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A computer system for images compression

A new approach to images compression is proposed. The approach involves the use of elements of tensor analysis based on singular decomposition. A feature of this approach used is the representation of the image by the matrix triad, which includes the tensor core and a pair of unitary matrices containing right and left singular vectors, respectively. Compression is achieved by one recurrent procedure, which involves lowering the rank of the triad to the level of allowable errors while maintaining the original image size. The result of semi-natural modeling the system components is provided.

Introduction

Compression is a popular technique for converting images to a form that is convenient for transmission over wired and wireless communication channels and also for their storage [1]. To date, several effective techniques have been developed based on different approaches. The most commonly used are considered hybrid techniques, including several compression methods, such as JPEG, JPEG2000. The main difference between the two is the replacement of low-performance transforms, such as Huffman and discrete-cosine, with more efficient arithmetic coding and wavelet transform, respectively. Analysis of the development of these methods shows the possibility of replacing individual stages of the method with more effecient units. The main goal of such methods is to improve the compression and recovery performance of images and minimize distortion. The best compression is achieving in the algorithms that involve losses, where frequency separation of the image components is assumed.

One of the effective methods that appeared at the end of the last century is vector quantization. The method assumes a high correlation between the individual elements of the image. However, there are a number of computational difficulties associated with the definition of a code word. The main output, as in the previously developed methods, is the discarding of a number of codewords, provided that it does not give significant distortion to the final product. A possible drop option can be constructed using the least squares method.

Problem Statement

The matrix $A \in R^{m \times n}$ of real values is algebraically represented by a three-dimensional tensor in the form of a bilinear form, obtained from the well-known one, represented by the product of three matrices in the form of a singular decomposition of the form

$$A = M \Lambda N^T, \tag{1}$$

where Λ is the kernel of a tensor of the same size as A, with nonnegative elements λ_i on the main diagonal (singular values) and the rest zero. The matrices $M \in R^{m\times m}$ and $N \in R^{n\times n}$ are unitary matrices consisting of left and right singular vectors, respectively.

Image compression size shrinks by grouping or dropping part of the pixels. If the grouping algorithms allow you to restore the quality of images, then discarding a number of pixels leads to a loss of its quality when scaled. Drop pixels in the compression process occurs on the basis of signs of repeatability or frequency. Then the problem of compression is reduced to the definition of a triplet of smaller matrices.

The resulting matrices are sequences of numbers, so the compression process is similar to the coding process, where the input n-dimensional vector $x \in R^n$ is converted by the encoder into a code word $\gamma(x)$ of a smaller size $l = n / K$ ($l < n$), where K is the compression ratio. The discarding of a part of the information is a source of image distortions, which, as a result of compression, with identical image sizes, is evaluated by various criteria, the most common among them are the average square of errors, the signal-to-noise ratio, and visual criteria. The average error square is written as

$$e_{mse} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}|\gamma(x_{ij}) - x_{ij}|^2, \tag{2}$$

where M, N are the vertical and horizontal dimensions of the image, or the signal-to-noise ratio is represented by the formula

$$R_{s/w} = 20\log_{10}\left(\frac{\gamma(x_{max})}{\sqrt{\varepsilon_{mse}}}\right), \tag{3}$$

where x_{max} is the maximum pixel value of the image, $R_{s/w}$ is the representation of the signal-to-noise ratio.

The purpose of the paper is to determine the matrices M, N and the kernel of the tensor D, which satisfy the specified image quality defined by formulas (2), (3).

Simulation

The simulation was carried out on a personal computer with an image of 384×384 pixels. The reference image was previously divided into sub-images of 8×8 pixels. Then, the procedure (5) was applied successively to each sub-image, so as to get an approximation to the original image with the specified quality, i.e.

$$x = \Lambda\gamma(x) = \sum_{i=1}^{p}\lambda_i m_i v_i^T + E = \hat{x} + E. \tag{4}$$

Here, E is the residual matrix obtained after obtaining p essential components of the svd decomposition, estimated by the Frobenius norm

$$e = \|E\|_F = \sqrt{\left\|S - M_p\Lambda_p N_p^T\right\|^2}. \tag{5}$$

Image quality was controlled by criteria (2), (3).

The proposed algorithm was estimated for the original image, the view of which is shown in Fig. 1. The algorithm was studied to test its effectiveness at different values ε, calculated criteria (2), (3), made a comparison with full decomposition and partial decomposition. Images on ε=0.1 are shown in Fig. 2.

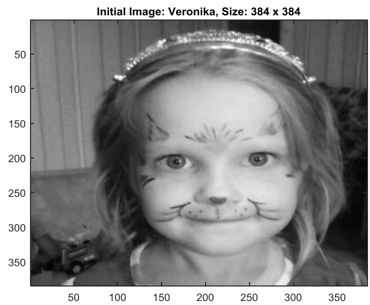

Fig. 1. Original image

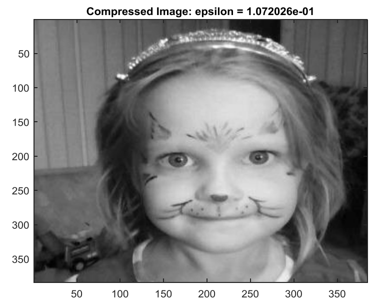

Fig. 2. Compressed image, ε=0.1

Conclusion

As a result of the study, it has been found that matrix methods based on decompositions that can be combined with other methods, such as grouping, traditionally used in hybrid compression methods, are quite effective for image compression. Higher image quality is achieved by increasing the accuracy of the truncated description approaching the original image. The developed algorithm has the property of adaptability. Further research is planned to be devoted to searching methods for finding the unknown tensor core for given unitary matrices *M*, *N*.

References

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