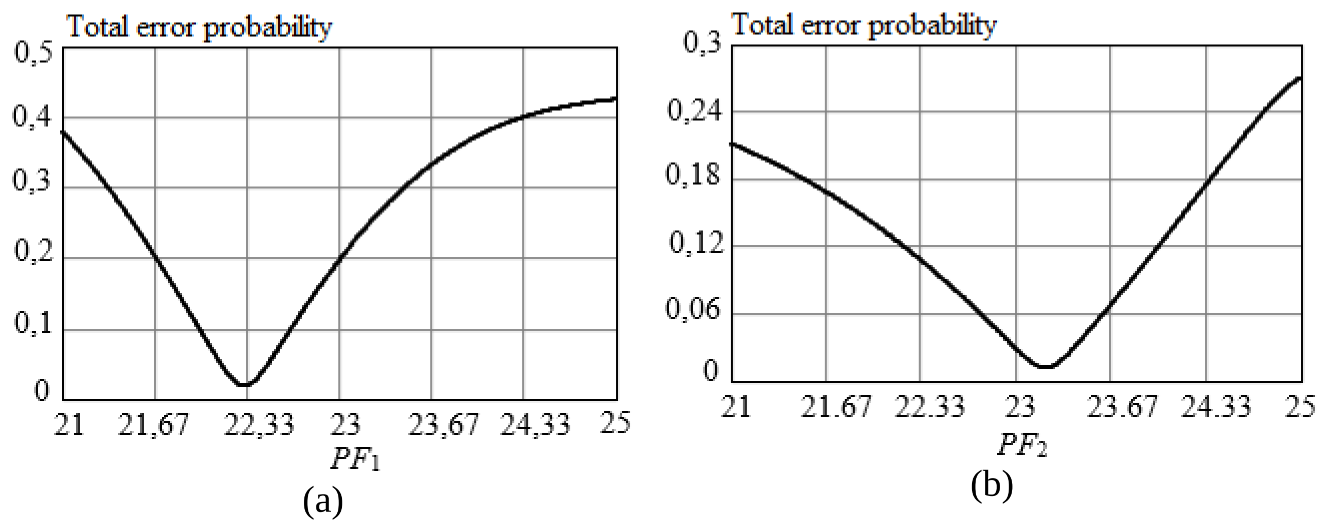


Fig. 3.7. Dependence of the total error probability on the threshold PF_1 for $k = 1$, $t_1 = 100$ h, and $t_2 = 200$ h (a), as well as on the threshold PF_2 for $k = 2$, $t_2 = 200$ h, $t_3 = 300$ h, and $PF_1 = 22.33$ kV (b)

Figure 3.7 (b) shows the dependence of the total error probability on the threshold PF_2 for $k = 2$, $t_2 = 200$ h, $t_3 = 300$ h, and $PF_1 = 22.33$ kV. As can be seen in Fig. 3.7 (b), the optimal replacement threshold is 23.21 kV, which exceeds the PF_1 value by 0.88 kV. Therefore, it may be concluded that the optimal *PF* threshold increases towards the *FF* threshold with an increase in the system operational time.

Figure 3.8 shows the dependence of the optimal value of the replacement threshold from the moment of suitability checking $t_k (k = \overline{1,7})$. As can be seen in Fig. 3.8, the optimal replacement threshold increases with the time of inspection, which may be explained by the increase in the mathematical expectation of the stochastic process (4.46) with time.

Table 3.1 gives the probabilities of correct and incorrect decisions when checking the system suitability at the time $t_2 = 200$ h and $PF_2 = FF$ for two values of the standard deviation σ_y, which differ by a factor of ten.

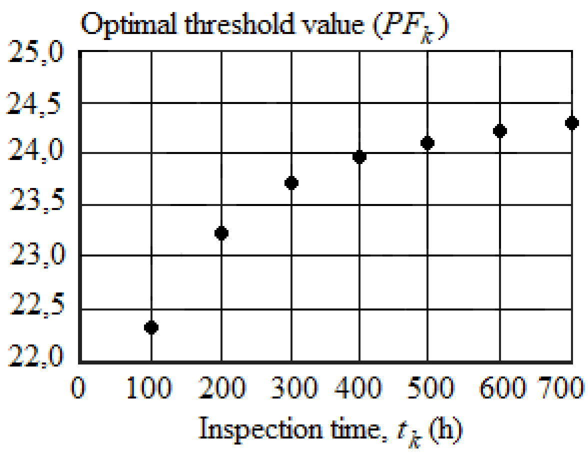


Fig. 3.8. Dependence of the replacement threshold, optimal by the criterion of the minimum total error probability, from the time of inspection $t_k (k = \overline{1,7})$

Table 3.1. The probabilities of correct and incorrect decisions when system suitability at time t_2 when $PF_2 = FF$

σ_y (kV)	$P\{\Gamma_1\}$	$P\{\Gamma_2\}$	$P\{\Gamma_3\}$	$P\{\Gamma_4\}$	$P\{\Gamma_5\}$	$P\{\Gamma_6\}$	P_{error} ($PF_2 = FF$)
1	0.260	0.0019	0.225	0.064	0.061	0.388	0.288
0.1	0.262	0	0.282	0.00671	0	0.449	0.282

It should be noted that with $PF_2 = FF$, the suitability checking turns into the operability checking.

As can be seen in Table 3.1, the total error probability (P_{error}) practically does not decrease with a 10-fold increase in the measurement accuracy of the system state parameter. Thus, at $PF = FF$ the probabilities of incorrect decisions made when checking system suitability are non-zero even for a zero-measurement error. The main contribution to the total error probability gives the probability $P\{\Gamma_3(t_1, t_2; t_3)\}$, i.e. the probability that the system being judged as suitable at inspection times t_1 и t_2 then fails in the interval (t_2, t_3).

Table 3.2 gives the probabilities of correct and incorrect decisions made when checking system suitability at time $t_2 = 200$ h and $PF_2 = PF_2^{opt}$ for the same values of the standard deviation σ_y as in Table 3.1.

Table 3.2. The probabilities of correct and incorrect decisions made when checking system suitability at time t_2 when $PF_2 = PF_2^{opt}$

σ_y (kV)	$P\{\Gamma_1\}$	$P\{\Gamma_2\}$	$P\{\Gamma_3\}$	$P\{\Gamma_4\}$	$P\{\Gamma_5\}$	$P\{\Gamma_6\}$	P_{error} $\left(PF_2^{opt}\right)$
1	0.203	0.059	0.046	0.243	0.00043	0.448	0.105
0.1	0.257	0.0052	0.0061	0.282	0	0.45	0.0113

As can be seen in Table 3.2, in the case of $PF_2 = PF_2^{opt}$, the probabilities of incorrect decisions $P\{\Gamma_2(t_1, t_2; t_3)\}$ and $P\{\Gamma_3(t_1, t_2; t_3)\}$ decrease by 11.3 and 7.5 times, respectively, with σ_y decreasing by 10 times, while the probability $P\{\Gamma_5(t_1, t_2; t_3)\}$ decreases almost to zero.

By comparing Tables 3.1 and 3.2, the following conclusions may be drawn: firstly, when introducing the optimal replacement threshold, the probability $P\{\Gamma_3(t_1, t_2; t_3)\}$ decreases by 5 and 46 times for the corresponding values of σ_y. Secondly, the probability of a false alarm $P\{\Gamma_2(t_1, t_2; t_3)\}$ increases by 31 times. Thirdly, the effect of decreasing the probability $P\{\Gamma_3(t_1, t_2; t_3)\}$ significantly exceeds the effect of increasing the probability $P\{\Gamma_2(t_1, t_2; t_3)\}$, which ultimately reduces the total error probability P_{error}. Fourthly, the probability $P\{\Gamma_4(t_1, t_2; t_3)\}$ increases almost fourfold, which is equivalent to an increase in the corresponding number of times the probability of the predicted failures.

Example 3.3. For the conditions of Example 3.2, determine the optimal values of the replacement thresholds by the criterion of a given a posteriori probability of the system's failure-free operation in the upcoming time interval (3.44). Let the given value of the a posteriori probability of the system failure-free operation is $P_A = 0{,}95$.

Figures 3.9 (a) and (b) show the dependences of the a posteriori probability of failure-free operation on the replacement threshold value for the first and seventh operation interval, respectively.

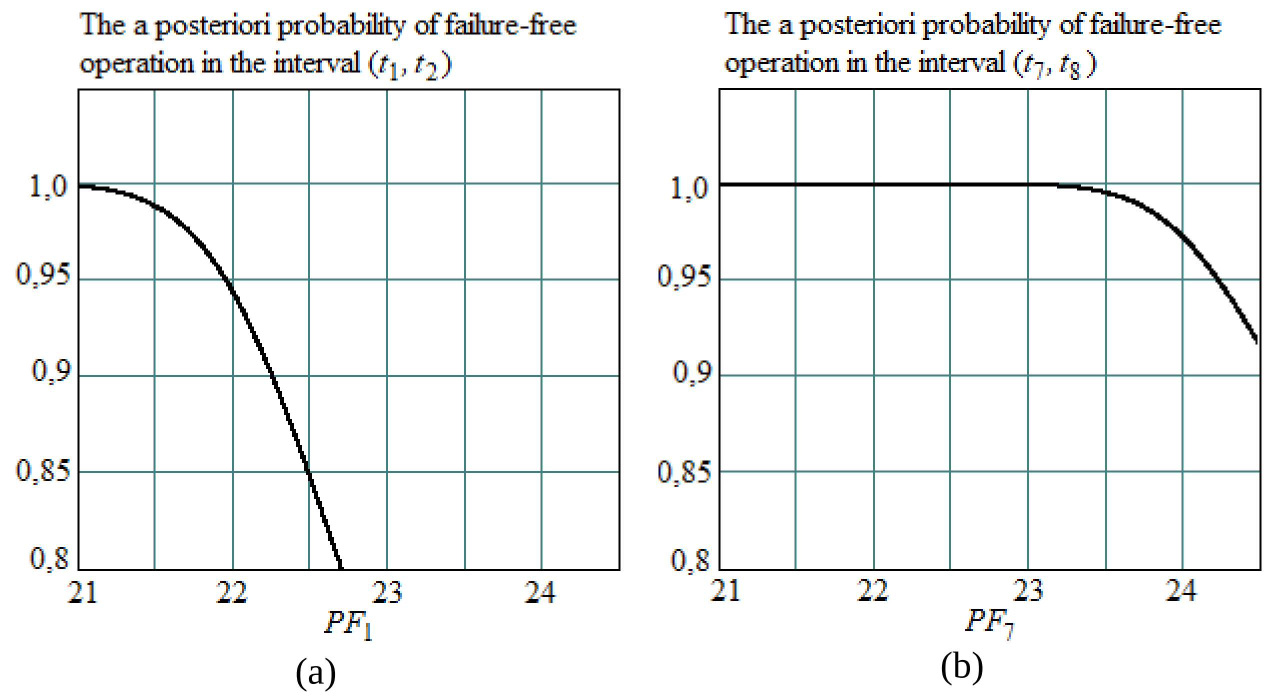


Fig. 3.9. Dependences of the a posteriori probability of failure-free operation on the replacement threshold value for the first (t_1, t_2) (a) and seventh (t_7, t_8) (b) system operation intervals

As can be seen in Fig. 3.9, the replacement threshold increases with increasing number of the operational interval. Thus, for the inspection times $t_1 = 100$ h and $t_7 = 700$ h, the values of the replacement thresholds, providing $P_A = 0.95$ for the intervals (t_1, t_2) and (t_7, t_8), are respectively $PF_1 = 21.95$ kV and $PF_7 = 24.23$ kV.

Figure 3.10 shows the dependence of the replacement threshold, optimal on the criterion of the given a posteriori probability, from the inspection time $t_k(k = \overline{1,7})$. As can be seen in Fig. 3.10, the character of the dependence of the optimal replacement threshold from the time-point of suitability checking is the same as in Fig. 3.8. However, all the points in Fig. 3.10 lie below the corresponding points in Fig. 3.8, which indicates a wider anticipatory tolerance in the case of using the criterion of a given a posteriori probability.

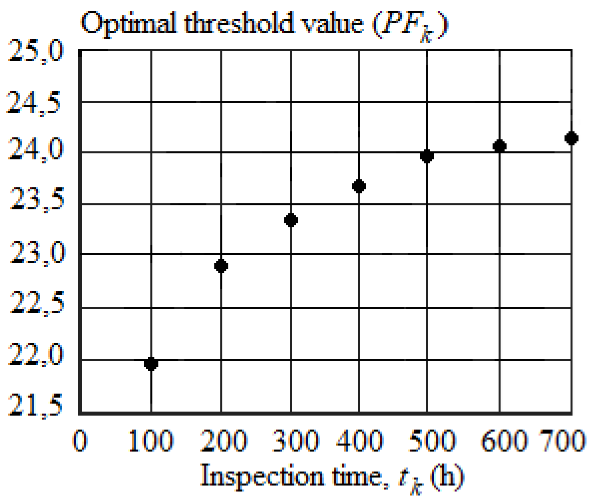


Fig. 3.10. Dependence of the replacement threshold, optimal by the criterion of a given a posteriori probability, from the time of inspection $t_k(k = \overline{1,7})$

3.3. A mathematical model of CBM for determining the optimal replacement thresholds and the inspection times on a finite interval of system operation

This section discusses the task of determining the optimal inspection schedule for the system, which is subject to gradual failures. Gradual failures are caused by internal degradation of the system, which can be detected by instrumental inspections. A mathematical model of condition monitoring with imperfect inspections and perfect repairs is proposed based on a regenerative process of system state changes. The system inspection consists in measuring the state parameter and comparing its value with the replacement threshold.

3.3.1. Inspection policy. Let us assume that the operation of the new system begins at time t_0, and sequential inspections are scheduled at times $t_1 < t_2 < \cdots t_M < T$, where T is the interval of the system operating time for which the condition monitoring is planned.

When checking the system suitability at time t_k $(k = \overline{1,M})$, the following decisions are possible:

- if the system is judged as suitable, then it is permitted for use during the operating interval (t_k, t_{k+1});
- if the system is judged as unsuitable, then it is repaired and permitted for use during the operating interval (t_0, t_1).

In the case of periodic inspections, the interval T is divided equally into M + 1 sub-intervals, and the system is inspected at times $k\tau$ $(k = \overline{1,M})$, where $M = T/\tau - 1$.

3.3.2. The space of system states. To determine the condition-based maintenance effectiveness indicators, we are going to use the well-known property of regenerative stochastic processes [10], according to which the fraction of time the system remains in the state S_i is the ratio of the mean time spent in the state S_i during the regeneration cycle to the average duration of this cycle. The process of system operation will be considered as a sequence of different states over a finite time interval T. Therefore, as in Section 2, the system behaviour over the time interval (0, T) will be described using a finite-state stochastic process $S(t)$. The process $S(t)$ includes the following states: S_1if the system is used for its intended purpose and is in the operable state at time t; S_2 if the system is not used for its intended purpose and is in the inoperable state (unrevealed failure) at time t; S_3 if the system is not used for its intended purpose because a scheduled suitability checking is carried out at time t; S_4 if the system is not used for its intended purpose at time t and preventive maintenance is performed, since a “false alarm” event (3.14) or a “true negative 1” event (3.16) occurred at the last suitability checking; S_5 if the system is not used for its intended purpose at time t and corrective repair is performed, since a “true negative 2” event (3.18) occurred at the last suitability checking.

We denote the time spent by the system in the state S_i $(i = \overline{1,5})$ by TS_i. It is evident that TS_i is a random variable with the expected mean time $E[TS_i]$. Since the process of state changes is regenerative, the system becomes as good as new after repairing in the state S_4 or S_5. Therefore, the average duration of the regeneration cycle is [4]

$$E\left[TS_0\right] = \sum_{i=1}^{5} E\left[TS_i\right]. \qquad (3.81)$$

3.3.3. Conditional probabilities of correct and incorrect decisions made when checking the system suitability. Let us consider the conditional probabilities of correct and incorrect decisions made when checking the system suitability, which are necessary to determine the expected mean times $\overline{E[TS_1], E[TS_5]}$. Suppose that the gradual failure of the system occurs at time η, where $t_k < \eta \le t_{k+1}$ $(k = \overline{0, M})$. To determine the conditional probabilities of correct and incorrect decisions, we use the time axis shown in Fig. 3.11, which demonstrates the location of inspection times $\overline{t_1, t_M}$ and gradual failure time η.

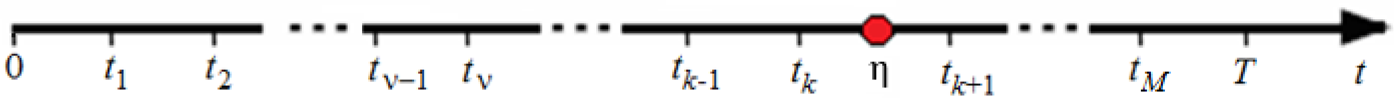


Fig. 3.11. The time-location of suitability checks $\overline{t_1, t_M}$ and gradual failure η

The conditional probability of a “false alarm” when checking the system suitability at the time t_n $(n = \overline{1, k-1})$ can be defined as follows:

$$P_{FA}\left(t_n \middle| \eta\right) = P\left\{\bigcap_{i=1}^{n-1} \mathrm{H}_i > t_i \bigcap \mathrm{H}_n \le t_n \middle| \mathrm{H} = \eta\right\}. \tag{3.82}$$

The conditional probability that operable system is correctly judged as unsuitable (“true negative 1”) when inspecting the system at the time t_k $(k = \overline{1, M})$ is formulated as follows:

$$P_{TN,1}\left(t_k \middle| \eta\right) = P\left\{\bigcap_{n=1}^{k-1} \mathrm{H}_n > t_n \bigcap \mathrm{H}_k \le t_k \middle| \mathrm{H} = \eta\right\}. \tag{3.83}$$

The conditional probability of the “missed detection 1” event when checking the system suitability at the time t_k $(k = \overline{1, M})$ can be defined as follows:

$$P_{MD,1}\left(t_k \middle| \eta\right) = P\left\{\bigcap_{n=1}^{k} \mathrm{H}_n > t_n \middle| \mathrm{H} = \eta\right\}. \tag{3.84}$$

The conditional probability that the system is correctly judged as suitable (“true positive”) when inspecting the system at the time t_n $(n = \overline{1, k-1})$ is formulated as follows:

$$P_{TP}\left(t_n \middle| \eta\right) = P\left\{\bigcap_{i=1}^{n} \mathrm{H}_i > t_i \middle| \mathrm{H} = \eta\right\}. \tag{3.85}$$

The conditional probability that inoperable system is judged as unsuitable ("true negative 2") when inspecting the system at the time t_j ($j = \overline{k+1, M}$) is formulated as follows:

$$P_{TN,2}\left(t_j|\eta\right) = P\left\{\bigcap_{i=k+1}^{j-1} \mathrm{H}_i > t_i \bigcap \mathrm{H}_j \le t_j \middle| \mathrm{H} = \eta\right\}. \tag{3.86}$$

The conditional probability of the "missed detection 2" event when checking the system suitability at the time t_j ($j = \overline{k+1, M}$) can be defined as follows:

$$P_{MD,2}\left(t_j|\eta\right) = P\left\{\bigcap_{i=1}^{j} \mathrm{H}_i > t_i \middle| \mathrm{H} = \eta\right\}. \tag{3.87}$$

As in subsection 3.2.3, the determination of each of the conditional probabilities (3.82) - (3.87) reduces to the calculation of the probability of the random point $\{\overline{\Delta_1, \Delta_m}\}$ hitting into the m-dimensional region formed by the limits of the variation of each random variable, and is equal to the m-fold integral over this region from the PDF $\psi_0(\overline{\delta_1, \delta_m}|\eta)$.

The conditional probability of the "false alarm" at time t_n ($n = \overline{1, k-1}$) corresponds to the n-dimensional region with the following limits: $-\infty < \Delta_n \le t_n - \eta$ and $t_l - \eta \le \Delta_l < \infty$ ($l = \overline{1, n-1}$). By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we obtain [4]:

$$P_{FA}\left(t_n|\eta\right) = \int_{-\infty}^{t_n-\eta} \int_{t_{n-1}-\eta}^{\infty} \ldots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_1, g_n}|\eta\right) \overline{dg_1 dg_n}. \tag{3.88}$$

The conditional probability of the "true negative 1" at time t_k ($k = \overline{1, N}$) corresponds to the k-dimensional region with the following limits: $-\infty < \Delta_k \le t_k - \eta$ and $t_i - \eta \le \Delta_i < \infty$ ($i = \overline{1, k-1}$). By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we obtain [4]:

$$P_{TN,1}\left(t_k|\eta\right) = \int_{-\infty}^{t_k-\eta} \int_{t_{k-1}-\eta}^{\infty} \ldots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_1, g_k}|\eta\right) \overline{dg_1 dg_k}. \tag{3.89}$$

The conditional probability of the "missed detection 1" at time t_k ($k = \overline{1, N}$) corresponds to the k-dimensional region with the following limits: $t_i - \eta \le \Delta_i < \infty$ ($i = \overline{1, k}$). By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we have [4]:

$$P_{MD,1}\left(t_k|\eta\right) = \int_{t_k-\eta}^{\infty} \ldots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_1, g_k}|\eta\right) \overline{dg_1 dg_k}. \tag{3.90}$$

The conditional probability of the "true positive" at time t_n $(n = \overline{1, k-1})$ corresponds to the n-dimensional region with the following limits: $t_l - \eta \leq \Delta_l < \infty$ $(l = \overline{1, n})$. By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we obtain [4]:

$$P_{TP}(t_n|\eta) = \int_{t_n-\eta}^{\infty} \dots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_1, g_n}|\eta\right)\overline{dg_1 dg_n}\,. \tag{3.91}$$

The conditional probability of the "true negative 2" at time t_j $(j = \overline{k+1, N})$ corresponds to the j-dimensional region with the following limits: $-\infty < \Delta_j \leq t_j - \eta$ and $t_i - \eta \leq \Delta_i < \infty$ $(i = \overline{1, j-1})$. By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we get [4]:

$$P_{TN,2}(t_j|\eta) = \int_{-\infty}^{t_j-\eta} \int_{t_{j-1}-\eta}^{\infty} \dots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_{k+1}, g_j}|\eta\right)\overline{dg_{k+1} dg_j}\,. \tag{3.92}$$

The conditional probability of the "missed detection 2" at time t_j $(j = \overline{k+1, N})$ corresponds to the j-dimensional region with the following limits: $t_i - \eta \leq \Delta_i < \infty$ $(i = \overline{1, j})$. By integrating the PDF $\psi_0(\overline{\delta_1, \delta_n}|\eta)$ over the indicated limits, we obtain [4]:

$$P_{MD,2}(t_j|\eta) = \int_{t_j-\eta}^{\infty} \dots \int_{t_1-\eta}^{\infty} \psi_0\left(\overline{g_1, g_j}|\eta\right)\overline{dg_1 dg_j}\,. \tag{3.93}$$

3.3.4. Mean time of staying the system in different states. Let us determine the expected mean times $\overline{E[TS_1], E[TS_5]}$. Using Fig. 3.11, we determine the conditional mathematical expectation of the time the system spends in the state S_1, provided that $\mathrm{H} = \eta$

$$E\left[TS_1|\eta\right] = \begin{cases} \sum_{n=1}^{k-1} t_n P_{FA}(t_n|\eta) + t_k P_{TN,1}(t_k|\eta) + \eta P_{MD,1}(t_k|\eta), \text{if } t_k < \eta \leq t_{k+1}, \\ \sum_{k=1}^{M} t_k P_{FA}(t_k|\eta) + T P_{TP}(t_M|\eta), \text{ if } \eta > T. \end{cases} \tag{3.94}$$

By applying the formula of the total mathematical expectation of a continuous random variable (2.8) to (3.94), we obtain [4]:

$$E\left[TS_1\right] = \sum_{k=0}^{M}\left[\int_{t_k}^{t_{k+1}} \sum_{n=1}^{k-1} t_n P_{FA}(t_n|x) + t_k P_{TN,1}(t_k|x) + x P_{MD,1}(t_k|x)\right]\omega(x)dx + \int_{T}^{\infty}\left[\sum_{k=1}^{M} t_k P_{FA}(t_k|x) + T P_{TP}(t_M|x)\right]\omega(x)dx\,. \tag{3.95}$$

The conditional mathematical expectation of the time spent by the system in the state S_2 provided that $\mathrm{H} = \eta$ is determined from the analysis of Fig. 3.11 as follows:

$$E\left[TS_2 | \eta\right]=\begin{cases}\sum\limits_{j=k+1}^{M}\left(t_j-\eta\right)P_{TN,2}\left(t_j | \eta\right)+(T-\eta)P_{MD,2}\left(t_M | \eta\right), \text{if } t_k<\eta\le t_{k+1}\left(k=\overline{0,M-1}\right),\\ (T-\eta)P_{MD,1}\left(t_M | \eta\right), \text{ if } t_M<\eta\le T,\\ 0, \text{if } \eta>T.\end{cases} \quad (3.96)$$

By applying the formula of the total mathematical expectation of a continuous random variable (2.8) to (3.96), we obtain [4]:

$$E\left[TS_2\right]=\sum_{k=0}^{M-1}\int_{t_k}^{t_{k+1}}\left[\sum_{j=k+1}^{M}\left(t_j-x\right)P_{TN,2}\left(t_j | x\right)+\left(T-x\right)P_{MD,2}\left(t_M | x\right)\right]\omega(x)dx+\int_{t_M}^{T}\left(T-x\right)P_{MD,1}\left(t_M | x\right)\omega(x)dx. \quad (3.97)$$

From the analysis of Fig. 3.11 it follows that the conditional mathematical expectation of the time spent by the system in the state S_3 under the condition that $\mathrm{H} = \eta$ is equal to

$$E\left[TS_3 | \eta\right]=\begin{cases}t_{SC}\sum\limits_{n=1}^{k-1}nP_{FA}\left(t_n | \eta\right)+kP_{TN,1}\left(t_k | \eta\right)+\sum\limits_{j=k+1}^{M}jP_{TN,2}\left(t_j | \eta\right)+MP_{MD,2}\left(t_M | \eta\right),\\ \text{if } t_k<\eta\le t_{k+1}\left(k=\overline{0,M-1}\right),\\ t_{SC}\sum\limits_{k=1}^{M-1}kP_{FA}\left(t_k | \eta\right)+MP_{TP}\left(t_{M-1} | \eta\right), \text{ if } \eta>t_M,\end{cases} \quad (3.98)$$

where t_{SC} is the average duration of the system suitability checking.

By applying the formula of the total mathematical expectation of a continuous random variable (2.8) to (3.98), we obtain [4]:

$$\begin{aligned}E\left[TS_3\right]=t_{SC}\sum_{k=0}^{M-1}\left[\int_{t_k}^{t_{k+1}}\sum_{n=1}^{k-1}nP_{FA}\left(t_n | x\right)+kP_{TN,1}\left(t_k | x\right)+\sum_{j=k+1}^{M}jP_{TN,2}\left(t_j | x\right)+MP_{MD,2}\left(t_M | x\right)\right]\omega(x)dx+\\ t_{SC}\int_{t_N}^{\infty}\left[\sum_{k=1}^{M-1}kP_{FA}\left(t_k | x\right)+MP_{TP}\left(t_{M-1} | x\right)\right]\omega(x)dx.\end{aligned} \quad (3.99)$$

On the base of Fig. 3.11, we determine the conditional mathematical expectation of the time spent by the system in the state S_4, provided that $\mathrm{H} = \eta$

$$E\left[TS_4 | \eta\right]=\begin{cases}t_{PR}\left[\sum\limits_{n=1}^{k-1}P_{FA}\left(t_n | \eta\right)+P_{TN,1}\left(t_k | \eta\right)\right], \text{if } t_k<\eta\le t_{k+1}\left(k=\overline{1,M}\right),\\ t_{PR}\left[\sum\limits_{k=1}^{M}P_{FA}\left(t_k | \eta\right)+P_{TP}\left(t_N | \eta\right)\right], \text{ if } \eta>T,\end{cases} \quad (3.100)$$

where t_{PR} is the average duration of preventive maintenance.

By applying the formula of the total mathematical expectation of a continuous random variable (2.8) to (3.100), we obtain [4]:

$$E\left[TS_4\right]=t_{PR}\sum_{k=0}^{M-1}\left[\int_{t_k}^{t_k+1}\sum_{n=1}^{k-1}P_{FA}\left(t_n|x\right)+P_{TN,1}\left(t_k|x\right)\right]\omega(x)dx+t_{PR}\int_{T}^{\infty}\left[\sum_{k=1}^{M}P_{FA}\left(t_k|x\right)+P_{TP}\left(t_M|x\right)\right]\omega(x)dx. \quad (3.101)$$

The conditional mathematical expectation of the time spent by the system in the state S_2 provided that $H = \eta$ is determined from the analysis of Fig. 3.11 as follows:

$$E\left[TS_5|\eta\right]=\begin{cases}t_{CR}\left[\sum\limits_{j=k+1}^{M}P_{TN,2}\left(t_j|\eta\right)+P_{MD,2}\left(t_M|\eta\right)\right], \text{if } t_k<\eta\le t_{k+1}\left(k=\overline{0,M}\right),\\ 0, \text{if } \eta>T,\end{cases} \quad (3.102)$$

where t_{CR} is the average duration of corrective maintenance.

By applying the formula of the total mathematical expectation of a continuous random variable (2.8) to (3.102), we get [4]:

$$E\left[TS_5\right]=t_{CR}\sum_{k=0}^{M}\int_{t_k}^{t_{k+1}}\left[\sum_{j=k+1}^{M}P_{TN,2}\left(t_j|x\right)+P_{MD,2}\left(t_M|x\right)\right]\omega(x)dx. \quad (3.103)$$

In the case of a periodic inspection schedule, the mean times $E[TS_1], E[TS_5]$ are determined by the following formulas:

$$\begin{aligned}E\left[TS_1\right]=&\sum_{k=0}^{M}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}n\tau P_{FA}\left(n\tau|x\right)+k\tau P_{TN,1}\left(k\tau|x\right)+xP_{MD,1}\left(k\tau|x\right)\right]\omega(x)dx+\\ &\int_{T}^{\infty}\left[\sum_{k=1}^{M}k\tau P_{FA}\left(k\tau|x\right)+TP_{TP}\left(M\tau|x\right)\right]\omega(x)dx,\end{aligned} \quad (3.104)$$

$$\begin{aligned}E\left[TS_2\right]=&\sum_{k=0}^{M-1}\int_{k\tau}^{(k+1)\tau}\left[\sum_{j=k+1}^{M}(j\tau-x)P_{TN,2}\left(j\tau|x\right)+(T-x)P_{MD,2}\left(N\tau|x\right)\right]\omega(x)dx+\\ &\int_{M\tau}^{T}(T-x)P_{MD,1}\left(M\tau|x\right)\omega(x)dx,\end{aligned} \quad (3.105)$$

$$\begin{aligned}E\left[TS_3\right]=&t_{SC}\sum_{k=0}^{M-1}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}nP_{FA}\left(n\tau|x\right)+kP_{TN,1}\left(k\tau|x\right)+\sum_{j=k+1}^{M}jP_{TN,2}\left(j\tau|x\right)+MP_{MD,2}\left(M\tau|x\right)\right]\omega(x)dx+\\ &t_{SC}\int_{M\tau}^{\infty}\left[\sum_{k=1}^{M-1}kP_{FA}\left(k\tau|x\right)+MP_{TP}\left((M-1)\tau|x\right)\right]\omega(x)dx,\end{aligned} \quad (3.106)$$

$$E\left[TS_4\right]=t_{PR}\sum_{k=0}^{M-1}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}P_{FA}\left(n\tau|x\right)+P_{TN,1}\left(k\tau|x\right)\right]\omega(x)dx+t_{PR}\int_{T}^{\infty}\left[\sum_{k=1}^{M}P_{FA}\left(k\tau|x\right)+P_{TP}\left(M\tau|x\right)\right]\omega(x)dx, \quad (3.107)$$

$$E\left[TS_5\right]=t_{CR}\sum_{k=0}^{M}\int_{k\tau}^{(k+1)\tau}\left[\sum_{j=k+1}^{M}P_{TN,2}\left(j\tau|x\right)+P_{MD,2}\left(M\tau|x\right)\right]\omega(x)dx. \quad (3.108)$$

3.3.5. The effectiveness indicators of CBM based on suitability checking. The typically used CBM effectiveness indicators are availability, average maintenance costs and ORF.

As in Chapter 2, we determine the availability by the formula (2.20). However, the mean times $\overline{E[TS_1], E[TS_5]}$ are calculated using the formulas (3.95), (3.97), (3.99), (3.101), and (3.103) in the case of a sequential inspection schedule or using the formulas (3.104)–(3.108) in the case of a periodic inspection schedule.

Average system operation costs per unit time are given by [4]:

$$E\left[C_{CBM,1}\right]=\frac{1}{E\left[TS_0\right]}\left\{C_{LF}E\left[TS_2\right]+C_{SC}E\left[TS_3\right]+C_{PR}E\left[TS_4\right]+C_{CR}E\left[TS_5\right]\right\}, \qquad (3.109)$$

where $C_{CBM,1}$ are the system operation costs per unit time; C_{LF} are the losses per unit time due to the system stay in the failed state; C_{SC} is the cost of system inspection per unit time; C_{PR} is the cost of preventive repair per unit time;; C_{CR} is the cost of corrective repair per unit time.

Average system operation costs over a time interval (0, T) is

$$E\left[C_{CBM,1}^{T}\right]=T\times E\left[C_{CBM,1}\right], \qquad (3.110)$$

where $C_{CBM,1}^{T}$ are the system operation costs in the interval (0, T).

As in Chapter 2 for the safety-critical systems, we will use ORF $R(t_k, t)$ as the primary maintenance effectiveness indicator, and as an additional indicator, the average system maintenance costs $E[C_{CBM,2}^{T}]$ over time T, which is determined by the following formulas:

$$E\left[C_{CBM,2}\right]=\frac{1}{E\left[TS_0\right]}\left\{C_{SC}E\left[TS_3\right]+C_{PR}E\left[TS_4\right]+C_{CR}E\left[TS_5\right]\right\}, \qquad (3.111)$$

$$E\left[C_{CBM,2}^{T}\right]=T\times E\left[C_{CBM,2}\right]. \qquad (3.112)$$

From a comparison of formulas (3.109) and (3.111) it is clear that in (3.111) the coefficient C_{LF} is zero. This is because, for systems that affect safety, it is usually not possible to quantify the consequences of failures, so the indicator $E[C_{CBM,2}^{T}]$ does not include losses due to the system being in an inoperable state.

When using CBM, under ORF we understand the probability that the system will not fail in the interval $(t_k, t), t_k < t \le t_{k+1}$, considering that at time points (t_1, t_k) the suitability checks and, if necessary the repairs, are carried out. On the finite operating time interval (0, T), ORF is determined as follows [11]:

$$R(t_k,t)=\sum_{j=0}^{k}\frac{P_R(t_j)}{\int_0^{T-t_j}\omega(x)dx}\int_{t-t_j}^{T-t_j}P_{TP}(t_k-t_j|h)\omega(h)dh,\ t\geq t_k, \tag{3.113}$$

$$P_R(t_j)=P_{PR}(t_j)+P_{CR}(t_j), \tag{3.114}$$

$$P_{PR}(t_j)=\sum_{\nu=0}^{j-1}\frac{P_R(t_\nu)}{\int_0^{T-t_\nu}\omega(x)dx}\int_{t_{j+1}-t_\nu}^{T-t_\nu}P_{FA}(t_j-t_\nu|h)\omega(h)dh, \tag{3.115}$$

$$P_{CR}(t_j)=1-P_{FR}(t_j)-\sum_{\nu=0}^{j-1}\frac{P_R(t_\nu)}{\int_0^{T-t_\nu}\omega(x)dx}\left\{\int_0^{t_j-t_\nu}P_{MD,2}(t_j-t_\nu|h)\omega(h)dh+\right.$$
$$\left.\int_{t_j-t_\nu}^{t_{j+1}-t_\nu}P_{MD,1}(t_j-t_\nu|h)\omega(h)dh+\int_{t_{j+1}-t_\nu}^{T-t_\nu}P_{TP}(t_j-t_\nu|h)\omega(h)dh\right\}, \tag{3.116}$$

where $P_R(t_j)$ is the probability of the system repair at the time t_j; $P_{PR}(t_{the\ j})$ is the probability of the system preventive repair at the time t_j due to the occurrence of the event (3.14); $P_{TRthe}\ (t_j)$ is the probability of the system corrective repair at the time t_j due to the occurrence of the event (3.16) or (3.18).

The proof of (3.113) - (3.116) we begin with expression (3.114). Let us introduce the following events: $W(t_j)$ is the event of the system being repaired at the time t_j after the *j*-th inspection; $W_{PR}(t_j)$ and $W_{CR}(t_j)$ are the events of the preventive and corrective system repair, respectively. The system will be repaired at the time t_j if one of the events $W_{PR}(t_j)$ or $W_{CR}(t_j)$ occurs. Consequently,

$$W(t_j)=W_{PR}(t_j)+W_{CR}(t_j). \tag{3.117}$$

Events $W_{PR}(t_j)$ and $W_{CR}(t_j)$ are mutually exclusive, since they are based on incompatible events (3.14), (3.16), and (3.18). Therefore, by applying the addition theorem of probability to (3.117), we obtain (3.114).

To prove (3.113), (3.115), and (3.116), we write the probabilistic definition of the indicators $R(t_k,t)$, $P_{FR}(t_j)$ and $P_{TR}(t_j)$:

$$R(t_k,t)=P\left\{\bigcup_{j=0}^{k}\left[W(t_j)\cap t-t_j<\mathrm{H}\leq T-t_j\cap\left(\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right)\right]\right\}, \tag{3.118}$$

$$P_{PR}\left(t_j\right)=P\left\{\bigcup_{\nu=0}^{j-1}\left[W(t_\nu)\bigcap\left(t_{j+1}-t_\nu<\mathrm{H}\le T-t_\nu\bigcap\mathrm{H}_j<t_j-t_\nu\right)\bigcap\left(\bigcap_{i=\nu+1}^{j-1}\mathrm{H}_i>t_i-t_\nu\right)\right]\right\}, \quad (3.119)$$

$$P_{CR}\left(t_j\right)=1-P_{FR}\left(t_j\right)-P\left\{\bigcup_{\nu=0}^{j-1}\left[W(t_\nu)\bigcap\left(\bigcap_{i=\nu+1}^{j}\mathrm{H}_i>t_i-t_\nu\right)\right]\right\}. \quad (3.120)$$

Since the scheduling of inspections is carried out over a finite time horizon (0, *T*), then considering that last repair of the system occurs at the time t_j, the random variable H exists in the interval $(0, T-t_j)$ with the conditional PDF:

$$\omega\left(\eta\middle|0<\mathrm{H}\le T-t_j\right)=\omega(\eta)\Big/\int_0^{T-t_j}\omega(x)dx\,. \quad (3.121)$$

Let us prove (3.113). Suppose that the last restoration of the system was at the time t_j. Assume the system failure occurs in the time interval from η to $\eta+d\eta$. Then the conditional probability of such an event provided that during previous inspections the system was correctly judged as suitable is equal to

$$P\left\{\eta<\mathrm{H}\le\eta+d\eta\middle|\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right\}=\omega\left(\eta\middle|0<\mathrm{H}\le T-t_j\right)d\eta\,. \quad (3.122)$$

The unconditional probability of the formulated event is given by

$$P\left\{\eta<\mathrm{H}\le\eta+d\eta\bigcap\left(\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right)\right\}=P_{TP}\left(t_k-t_j|\eta\right)\omega\left(\eta\middle|0<\mathrm{H}\le T-t_j\right)d\eta\,. \quad (3.123)$$

The probability of the event

$$t-t_j<\mathrm{H}\le T-t_j\bigcap\left(\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right) \quad (3.124)$$

is determined by integrating (3.123) over the region of existence of the random variable H:

$$P\left\{t-t_j<\mathrm{H}\le T-t_j\bigcap\left(\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right)\right\}=\int_{t-t_j}^{T-t_j}P_{TP}\left(t_k-t_j|x\right)\omega\left(x\middle|0<\mathrm{H}\le T-t_j\right)dx\,. \quad (3.125)$$

Considering (3.121), relation (3.125) reduces to the following form:

$$P\left\{t-t_j<\mathrm{H}\le T-t_j\bigcap\left(\bigcap_{i=j+1}^{k}\mathrm{H}_i>t_i-t_j\right)\right\}=\frac{1}{\int_0^{T-t_j}\omega(x)dx}\int_{t-t_j}^{T-t_j}P_{TP}\left(t_k-t_j|x\right)\omega(x)dx\,. \quad (3.126)$$

Using the multiplication theorem of probability, we determine the joint probability of system recovery at the time t_j and event (3.124)

$$P\left\{W_j \cap \left[t-t_j < \mathrm{H} \le T-t_j \cap \left(\bigcap_{i=j+1}^{k} \mathrm{H}_i > t_i - t_j\right)\right]\right\} = \frac{P_R\left(t_j\right)}{\int_0^{T-t_j} \omega(x)dx} \int_{t-t_j}^{T-t_j} P_{TP}\left(t_k - t_j | x\right)\omega(x)dx. \quad (3.127)$$

Since the system can be repaired at any of the moments $\overline{t_0, t_k}$ and after repair the system becomes as good as new, the events $\overline{W_0, W_k}$ are independent, then the sum of the probabilities (3.127) with the change of *j* from 0 to *k* yields formula (4.113) , where $P_R(t_0) = P[W(t_0)] = 1$. Q.E.D.

The proof of relations (3.115) and (3.116) is similar.

3.3.6. Determination of the optimal inspection times and replacement thresholds. The task of determination of the optimal inspection times and replacement thresholds over a finite interval of system operation can be formulated according to various criteria. Optimization criteria may include such measures as maximum availability, minimum average system operation costs over time *T*, the provision of the required level of ORF with a minimum of average maintenance costs over time *T* or maximum of ORF with a restriction on the average maintenance costs over time *T*.

The criterion of minimum average system operation costs over time *T* is presented in the following form:

$$E\left[C_{CBM,1}^{T}\left(\overline{t_1^{opt}, t_M^{opt}}; \overline{PF_1^{opt}, PF_M^{opt}}\right)\right] = \min_{PF_1, PF_M \cap t_1, t_M} E\left[C_{CBM,1}^{T}\left(\overline{t_1, t_M}, \overline{PF_1, PF_M}\right)\right], \qquad (3.128)$$

where $\overline{t_1^{opt}, t_M^{opt}}$ are the optimal inspection times over the operating time interval (0, *T*); $\overline{PF_1^{opt}, PF_M^{opt}}$ are the optimal replacement thresholds at times $\overline{t_1^{opt}, t_M^{opt}}$.

In the case of a periodic inspection schedule, the criterion (3.128) takes the form

$$E\left[C_{CBM,1}^{T}\left(\tau_{opt}; \overline{PF_{\tau_{opt}}^{opt}, PF_{M\tau_{opt}}^{opt}}\right)\right] = \min_{\tau; PF_\tau, PF_{M\tau}} \left\{E\left[C_{CBM,1}^{T}\left(\tau; \overline{PF_\tau, PF_{M\tau}}\right)\right]\right\}, \qquad (3.129)$$

where τ_{opt} is the optimal periodicity of inspection; $\overline{PF_{\tau_{opt}}^{opt}, PF_{M\tau_{opt}}^{opt}}$ are the optimal replacement thresholds at times $\overline{\tau_{opt}, M\tau_{opt}}$.

In the case of a sequential inspection schedule, the maximum availability criterion is formulated as follows:

$$A^{\max}\left(\overline{t_1^{opt}, t_M^{opt}}; \overline{PF_1^{opt}, PF_M^{opt}}\right) = \max_{t_1, t_M; PF_1, PF_M} \left\{K_\Gamma\left(\overline{t_1, t_M}; \overline{PF_1, PF_M}\right)\right\}. \qquad (3.130)$$

In the case of a periodic inspection schedule, the criterion (3.130) is converted into the following form:

$$A^{\max}\left(\tau_{opt};\overline{PF_{\tau_{opt}}^{opt},PF_{M\tau_{opt}}^{opt}}\right)=\max_{\tau;PF_{\tau},PF_{M\tau}}\left\{A\left(\tau;\overline{PF_{\tau},PF_{M\tau}}\right)\right\}. \tag{3.131}$$

The task (3.131) may be simplified if, instead of the optimal replacement threshold for each inspection time, determine one optimal threshold PF^{opt} for all checking times. In such case the optimization criterion can be formulated as follows:

$$A^{\max}\left(\tau_{opt},PF^{opt}\right)=\max_{\tau,PF}\left\{A\left(\tau,PF\right)\right\}. \tag{3.132}$$

Since two indicators evaluate the effectiveness of the safety-critical systems, we may formulate two optimization criteria. If the minimum allowable ORF R^{*} is specified, then one can minimize the average maintenance cost $E\left[C_{CBM,2}^{T}\right]$. The optimization criterion, in this case, has the following form [11]:

$$\begin{cases}\overline{PF_{1}^{opt},PF_{M}^{opt}}\bigcap\overline{t_{1}^{opt},t_{M}^{opt}}\Rightarrow\min\limits_{\overline{PF_{1},PF_{M}}\bigcap\overline{t_{1},t_{M}}}E\left[C_{CBM,2}^{T}\left(\overline{t_{1},t_{M}},\overline{PF_{1},PF_{M}}\right)\right]\\ R\left(t_{k}^{opt},t_{k+1}^{opt}\right)\geq R^{*}\ \left(k=\overline{1,M}\right).\end{cases} \tag{3.133}$$

If the maximum allowed average maintenance cost $E\left[C^{*}\right]$ is specified, the optimization criterion is as follows [11]:

$$\begin{cases}\overline{PF_{1}^{opt},PF_{M}^{opt}}\bigcap\overline{t_{1}^{opt},t_{M}^{opt}}\Rightarrow\max\limits_{\overline{RT_{1},RT_{M}}\bigcap\overline{t_{1},t_{M}}}R\left(t_{k},t_{k+1}\right)\left(k=\overline{1,M}\right)\\ E\left[C_{CBM,2}^{T}\left(\overline{t_{1}^{opt},t_{M}^{opt}},\overline{PF_{1}^{opt},PF_{M}^{opt}}\right)\right]\leq E\left[C^{*}\right].\end{cases} \tag{3.134}$$

In the case of periodic inspections, the criteria (3.132) and (3.133) have the following form [11]:

$$\begin{cases}\overline{PF_{1}^{opt},PF_{M}^{opt}}\bigcap\tau_{opt}\Rightarrow\min\limits_{\overline{PF_{1},PF_{M}}\bigcap\tau}E\left[C_{CBM,2}^{T}\left(\tau_{opt},\overline{PF_{1},PF_{M}}\right)\right],\\ R\left[k\tau_{opt},(k+1)\tau_{opt}\right]\geq R^{*}\left(k=\overline{1,M}\right),\end{cases} \tag{3.135}$$

$$\begin{cases}\overline{PF_{1}^{opt},PF_{M}^{opt}}\bigcap\tau^{opt}\Rightarrow\max\limits_{\overline{PF_{1},PF_{M}}\bigcap\tau}R\left[k\tau,(k+1)\tau\right]\left(k=\overline{1,M}\right),\\ E\left[C_{CBM,2}^{T}\left(\tau^{opt},\overline{PF_{1}^{opt},PF_{M}^{opt}}\right)\right]\leq E\left(C^{*}\right).\end{cases} \tag{3.136}$$

Example 3.3. Assume that as in Example 3.1, the system state parameter is the output voltage of the radar transmitter power supply. It is necessary to solve the problem (3.132) with the following initial data: $T = 3{,}000$ h; $FF = 20$ kV; $a_0 = 16$ kV; $m_1 = 0.002$ kV; $\sigma_1 = 0.00085$ kV/h; $\sigma_y = 0.25$ kV; $t_{SC} = t_{PR} = 3$ h; $t_{CR} = 10$ h.

Figure 3.12 shows the dependence of the system availability on the number of inspections in the interval (0, *T*). From Fig. 3.12 follows that the optimal solution for the case of suitability checking has the following form: $PF^{opt} = 18.5$ kV, $M_{opt} = 5$, $\tau_{opt} = 500$ h, and $A^{\max}\left(\tau_{opt}, PF^{opt}\right) = 0.99$. In the case of operability checking, i. e. at $PF = FF = 20$ kV, the following solution is optimal: $M_{opt} = 25$, $\tau_{opt} = 115.4$ h, and $A^{\max}\left(\tau_{opt}, PF = FF\right) = 0.928$. Thus, the use of the optimal replacement threshold $PF^{opt} < FF$ substantially increases the availability and significantly reduces the number of inspections.

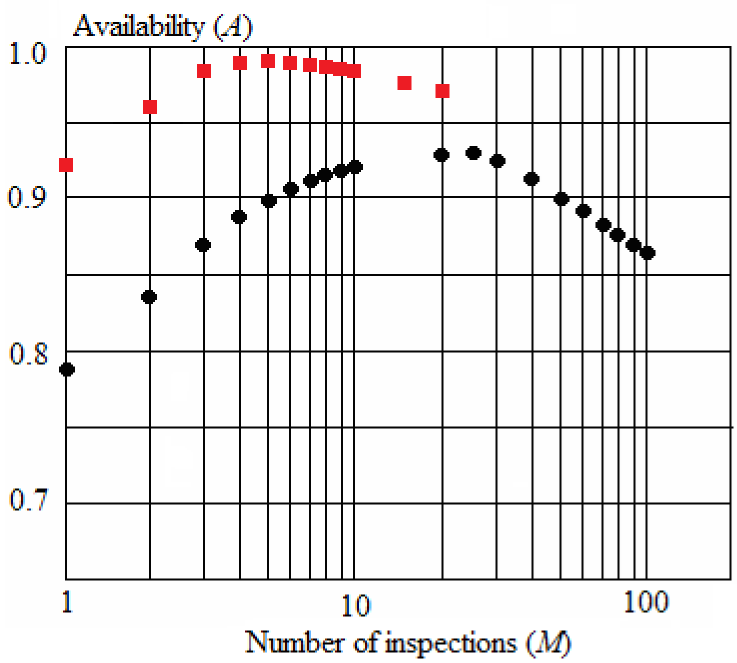


Fig. 3.12. Dependence of availability on the number of inspections at $PF^{opt} = 18.5$ kV $< FF$ (red squares) and at $PF = FF = 20$ kV (black circles).

Example 3.4. Assume again that as in Example 3.1, the system state parameter is the output voltage of the radar transmitter power supply. It is necessary to solve the problem

(3.133) for the same initial data as in Example 3.3. Let us assume that the minimum allowable value of ORF is $R^* = 0.95$.

We solve the problem (3.133) using Mathcad. Table 3.3 presents the values of optimal suitability checking times and replacement thresholds. As can be seen in Table 3.3, the time interval between suitability checking decreases and tends to approximately 500 h value. At the same time, the optimal replacement threshold value gets stable starting with the 2nd inspection.

Table 3.3. The values of optimal suitability checking times and replacement thresholds

Inspection time (h)	t_1	t_2	t_3	t_4	t_5	t_6	t_7
	1165	1890	2475	3015	3535	4040	4535
Replacement threshold (kV)	PF_1	PF_2	PF_3	PF_4	PF_5	PF_6	PF_7
	18.5	19	19	19	19	19	19

Figure 3. 13 shows the dependence of the ORF on the system operational time with the indication of suitability checking times over the interval $(0, 5{,}000 \text{ h})$. As can be seen in Fig. 3.13, with the course of the running time, the ORF changes from a maximum value close to one, after the suitability checking and, if necessary, restoration works to a minimum of 0.95, immediately preceding the next inspection time. In this example, the maximum ORF value $R(t_{k,}t_k), k = \overline{1,7}$ virtually does not change with the inspection number and is approximately equal to 0.99. Apparently, the higher the minimum allowed ORF level R^*, the more inspections are required over a specified interval (0, T).

The minimum value of the average maintenance cost when checking the system suitability in the time interval (0, 5,000 h) is equal to

$$E\left[C^T\right]=C_{FR}\sum_{j=1}^{7}P_{FR}\left(t_j\right)+C_{TR}\sum_{j=1}^{7}P_{TR}\left(t_j\right)+7\times C_{CM}=3000\times 0.36+1.98\times 10000+7\times 500=\$24{,}370. \quad (3.137)$$

Suppose now that $PF_k = FF$ $(k = \overline{1,M})$, that is, instead of checking the system suitability, an operability check is applied. The optimal operability checking times are given in Table 3.4.

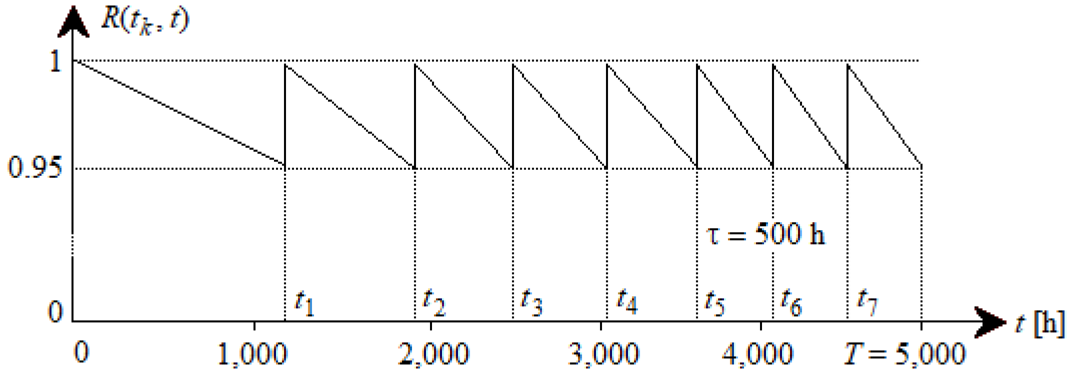


Fig. 3.13. Dependence of ORF on the system operating time with the indication of suitability checking times over the interval $(0, 5{,}000\text{ h})$

As can be seen in Table 3.4, the time interval between operability checking decreases and tends to an approximately 80 h. In this case, the number of operability checks necessary to ensure the minimum ORF-value of 0.95 increases to 43.

Table 3.4. Optimal operability checking times

Inspection time (h)	t_1	t_2	t_3	t_4	t_5	…	t_{43}
	1165	1285	1380	1470	1555	…	4920
Replacement threshold (kV)	*FF*	*FF*	*FF*	*FF*	*FF*	…	*FF*
	20	20	20	20	20	…	20

Figure 3. 14 shows the dependence of the ORF on the system operational time with the indication of operability checking times over the interval $(0, 5{,}000\text{ h})$. In this case, the minimum value of the average maintenance cost is equal to

$$E\left[C^T\right]=C_{FR}\sum_{j=1}^{43}P_{FR}\left(t_j\right)+C_{TR}\sum_{j=1}^{43}P_{TR}\left(t_j\right)+43\times C_{CM}=3000\times 0.258+2.365\times 10000+43\times 500=\$\ 45{,}924. \quad (3.137)$$

By comparing (3.136) and (3.137), we can see that the minimum average cost of maintenance based on suitability checking is less by almost half of the average cost of maintenance based on operability checking.

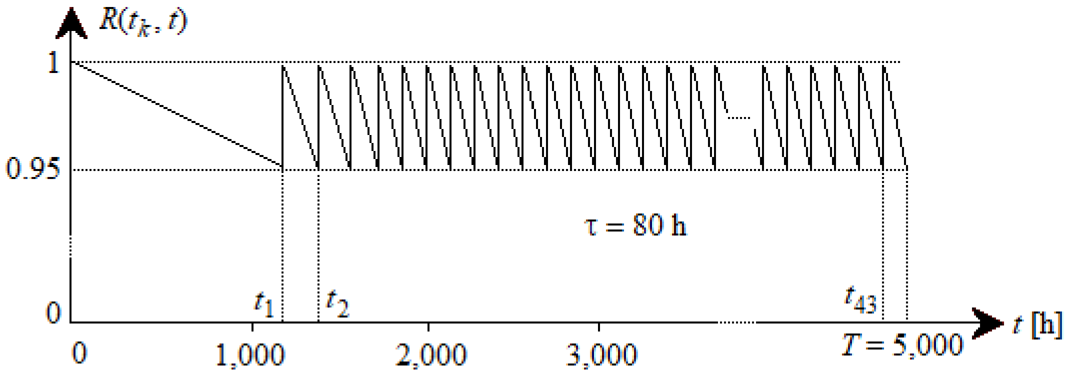


Fig. 3.14. Dependence of ORF on the system operating time with the indication of operability checking times over the interval (0, 5,000 h)

Therefore, for safety-critical systems, the maintenance strategy based on suitability checking is more efficient than the maintenance strategy based on operability checking.

3.4. Conclusions

1. A mathematical model of CBM has been developed which, unlike the known models, considers the probabilities of correct and incorrect decisions made when checking system suitability at scheduled times, to significantly reduce the probability of system failure in the interval between inspections due to the rejection of potentially unreliable systems.

2. Generalized expressions (3.25)–(3.30) and (3.35)–(3.40) have been developed to calculate the probabilities of correct and incorrect decisions made when checking system suitability, considering the results of previous inspections and the possibility of discarding potentially unreliable systems.

3. The criteria for determining the optimal replacement thresholds such as maximum net income (3.41), minimum average risk (3.42), minimum total error probability (3.43), and given a posteriori probability of the system's failure-free operation in the forthcoming time interval (3.44) have been formulated, which can significantly reduce the likelihood of system failure in the intervals between inspections.

4. Expression (3.47) is obtained that allows for a linear random process of degradation to calculate the conditional joint PDF of the errors in the evaluation of the operating time to failure through the known PDF of the measurement error of the system state parameter and the values of the functional failure and replacement thresholds.

5. With the use of 3D images of the conditional PDF of errors in the evaluation of time to failure, it has been shown that the introduction of a small anticipatory tolerance significantly weakens the requirements to the accuracy of measuring the system state parameter.

6. Numerical examples show that in the absence of an anticipatory tolerance, the total error probability practically does not decrease with a 10-fold increase in the accuracy of measuring the system state parameter. While with the introduction of the optimal anticipatory tolerance, the total error probability decreases by almost ten times with a 10-fold increase in the measurement accuracy of the state parameter due to a significant increase in the likelihood of rejecting potentially unreliable systems.

7. Numerical examples have shown that when using the criteria of the minimum total error probability and given a posteriori probability, the optimum value of the replacement threshold increases with the time of inspection, and the value of the anticipatory tolerance decreases, which is explained by the increase in the mathematical expectation of the stochastic degradation process with time.

8. The generalized analytical expressions (3.95), (3.97), (3.99), (3.101), and (3.104)–(3.108) have been obtained for the mean time spent by the system in different states of operation and maintenance for sequential and periodic inspection schedule, considering the conditional probabilities of correct and incorrect decisions made when checking system suitability.

9. Cost-related effectiveness indicators have been developed for CBM based on suitability checking of systems not affecting safety (3.109) and (3.110), and for safety-critical systems (3.111) and (3.112).

10. For systems that affect safety, the generalized relations (3.113) - (3.116) have been proved to calculate the ORF and the probability of an unplanned recovery in a finite time interval.

11. The tasks of determining the optimal scheduling of suitability checks and replacement thresholds according to the criteria of minimum average system operation costs (3.128) and (3.129), maximum availability (3.130) - (3.132), minimum average maintenance

cost with restriction on the ORF (3.133) and (3.135), and maximum ORF with a limit on the average maintenance cost (3.134) and (3.136), have been formulated.

12. The results of numerical calculations show that the CBM based on suitability checking is more effective than the maintenance based on operability checking, as it ensures higher availability and lower maintenance costs with fewer inspections.

REFERENCES

1. Raza, A. Optimal thresholds for stochastically deteriorating systems/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering and computer science 2015, WCECS 2015, 21-23 October, 2015, San Francisco, USA. – V. II. – P. 934-939.
2. Raza, A. A Probabilistic model of periodic condition monitoring with imperfect inspections/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering 2015, WCE 2015, 1–3 July, 2015, London, UK. – V. II. – P. 999–1005.
3. Raza, A. Optimal policies of condition-based maintenance under multiple imperfect inspections/ A. Raza, V. Ulansky// Transactions on engineering technologies: collective monograph. – Springer, 2016. – P. 285–299.
4. Raza, A. Modelling condition monitoring inspection intervals/ A. Raza, V. Ulansky// Electronics and electrical engineering: Proceedings of the 2014 Asia-Pacific Electronics and Electrical Engineering Conference (EEEC 2014), December 27-28, 2014, Shanghai, China. – London: CRC Press, Taylor & Francis Group, 2015. – P. 45–51.
5. Ulansky, V. Modelling of condition monitoring with imperfect inspections/ V. Ulansky, A. Raza// Proceedings of the 19th World conf. on nondestructive testing 2016, WCNDT 2016, 13-17 June, 2016, Munich, Germany. – P. 1–9.
6. Ma, C. Analysis of equipment fault prediction based on metabolism combined model/ C. Ma, Y. Shao, R. Ma// Journal of machinery manufacturing and automation. – 2013. – V. 2(3). – P. 58–62.

7. Kallen, M. Optimal maintenance decisions under imperfect inspection/ M. Kallen, J. Noortwijk// Reliability engineering and system safety. – 2005. – V. 90 (2-3), P. 177-185.

8. Raza, A. Modelling of predictive maintenance for a periodically inspected system/ A. Raza, V. Ulansky// Procedia CIRP. – 2017. – V. 59. – P. 95–101.

9. Игнатов, В. А. Прогнозирование оптимального обслуживания технических систем/ В. А. Игнатов, В. В. Уланский, Т. Тайсир. – К.: Знание, 1981. – 20 с.

10. Barlow, R. E. Statistical theory of reliability and life testing/ R. E. Barlow, F. Proschan. – New York: Holt, Rinehart and Winston, 1975. – 290 p.

11. Ulansky, V. Determination of the optimal maintenance threshold and periodicity of condition monitoring/ V. Ulansky, A. Raza//First world congress on condition monitoring (WCCM), 13-16 June 2017, London, UK. – WCCM proceedings, 2017. – P. 1-12.

CHAPTER 4:

TECHNIQUES OF OPTIMIZING THE MAINTENANCE OF VEHICLES' EQUIPMENT

4.1. The architecture of modern avionics

Avionics systems must meet a large number of requirements for safety, reliability, performance standards, overall dimensions, power consumption, weight, etc. Over the past 40 years, in response to these requirements, manufacturers have proposed two digital avionics architectures [1, 2].

Federated avionics (FA) architecture implies that each system has a dedicated controller, and different systems do not use the controller hardware. These controllers have a weak association with the controllers of other functions. Therefore, the propagation of errors from function to function is possible only in the case of an interaction, which can be detected and allowed by software.

Figure 4.1 shows an example of FA architecture.

The advantages of this architecture compared to the independent avionics architecture used in analogue avionics includes a reduction in the amount of hardware due to the principle of complex information processing and reduction of the total weight of the cable network by using multiplexed communication links between on-board sources and receivers of information. The disadvantages include the possibility of failure spreading from one system to other systems due to mutual data exchange. Failure diagnostics is done automatically by hardware and software system monitoring means. Boeing 767 and 757 series were the first commercial aircraft with the digital FA architecture.

As can be seen in Fig. 4.1, the FA architecture assumes the use of dedicated LRUs for each avionics function and interconnection of each LRU with others via point-to-point data buses such as ARINC 429 or ARINC 629. However, very soon it became apparent that ARINC 429 capabilities were not sufficient to transfer the volumes of digital data exchanged between different LRUs. The original ARINC 429 standard (1978) included the definition of about one hundred (32-bit) data words identified by unique "labels." By the early 1990s, the number of such data words had increased to such a level that the ARINC 429 standard was divided into three parts. Part 2 contained definitions for labels and data words and was approaching 200 pages in volume for the latest edition published in 2004. Part 3 defined the data file sending method since the transfer of significant data volumes became extremely important.

One of the drawbacks of the FA architecture is that it is difficult to expand. Any additional LRUs require additional cable links with each other and with existing LRUs. This architecture requires long cable runs to connect remote LRUs that increase weight and may lead to reliability problems.

It should be noted that until the late 1990s, digital avionics of most civil aircraft, including Airbus A320, A330, and A340 series, was based on the FA architecture philosophy.

Integrated modular avionics was developed to create a modular, open, fault-tolerant and flexible digital avionics architecture. The IMA architecture presents open network architecture with the common computing platform. This architecture uses standard computer systems as a common platform for multiple functions. These so-called IMA modules are connected via data buses. Since all functions located on one module share the computational resource and memory of the corresponding platform, propagation of failures is suppressed using time division mechanisms. By now, several generations of IMA architecture have been developed.

Starting with the Boeing 777 series, the FA architecture began to progressively move to IMA with the Airplane Information Management System (AIMS). Several vital functions (for example, flight control, communication management, aircraft condition monitoring) previously performed by independent LRUs were implemented using IMA. The IMA architecture was developed for most functions of A380 avionics.

Figure 4.2 shows an example of the IMA architecture. Dedicated avionics applications perform most functions of conventional LRUs in IMA architecture. Generic IMA modules, called CPIOMs run these dedicated applications.

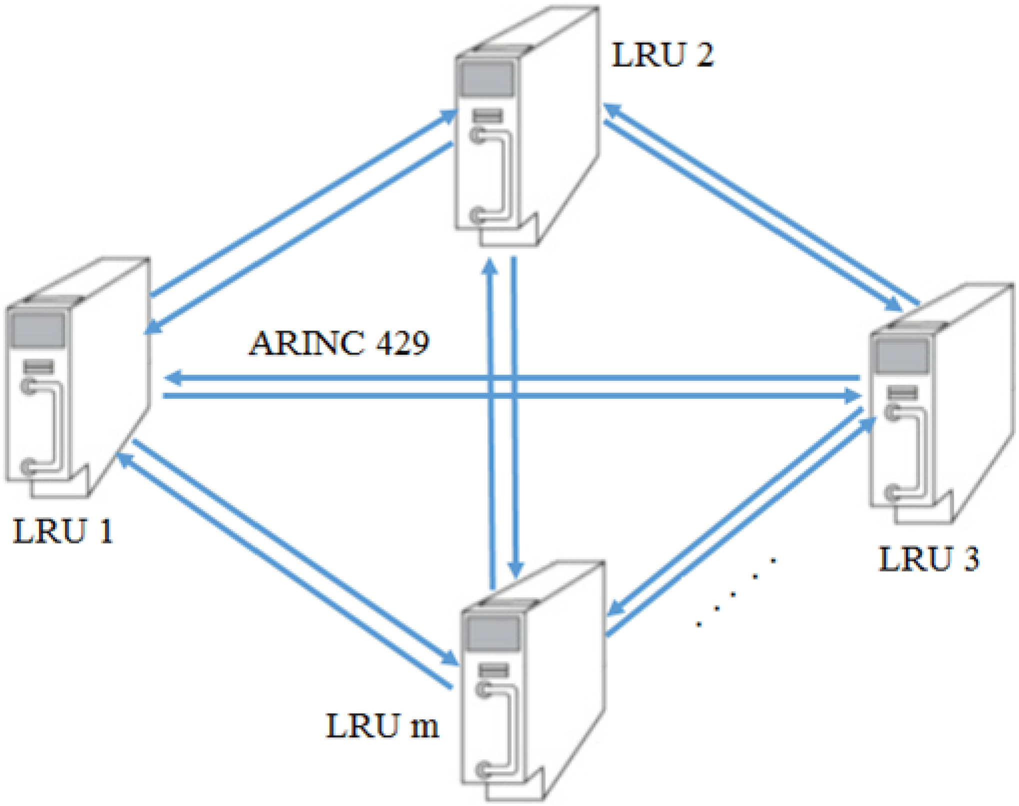


Fig. 4.1. Example of federated avionics architecture

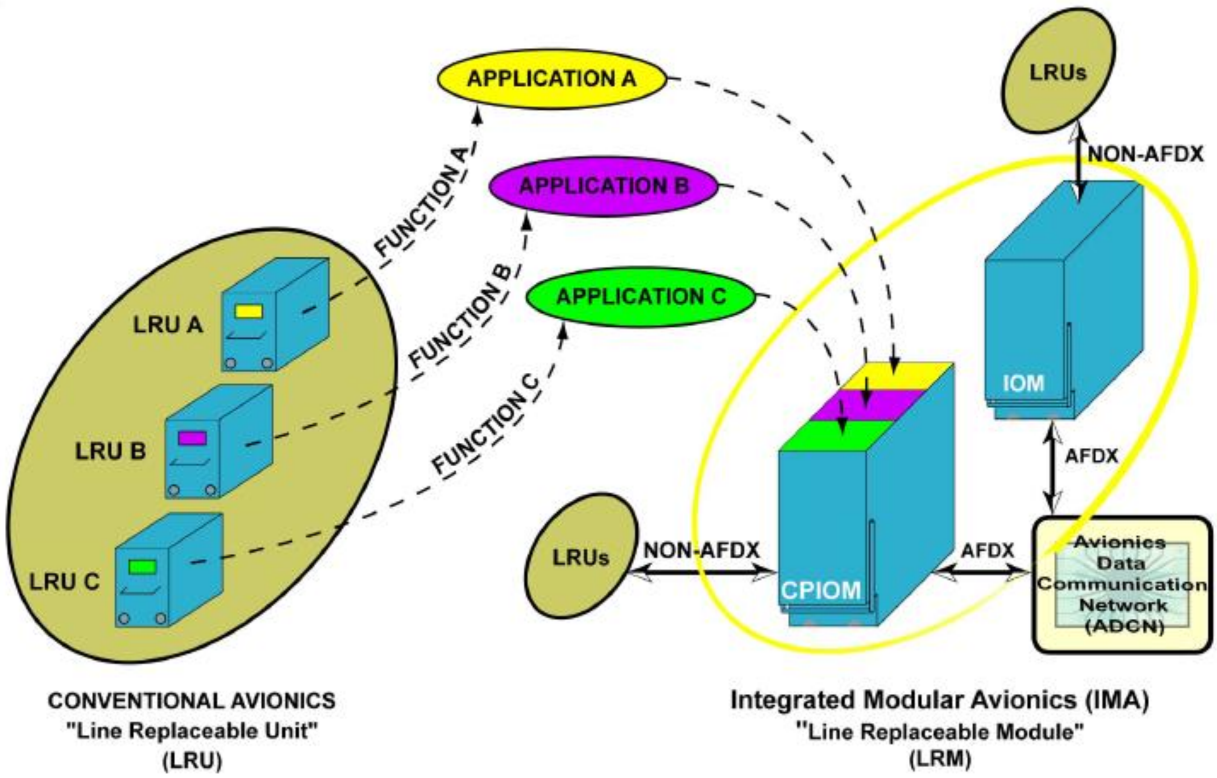


Fig. 4.2. Example of integrated avionics architecture [3]

Consequently, the IMA concept reduces operating costs due to fewer computers needed. Each CPIOM integrates new hardware and software technologies and runs these dedicated applications on common computing and memory resource while providing an I/O interface for specific conventional avionics system LRUs. Moreover, to meet the high demand for connection to conventional avionics systems, additional IOMs are included in the IMA structure. CPIOMs and IOMs are LRMs. Communication between LRMs is carried out via the ADCN through communication technologies developed by a non-aeronautical standard, which is adapted to aviation restrictions. This technology is called AFDX, and it complies with the ARINC 664 standard. The IMA architecture involves connecting all modules (CPIOM and IOM) into ADCN, while all information is routed through the AFDX switches to the required LRMs.

A380 aircraft have seven avionics CPIOMs that perform different types of functions, each being identified by the letter (from A to G): A — pneumatic + air conditioning (optional); B — air conditioning; C — cockpit + flight control; D — data transmission channel; E — energy; F — fuel; G — chassis.

Each CPIOM type is associated with a specific number. CPIOMs with the same number are interchangeable but may require software reconfiguration.

The A380 avionics has 30 interchangeable LRMs and 22 programming functions, located in CPIOMs [4]. The ACDN consists of 16 switches (8 per network) and the corresponding AFDX cables. These switches connect the following aviation system components: 8 IOMs; 22 CPIOMs; 50 LRUs with AFDX interface.

To reduce the number of connecting wires from the control panels in the cockpit to the system computers in the avionics compartment, the controller area network (CAN) bus is used in the A380 [6, 7]. The CAN bus is the standard vehicle bus designed to enable communication between microcontrollers and devices inside the vehicle without using the host computer. Although Airbus began to widely use the CAN bus for A380 to reduce the amount of wiring, the popular ARINC 429 bus is still used in this aircraft series to connect the radio control panels in the cockpit with the LRUs in the avionics compartments. Many LRUs of radio communication and navigation systems, including VHF/HF transceivers, ATC transponders, weather radar, ILS receivers, VOR and ADF receivers, are currently manufactured only with ARINC 429 interfaces, while there are no LRUs of radio communication and navigation systems designed for interaction with the CAN bus [6]. Besides, many electronic engine control units also have ARINC 429 interfaces. The latter is because the ARINC 429 bus has a clear data structure suitable for aircraft systems. Therefore, electronic LRUs are connected to ADCN via CPIOMs or a common remote data concentrator (CRDC). The CRDCs collect, convert and exchange data between ADCN and LRUs that do not have AFDX interfaces and installed outside the avionics compartment.

Therefore, the IMA architecture is more widespread than the FA architecture due to a reduction in weight, size, power consumption and operating costs.

4.2. A technique for calculation of the probabilistic and time-related maintenance effectiveness indicators of digital avionics LRUs/LRMs

A technique to calculate maintenance effectiveness indicators may be used for any LRUs that meet the ARINC 700 (FA architecture) specifications, as well as for LRUs and LRMs that meet the specifications of ARINC 651/653 (IMA architecture). The technique uses the analytical expressions derived in Chapter 2 for the case of exponential distribution of operating time to permanent and intermittent failure. Two versions of the technique for calculating maintenance effectiveness indicators are considered, differing in the duration of the operating time T. The first version corresponds to the finite operating time interval, i. e. $T < \infty$. In this case, the interval T may correspond to the warranty maintenance period or, for example, the duration of the outsourcing contract with the repair station. In the second case, $T = \infty$, which usually corresponds to the duration of post-warranty maintenance or the operating time intervals much longer than the MTBUR of LRUs/LRMs.

4.2.1. A technique for calculation of the probabilistic and time-related maintenance effectiveness indicators over a finite time interval.

1. The following initial data should be known: the time interval T; the rate of permanent (λ) and intermittent (θ) failure of LRU/LRM; the average flight time (τ); the average duration of maintenance operations at the O-level ($t_M^{O-level}$); the average time of waiting for a spare LRU/LRM from the airline warehouse in the situation “aircraft on ground” (t_{spare}); the average scheduled stop time of the aircraft at the base airport (t_{stop});

the average time of shipping the failed LRU/LRM to the repair and back ($t_{shipping}$); the average time of the LRU/LRM repair due to intermittent (t_{IFR}) and permanent failure (t_{PFR}).

2. Calculation of the LRU/LRM availability over a finite time interval:

2.1. Determination of the mean time spent by the LRU/LRM in the operable state by formula (2.16):

$$E\left[TS_1\right]=\frac{\tau}{1-e^{-\theta\tau}}\left[1-e^{-(\lambda+\theta)T}\right]+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}+\tau e^{-(\lambda+\theta)T}.$$

2.2. Determination of the mean time spent by the LRU/LRM in the inoperable state by formula (2.17):

$$E\left[TS_2\right]=\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right].$$

2.3. Determination of the mean time spent by the LRU/LRM in the O-level maintenance by formula (2.7):

$$E\left[TS_3\right]=t_M^{O-level}.$$

2.4. Determination of the mean time spent by the LRU/LRM in the state of waiting for a spare LRU/LRM from the airline warehouse by formula (2.8):

$$E\left[TS_4\right]=\left(t_{spare}+t_M^{O-level}-t_{stop}\right).$$

2.5. Determination of the mean time spent by the LRU/LRM in the state of shipping to repair or from repair by formula (2.9):

$$E\left[TS_5\right]=t_{shipping}.$$

2.6. Determination of the mean time spent by the LRU/LRM in the state of repairing due to an intermittent failure by formula (2.18):

$$E\left[TS_6\right]=t_{IFR}\left\{\left(1-e^{-\lambda\tau}\right)\left[\frac{e^{-\lambda\tau}-e^{-\lambda T}}{1-e^{-\lambda\tau}}-\frac{e^{-(\lambda+\theta)\tau}-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]+\left(1-e^{-\theta T}\right)e^{-\lambda T}\right\}.$$

2.7. Determination of the mean time spent by the LRU/LRM in the state of repairing due to a permanent failure by formula (2.19):

$$E\left[TS_7\right]=t_{PFR}\left\{\left(1-e^{-\lambda\tau}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]+e^{-(\lambda+\theta)T}\right\}.$$

2.8. Determination of the mean time between repairing the LRU/LRM using (2.1):

$$E[T_0]=\sum_{i=1}^{7}E[TS_i].$$

2.9. Determination of the LRU/LRM availability over a finite time interval T by formula (2.20):

$$A=E[TS_1]/E[T_0].$$

3. Determination of the MTBUR over a finite time interval using formula (2.23):

$$E[TBUR_T]=\frac{\tau}{1-e^{-\theta\tau}}\left[1-e^{-(\lambda+\theta)T}\right]+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}+\tau e^{-(\lambda+\theta)T}+$$
$$\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right].$$

4. Calculation of the operational reliability function (ORF) on a finite time interval:

4.1. Determination of the probability of repairing the LRU/LRM with an intermittent failure at time $j\tau$ ($j=\overline{1,N}$; $N=T/\tau-1$) by formula (2.43):

$$P_{IF}(j\tau)=\left(1-e^{-\theta\tau}\right)\sum_{\nu=0}^{j-1}\frac{P_R(\nu\tau)}{1-e^{-\lambda(T-\nu\tau)}}e^{-(j-\nu-1)\theta\tau}\left[e^{-(j-\nu)\lambda\tau}-e^{-\lambda T}\right].$$

4.2. Determination of the probability of repairing the LRU/LRM with a permanent failure at time $j\tau$ by formula (2.44):

$$P_{PF}(j\tau)=1-\sum_{\nu=0}^{j-1}\frac{P_R(\nu\tau)}{1-e^{-\lambda(T-\nu\tau)}}e^{-(j-\nu-1)\theta\tau}\left[e^{-(j-\nu)\lambda\tau}-e^{-\lambda T}\right].$$

4.3. Determination of the total probability of LRU/LRM repairing at time $j\tau$ by formula (2.33):

$$P_R(j\tau)=P_{IF}(j\tau)+P_{PF}(j\tau).$$

4.4. Determination of the ORF of LRU over the operating time interval $(k\tau,t)$, $k\tau<t\leq(k+1)\tau$ by formula (2.42):

$$R(k\tau,t)=\sum_{j=0}^{k}\frac{P_R(j\tau)}{1-e^{-\lambda(T-j\tau)}}e^{-(k-j)\theta\tau}\left[e^{-\lambda(t-j\tau)}-e^{-\lambda T}\right].$$

4.2.2. A technique for calculation of the probabilistic and time-related maintenance effectiveness indicators over an infinite time interval.

1. In the occasion of an infinite operating time interval ($T=\infty$), the initial data is the same as for the case of a finite time horizon, i. e. we assume that parameters λ, θ, τ, $t_M^{O-level}$, t_{spare}, t_{stop}, t_{IFR}, t_{PFR}, and $t_{shipping}$ are known.

2. Calculation of the LRU/LRM availability on an infinite time interval:

- Determination of the mean time spent by the LRU/LRM in the operable state by formula (2.48):

$$E\left[TS_1\right]=\frac{\tau}{1-e^{-\theta\tau}}+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1}{1-e^{-(\lambda+\theta)\tau}}\,.$$

- Determination of the mean time spent by the LRU/LRM in the inoperable state by formula (2.49):

$$E\left[TS_2\right]=\frac{1}{1-e^{-(\lambda+\theta)\tau}}\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right).$$

- Determination of the mean time spent by the LRU/LRM in the states S_3, S_4 and S_5 by formulas (2.7) - (2.9):

$$E\left[TS_3\right]=t_M^{O-level},\ E\left[TS_4\right]=\left(t_{spare}+t_M^{O-level}-t_{stop}\right),\ E\left[TS_5\right]=t_{shipping}.$$

- Determination of the mean time spent by the LRU/LRM in the state of repairing due to an intermittent failure by formula (2.50):

$$E\left[TS_6\right]=t_{IFR}\left(1-e^{-\lambda\tau}\right)\left[\frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}}-\frac{e^{-(\lambda+\theta)\tau}}{1-e^{-(\lambda+\theta)\tau}}\right].$$

- Determination of the mean time spent by the LRU/LRM in the state of repairing due to an intermittent failure by formula (2.51):

$$E\left[TS_7\right]=t_{PFR}\left[\frac{1-e^{-\lambda\tau}}{1-e^{-(\lambda+\theta)\tau}}\right].$$

- Determination of the mean time between repairing the LRU/LRM using (2.1):

$$E[T_0]=\sum_{i=1}^{7}E\left[TS_i\right].$$

- Determination of the LRU/LRM availability over an infinite time interval T by formula (2.20):

$$A=E\left[TS_1\right]/E[T_0].$$

3. Determination of the MTBUR over an infinite time interval using formula (2.53):

$$E[TBUR]=\frac{\tau}{1-e^{-\theta\tau}}+\left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda}-\frac{\tau}{1-e^{-\theta\tau}}\right)-\tau e^{-\lambda\tau}\right]\frac{1}{1-e^{-(\lambda+\theta)\tau}}+\frac{1}{1-e^{-(\lambda+\theta)\tau}}\left(\tau-\frac{1-e^{-\lambda\tau}}{\lambda}\right).$$

4. Calculation of the ORF on an infinite time interval:

- Determination of the probability of repairing the LRU/LRM with an intermittent failure at time $j\tau$ (j = 1, 2, ...) by formula (2.58):

$$P_{IF}(j\tau)=\left(1-e^{-\theta\tau}\right)\sum_{\nu=0}^{j-1}P_R(\nu\tau)\exp\left\{-\left[(j-\nu-1)\theta+(j-\nu)\lambda\right]\tau\right\}.$$

- Determination of the probability of repairing the LRU/LRM with a permanent failure at time $j\tau$ (j = 1, 2, ...) by formula (2.59):

$$P_{PF}(j\tau)=1-\sum_{\nu=0}^{j-1}P_R(\nu\tau)\exp\left\{-\left[(j-\nu-1)\theta+(j-\nu)\lambda\right]\tau\right\}.$$

- Determination of the total probability of LRU/LRM repairing at time $j\tau$ by formula (2.33):

$$P_R(j\tau)=P_{IF}(j\tau)+P_{PF}(j\tau).$$

- Determination of the ORF of LRU over the operating time interval $(k\tau, t)$, $k\tau < t \le (k+1)\tau$ by formula (2.57):

$$R(k\tau,t)=\sum_{j=0}^{k}P_R(j\tau)\exp\left\{-\left[(k-j)\theta\tau+\lambda(t-j\tau)\right]\right\}.$$

4.3. A technique for calculation of the probabilistic and time-related maintenance effectiveness indicators of redundant digital avionics systems

1. The initial data for the calculation include the mean time spent by the LRU/LRM in the operable state ($E[TS_1]$), inoperable state ($E[TS_2]$), in the state of waiting for a spare LRU/LRM from the airline warehouse ($E[TS_4]$), the steady-state value of LRU operational reliability function ($R(\tau)^*$), as well as the type of redundancy and the total number of LRUs/LRMs in the system (m).

2. Calculation for the case of a parallel redundancy structure:

- Determination of the availability and unavailability using formulas (2.66) and (2.68):

$$A=1-\left\{1-E(TS_1)/\left[E(TS_1)+E(TS_2)+E(TS_4)\right]\right\}^m,$$

$$\overline{A}=\left\{1-E(TS_1)/\left[E(TS_1)+E(TS_2)+E(TS_4)\right]\right\}^m.$$

- Determination of the steady-state values of the ORF and the probability of system failure in flight using formulas (2.67) and (2.69):

$$R_m^*(\tau)=1-\left[1-R^*(\tau)\right]^m,$$

$$Q_m^*(\tau)=\left[1-R^*(\tau)\right]^m.$$

3. Calculation for the case of a “k-out-of-m” redundancy structure:

- Determination of the availability and unavailability using formula (2.70), as well as the complementary formula:

$$A=\sum_{i=k}^{m}\binom{m}{i}\left[\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^i\left[1-\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^{m-i},$$

$$\overline{A}=1-\sum_{i=k}^{m}\binom{m}{i}\left[\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^i\left[1-\frac{E(TS_1)}{E(TS_1)+E(TS_2)+E(TS_4)}\right]^{m-i}.$$

- Determination of the steady-state values of the ORF and the probability of system failure in flight using formula (2.71), as well as the complementary formula:

$$R_m^*(\tau)=\sum_{i=k}^{m}\binom{m}{i}\left[R^*(\tau)\right]^i\left[1-R^*(\tau)\right]^{m-i},$$

$$Q_m^*(\tau)=1-\sum_{i=k}^{m}\binom{m}{i}\left[R^*(\tau)\right]^i\left[1-R^*(\tau)\right]^{m-i}.$$

4. Calculation for the case of a parallel-series redundancy structure:

- Determination of the availability and unavailability using formula (2.72), as well as the complementary formula:

$$A=1-\left\{1-\prod_{i=1}^{l}\frac{E(TS_{1,i})}{E(TS_{1,i})+E(TS_{2,i})+E(TS_{4,i})}\right\}^m,$$

$$\overline{A}=\left\{1-\prod_{i=1}^{l}\frac{E(TS_{1,i})}{E(TS_{1,i})+E(TS_{2,i})+E(TS_{4,i})}\right\}^m.$$

- Determination of the steady-state values of the ORF and the probability of system failure in flight using formula (2.73), as well as the complementary formula:

$$R_m^*(\tau)=1-\left[1-\prod_{i=1}^{l}R_i^*(\tau)\right]^m,$$

$$Q_m^*(\tau)=\left[1-\prod_{i=1}^{l}R_i^*(\tau)\right]^m.$$

4.4. A technique for minimizing the costs of warranty maintenance of redundant digital avionics systems

This technique uses the mathematical model of LRU/LRM operation over a finite time interval and the warranty maintenance model of redundant avionics systems considered in subsections 2.2 and 2.5, respectively.

1. The following initial data should be known: the warranty period expressed in flight hours (T_W); the number of aircraft with a supplier's warranty (N_W); the total number of LRUs/LRMs in the avionics system (m); the maintenance labour cost per hour (LC); the rate of permanent (λ) and intermittent (θ) failures of LRU/LRM; the average flight time (τ); the average duration of maintenance operations at the O-level ($t_M^{O-level}$); the cost of a spare LRU/LRM (C_{LRU}); the guaranteed LRU/LRM repair time (T_{RS}); the guaranteed expedited delivery time of a spare LRU/LRM (T_{ED}); the average time of rechecking the dismantled LRU by ground test equipment at the I-level maintenance ($t_{TE}^{I-level}$); the number of types of LRUs that can be rechecked by the test equipment at the I-level ($F_{TE}^{I-level}$); the cost of the ground test equipment at the I-level maintenance ($C_{TE}^{I-level}$).

2. Determination of the steady-state values of the probabilities of repairing the LRU/LRM with an intermittent and permanent failures using formulas (2.45) and (2.46):

$$P_{IF}^{*}(\tau) = \left(1-e^{-\theta\tau}\right)\left(e^{-\lambda\tau}-e^{-\lambda T}\right) \Big/ \left[\left(1-e^{-\lambda T}\right)\left(1-e^{-\theta T}\right)\right],$$

$$P_{PF}^{*}(\tau) = 1-\left(e^{-\lambda\tau}-e^{-\lambda T}\right) \Big/ \left(1-e^{-\lambda T}\right).$$

3. Determination of the steady-state value of the total probability of LRU/LRM repairing using formulas (2.33), (2.45) and (2.46):

$$P_R^{*}(\tau) = P_{IF}^{*}(\tau) + P_{PF}^{*}(\tau) = \left(1-e^{-\theta\tau}\right)\left(e^{-\lambda\tau}-e^{-\lambda T}\right) \Big/ \left[\left(1-e^{-\lambda T}\right)\left(1-e^{-\theta T}\right)\right] +$$

$$1-\left(e^{-\lambda\tau}-e^{-\lambda T}\right) \Big/ \left(1-e^{-\lambda T}\right).$$

4. Determination of the average number of unscheduled removals of LRUs/LRMs due to permanent and intermittent failures over the time interval (0, T_W) by formula (2.64):

$$N_R(T_W) = T_W \, P_R^{*}(\tau) / \tau .$$

5. Determination of the LRU/LRM unscheduled removal rate $\lambda_{i,i+1}$ by formula (A.2.1):

$$\lambda_{i,i+1}=\begin{cases}q\tau/P_R^*(\tau), \text{ if } i=\overline{1,PS+1},\\ (q+PS+1-i)\tau/P_R^*(\tau), \text{ if } i=\overline{PS+2,PS+q}.\end{cases}$$

6. Determination of the mean LRU/LRM repair time by formula (A.2.3):

$$E[T_R]=\begin{cases}T_{RS}, \text{ for the first WMO},\\ T_{RS}P_{PF}+t_{TE}^{I-level}, \text{ for the second WMO}.\end{cases}$$

7. Determination of the LRU/LRM repair rate $\mu_{i+1,\,i}$ by formula (A.2.2):

$$\mu_{i+1,i}=\begin{cases}i\mu_{RS}=i/E[T_R], \text{ if } i=\overline{1,PS},\\ i\mu_{ED}=i/T_{ED}, \text{ if } i=\overline{PS+1,PS+q}.\end{cases}$$

8. Determination of the mean time spent by the LRU/LRM in the state of repairing due to a permanent failure by formula (2.19):

$$E\left[TS_7\right]=t_{PFR}\left\{\left(1-e^{-\lambda\tau}\right)\left[\frac{1-e^{-(\lambda+\theta)T}}{1-e^{-(\lambda+\theta)\tau}}\right]+e^{-(\lambda+\theta)T}\right\}.$$

9. Determination of the a posteriori probability that dismantled LRU/LRM has a permanent failure by formula (A.2.6):

$$P_{PF}=E\left[TS_7\right]/t_{PFR}\,.$$

10. Determination of the failure flow parameter of a set mN_W of the same type LRUs/LRMs by formula (A.2.11):

$$\Lambda=\frac{mN_W P_R^*(\tau)}{\tau}.$$

11. Determination of the average time of waiting for a spare LRU/LRM from the airline warehouse in the situation “aircraft on ground” by formula (A.2.9):

$$t_{spare}\left(PS,US\right)=\Lambda^{-1}\sum_{i=1}^{q}iP_{PS+i+1}\,.$$

12. Determination of the optimal number of spare LRUs/LRMs by the criterion (2.77):

$$E\left[TS_4\right]=\left[t_{spare}\left(PS,US\right)+t_M^{O-level}-t_{stop}\right]\to 0.$$

13. Determination of the $WTEC_1$ using formula (2.74):

$$WTEC_1=m\times LC\times t_M^{O-level}N_R(T_W)+(PS+US)\times C_{LRU}/N_W\,.$$

14. Determination of the $WTEC_2$ using formula (2.76):

$$WTEC_2 = m \times LC\left(t_M^{O-level} + t_{TE}^{I-level}\right) N_R(T_W) + \frac{C_{TE}^{I-level}}{N_W \times F_{TE}^{I-level}} + (PS + US)C_{LRU} / N_W$$

15. Choice of the optimal WMO by criterion (2.78):

$$WTEC_{opt} = \min\left(WTEC_i,\ i = 1,2\right).$$

Let us consider an example of minimizing the costs of warranty maintenance of the A380 airborne inertial reference system (ADIRS). In 2012, the airline Emirates (UAE) received eight aircraft A380 [8]. Each aircraft has three air data inertial reference units (ADIRU) of HG2030BE (Honeywell) type that form the redundant ADIRS system. The appearance and main characteristics of the HG2030BE ADIRUs are shown in Fig. 4.3. Figure 4.3 shows the appearance and main characteristics of the HG2030BE unit.

As shown in Fig. 4.4-4.6, the ADIRS provides air data (the airspeed, angle of attack and altitude) and inertial control information (the position and altitude) to the pilot's displays, as well as to other aircraft systems such as engines, autopilot, flight control system and chassis. The ADIRS comprises three fault-tolerant ADIRUs located in the electronic rack of the aircraft. The third ADIRU is a standby unit that may be selected to provide data to the first or second pilot's displays in the case of a partial or total failure of ADIRUs № 1 or № 2.

Characteristics

Part Number	HG2030BE
Size	4 MCU
Weight (typical)	15.5 lbs (7 Kg)
Power Input	115 VAC or 28 VDC
Power Dissipation (Typical)	39 WATTS
Cooling	Passive
Interfaces	AFDX and ARINC 429
Environmental	DO-160D +
Software	DO-178B Level A
Gyros	Digital Ring Laser Gyro Fleet wide reliability experience >400,000 MTBF
Accelerometers	Quartz Fleet wide reliability experience >5,000,000 MTBF
Reliability	40,000 MTBF / 23,500 MTBUR
Processor	50X increase in processor MIPS
Outputs	Air Data, Inertial, and Hybrid GPS

Fig. 4.3. Appearance and main characteristics of HG2030BE ADIRU [8]

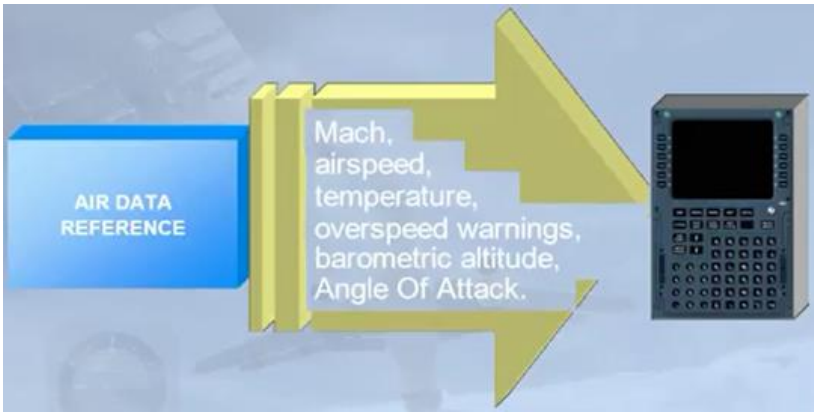


Fig. 4.4. Information provided by the ADIRS to on-board computers [9]

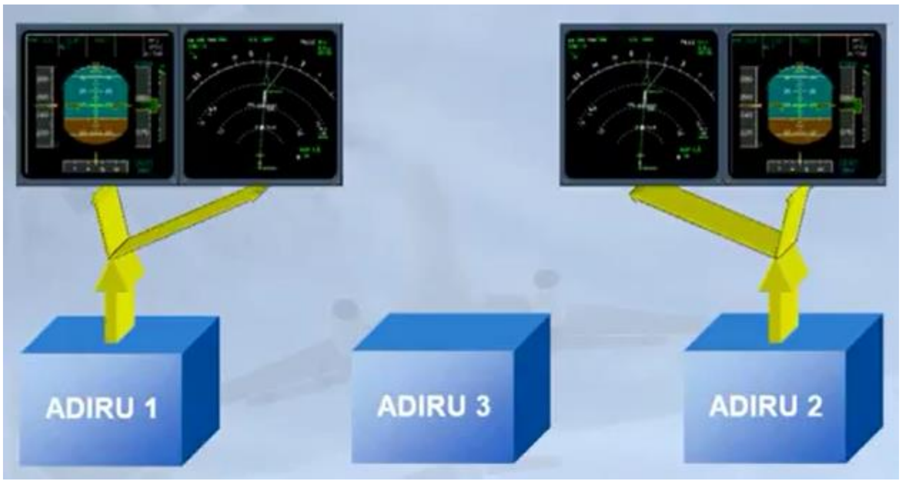


Fig. 4.5. ADIRUs № 1 and № 2 independently display information on the first or second pilot’s displays [9]

Fig. 4.6. ADIRU № 3 is a standby unit [9]

As can be seen in Fig. 4.5 and 4.6, the ADIRS does not have crossover redundancy between ADIRU № 1 and ADIRU № 2, since ADIRU № 3 is the only alternative source of air and inertial data. Failure of air data stream from ADIRU № 1 or ADIRU № 2 will result in a loss of airspeed and altitude information on the corresponding display In any case, the information can only be restored by selecting ADIRU № 3.

From the description of the principle of ADIRU № 1, ADIRU № 2 and ADIRU № 3 connection to the on-board computers and the 1st and 2nd pilot's displays, it follows that the following formula calculates the steady-state ORF of the ADIRS:

$$P^*_{ADIRS}(\tau)=\left[P^*_{ADIRU}(\tau)\right]^3+3\left[P^*_{ADIRU}(\tau)\right]^2\left[1-P^*_{ADIRU}(\tau)\right]+P^*_{ADIRU}(\tau)\left[1-P^*_{ADIRU}(\tau)\right]^2, \qquad (4.1)$$

where $P^*_{ADIRU}(\tau)$ is the steady-state value of the ADIRU operational reliability function.

Similarly, the ADIRS availability is determined from

$$A_{ADIRS}=\left[\frac{E[TS_1]}{E[TS_1]+E[TS_2]+E[TS_4]}\right]^3+3\left[\frac{E[TS_1]}{E[TS_1]+E[TS_2]+E[TS_4]}\right]^2\times$$
$$\left[1-\frac{E[TS_1]}{E[TS_1]+E[TS_2]+E[TS_4]}\right]+\frac{E[TS_1]}{E[TS_1]+E[TS_2]+E[TS_4]}\left[1-\frac{E[TS_1]}{E[TS_1]+E[TS_2]+E[TS_4]}\right]^2. \qquad (4.2)$$

As in subsection 2.5.4, we will consider two alternative WMOs for ADIRUs.

The main characteristics of HG2030BE ADIRU (Figure 4.3) show that *MTBF* = 40,000 h and $E[TBUR]$ = 23,500 h. According to the data given in [11], the approximate price of ADIRU is $ 50,000. According to [12], the average flight time (τ) of the A380 is 8 h, and the average flying hours per year for a single A380 is approximately 5,000 h. Data on testing and setting up ADIRU in the laboratory are given in [13]. The rest of the initial data is taken from various instructions and technical descriptions and is given in Table. 4.1.

Table 4.1. Initial data for warranty maintenance optimization

Parameter name (unit)	Symbol	Parameter value
1	2	3
Warranty period expressed in flight hours (h)	T_W	5,000
The average duration of maintenance operations at the O-level for A380 (h)	$t_M^{O-level}$	1

1	2	3
The average scheduled stop time of A380 at the base airport (h)	t_{stop}	1.5
Maintenance labour cost per hour ($/h)	LC	15
Cost of a spare ADIRU ($)	C_{LRU}	50,000
Guaranteed ADIRU repair time by the manufacturer (days)	T_{RS}	15
Guaranteed expedited delivery time of a spare ADIRU (days)	T_{ED}	1
ADIRU permanent failure rate (h^{-1})	λ	2.5×10^{-5}
ADIRU intermittent failure rate (h^{-1})	θ	1.76×10^{-5}
The average time of rechecking the ADIRU using test equipment located at the I-level maintenance (h)	$t_{TE}^{I-level}$	2
Number of LRUs types that can be rechecked by the test equipment located at I-level maintenance (pcs)	$F_{TE}^{I-level}$	1
Cost of the test equipment located at I-level maintenance ($)	$C_{TE}^{I-level}$	20,000

Table. 4.2 presents the results of the calculations.

Table 4.2. The results of calculations for the 1st and 2nd warranty maintenance options

Warranty maintenance option	Optimal number of planned spare ADIRUs (PS_i)	Number of unplanned spare ADIRUs (US_i)	$WTEC_i$ ($)	ADIRS unavailability ($\bar{A}$)
i = 1	3	0	18,830	2×10^{-8}
i = 2	2	0	13,600	2×10^{-8}

As can be seen in Table 4.2, the average maintenance costs per aircraft for the 2nd WMO are 28 % less than for the 1st option. Therefore, the 2nd WMO is preferable.

4.5. A technique for minimizing the costs of post-warranty maintenance of redundant digital avionics systems

This technique uses the mathematical model of LRU/LRM operation over an infinite time interval and the post-warranty maintenance model of redundant avionics systems considered in subsections 2.3 and 2.6.

1. The initial data includes the same parameters as given in item 1 of Subsection 4.4, except T_W and N_W, as well as the following additional parameters: the post-warranty maintenance period (T_{PW}); the number of aircraft that do not have the supplier's warranty (N_{PW}); the average cost of shipping the LRU / LRM for repairing and back to the airline (C_{TR}); the average repair cost of LRU/LRM with intermittent failure (C_{IF}); the average repair cost of LRU/LRM with permanent failure (C_{PF}); the average repair cost of the SRU with permanent failure at the manufacturer ($C_{PF,R}$); the average cost of shipping the SRU to the manufacturer and back to the airline ($C_{TR,SRU}$); the cost of ATE used at the I-level maintenance ($C_{ATE}^{I-level}$); the number of LRU/LRM types that can be checked using ATE at the I-level maintenance ($F_{ATE}^{I-level}$); the cost of the *j*-th SRU ($j = \overline{1,n}$) (C_j); the number of spare SRUs of the *j*-th type (SRU_j); the number of different SRUs in the LRU/LRM (n); the required probability that all LRUs/LRMs will be provided by SRUs of the *j*-th type ($P(\mathrm{H}_j)$); the permanent failure rate for the *j*-th type SRU (λ_j); the intermittent failure rate for the *j*-th type SRU (θ_j); the average repair time of the *j*-th type SRU at the manufacturer ($t_{RS,j}$); the average cost of SRU repair with intermittent failure at the manufacturer ($C_{IF,R}$); the average time to detect place of the intermittent failure in the dismantled LRU/LRM with depth to SRU using IFD ($t_{IF,D}^{I-level}$); the IFD cost (C_{IFD}); the number of LRU/LRM types tested with IFD to detect intermittent failures ($F_{IFD}^{I-level}$); the average time to detect a permanent failure in the LRU/LRM with a depth to SRU and to replace the failed SRU at the I-level maintenance ($t_{PF,D}^{I-level}$); the average time to detect an intermittent failure in the LRU/LRM with a depth to SRU and to replace the failed SRU at the I-level maintenance ($t_{IF,D}^{I-level}$); the

average time to detect the place of a permanent failure in the SRU with a depth to one or more non-repairable electronic components and replace them at D-level maintenance ($t_{PF,D}^{D-level}$); the average time to detect the place of an intermittent failure in the SRU with a depth to one or more non-repairable electronic components and replace them at D-level maintenance ($t_{IF,D}^{D-level}$); the average cost of replaceable non-repairable electronic components used when repairing the SRU with a permanent failure at D-level maintenance ($C_{PF,R}^{D-level}$); the average cost of replaceable non-repairable electronic components used when repairing the SRU with an intermittent failure at D-level maintenance ($C_{IF,R}^{D-level}$); the cost of diagnostics and repair equipment used at D-level maintenance ($C_{DRT}^{D-level}$); the number of SRU types repaired at D-level maintenance ($Z_{DRT}^{D-level}$); the cost of a spare electronic component of the q-th type in the l-th SRU ($C_{l,q}, l = \overline{1,n}, q = \overline{1,SRU_l}$); the number of spare non-repairable electronic components of the q-th type in the l-th SRU ($X_{l,q}$).

2. Determination of the steady-state value of the total probability of LRU/LRM repair using formula (2.61):

$$P_R^*(\tau) = \lim_{j\to\infty} P_R(j\tau) = 1 - e^{-(\lambda+\theta)\tau}.$$

3. Determination of the LRU/LRM unscheduled removal rate $\lambda_{i,i+1}$ by formula (A.2.1):

$$\lambda_{i,i+1} = \begin{cases} q\tau / P_R^*(\tau), \text{ if } i = \overline{1, PS+1}, \\ (q + PS + 1 - i)\tau / P_R^*(\tau), \text{ if } i = \overline{PS+2, PS+q}. \end{cases}$$

4. Determination of the average LRU/LRM repair time by formula (A.2.3):

$$E[T_R] = \begin{cases} T_{RS}, \text{ for the first PWMO,} \\ T_{RS}P_{PF} + t_{TE}^{I-level}, \text{ for the second PWMO.} \end{cases}$$

5. Determination of the LRU/LRM repair rate $\mu_{i+1,i}$ by formula (A.2.2):

$$\mu_{i+1,i} = \begin{cases} i\mu_{RS} = i / E[T_R], \text{ if } i = \overline{1, PS}, \\ i\mu_{ED} = i / T_{ED}, \text{ if } i = \overline{PS+1, PS+q}. \end{cases}$$

6. Calculation the MTBUR by formula (2.53):

$$E[TBUR] = \frac{\tau}{1-e^{-\theta\tau}} + \left[\left(1-e^{-\lambda\tau}\right)\left(\frac{1}{\lambda} - \frac{\tau}{1-e^{-\theta\tau}}\right) - \tau e^{-\lambda\tau}\right]\frac{1}{1-e^{-(\lambda+\theta)\tau}} + \frac{1}{1-e^{-(\lambda+\theta)\tau}}\left(\tau - \frac{1-e^{-\lambda\tau}}{\lambda}\right).$$

7. Determination of the mean time spent by the LRU/LRM in the state of repairing due to a permanent failure by formula (2.51):

$$E\left[TS_7\right]=t_{PFR}\left[\frac{1-e^{-\lambda\tau}}{1-e^{-(\lambda+\theta)\tau}}\right].$$

8. Determination of the a posteriori probability that dismantled LRU/LRM has a permanent failure by formula (A.2.6):

$$P_{PF}=E\left[TS_7\right]/t_{PFR} .$$

9. Determination of the failure flow parameter of a set mN_W of the same type LRUs/LRMs by formula (A.2.11):

$$\Lambda=\frac{mN_{PW}P_R^*(\tau)}{\tau}.$$

10. Determination of the average time of waiting for a spare LRU/LRM from the airline warehouse in the situation “aircraft on ground” by formula (A.2.9):

$$t_{spare}\left(PS,US\right)=\Lambda^{-1}\sum_{i=1}^{q}iP_{PS+i+1} .$$

11. Determination of the optimal number of spare LRUs/LRMs by the criterion (2.77):

$$E\left[TS_4\right]=\left[t_{spare}\left(PS,US\right)+t_M^{O-level}-t_{stop}\right]\to 0.$$

12. Determination of the average total number of removals due to permanent and intermittent failures over time T_{PW} by formula (2.64):

$$N_R\left(T_{PW}\right)=T_{PW}\,P_R^*\left(\tau\right)/\tau .$$

13. Determination of the $PWTEC_1$ using formula (2.79):

$$PWTEC_1=m\left[LCt_M^{O-level}+C_{TR}+C_{IF}P_{IF}+C_{PF}P_{PF}\right]N_R(T_{PW})+C_{LRU}\left(PS+US\right)/N_{PW} .$$

14. Determination of the $PWTEC_2$ using formula (2.80):

$$PWTEC_2=m\left[\left(C_{TR}+C_{PF}\right)P_{PF}+LC\left(t_M^{O-Level}+t_{TE}^{I-level}\right)\right]N_R(T_{PW})+$$
$$C_{TE}^{I-level}/\left(N\times F_{TE}^{I-level}\right)+C_{LRU}\left(PS+US\right)/N_{PW} .$$

15. Determination of the $PWTEC_3$ using formula (2.81):

$$PWTEC_3=m\left[LC\left(t_M^{O-level}+t_{ATE}^{I-level}\right)+\left(C_{PF,R}+LC\times t_{PF,D}^{I-level}\right)P_{PF}+C_{TR,SRU}\right]N_R\left(T_{PW}\right)+$$
$$C_{ATE}^{I-level}/\left(N\times F_{ATE}^{I-level}\right)+\left[C_{LRU}\left(PS+US\right)+\sum_{j=1}^{n}C_jSRU_j\right]\Big/N_{PW} .$$

16. Determination of the optimal number of spare SRUs of the j-th type for the 3rd PWMO by formula (2.82):

$$1-P\left(\mathrm{H}_j\right)>\frac{\left(\mathrm{H}_j\lambda_j t_{RS,j}\right)^{(SRU_j+1)}}{\left(SRU_j+1\right)!}\exp\left(-\mathrm{H}_j\lambda_j t_{RS,j}\right).$$

17. Determination of the $PWTEC_4$ using formula (2.83):

$$PWTEC_4=m\Big[LC\left(t_M^{O-level}+t_{ATE}^{I-level}\right)+\left(C_{PF,R}+LC\times t_{PF,D}^{I-level}\right)P_{PF}+\left(C_{IF,R}+LC\times t_{IF,D}^{I-level}\right)P_{IF}+C_{TR,SRU}\Big]N_R\left(T_{PW}\right)+$$
$$C_{ATE}^{I-level}\Big/\left(N_{PW}\times F_{ATE}^{I-level}\right)+C_{IFD}\Big/\left(N_{PW}\times F_{IFD}^{I-level}\right)+\Big[C_{LRU}\left(PS+US\right)+\sum_{j=1}^{n}C_jSRU_j\Big]\Big/N_{PW}.$$

18. Determination of the optimal number of spare SRUs of the j-th type for the 4th PWMO by formula (2.84):

$$1-P\left(\mathrm{H}_j\right)>\frac{\left[\mathrm{H}_j\left(\lambda_j+\theta_j\right)t_{RS,j}\right]^{(SRU_j+1)}}{\left(SRU_j+1\right)!}\exp\left[-\mathrm{H}_j\left(\lambda_j+\theta_j\right)t_{RS,j}\right].$$

19. Determination of the $PWTEC_5$ using formula (2.96):

$$PWTEC_5=m\Big\{LC\left(t_M^{O-level}+t_{ATE}^{I-level}\right)+\Big[LC\left(t_{PF,D}^{I-level}+t_{PF,D}^{D-level}\right)+C_{PF,R}^{D-level}\Big]P_{PF}+$$
$$\Big[\left(LC\left(t_{IF,D}^{I-level}+t_{IF,D}^{D-level}\right)+C_{IF,R}^{D-level}\right)P_{IF}\Big]\Big\}N_R\left(T_W\right)+C_{ATE}^{I-level}\Big/\left(N_{PW}\times F_{ATE}^{I-level}\right)+$$
$$C_{IFD}^{I-level}\Big/\left(N_{PW}\times F_{IFD}^{I-level}\right)+C_{DRT}^{D-level}\big/Z_{DRT}^{D-level}+\Big[C_{LRU}\left(PS+US\right)+\sum_{j=1}^{n}C_jSRU_j+\sum_{l=1}^{n}\sum_{q=1}^{SRU_l}C_{l,q}X_{l,q}\Big]\Big/N_{PW}.$$

20. Determination of the optimal number of spare SRUs of the j-th type for the 5th PWMO by formula (2.88):

$$1-P\left(\mathrm{H}_j\right)>\frac{\left[\mathrm{H}_j\left(\lambda_j+\theta_j\right)\left(t_{PF,D}^{D-level}P_{PF}+t_{IF,D}^{D-level}P_{IF}\right)\right]^{(SRU_j+1)}}{\left(SRU_j+1\right)!}\exp\left[-\mathrm{H}_j\left(\lambda_j+\theta_j\right)\left(t_{PF,D}^{D-level}P_{PF}+t_{IF,D}^{D-level}P_{IF}\right)\right]..$$

Let us consider an example of minimizing the cost of post-warranty maintenance for the ADIRS of the aircraft Airbus A380. By September 2017, the Emirates (UAE) airline received 97 A380 aircraft for operation [14]. Let us determine the average costs per one ADIRS of the A380 for various PWMOs by the developed technique. According to [14, 15], each ADIRU includes the following SRUs: an air data computer (ADC); a multi-mode receiver (MMR); 3 digital ring laser gyros; 3 quartz accelerometers; and a power supply module. According to [17], the cost of three Honeywell (USA) ring laser gyroscopes is about $ 15,000.

Tables 4.1 and 4.3 present the data required to calculate the average costs per aircraft for different PWMOs.

Table 4.3. Initial data for post-warranty maintenance optimization

Parameter name (unit)	Symbol	Parameter value
1	2	3
Post-warranty maintenance period, expressed in flight hours (h)	T_{PW}	50,000
Number of the Airbus A380 aircraft without warranty of the supplier (pcs)	N_{PW}	97
The average cost of LRU shipping to repair and back ($)	C_{TR}	300
The average cost of repairing the LRU with an intermittent failure ($)	C_{IF}	5,000
The average cost of repairing the LRU with a permanent failure ($)	C_{PF}	10,000
The average cost of repairing the SRU with a permanent failure at the manufacturer ($)	$C_{PF,R}$	1,000
The average cost of repairing the SRU with an intermittent failure at the manufacturer ($)	$C_{IF,R}$	1,500
The average cost of shipping the SRU to the manufacturer and back ($)	$C_{TR,SRU}$	100
Cost of ATE used at the I-level maintenance ($)	$C_{ATE}^{I-level}$	2,000,000
Number of LRU types that can be tested by ATE at the I-level maintenance (pcs)	$F_{ATE}^{I-level}$	120

1	2	3
Number of different SRUs in the LRU under consideration (pcs)	n	9
Cost of the *j*-th SRU ($)	C_j	5,000
The required probability that all LRUs will be provided with SRUs of the *j*-th type	$P(\mathrm{H}_j)$	0.99
Average repair time of the *j*-th type SRU at the manufacturer (days/h)	$t_{RS,j}$	15/195
Average time to detect the location of an intermittent failure in the dismantled LRU with depth to SRU and replace the faulty SRU at I-level maintenance (h)	$t_{IF,D}^{I-Level}$	1
Cost of IFD ($)	C_{IFD}	100,000
Number of LRU types tested by IFD (pcs)	$F_{IFD}^{I-Level}$	120
Average time to detect the location of a permanent failure in the dismantled LRU with depth to SRU and replace the failed SRU at I-level maintenance (h)	$t_{PF,D}^{I-Level}$	0.25
Average time to detect a permanent failure in the SRU with depth to one or more non-repairable electronic components and replace them at D-level maintenance (h)	$t_{PF,D}^{D-Level}$	2
Average time to detect an intermittent failure in the SRU with depth to one or more non-repairable electronic components and replace them at D-level maintenance (h)	$t_{IF,D}^{D-Level}$	3

1	2	3
The average cost of replaced non-repairable components when repairing the SRU with a permanent failure at D-level maintenance ($)	$C_{PF,R}^{D-Level}$	200
The average cost of replaced non-repairable components when repairing the SRU with an intermittent failure at D-level maintenance ($)	$C_{IF,R}^{D-Level}$	200
Cost of diagnostics and repair equipment used at D-level maintenance ($)	$C_{DRT}^{D-Level}$	200,000
Number of SRU types repaired at D-level maintenance ($)	$Z_{DRT}^{D-Level}$	500
Cost of spare electronic components in all SRUs of the LRU under consideration ($)	$\sum_{l=1}^{9}\sum_{q=1}^{SRU,l} C_{l,q}X_{l,q}$	40,000

Table 4.4 presents the results of calculations.

Table 4.4. Calculation results for five different post-warranty maintenance options

PWMO type (i)	$E[TBUR]$ (h)	Optimal number of spare ADIRUs (PS_i)	$PWTEC_i$ ($)	ADIRS unavailability ($\bar{A}$)
$i = 1$	23,500	4	110,600	2×10^{-8}
$i = 2$	23,500	3	86,150	2×10^{-8}
$i = 3$	23,500	1	11,820	2×10^{-8}
$i = 4$	30,800	1	13,300	2×10^{-8}
$i = 5$	30,800	1	10,070	2×10^{-8}

Since the 4th and 5th PWMO reckon for the use of the IFD, the rate of intermittent failures in repaired LRUs should significantly reduce. For example, as described in [18], after repairing the AN/APG-68 on-board radars of F-16 using IFD, MTBUR increased by more than three-fold. Therefore, in calculations we assumed that the intermittent failure rate is less by a factor of 3, i. e. $\theta = 0.6\times10^{-5}$ h^{-1}, for the 4th and 5th PWMO.

Number of spare SRU_j $(j = \overline{1,9})$ was calculated for $P(\mathrm{H}_j) = 0.99$. Using inequalities (2.93) and (2.95), we obtain that for the 3rd and 4th post-warranty maintenance options $SRU_j = 2$ $(j = \overline{1,9})$, and for the 5th maintenance option $SRU_j = 1$ $(j = \overline{1,9})$.

Table 4.4 shows that the 5th PWMO, which uses IFD at I- and D-level of maintenance, has the lowest value of ADIRS maintenance costs, and, therefore, is the best. Indeed, $PWTEC_5$ is 11 times less than $PWTEC_1$ and over 8.5 times less than $PWTEC_2$. Note also that the use of IFD at I-level maintenance only is inadvisable, since $PWTEC_4 > PWTEC_3$ by 12.5 %. The calculations show that the 5th PWMO requires the least number of spare parts, namely one LRU and one SRU of each type.

4.6. A technique of determining the optimal replacement thresholds for a known inspection schedule of a deteriorating system

This technique uses the mathematical model of CBM for deteriorating equipment of vehicles discussed in subsection 3.2.

4.6.1. Categories of instrumental measurements used in condition monitoring. A sufficiently long period of degradation usually precedes failures of deteriorating vehicles' equipment, so the condition monitoring is necessary to prevent or detect failures. Monitoring of equipment condition is carried out by instrumental measurements of one or several degradation parameters. According [19], the following categories of instrumental measurements are distinguished for condition monitoring:

1. *Temperature measurements* (e. g., thermography) help detect potential failures associated with temperature changes in equipment.

2. *Dynamic monitoring* (e. g., spectrum analysis, and impact pulse analysis) involves measuring and analysing the energy radiated by mechanical equipment in the form of waves, such as vibration, pulses and acoustic effects.

3. *Oil analysis* (e. g., ferrography, particle counter testing) is performed for various types of oils, such as lubricants, hydraulic or insulating oils. This type of monitoring can

indicate such problems as equipment degradation (e. g., due to wear), oil contamination, incorrect type of oil (e. g., improper amounts of additives) and deterioration of oil quality.

4. *Corrosion monitoring* (e. g., coupon testing, corrosion testing) helps determine the corrosion degree, corrosion rate and corrosion condition (e. g., active or passive condition) of the material.

5. *Non-destructive testing* includes tests (e. g., x-ray, ultrasound) that are non-invasive for the equipment under test.

6. *Methods of electrical condition monitoring* (e. g., high-potential testing or power signature analysis) include measuring changes in such system properties as resistance, conductivity, electrical strength, and potential.

7. *Equipment performance monitoring* is a condition monitoring method that predicts problems by monitoring changes in such variables as pressure, flow rate, power consumption, and equipment capacity.

4.6.2. General procedure for determining the optimal replacement thresholds. The procedure assumes that a single system state parameter describes the system degradation process. Determination of the optimal replacement thresholds is considered using the example of a stochastic degradation process described by equation (3.46), in which A_0 and A_1 are independent random variables with a Gaussian distribution.

1. The following initial data should be known: the mathematical expectation (m_0) and standard deviation (σ_0) of the initial value of the system state parameter; the mathematical expectation (m_1) and standard deviation (σ_1) of the degradation rate of the system state parameter; the standard deviation of the measurement error (σ_y); the system functional failure threshold (FF); the condition monitoring periodicity (τ); the average profit per unit time of system operation (C_{profit}); the average cost of preventive replacement (repair) of the system (C_{pr}); the average cost of additional spare parts due to untimely preventive replacement (repair) of the system (C_{sp}); the average cost of corrective replacement (repair) of the system (C_{cr}); the average loss due to missed failure detection when checking the system suitability (C_{uf}), the minimum allowable value of the a posteriori probability of the system failure-free operation (P_A).

2. Determination of the PDF of the system's operating time to failure by the following formula [20]:

$$\omega(t)=\frac{m_1\left(\sigma_0^2+\sigma_1^2t^2\right)+\sigma_1^2t\left(FF-m_0-m_1t\right)}{\sqrt{2\pi}\left(\sigma_0^2+\sigma_1^2t^2\right)^{3/2}}\exp\left\{-\frac{\left(FF-m_0-m_1t\right)^2}{2\left(\sigma_0^2+\sigma_1^2t^2\right)}\right\}.$$

3. Determination of the conditional PDF the system's operating time to failure provided that $A_0=a_0$ by formula (3.80):

$$\omega\left(\eta|a_0\right)=\frac{m_1\sigma_1^2\eta^2+\sigma_1^2\eta\left(FF-a_0-m_1\eta\right)}{\sqrt{2\pi}\sigma_1^3\eta^3}\exp\left\{-\frac{\left(FF-a_0-m_1\eta\right)^2}{2\sigma_1^2\eta^2}\right\}.$$

4. Determination of the conditional joint PDF of the errors in evaluating the operating time to failure using formula (3.69):

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^k\int_0^{FF}f\left(a_0\right)\omega\left(\eta\middle|a_0\right)\left(\frac{FF-a_0}{\eta}\right)^k\times$$

$$\prod_{i=1}^{k}\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{\left(a_0-FF\right)\delta_i}{\eta}+PF_i-FF\right]^2\right\}\frac{da_0}{\omega\left(\eta\right)}.$$

5. Determination of the probabilities of correct and incorrect decisions made when checking the system suitability at time $k\tau$ (k = 1, 2, ...) by formulas (3.35)–(3.40):

$$P\left[\Gamma_1\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{(k+1)\tau}^{\infty}\omega(\vartheta)\int_{k\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta,$$

$$P\left[\Gamma_2\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{(k+1)\tau}^{\infty}\omega(\vartheta)\int_{-\infty}^{k\tau-\vartheta}\int_{(k-1)\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta,$$

$$P\left[\Gamma_3\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{k\tau}^{(k+1)\tau}\omega(\vartheta)\int_{k\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta,$$

$$P\left[\Gamma_4\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{k\tau}^{(k+1)\tau}\omega(\vartheta)\int_{-\infty}^{k\tau-\vartheta}\int_{(k-1)\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta,$$

$$P\left[\Gamma_5\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_0^{k\tau}\omega(\vartheta)\int_{k\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta,$$

$$P\left[\Gamma_6\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_0^{k\tau}\omega(\vartheta)\int_{-\infty}^{k\tau-\vartheta}\int_{(k-1)\tau-\vartheta}^{\infty}\ldots\int_{\tau-\vartheta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\vartheta\right)\overline{dg_1dg_k}d\vartheta.$$

6. Solving the problem of finding the optimal replacement threshold at time $k\tau$ by the criterion of maximum net income (3.41):

$$PF_k^{opt} \Rightarrow \max_{PF_k}\Big\{C_{profit}\left(t_{k+1}-t_k\right)P\Big[\Gamma_1\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]-C_{pr}P\Big[\Gamma_4\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]-$$

$$\left(C_{pr}+C_{sp}\right)P\Big[\Gamma_2\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]-C_{cr}P\Big[\Gamma_6\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]-$$

$$C_{uf}\Big\{P\Big[\Gamma_3\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]+P\Big[\Gamma_5\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]\Big\}\Big\},$$

or by the minimum Bayes risk criterion (3.42):

$$PF_k^{opt} \Rightarrow \min_{PF_k}\Big\{\left(C_{pr}+C_{sp}\right)P\Big[\Gamma_2\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]+C_{uf}\Big\{P\Big[\Gamma_3\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]+P\Big[\Gamma_5\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]\Big\}\Big\},$$

or by the criterion of minimum total error probability (3.43):

$$PF_k^{opt} \Rightarrow \min_{PF_k}\Big\{P\Big[\Gamma_2\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]+P\Big[\Gamma_3\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]+P\Big[\Gamma_5\Big(\overline{\tau,k\tau};(k+1)\tau\Big)\Big]\Big\},$$

or by the criterion of a given a posteriori probability of failure-free operation (3.44)

$$PF_k^{opt} \Rightarrow P\left\{\mathrm{H} > t_{k+1} | \mathrm{H}_k > t_k\right\} = P_A .$$

4.6.3. The procedure for determining the optimal replacement thresholds by the minimum total error probability criterion for a non-random initial value of the system state parameter.

1. The following initial data should be known: the initial value of the system state parameter (a_0); the mathematical expectation (m_1) and standard deviation (σ_1) of the system state parameter degradation rate; the standard deviation of the system state parameter measurement error (σ_y); the functional failure threshold of the system (FF); the periodicity of condition monitoring (τ).

2. Determination of the PDF of the system's operating time to failure by formula (3.80):

$$\omega(\eta)=\frac{m_1\sigma_1^2\eta^2+\sigma_1^2\eta\left(FF-a_0-m_1\eta\right)}{\sqrt{2\pi}\sigma_1^3\eta^3}\exp\left\{-\frac{\left(FF-a_0-m_1\eta\right)^2}{2\sigma_1^2\eta^2}\right\}.$$

3. Determination of the conditional joint PDF of the errors in evaluating the operating time to failure using formula (3.70):

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^k\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{\left(a_0-FF\right)\delta_i}{\eta}+PF_i-FF\right]^2\right\}.$$

4. Determination of the probabilities of "false alarm", "missed detection 1" and "missed detection 2" at time $k\tau$ (k = 1, 2, ...) by formulas (3.75), (3.76) and (3.78):

$$P\left[\Gamma_2\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{(k+1)\tau}^{\infty}\omega(\vartheta)\left[\prod_{i=1}^{k}\int_{\frac{(a_0-FF)(i\tau-\vartheta)}{\vartheta}+PF_i-FF}^{\infty}\Omega(x_i)\,dx_i\right]d\vartheta,$$

$$P\left[\Gamma_3\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{k\tau}^{(k+1)\tau}\omega(\vartheta)\left[\prod_{i=1}^{k}\int_{-\infty}^{\frac{(a_0-FF)(i\tau-\vartheta)}{\vartheta}+PF_i-FF}\Omega(x_i)\,dx_i\right]d\vartheta,$$

$$P\left[\Gamma_5\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]=\int_{0}^{k\tau}\omega(\vartheta)\left[\prod_{i=1}^{k}\int_{-\infty}^{\frac{(a_0-FF)(i\tau-\vartheta)}{\vartheta}+PF_i-FF}\Omega(x_i)\,dx_i\right]d\vartheta,$$

where

$$\Omega(y)=\frac{1}{\sqrt{2\pi}\sigma_y}\exp\left[-\frac{(y-m_y)^2}{2\sigma_y^2}\right].$$

5. Solving the problem of finding the optimal replacement threshold at time $k\tau$ by the minimum total error probability criterion (3.43):

$$PF_k^{opt}\Rightarrow\min_{PF_k}\left\{P\left[\Gamma_2\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]+P\left[\Gamma_3\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]+P\left[\Gamma_5\left(\overline{\tau,k\tau};(k+1)\tau\right)\right]\right\}.$$

Let us consider an example of determining the optimal replacement thresholds for the output voltage of a radar transmitter power supply $V_{out}(t)$. Following [21], the periodical inspection schedule of the output voltage supposes that τ = 50 h. According to [21], the radar transmitter is operable, if $V_{out}(t)\leq 25$ kV, that is, FF = 25 kV. The initial data is as follows: $a_0=19.645$ kV, $m_1=0.01$ kV/h, $\sigma_1=0.0043$ kV/h, $\sigma_y=0.1$ kV, and $m_y=0$.

We determine the optimal replacement thresholds PF_k for inspection times $k\tau$ ($k=\overline{1,7}$). By the classification of instrumental measurements given in paragraph 3.6.1, this type of measurement refers to the equipment performance monitoring.

Table 4.5 shows the optimal replacement thresholds for condition monitoring at times $k\tau$ ($k=\overline{1,7}$).

Table 4.5. Optimal replacement thresholds

PF_1	PF_2	PF_3	PF_4	PF_5	PF_6	PF_7
21.90 kV	23.22 kV	23.67 kV	23.90 kV	24.11 kV	24.22 kV	24.33 kV

Figure 4.7 shows the dependence of the total error probability on the threshold PF_4.

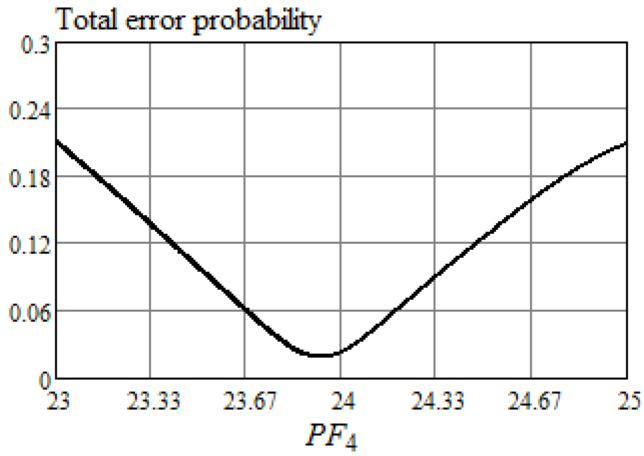


Fig. 4.7. Dependence of the total error probability dependence on the threshold PF_4 for $k = 4$ and $PF_1 = 21.90$ kV, $PF_2 = 23.22$ kV and $PF_3 = 23.67$ kV

As can be seen in Fig. 4.7, the use of the optimal replacement threshold PF_4 reduces the total error probability by a factor of 10 compared with the case when $PF_4 = FF = 25$ kV.

4.7. A technique for determining the optimal replacement threshold and periodicity of suitability checking when monitoring the system condition

As in subsection 4.6.2, we assume that one state parameter describes the process of degradation of the system. Let us consider the determination of the optimal replacement threshold and optimal periodicity of suitability checking using a stochastic degradation model (3.46), in which the random variable A_1 has a Gaussian distribution and $A_0 = a_0$.

1. The following initial data should be known: the initial value of the system state parameter (a_0); the mathematical expectation (m_1) and standard deviation (σ_1) of the system state parameter degradation rate; the standard deviation of the system state parameter measurement error (σ_y); the functional failure threshold of the system (FF); the scheduled CBM interval (T); the average duration of the system suitability checking (t_{SC}), preventive maintenance (t_{PR}) and corrective maintenance (t_{CR}).

2. Determination of the PDF of the system's operating time to failure by formula (3.80):

$$\omega(\eta) = \frac{m_1\sigma_1^2\eta^2 + \sigma_1^2\eta(FF - a_0 - m_1\eta)}{\sqrt{2\pi}\sigma_1^3\eta^3}\exp\left\{-\frac{(FF - a_0 - m_1\eta)^2}{2\sigma_1^2\eta^2}\right\}.$$

3. Determination of the conditional joint PDF of the errors in evaluating the operating time to failure using formula (3.70):

$$\psi_0\left(\overline{\delta_1,\delta_k}|\eta\right)=\left(\frac{1}{\sigma_y\sqrt{2\pi}}\right)^k\left(\frac{FF-a_0}{\eta}\right)^k\prod_{i=1}^{k}\exp\left\{-\frac{1}{2\sigma_y^2}\left[\frac{(a_0-FF)\delta_i}{\eta}+PF_i-FF\right]^2\right\}.$$

4. Determination of the conditional probabilities of correct and incorrect decisions made when checking the system suitability by formulas (3.88)–(3.93):

$$P_{FA}\left(n\tau|\eta\right)=\int_{-\infty}^{n\tau-\xi}\int_{(n-1)\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_1,g_n}|\eta\right)\overline{dg_1dg_n},\quad P_{TN,1}\left(k\tau|\eta\right)=\int_{-\infty}^{k\tau-\eta}\int_{(k-1)\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\eta\right)\overline{dg_1dg_k},$$

$$P_{MD,1}\left(k\tau|\eta\right)=\int_{k\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_1,g_k}|\eta\right)\overline{dg_1dg_k},\quad P_{TP}\left(n\tau|\eta\right)=\int_{n\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_1,g_n}|\eta\right)\overline{dg_1dg_n},$$

$$P_{TN,2}\left(j\tau|\eta\right)=\int_{-\infty}^{j\tau-\eta}\int_{(j-1)\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_{k+1},g_j}|\eta\right)\overline{dg_{k+1}dg_j},\quad P_{MD,2}\left(j\tau|\eta\right)=\int_{j\tau-\eta}^{\infty}\ldots\int_{\tau-\eta}^{\infty}\psi_0\left(\overline{g_1,g_j}|\eta\right)\overline{dg_1dg_j}.$$

5. Determination of the mean time spent by the system in various states by formulas (3.104)–3.108):

$$E\left[TS_1\right]=\sum_{k=0}^{M}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}n\tau P_{FA}\left(n\tau|x\right)+k\tau P_{TN,1}\left(k\tau|x\right)+xP_{MD,1}\left(k\tau|x\right)\right]\omega(x)dx+$$
$$\int_{T}^{\infty}\left[\sum_{k=1}^{M}k\tau P_{FA}\left(k\tau|x\right)+TP_{TP}\left(M\tau|x\right)\right]\omega(x)dx,$$

$$E\left[TS_2\right]=\sum_{k=0}^{M-1}\int_{k\tau}^{(k+1)\tau}\left[\sum_{j=k+1}^{M}(j\tau-x)P_{TN,2}\left(j\tau|x\right)+(T-x)P_{MD,2}\left(M\tau|x\right)\right]\omega(x)dx+$$
$$\int_{M\tau}^{T}(T-x)P_{MD,1}\left(M\tau|x\right)\omega(x)dx,$$

$$E\left[TS_3\right]=t_{SC}\sum_{k=0}^{M-1}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}nP_{FA}\left(n\tau|x\right)+kP_{TN,1}\left(k\tau|x\right)+\sum_{j=k+1}^{M}jP_{TN,2}\left(j\tau|x\right)+MP_{MD,2}\left(M\tau|x\right)\right]\omega(x)dx+$$
$$t_{SC}\int_{t_M}^{\infty}\left[\sum_{k=1}^{m-1}kP_{FA}\left(k\tau|x\right)+MP_{TP}\left((M-1)\tau|x\right)\right]\omega(x)dx,$$

$$E\left[TS_4\right]=t_{PR}\sum_{k=0}^{M-1}\left[\int_{k\tau}^{(k+1)\tau}\sum_{n=1}^{k-1}P_{FA}\left(n\tau|x\right)+P_{TN,1}\left(k\tau|x\right)\right]\omega(x)dx+t_{PR}\int_{T}^{\infty}\left[\sum_{k=1}^{M}P_{FA}\left(k\tau|x\right)+P_{TP}\left(M\tau|x\right)\right]\omega(x)dx,$$

$$E\left[TS_5\right]=t_{CR}\sum_{k=0}^{M}\int_{k\tau}^{(k+1)\tau}\left[\sum_{j=k+1}^{M}P_{TN,2}\left(j\tau|x\right)+P_{MD,2}\left(M\tau|x\right)\right]\omega(x)dx.$$

6. Calculation of the system availability by formula (2.20):

$$A=E\left[TS_1\right]/E\left[TS_0\right].$$

7. Solving the problem (3.131) to find the optimal inspection periodicity and replacement threshold:

$$A^{\max}\left(\tau_{opt}, PF^{opt}\right) = \max_{\tau, PF}\left\{A\left(\tau, PF\right)\right\}.$$

Let us consider an example of determining the optimal replacement threshold and periodicity of suitability checking for the output voltage of a radar transmitter power supply [21] with the following initial data: FF = 25 kV; T = 1,000 h; t_{CR} = 10 h; $t_{SC} = t_{PR}$ = 3 h; a_0 = 19.645 kV; m_1 = 0.01 kV/h; σ_1 = 0.0043 kV/h; σ_y = 0.1 kV; and m_y = 0.

Figure 4.8 shows the availability dependence on the number of inspections for two maintenance types. Curve 1 corresponds to CBM, which requires 10 suitability checks with a periodicity of 91 h over the 1,000 h interval to provide the availability value of 0.965, i. e. M_{opt} = 10, $\tau_{opt} \approx$ 91 h, PF^{opt} = 24 kV, and $A^{\max}\left(\tau_{opt}, PF^{opt}\right) = 0.965$. Curve 2 corresponds to maintenance strategy based on operability checking, i.e. corrective maintenance. For the corrective maintenance, the optimal solution has the following form: M_{opt} = 17, $\tau_{opt} \approx$ 56 h and $A^{\max}\left(\tau_{opt}, PF = FF\right) = 0.89$. Therefore, using the CBM increases availability and reduces the number of inspections.

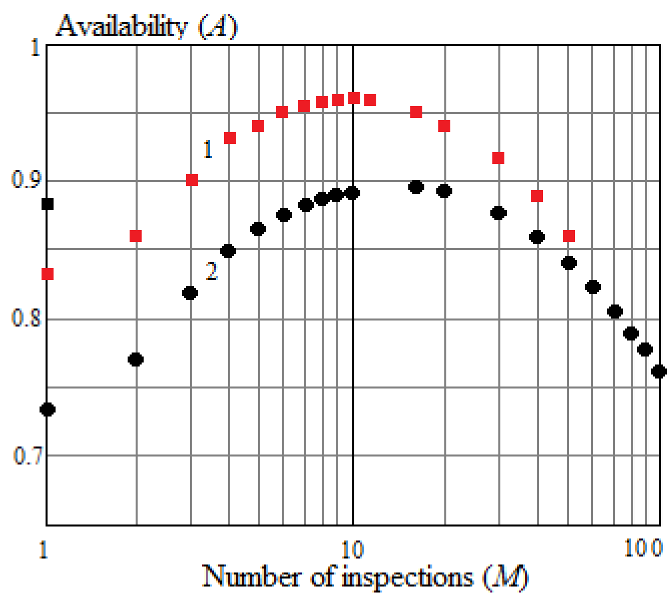


Fig. 4.8. Dependence of the system availability on the number of inspections over the interval of 1,000 h for PF^{opt} = 24 kV < FF (dependence 1) and for $PF = FF$ = 25 kV (dependence 2)

4.8. Checking the adequacy of mathematical models

To check the adequacy of the mathematical models developed in chapters 2 and 3, we will simulate the corresponding maintenance processes using the Monte Carlo method [21] and compare the simulation results with the theoretical calculations.

4.8.1. Simulation of the avionics LRU/LRM operation and maintenance. Let us simulate the operation of the ADIRU HG2030BE [8] used in the Airbus A380 aircraft and compare the simulated mean times $E[TS_1]$, $E[TS_2]$, $E[TS_5]$, $E[TS_6]$, and $E[TBUR]$ with the calculated results by formulas (2.48) - (2.51) and (2.53).

The simulation algorithm of avionics LRM/LRU operation and maintenance includes the following steps:

Step 0. Set the initial data and the number of simulations *NI*.

Step 1. Set $i = 0$.

Step 2. Set $i = i + 1$.

Step 3. Generation of the i-th realization of the operating time to failure of the LRU/LRM with fixating the time to permanent or intermittent failure.

Step 4. Statistical evaluation of $E[TS_1]$, $E[TS_2]$, $E[TS_5]$, $E[TS_6]$, and $E[TBUR]$.

Step 5. If $i < NI$, then go to step 2. Otherwise, go to step 6.

Step 6. The output of the simulation results.

The simulation program is written in the Visual FoxPro 9.0 object-oriented language.

The following initial data are indicated in the input interface of the program for the simulation of ADIRU operation and maintenance:

National Aviation University, Department of Electronics, Ahmed Raza

Pseudo-random non-integer number generator: built-in FOX

Distribution law of time to failure: EXP

A variant of generator start-up: without displacement

(each time you run the test after logging in, the sequence of pseudo-random non-integer numbers is repeated)

Without decoding by cycles (because > 1,000)

Simulation of LRM/LRU: ADIRU

Operation and maintenance for aircraft: A380

Mean time between permanent failures (MTBPF) in flight hours: 40,000

Mean time between intermittent failures (MTBIF) in flight hours: 56,818

Average flight duration in hours: 8

Average repair time due to permanent failure in hours: 1

Average repair time due to intermittent failure in hours: 1

Table 4.6 presents the simulation results and Fig. 4.9 shows the relative deviation of the simulated $E[TBUR]$ from the theoretical value expressed in percentage.

As can be seen in Table 4.6 and Fig. 4.10, the statistical estimates of the mean times $[TS_1]$, $E[TS_2]$, $E[TS_5]$, $E[TS_6]$, and $E[TBUR]$ quickly converge to the theoretical values of these indicators, which proves the adequacy of the developed mathematical model to the real process of ADIRU operation and maintenance.

Table 4.6. Simulation results of ADIRU operation and maintenance

Number of cycles	Mean times				
	E(TS1)	E(TS2)	E(TS5)	E(TS6)	E(TBUR)
1	2	3	4	5	6
10	19338,4700	3,1300	0,2000	0,8000	19341,6000
50	21505,6100	2,5500	0,3200	0,6800	21508,1600
100	23339,3200	2,2000	0,4200	0,5800	23341,5200
500	24486,7000	2,4700	0,3700	0,6300	24489,1700
1000	23721,8700	2,3000	0,4400	0,5700	23724,1700
5000	23345,6400	2,3900	0,4100	0,5900	23348,0200
10000	23324,7500	2,3300	0,4200	0,5800	23327,0800
50000	23444,8500	2,3600	0,4100	0,5900	23447,2100
100000	23443,2700	2,3400	0,4100	0,5900	23445,6100
500000	23532,8500	2,3500	0,4100	0,5900	23535,1900
1000000	23478,7100	2,3500	0,4100	0,5900	23481,0600
Theoretical results:	**23475,8000**	**2,3500**	**0,4100**	**0,5900**	**23478,1500**

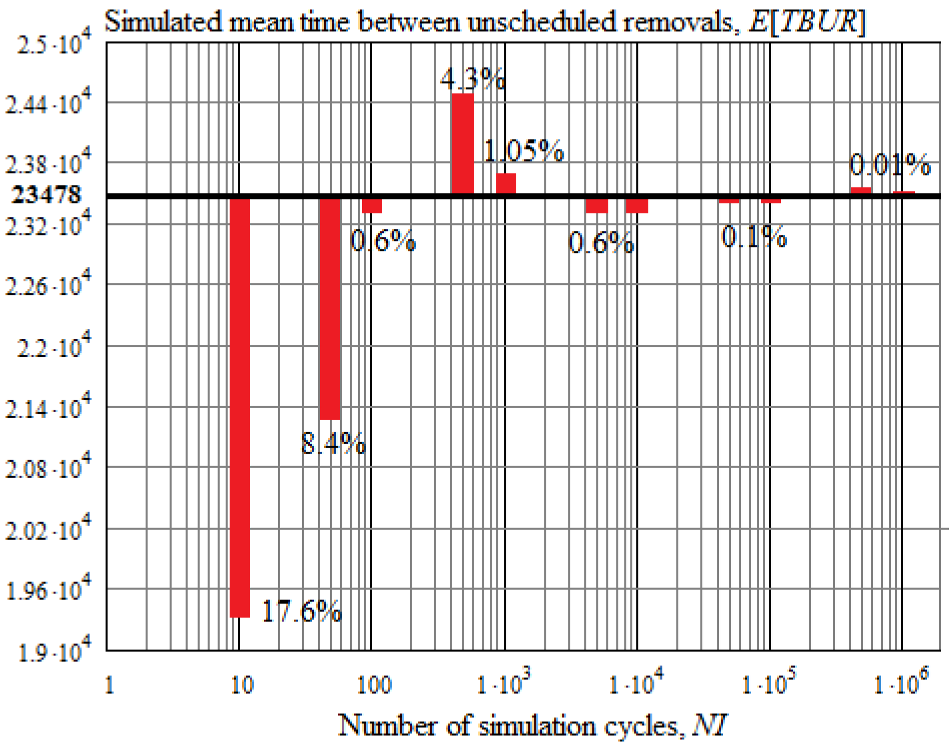


Fig. 4.9. A graphical presentation of simulation results for MTBUR

4.8.2. Simulation of condition-based maintenance of deteriorating vehicles' equipment. Let us perform a simulation of CBM for the power supply voltage of a radar transmitter and compare the simulated probabilities $\overline{P[\Gamma_1(\overline{t_1, t_k}; t_{k+1})], \mathrm{P}[\Gamma_6(\overline{t_1, t_k}; t_{k+1})]}$ with the calculated values of these probabilities by formulas (3.74) - (3.79).

The algorithm for simulating the probabilities of correct and incorrect decisions made when checking the suitability of a system includes the following steps:

Step 0. Set the initial data and the number of simulations *NI*.

Step 1. Set $i = 0$.

Step 2. Set $i = i + 1$.

Step 3. Generation of the i-th realization of a stochastic process of system degradation.

Step 4. Statistical evaluation of the probabilities of correct and incorrect decisions when checking the system suitability $\overline{P[\Gamma_1(\overline{t_1, t_k}; t_{k+1})], \mathrm{P}[\Gamma_6(\overline{t_1, t_k}; t_{k+1})]}$.

Step 5. If $i < NI$, then go to step 2. Otherwise, go to step 6.

Step 6. The output of the simulation results.

The program of simulation of probabilities of correct and incorrect decisions made when checking the suitability of a system is written in the Visual FoxPro 9.0 object-oriented language.

When simulating the CBM process of the radar transmitter power supply, the following initial data were recorded in the input interface of the program:

- National Aviation University, Department of Electronics, Ahmed Raza
- Pseudo-random non-integer number generator: built-in FOX
- A variant of generator start-up: without displacement
- Simulation of: power supply of radar transmitter
- Distribution law of degradation speed coefficient: NORMAL
- Distribution law of measurement error: NORMAL
- The initial value of degradation parameter: 19.645
- Mathematical expectation of degradation speed: 0.01
- The standard deviation of degradation speed: 0.0043
- Mathematical expectation of measurement error: 0
- The standard deviation of measurement error: 0.5
- Functional failure threshold: 25
- Replacement threshold: 22.2
- Inspection time tk: 300
- Inspection time tk+1: 500
- Number of simulation cycles: 10, 50, 100, 500, 1000, 5000, 10000

Table 4.7 and Fig. 4.10 present the simulation results. As can be seen in Table. 4.7 and Fig. 4.10, statistical estimates of the probabilities $\overline{P[\Gamma_1(\overline{t_1, t_k}; t_{k+1})], \mathrm{P}[\Gamma_6(\overline{t_1, t_k}; t_{k+1})]}$ quickly converge to the theoretical values of these indicators, which proves the adequacy of the developed CBM mathematical model.

Table 4.7. The simulation results of the probabilities of correct and incorrect decisions made when checking the suitability of the radar transmitter power supply

Number of simulation cycles	Statistical estimates of the probabilities					
	P(Γ1)	P(Γ2)	P(Γ3)	P(Γ4)	P(Γ5)	P(Γ6)
1	2	3	4	5	6	7
10	0.60000	0.00000	0.00000	0.30000	0.00000	0.10000
50	0.40000	0.20000	0.02000	0.32000	0.00000	0.06000
100	0.35000	0.16000	0.01000	0.46000	0.00000	0.02000
500	0.30000	0.20600	0.01000	0.44200	0.00000	0.04200
1000	0.37000	0.20400	0.00500	0.38300	0.00000	0.03800
5000	0.36860	0.19180	0.00680	0.40200	0.00000	0.03080
10000	0.36160	0.19610	0.00660	0.40130	0.00000	0.03440
Theoretical results:	**0.3610**	**0.2000**	**0.0065**	**0.3980**	**0.0000**	**0.0340**

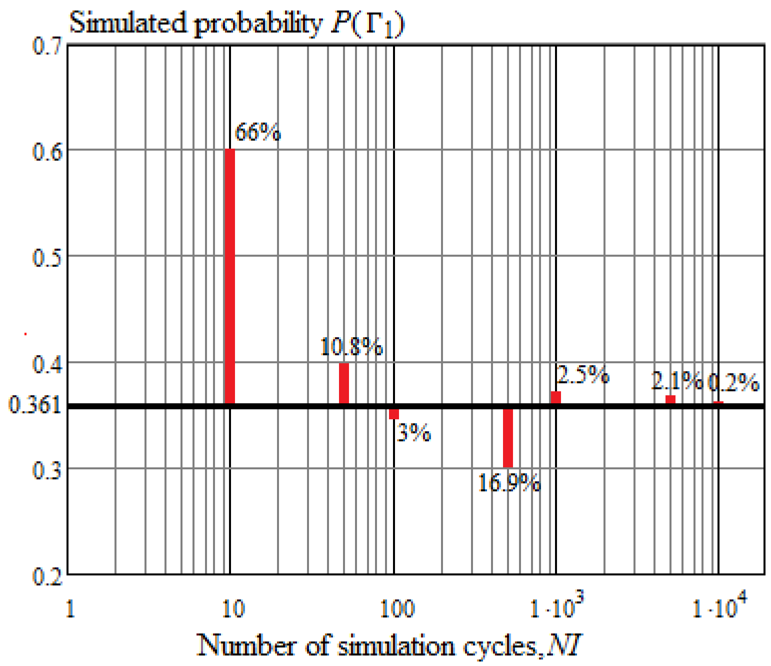


Fig. 4.10. A graphical presentation of simulation results of the probability $P(\Gamma_1)$

4.9. Conclusions

1. The analysis of modern avionics architectures shows that digital avionics uses the principle of federated (Boeing 757, Boeing 767, A318-A321, A330, and A340) or integrated modular architecture (Boeing 777 and 787, and A380).

2. Based on a comparative analysis, it has been shown that the IMA replaces a number of separate processors and LRUs used in the FA architecture, by a fewer number of centralized processors and LRMs (CPIOMs and IOMs), reducing the total number of LRUs, thus saving the weight and reducing maintenance costs in new commercial aircraft.

3. The analysis of the IMA of the Airbus A380 shows that the LRUs of radio communication and navigation systems, including VHF/HF transceivers, ATC transponder, weather radar, ILS, VOR and ADF receivers, have only the ARINC 429 interface. Therefore, electronic LRUs use CPIOM or CRDC for connecting to the ADCN.

4. A380 avionics comprises 30 LRMs (8 IOMs and 22 CPIOM modules), 50 LRUs with AFDX interface and about 30 LRUs with ARINC 429 interface. With a large number of LRMs and LRUs, a reduction in the maintenance cost of A380 avionics is possible due to the optimal combination of O, I and D maintenance levels and the use of modern diagnostic tools for detecting permanent and intermittent failures.

5. The techniques for calculating the probabilistic and time-related indicators of the avionics LRUs/LRMs maintenance effectiveness on a finite and infinite time interval have been developed. The proposed procedures allow determining the availability, ORF, and MTBUR, considering the rates of permanent and intermittent failures.

6. A technique for calculating the probabilistic indicators of maintenance effectiveness of redundant digital avionics systems for such structures as parallel, a majority, and parallel-series considering permanent and intermittent failures, has been developed.

7. A technique for minimizing the warranty maintenance cost of the redundant digital avionics systems has been developed, demonstrating (through the example of the ADIRS system of the Airbus A380 aircraft) that in the case of the optimal option of warranty maintenance, the average maintenance cost per aircraft decreases by 28 %.

8. A technique for minimizing the cost of post-warranty maintenance for redundant digital avionics systems has been developed, which allows choosing the optimal maintenance option after expiring the supplier's warranty. The technique has demonstrated (through the example of the ADIRS system of the Airbus A380 fleet of the Emirates Airline) that a three-level maintenance option with IFD at the I- and D-levels was optimal. Because it reduced the total expected maintenance cost by 1100 % compared to a one-level option,

by over 850 % compared to a two-level choice without IFD, by 15 % compared to a three-level option without using IFD, and by 24 % compared to the three-level option with IFD only at the I-level.

9. A technique for determining the optimal replacement thresholds when monitoring the condition of the degrading system at scheduled times has been developed, which allows reducing the total error probability by a factor of 10 compared with the case of using only the functional failure threshold.

10. A technique for joint optimization of replacement threshold and periodicity of suitability checking when monitoring the system condition has been developed. A real example of monitoring the output voltage of the radar transmitter power supply has shown that the joint optimization of the replacement threshold and inspection periodicity allows reducing the unavailability by 68% and the number of inspections by 41 %.

11. Algorithms and simulation programs to test the adequacy of the proposed maintenance models of vehicles' equipment have been developed. It has been shown that the statistical estimates of the mean times spent by LRU/LRM in different states and the probabilities of correct and incorrect decisions, when checking system suitability, converge rapidly to the theoretical values of these indicators, which proves the adequacy of the developed mathematical models.

REFERENCES

1. New challenges for future avionic architectures/ P. Bieber, F. Boniol, M. Boyer et al.// Aerospace lab journal. - 2012. - № 4. - P. 1-10.

2. Aliki, O. System testing in the avionics domain: dissertation zur erlangung des grades einer doktorin der ingenieurwissenschaften/ Aliki Ott; Universität Bremen. – Germany: Bremen, 2007. – 434 p.

3. A380 Technical training manual. LEVEL I - ATA 42 integrated modular avionics & avionics data communication network. - 2006. - Airbus. - 20 p.

4. Itier, J. B. A380 integrated modular avionics/ J. B. Itier// ARTIST2 meeting on integrated modular avionics. – Available: http://www.artist-embedded.org /docs/ Events/ 2007/IMA/Slides/ARTIST2_IMA_Itier.pdf – 12.11.2007

5. Module 11.19 integrated modular avionics (ATA 42)/ AFAQ institute of aviation technology. – 2012. – № 2. – 64 p.

6. Stock, M. ARINC 825 specification for CAN in airborne applications/ M. Stock// CAN Newsletter. – 2009. – № 4. – P. 40–42.

7. ARINC Specification 825: general standardization of CAN (controller area network) bus protocol for airborne use. – 01.07.2015.

8. Air data inertial reference system (ADIRS). ADIRS for Airbus aircraft. - https://aerocontent.honeywell.com/aero/common/documents/ADIRS.pdf – 01.05.2007

9. Aviation English. club. Navigation ADIRS presentation. – https://www.youtube.com/watch?v=fLj3xeaRo_E – 21.10.2015

10. ARINC 738–3. Air Data and Inertial Reference System (ADIRS). – 2008. – Retrieved 14.07.2008.

11. Report from the ADS–B aviation rulemaking committee to the federal aviation administration. Recommendations on aviation administration notice No. 7–15. – Washington, D.C.: USA. – 360 p. – 26.08.2008.

12. Flottau, J. Report card: Airbus A380 after eight years in service/ J. Flottau// Aviation week network. – Available: http://aviationweek.com/airbus-a380/report-card-airbus-a380-after-eight-years-service - 29.10.2015

13. Aligning the Airbus ADIRU. Training. Honeywell. – Available: https://www.youtube.com/watch?v=t2yzsc3y1R8 - 18.11.2013

14. Honeywell's inertial navigation system becomes standard equipment on Airbus. Honeywell press release. – Available: http://www51.honeywell.com/honeywell/news-events/press-releases-details/ 04.30.10ADIRS.html – 30.04.2010

15. Gauber, J. M. Currently developing and future communications and technology impact on AMDAR/ J. M. Gauber// Instruments and observing methods. Report No. 123. – Geneva: World Meteorological Organization. – 2016. -52 p.

16. GG1320AN digital ring laser gyroscope. Honeywell. – Available: https://aerospace.honeywell.com/en/products/navigation-and-sensors/gg1320an-digital-ring-laser-gyroscope – 09.06.2015

17. Keller, J. Boeing to provide ring laser gyro navigation avionics from Honeywell for U.S. Navy F/A-18 combat jets/ J. Keller// Military & aerospace. – Available: http://www.militaryaerospace.com/articles/2011/03/boeing-to-provide.html 2011 – 27.03.2011

18. Anderson, K. IFDIS - expanding role across the DoD maintenance enterprise”/ K. Anderson// Presentation of the 2012 Department of Defence Maintenance Symposium, November 13-16, Grand Rapids, Michigan, USA, pp. 1–29.

19. ABS guidance notes on reliability-centered maintenance. – Houston: USA, 2004. – 156 p.

20. Игнатов, В.А. Прогнозирование оптимального обслуживания технических систем/ В.А. Игнатов В.А., В.В. Уланский В.В., Т. Тайсир. – К.: Знание, 1981. – 20 с.

21. Fishman, G.S. Monte Carlo: concepts, algorithms, and applications/ G.S. Fishman. – New York: Springer, 1996. – 698 p.

CONCLUSIONS

In the thesis, the actual scientific task of developing mathematical maintenance models of digital avionics and deteriorating equipment of vehicles has been solved. The developed mathematical models make it possible to substantially reduce the maintenance costs of digital avionics systems and degrading equipment of vehicles while ensuring the required regularity and safety of flights. The following key results of the thesis research have been obtained:

1. The analysis of mathematical models of multilevel maintenance of digital avionics systems has been carried out. The need for developing mathematical maintenance models of continuously monitored redundant avionics systems, considering the alternating mode of their use, the continuous character of in-flight testing, the possibility of the appearance of both permanent and intermittent failures in-flight, and the possibility of organizing several levels of ground handling, has been justified.

2. The analysis of modern trends and mathematical models of condition-based maintenance of vehicles' equipment has been conducted. The necessity of developing CBM mathematical models of degrading vehicles' equipment, considering the probabilities of correct and incorrect decisions when checking the system suitability, and joint optimization of inspection schedule and replacement thresholds for systems that affect and do not affect safety, have been justified.

3. For the first time, mathematical models for evaluating the operational reliability of continuously monitored LRMs/LRUs and redundant avionics systems on finite and infinite time intervals, considering the effect of both permanent and intermittent failures, have been developed. Numerical analysis has shown that the MTBUR decreases by a factor of two with the intermittent failure rate increase from 10^{-5} to 10^{-4} h^{-1} and by a factor of 5 with a rise from 10^{-4} to 10^{-3} h^{-1}, while the average number of unscheduled removals and average cost of LRU/LRM repair increased sharply when the intermittent failure rate exceeded 10^{-4} h^{-1}.

4. For the first time, generalized relationships for calculating the average maintenance costs during the warranty and post-warranty periods of operation of redundant

avionics systems, considering the impact of both permanent and intermittent failures, have been developed. Numerical analysis has shown that the use of ATE at the I-level maintenance virtually eliminates the harmful effect of intermittent failures on the number of spare LRUs and average maintenance costs during the warranty period of operation. Besides, the three-level post-warranty maintenance system was optimal over a wide range of initial data because it provided a minimum number of spare LRUs and SRUs and minimum maintenance costs, which was 4.6 times less than for a one-level system, and more than 30% less than in any two-level maintenance system.

5. For the first time, a mathematical model of CBM, based on condition monitoring at scheduled times, has been developed, which considers the probabilities of correct and incorrect decisions made when checking system suitability. Numerical analysis has shown that in the absence of a replacement threshold, the total error probability when checking system suitability practically does not decrease at a 10-fold increase in the accuracy of measuring the system state parameter. When the optimal replacement threshold introduced, the total error probability decreased by almost ten times with a 10-fold increase in the measurement accuracy due to the rejection of potentially unreliable systems.

6. For the first time, the tasks of determining the optimal replacement threshold for each inspection time based on the criteria of the maximum net income, minimum Bayes risk, minimum total error probability, and given a posteriori probability of the system's failure-free operation in the forthcoming period have been formulated. Numerical analysis has shown that when using the criteria of the minimum total error probability and the given a posteriori probability of a system's failure-free operation, the optimum value of the replacement threshold increases with the time of inspection, and the value of the anticipatory tolerance decreases accordingly. The total error probability decreases from 27% to 1% when using the optimal replacement threshold for each checking time.

7. For the first time, general mathematical expressions for calculating effectiveness indicators of CBM based on suitability checking and the criteria for joint determination of optimum inspection schedule and replacement thresholds for systems that affect and do not affect safety have been developed. Numerical analysis has shown that the maintenance

based on suitability checking is more effective than the maintenance based on operability checking, as it ensures higher availability and ORF with significantly fewer inspections.

8. A technique for minimizing the cost of warranty maintenance for redundant digital avionics systems has been developed, which allows choosing the optimal maintenance option during the warranty period of the supplier. For the ADIRS system of the Airbus A380 (Emirates Airline), it has been shown that using ground test equipment at the I-level maintenance reduces average maintenance costs per aircraft by 28 %.

9. A technique for minimizing the cost of post-warranty maintenance for redundant digital avionics systems has been developed, which allows choosing the optimal maintenance option after expiring the supplier’s warranty. The technique has demonstrated (for the ADIRS system of the Airbus A380 fleet of the Emirates Airline) that a three-level maintenance option with IFD at the I- and D-levels was optimal. Because it reduced the total expected maintenance cost by 1100 % compared to a one-level option, by over 850 % compared to a two-level choice without IFD, by 15 % compared to a three-level option without IFD, and by 24 % compared to the three-level option with IFD only at the I-level.

10. A technique for joint optimization of replacement threshold and periodicity of suitability checking, when monitoring the system condition, has been developed. Numerical analysis has shown that when monitoring the output voltage of the radar transmitter power supply the joint optimization of the replacement threshold and inspection periodicity allows reducing the unavailability by 68% and the number of inspections by 41 %.

The results of the thesis can be used in the development and maintenance of federated and integrated modular avionics, as well as degrading equipment of vehicles.

Appendix 1:

List of the candidate's publications:

1. Раза, А. Математическая модель обслуживания цифровых систем авионики с учетом перемежающихся отказов/ А. Раза// Математические машины и системы. – 2018. – № 1. – С. 138–147.

2. Raza, A. Mathematical model of corrective maintenance based on operability checks for safety critical systems/ A. Raza// American journal of applied mathematics. – 2018. – V. 6(1). – P. 8–15.

3. Raza, A. Maintenance model of digital avionics/ A. Raza// Aerospace. – 2018. – V. 5(2). – P. 1–16.

4. Raza, A. Cost model for assessing losses to avionics suppliers during warranty period/ A. Raza, V. Ulansky// Advances in through-life engineering services. Decision Engineering: a collective monograph. – Springer, 2017. – P. 291–307.

5. Raza, A. Optimal policies of condition-based maintenance under multiple imperfect inspections/ A. Raza, V. Ulansky// Transactions on engineering technologies: a collective monograph. – Springer, 2016. – P. 285–299.

6. Raza, A. Modelling of predictive maintenance for a periodically inspected system/ A. Raza, V. Ulansky// Procedia CIRP. – 2017. – V. 59. – P. 95–101.

7. Raza, A. Minimizing total lifecycle expected costs of digital avionics' maintenance/ A. Raza, V. Ulansky// Procedia CIRP. – 2015. – V. 38. – P. 118–123.

8. Ulansky, V. Determination of the optimal maintenance threshold and periodicity of condition monitoring/ V. Ulansky, A. Raza// First world congress on condition monitoring (WCCM), 13-16 June 2017, London, UK. – WCCM proceedings, 2017. – P. 1–12.

9. Raza, A. Generalized cost functions of avionics breakdown maintenance strategy/ A. Raza, V. Ulansky, K. Augustynek, K. Warwas// 2017 IEEE Aerospace conference, 4-11 March, 2017, Big Sky, Montana, USA. – Conference proceedings, 2017. – P. 1–15.

10. Raza, A. Modelling condition monitoring inspection intervals/ A. Raza, V. Ulansky// Electronics and electrical engineering: Proceedings of the 2014 Asia-Pacific Electronics and Electrical Engineering Conference (EEEC 2014), December 27-28, 2014, Shanghai, China. — London: CRC Press, Taylor & Francis Group, 2015. – P. 45–51.

11. Raza, A. A Probabilistic model of periodic condition monitoring with imperfect inspections/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering 2015, WCE 2015, 1-3 July, 2015, London, UK. – V. II. – P. 999–1005.

12. Raza, A. Optimal thresholds for stochastically deteriorating systems/ A. Raza, V. Ulansky// Lecture notes in engineering and computer science: Proceedings of the World congress on engineering and computer science 2015, WCECS 2015, 21-23 October, 2015, San Francisco, USA. – V. II. – P. 934–939.

13. Ulansky, V. Modelling of condition monitoring with imperfect inspections/ V. Ulansky, A. Raza// Proceedings of the 19th World conf. on nondestructive testing 2016, WCNDT 2016, 13-17 June, 2016, Munich, Germany. – P. 1–9.

14. Raza, A. Assessing the impact of intermittent failures on the cost of digital avionics' maintenance/ A. Raza, V. Ulansky// 2016 IEEE Aerospace conference, 5-12 March, 2016, Big Sky, Montana, USA. – Conference proceedings, 2016. – P. 1–16.

15. Raza, A. Modelling of operational reliability and maintenance cost for avionics systems with permanent and intermittent failures/ A. Raza, V. Ulansky// Proceedings of the 9th IMA international conference on modelling in industrial maintenance and reliability, 12-14 July, 2016, London, UK. – P. 186–192.

16. Raza, A. Modeling of discrete condition monitoring for radar equipment/ A. Raza, V. Ulansky// 2014 IEEE Microwaves, radar and remote sensing symp., 23-25 September 2014, Kyiv, Ukraine. – MRRS proceedings, 2014. – P. 88–91.

Appendix 2:

Modelling of warehouse management system of spare LRUs/LRMs

To determine the delay in meeting the requirement for a spare LRU/LRM at the base airport, we use, as in [1], the continuous-time Markov chain to describe the operation of the warehouse of spare LRUs/LRMs. Let q be the number of identical LRUs / LRMs mounted on the aircraft boards that have a supplier warranty. It is evident that $q = mN$, where N is the number of aircraft with the supplier's warranty and m is the number of identical LRUs/LRMs on the board of one plane. Figure A.2.1 shows the graph of the system consisting of q main and PS spare LRUs/LRMs (in the future referred to as "q, PS system").

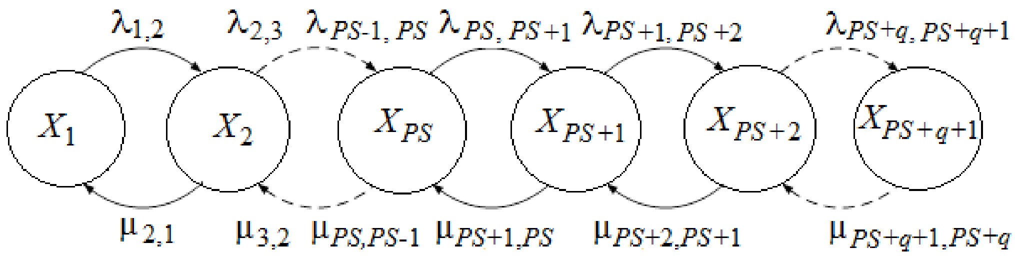


Fig. A.2.1. State graph of "q, PS system" of spare LRUs/LRMs

Fig. A.2.1 includes the following notation:

X_1 is the state of "q, PS system", in which q main LRUs/LRMs mounted in aircraft are operable and PS spare LRUs/LRMs are in the exchange fund in the warehouse;

X_2 is the state of "q, PS system", in which q main LRUs/LRMs mounted in aircraft are operable and PS - 1 spare LRUs/LRMs are in the exchange fund in the warehouse;

…………………………………………………………………………………...……

X_{PS} is the state of "q, PS system", in which q main LRUs/LRMs mounted in aircraft are operable, and one spare LRU/LRM is in the exchange fund in the warehouse;

X_{PS+1} is the state of "q, PS system", in which q main LRUs/LRMs mounted in aircraft are operable, and no spare LRUs/LRMs are in the warehouse;

X_{PS+2} is the state of "q, PS system", in which q – 1 main LRUs/LRMs mounted in aircraft are operable, and no spare LRUs/LRMs are in the warehouse;

………………………………………………………………………………….....

X_{PS+q+1} is the state of "q, PS system", in which all the main LRUs/LRMs mounted in aircraft are in failed state, and no spare LRUs/LRMs are in the warehouse;

$\lambda_{i,\,i+1}$ is the unscheduled LRU/LRM removal rate carrying the "q, PS system" from the i-th ($i = \overline{1, PS+q}$) into the (i + 1)-th state;

$\mu_{i+1,i}$ is the LRU/LRM repair rate carrying the "q, PS system" from the (i + 1)-th to the i-th state.

The unscheduled LRU/LRM removal rate $\lambda_{i,\,i+1}$ is determined as follows [2]:

$$\lambda_{i,i+1} = \begin{cases} q\tau / P_R^*(\tau) \text{ if } i = \overline{1, PS+1}, \\ (q+PS+1-i)\tau / P_R^*(\tau) \text{ if } i = \overline{PS+2, PS+q}. \end{cases} \tag{A.2.1}$$

The LRU repair rate $\mu_{i+1,\,i}$ depends both on T_{RS} and T_{ED} and is determined as [2]:

$$\mu_{i+1,i} = \begin{cases} i\mu_{RS} = i / E[T_R] \text{ if } i = \overline{1, PS}, \\ i\mu_{ED} = i / T_{ED} \text{ if } i = \overline{PS+1, PS+q}, \end{cases} \tag{A.2.2}$$

where $E[T_R]$ is the average repair time of the LRU/LRM.

The LRU/LRM repair time, presented by formula (A.2.2), corresponds to the case of unlimited recovery, which is typical for warranty maintenance.

The average repair time depends on WMO and is determined as follows [2]:

$$E[T_R] = \begin{cases} T_{RS}, \text{ for the } 1^{\text{st}} \text{ WMO}, \\ T_{RS}P_{PF} + t_{TE}^{I-Level}, \text{ for the } 2^{\text{nd}} \text{ WMO}, \end{cases} \tag{A.2.3}$$

where P_{PF} is the a posteriori probability that the removed LRU/LRM has a permanent failure.

Since the removed LRU/LRM can have either a permanent or intermittent failure, then there are two a posteriori probability, the sum of which is equal to one:

$$P_{PF} + P_{IF} = 1, \tag{A.2.4}$$

where P_{PF} and P_{IF} are, respectively, the a posteriori probability that the removed LRU/LRM has a permanent or intermittent failure.

Since the process of changing the LRU/LRM states is regenerative, the probability of LRU/LRM restoration is equal to one within the regeneration cycle. Consequently,

$$P_{IF} = E[TS_6] / t_{IFR}, \tag{A.2.5}$$

$$P_{PF} = E[TS_7] / t_{PFR}. \tag{A.2.6}$$

For example, from equations (2.50) and (2.51) follows that at $T = \infty$ and the exponential distribution of time to permanent and intermittent failure:

$$P_{IF} = \left(1 - e^{-\lambda\tau}\right)\left[\frac{e^{-\lambda\tau}}{1 - e^{-\lambda\tau}} - \frac{e^{-(\lambda+\theta)\tau}}{1 - e^{-(\lambda+\theta)\tau}}\right], \tag{A.2.7}$$

$$P_{PF} = \frac{1 - e^{-\lambda\tau}}{1 - e^{-(\lambda+\theta)\tau}}. \tag{A.2.8}$$

It is easy to see that for relations (A.2.7) and (A.2.8) the equality (A.2.4) is valid.

The average waiting time for a spare LRU/LRM from the warehouse at the base airport is determined by the following formula [1]:

$$t_{spare}\left(PS, US\right) = \Lambda^{-1} \sum_{i=1}^{q} i P_{PS+i+1}, \tag{A.2.9}$$

where Λ is the average rate of requests to the warehouse for a spare LRU/LRM delivery; P_j ($j = \overline{PS + 2, PS + q + 1}$) is the probability that the "q, PS system" being in the state X_j.

Since LRUs/LRMs are repairable items, the average rate of requests to the warehouse for delivery of a spare LRU/LRM is essentially a failure flow parameter. Following [3], the failure flow parameter is the ratio of the mathematical expectation of the number of failures of the repairable item for a sufficiently small period of operation to the value of this period.

If the airline has N aircraft in operation, the failure flow parameter (in respect to both permanent and intermittent failures) of the mN identical LRUs/LRMs is given by

$$\Lambda = \frac{mNN_R\left(\Delta t\right)}{\Delta t}. \tag{A.2.10}$$

By substituting $N_R(\Delta t)$ from (2.64) into (A.2.10), we obtain

$$\Lambda = \frac{mNP_R^*(\tau)}{\tau}. \tag{A.2.11}$$

The following relation from (A.2.9):

$$US = \sum_{i=1}^{q} i P_{PS+i+1} \tag{A.2.12}$$

is the mathematical expectation of the number of unplanned spare LRUs/LRMs delivered by the manufacturer in response to an emergency request from the aircraft buyer.

REFERENCES

1. Игнатов, В.А. Прогнозирование оптимального обслуживания технических систем/ В. В. Уланский, Г. Ф. Конахович, И. А. Мачалин. – К.: Знание, 1981. – 20 с.
2. Raza, A. Assessing the impact of intermittent failures on the cost of digital avionics' maintenance/ A. Raza, V. Ulansky// 2016 IEEE Aerospace conference, 5-12 March, 2016, Big Sky, Montana, USA. – Conference proceedings, 2016. – P. 1-16.
3. Козлов, Б. А. Справочник по расчету надежности аппаратуры радиоэлектроники и автоматики/ Б. А. Козлов, И. А.Ушаков. — М.: Советское радио, 1975. - 472 с.

Appendix 3:

The act of confirmation of using the results of the thesis work

ЗАТВЕРДЖУЮ
Проректор Національного авіаційного
університету з наукової роботи
В. Харченко
"19" 04 2018 р.

АКТ ВПРОВАДЖЕННЯ

результатів кандидатської дисертації Рази А. в навчальний процес
Національного авіаційного університету

Ми, що нижче підписались, директор Навчально-наукового інституту аеронавігації, електроніки та телекомунікацій, д.т.н., проф. Мачалін І.О. та завідуючий кафедрою електроніки, д.т.н., проф. Яновський Ф.Й. склали цей акт про те, що результати кандидатської дисертації Рази Ахмеда «Математичні моделі технічного обслуговування обладнання засобів транспорту» використовуються в навчальному процесі Національного авіаційного університету.

Найменування впровадженого результату	Досягнутий фактичний ефект і форма впровадження
1. Математичні моделі для розрахунку показників ефективності технічного обслуговування електронних систем авіоніки з урахуванням стійких і переміжних відмов. 2. Математичні моделі для розрахунку середніх експлуатаційних витрат впродовж гарантійного і післягарантійного періодів експлуатації резервованих електронних систем авіоніки для альтернативних варіантів технічного обслуговування, що відрізняються наявністю одного, двох і трьох рівнів обслуговування. 3. Методика мінімізації вартості технічного обслуговування резервованих електронних систем авионіки.	Впровадження зазначених матеріалів дозволило студентам вивчати моделі технічного обслуговування сучасних електронних систем авіоніки, способів вибору оптимального варіанта обслуговування таких систем на лекційних і практичних заняттях, а також використовувати зазначені матеріали при виконанні бакалаврських робіт. Забезпечило оволодіння студентами навичками математичного і статистичного моделювання сучасних електронних систем авіоніки. Матеріли дисертації використовувались в навчальній дисципліні «Електронні системи» для спеціальності 171 «Електроніка» спеціалізації «Електронні системи».

Директор НН ІАН І.О. Мачалін
Зав. кафедрою електроніки Ф.Й. Яновський