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AN ANALYSIS OF SINGULARITY OF THE MATRICES OF PRIORITIES AND SENSITIBILITY OF DECISIONS AS KEY PERFORMANCE INDICATORS OF THE ANALYTIC HIERARCHIES PROCESS

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ABSTRACT

The key performance indicators of the method of multiobjective optimisation – analytic hierarchy process – are considered. One of the main properties of priority matrix of analytic hierarchy process is its opposite symmetry in the case of strict coordination of preferences. It is proved, that determinant of strictly opposite symmetrical matrix is identically equal to zero for any order of matrix. The sensibility of decision of eigenvalue problem for priority matrix is researched. It is shown, that the spectral radius of matrix is sufficient key parameter for estimation of sensibility of decision and risk of passage from correct to false decision. The noisiness of elements of priority matrix for reduction of sensibility of decision and development of the methods of realisation of analytic hierarchy process which are tolerant to disturbances and errors in judgements is proposed. The method of analysis sensibility of decision "in large" which is grounded on estimates of relation between spectral radius of matrix and the value of Gershgorin's circle is developed. The results received in this work may be useful for wide range of scientific and technical applications.

Key words: analytic hierarchy process, priority matrix, matrix norm, eigenvalue problem, opposite symmetry, spectral radius, Gershgorin's circle.

АНАЛИЗ ОСОБЕННОСТЕЙ МАТРИЦЫ ПРИОРИТЕТОВ И РЕШЕНИЙ КАК ГЛАВНЫЕ ПОКАЗАТЕЛИ АНАЛИТИЧЕСКОГО ИЕРАРХИЧЕСКОГО ПРОЦЕССА

РЕЗЮМЕ

Рассматриваются ключевые параметры эффективности метода многокритериальной оптимизации – метода анализа иерархий. Одно из главных свойств матрицы предпочтений, которая применяется в методе анализа иерархий-это ее обратная симметричность в случае строгой согласованности предпочтений. Доказано, что детерминант строго обратно-симметричной матрицы тождественно равен нулю для любого порядка матрицы. Исследована чувствительность решения проблемы собственных значений для матрицы приоритетов. Показано, что спектральный радиус матрицы - это достаточный ключевой параметр для оценивания чувствительности решения и риска перескока от правильного решения к ложному. Для уменьшения чувствительности решения и разработки вариантов реализации метода анализа иерархий, толерантных к возмущениям и ошибкам в суждениях, предложено применять зашумлегие элементов матрицы приоритетов. Разработано метод анализа чувствительности решения "в большом", который основан на оценках отношения между спектральным радиусом матрицы и величиной круга Гершгорина. Результаты, полученные в этой работе, будут полезны для широкого ряда научных и технических приложений.

Ключевые слова: метода анализа иерархий, матрица приоритетов, норма матрицы, проблема собственных значений, обратная симметрия, спектральный радиус, круги Гершгорина.

ANALİTİK İYERARXİYA PROSESİNİN ƏSAS İCRA GÖSTƏRİCİLƏRİ KİMİ PRİORİTET VƏ QƏRARLARIN TƏRCİH MATRİSLƏRİNİN SPESİFİKLİYİNİN TƏHLILI

XÜLASƏ

Çoxkriteriyalı optimizasiya-iyerarxiyaların analizi üsulunun effektivliyinin açar parametrlərinə baxılmışdır. İyerarxiyaların analizi üsulunda tətbiq edilən tərcih matrisinin əsas xassələrindən biri tərcihlərin ciddi uyğunluğu zamanı onun tərs simmetrik olmasıdır. İsbat olunmuşdur ki, ciddi tərs-simmetrik matrisin determinantı istənilən tərtibli matris üçün eyniliklə sıfıra bərabərdir. Prioritetlər matrisi üçün məxsusi ədəd probleminin həllinin həssaslığı tədqiq olunmuşdur. Göstərilmişdir ki, matrisin spektral radiusu həllin həssaslığının və düzgün həldən yanlış həllə keçid riskinin qiymətləndirilməsi üçün kifayət qədər vacib parametrdir. Həllin həssaslığının azaldılması üçün və iyerarxiyaların analizi üsulunun reallaşdırmaq variantlarının işlənməsi üçün prioritetlərin matrisinin elementlərinin küylənməsini tətbiq etmək təklif edilmişdir. Matrisin spektral radiusu ilə Qerşqorin dairəsinin kəmiyyəti arasındakı münasibətin qiymətləndirilməsinə əsaslanan "böyük" həllin həssaslığının analizi üsulu işlənmişdir. İşdə alınmış nəticələr geniş sırada elmi və texniki tətbiqlərdə faydalı olacaq.

Açar sözlər: iyerarxiyaların analizi üsulu, prioritetlər matrisi, matrisin norması, məxsusi ədəd problemi, tərs simmetriklik, spektral radius, Qerşqorin dairəsi.

I. Introduction

The analytic hierarchy process (AHP) is widely used in the tasks of optimization and acceptance of decisions on a few, including to the contradictory criteria, and also in conflict situations [1, 2]. He is used for the conclusion of scales of relations on results of pair comparisons in the complex hierarchical systems. While pair comparing the structural elements of the system, we get the matrices of preferences (priorities). The scales of relative importance of priorities conclude from these matrices (in continuous case – from the kernels of linear operators) by the dissolving of the eigenvalues problem.

The matrices of preferences are positive and back symmetric:

$$\mathbf{A} = \|a_{ij}\|, \;\; a_{ij} > 0; \;\; i, j = \overline{1, N}; \;\; a_{ij} = 1/a_{ji}.$$

Researches of properties of these matrices were conducted in works [1, p.154], however, some important questions, in particular, property of singularity of matrices of the such special kind and problems of sensitiveness and stability of decisions effluent from it, are not considered. These parameters of method AHP can be considered as so called "Key performance indicators" – the most important parameters of efficiency and accuracy of analytic hierarchy process.

Just the problem statement was made in [3]. An attempt to make up for a deficiency is done in this work.

II. Properties of matrix of priorities in the case of ideal coordination

Let $\{a_1, a_2, ..., a_n\}$ $\{a_1, a_2, ..., a_n\}$ is set of objects subject to comparative estimation. The quantitative results of evaluation of pair of objects $\{a_i, a_j\}$ $\{a_i, a_j\}$ are written down as a matrix, thus relative superiority of object *i* above object *j is* expressed by a fraction $a_{ij} = a_i/a_j$. It is assumed that $a_i \neq 0, \; i = 1, N$; otherwise object with zeroing quality simply there is no sense to plug in the aggregate of system descriptions.

In ideal case of absolutely exact judgements about relative importance of these objects and, accordingly, faultless quantitative estimations on results these judgements the matrix of priorities will be strict opposite symmetric: $\mathbf{A} = \|a_{ij}\|, \;\; a_{ij} = 1/a_{ji}$ or , $a_{ij} = a_i/a_j$. $a_{ji} = a_j/a_i$ We will mark that in all cases $a_{ii} = 1$. Consequently, the matrix of priorities looks like in the case of ideal coordination

$$\mathbf{A} = \begin{pmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & 1 & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & 1 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & 1 \end{pmatrix} = \begin{pmatrix} 1 & a_1/a_2 & a_1/a_3 & \cdots & a_1/a_n \\ a_2/a_1 & 1 & a_2/a_3 & \cdots & a_2/a_n \\ a_3/a_1 & a_3/a_2 & 1 & \cdots & a_3/a_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_n/a_1 & a_n/a_2 & a_n/a_3 & \cdots & 1 \end{pmatrix}$$

At the decision of matrix equalizations of any kind (systems of algebraic equations, complete or partial problems of eigenvalues) special part is acted by the size of determinant of matrix. If the determinant of matrix det A is equal to zero (a singular matrix), the decisions of matrix equalizations are unstable, i.e. sensible to small declinations of basic data and errors of calculations.

We explore property of determinant of positive strictly back symmetric matrix. We will prove, that a determinant opposite symmetric matrix of any order is always equal to the zero. We conduct proof on induction.

Proof.

1. n=2.

$$\mathbf{A} = \begin{pmatrix} 1 & a_1/a_2 \\ a_2/a_1 & 1 \end{pmatrix} \tag{1}$$

It is clear that $\det \mathbf{A} = 1 - \frac{a_1}{a_2} \cdot \frac{a_2}{a_1} = 1 - 1 = 0$.

2. n=3.

$$\mathbf{A} = \begin{pmatrix} 1 & a_1/a_2 & a_1/a_3 \\ a_2/a_1 & 1 & a_2/a_3 \\ a_3/a_1 & a_3/a_2 & 1 \end{pmatrix} \tag{2}$$

We will expand a determinant on the first column

$$\det \mathbf{A} = \begin{vmatrix} 1 & a_2/a_3 \\ a_3/a_2 & 1 \end{vmatrix} - \frac{a_2}{a_1} \begin{vmatrix} \frac{a_1}{a_2} & \frac{a_1}{a_3} \\ \frac{a_3}{a_2} & 1 \end{vmatrix} + \frac{a_3}{a_1} \begin{vmatrix} \frac{a_1}{a_2} & a_1/a_3 \\ 1 & \frac{a_2}{a_3} \end{vmatrix} \tag{3}$$

In the second element we multiply the first column on a coefficient a_2/a_1, and in the third element we multiply the first column on a coefficient a_3/a_1. As is generally known [4, p. 41], such operation causes change of size of determinant in the proper number one time (for the second element – in a_2/a_1, and for the third – in a_3/a_1).

We will get such sum of determinants:

$$\det \mathbf{A} = \begin{vmatrix} 1 & a_2/a_3 \\ a_3/a_2 & 1 \end{vmatrix} - \begin{vmatrix} 1 & a_1/a_3 \\ a_3/a_1 & 1 \end{vmatrix} + \begin{vmatrix} a_3/a_2 & a_1/a_3 \\ a_3/a_1 & a_2/a_3 \end{vmatrix} \tag{4}$$

As all elements in (4) are the symmetric matrices of the second order of kind (1) back, their determinants are equal to the zero. Accordingly, the determinant of matrix (2) is equal to the zero.

3. n=4.

$$\mathbf{A} = \begin{pmatrix} 1 & a_1/a_2 & a_1/a_3 & a_1/a_4 \\ a_2/a_1 & 1 & a_2/a_3 & a_2/a_4 \\ a_3/a_1 & a_3/a_2 & 1 & a_3/a_4 \\ a_4/a_1 & a_4/a_2 & a_4/a_3 & 1 \end{pmatrix} \tag{5}$$

We can let $a_1 = 1$ with any loss of generality. As earlier we decompose the determinant of matrix (5) on the elements of the first column.

$$\det \mathbf{A} = \begin{vmatrix} 1 & 1/a_2 & 1/a_3 & 1/a_4 \\ a_2 & 1 & a_2/a_3 & a_2/a_4 \\ a_3 & a_3/a_2 & 1 & a_3/a_4 \\ a_4 & a_4/a_2 & a_4/a_3 & 1 \end{vmatrix} = (-1)^{1+1} \cdot \begin{vmatrix} 1 & a_2/a_3 & a_2/a_4 \\ a_3/a_2 & 1 & a_3/a_4 \\ a_4/a_2 & a_4/a_3 & 1 \end{vmatrix} + (-1)^{1+2} \cdot \frac{1}{a_2} \begin{vmatrix} a_2 & a_2/a_3 & a_2/a_4 \\ a_3 & 1 & a_3/a_4 \\ a_4 & a_4/a_3 & 1 \end{vmatrix} +$$

$$+ (-1)^{1+3} \cdot \frac{1}{a_3} \begin{vmatrix} a_2 & 1 & a_2/a_4 \\ a_3 & a_3/a_2 & a_3/a_4 \\ a_4 & a_4/a_2 & 1 \end{vmatrix} + (-1)^{1+4} \cdot \frac{1}{a_4} \begin{vmatrix} a_2 & a_2/a_3 & a_2/a_4 \\ a_3 & a_3/a_2 & 1 \\ a_4 & a_4/a_2 & a_4/a_3 \end{vmatrix}.$$

After obvious mathematic transformations we get final result as shown below:

$$\det \mathbf{A} = \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} + \begin{vmatrix} \frac{a_2}{a_3} & 1 & \frac{a_2}{a_4} \\ 1 & \frac{a_3}{a_2} & \frac{a_3}{a_4} \\ \frac{a_4}{a_3} & \frac{a_4}{a_2} & 1 \end{vmatrix} - \begin{vmatrix} \frac{a_2}{a_3} & 1 & \frac{a_2}{a_3} \\ \frac{a_3}{a_4} & \frac{a_3}{a_2} & 1 \\ 1 & \frac{a_4}{a_2} & \frac{a_4}{a_3} \end{vmatrix} \quad (6)$$

The first and second elements are the opposite-symmetric matrices of the third order of kind (2). Their determinants, as shown in point 2, are equal to zero. In the third element we will do transposition of the first and second columns, that, as is generally known [4, p. 40], will result just in the change of sign.

In a fourth element we do two transpositions: we move the first column with the second, and then will move a new second column with third one. Thus the sign of determinant changes twice, i.e., actually, remains former.

$$\det \mathbf{A} = \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} - \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} - \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} - \begin{vmatrix} 1 & \frac{a_2}{a_3} & \frac{a_2}{a_4} \\ \frac{a_3}{a_2} & 1 & \frac{a_3}{a_4} \\ \frac{a_4}{a_2} & \frac{a_4}{a_3} & 1 \end{vmatrix} \quad (7)$$

As is clear from expression (7), all elements are the opposite symmetric matrices of the third order, the determinants of which are equal to zero.

Chain of arguments for the matrices of order of n=5, 6, 7, … it will be similar.

End of proof.

It should be noted that $\det \mathbf{A} = 0$ not only at reverse symmetry but also at equality of priorities. At that rate the identical rows appear, that has negative impact, as the system becomes indefinite.

III. Sensibility of rte matrices of hierarchies to disturbances of judgements

In work [5] considered applied tasks of multiobjective optimisation-choice of optimum class of the architecture of software of complex simulators for flight dispatchers. In work [6] learned the system approach to various problems of multiobjective optimization with application analytic hierarchies process, including the choice of optimal route for sending of information in a network with the heterogeneous streams of traffic. The relations of preferences are based on the results of the exact measuring, probabilistic estimations and subjective agreement. The modified method of analysis of hierarchies with the exact calculations of eigenvalues of matrix of priorities is applied. Comparative estimations are given to exactness, stability and asymptotic sensitiveness of algorithms of search of decision.

The matrix of pair comparisons for criteria is made on the basis of results of basic data analysis. Pair comparisons are used in order to disengage oneself from the concrete values of descriptions of route, which act part of private criteria, and from dimensions these descriptions. It allows to those or by other method to take the task of multicriterion (vectorial) optimisation to the associate scalar job mix of однокритериальной optimisation. The value has only importance of one criterion as compared to other. Degree of importance of one criterion in relation to other, in accordance with the theoretical ground of analytic hierarchy process [7], is determined by the method of expert estimations. In wide sense in the examined task can come forward experts:

– administrator of network;
– users are the customers of services;
– consulting model, in which accumulates and information about current status of network and commutation knots is processed.

The origin of records in the databases of routing can be different:

– from stack software of communications protocols (creation of minimum routing, records directories about the special destination addresses of type of addresses of the local testing, multicast or broadcast addresses);
– from the administrator of network (static records without limitation of term of life, saved at the restart, and sometimes – after the shutdown and repeated including of device of routing);
– from standard protocols of routing (dynamic records with the limited term of life).

Presently, in connection with the huge increase of calculable resources and festering of calculable powers in the network equipment even such tasks can decide in the process of current management by a computer network. Therefore in work [6, p. 101] the algorithm of acceptance of decisions about the choice of route is offered, based on more exact method of calculation of eigenvalues and eigenvectors, than methods of calculus of approximations, offered in [1, p. 26-30]. For realization of algorithm the programs developed on the basis of standard methods of calculable mathematics were used.

The fundamental problem of applicability of method consists in the search and ground of answer for a question: what real function of complex eigenvalue and eigenvector to choose as the synonymous parameter of decision?

For an answer for this question we will take into account that in accordance with the basic theorem of higher algebra the polynomial of the N^{th} order has exactly the N roots, which can either be material or make the complex-conjugate pair. If the order of the N polynomial is an even number, in theory all roots can be complex-attended. At N odd obligatory there will be even one material root. As the eigenvalues of characteristic polynomial of matrix are his roots, dependences between these roots and some comparative quantitative estimations of the generalized size (norms) of matrix can give sufficient information about stability of decision and his sensitiveness to the changes of elements of matrix.

As is generally known from the theory of matrices [8 – 11], as the most widespread norms matrices use the following types of norms:

- $\|\mathbf{A}\|_m = \max_i \sum_j |a_{ij}|$ (*m*- norm; adding up on lines and choice of maximal value of sum);
- $\|\mathbf{A}\|_l = \max_j \sum_i |a_{ij}|$ (*l*- norm; adding up on columns and choice of maximal value of sum);
- $\|\mathbf{A}\|_l = \sqrt{\sum_{i,j} |a_{ij}|^2}$ (*k*- norm or Euclid norm (sometimes called by the Frobenius norm).

As essence of the examined task makes the analysis of eigenvalues, in work [5, p.43] as the norm of matrix a spectral radius is chosen. (If $\lambda_1, \ldots, \lambda_n$ are eigenvalues of matrix **A,** then $\lambda_A = \max_j |\lambda_j|$, where ($1 \le j \le n$), is a spectral radius of **A**.)

Strictly speaking, a spectral radius not always satisfies to the axioms of matrix norms, in particular, when a matrix is not zeroing, and all its eigenvalues here equal to the zero. However associated he closely is with the size of matrix norm as high bound of spectral radius [12].

Therefore and one or another norm of matrix, and spectral radius can be used for the analysis of stability of problem of eigenvalues taking into account concrete terms.

In works [5 – 6] the results of calculations are analysed enough large volume for the matrices of priorities of both even and odd order. For positive strictly back-symmetric matrices by a spectral radius, in accordance with the Perron-Frobenius theorem, essentially, there is the first eigenvalue which is always material. Both material the other eigenvalues can be and complex-attended, but also those, et al, as a rule, there is a considerably less spectral radius on the module.

There is opened a question about stability of the chosen numeral method of decision and his sensitiveness to declinations of basic data [13]. This question is very actual, as in the conditions of high degree of heterogeneity of network the matrix of priorities can be badly conditioned. It is explained a bad conditionality by low exactness of measuring or estimations of the real descriptions of network, such as reliability or instantaneous carrying capacity. In addition, always the known tyranny in determination by the experts of relative importance of one or another descriptions takes place.

We will mark that it is necessary to distinguish the conditionality of matrix in the decision of complete or partial problem of eigenvalues $\mathbf{A}\vec{\mathbf{X}} = \lambda\vec{\mathbf{X}}$ and in the tasks of type of appeal of matrix for the decision of the system of linear equalizations $\mathbf{A}\vec{\mathbf{X}} = \mathbf{B}$. Thus the conditionality of matrix in sense of problem of eigenvalues is straight unconnected with the conditionality of matrix in the task of appeal.

And in that, and in other the cases quantitative estimations of stability of decision give the numbers of conditionality [3–6]. For the analysis of problem of eigenvalues the spectral number of conditionality of kind is taken

$$k(\mathbf{H}) = \|\mathbf{H}^{-1}\| \|\mathbf{H}\| \tag{8}$$

where $\|\mathbf{H}\|$ is matrix of right eigenvectors $\vec{\mathbf{X}}_i,\ i = \overline{1,n}$ of equalization, or

$$k(\mathbf{H}) = \sqrt{\mu_{\max} / \mu_{\min}}\ , \tag{9}$$

where $\mu_{\max}, \mu_{\min}$ - accordingly most and the least eigenvalue of matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$, T is symbol of transposition.

In second case the theoretical decision is given by a formula. $\vec{\mathbf{X}} = \mathbf{A}^{-1}\mathbf{B}$ It exists in that and only in case that determinant of matrix. $\det \mathbf{A} \neq 0$ However at the calculation of determinant it can happen by even an exact method, that from the errors of task of elements of matrix a result will be got $\det \mathbf{A} = 0$. The system appears inconsistent.

The number of conditionality looks like for the task of appeal of matrix

$$k(\mathbf{A}) = \left\|\mathbf{A}^{-1}\right\| \left\|\mathbf{A}\right\| \quad (10)$$

(Here we do not specify the type of norm of matrix on a reason which will be indicated below.)

The numbers of conditionality not always give exhaustive description to the conditionality of matrix. Therefore as additional description of stability of the examined task we will enter the certain component criterion of evaluation of numbers of conditionality and size of determinant.

An inverse matrix is steady, if small the small changes of elements of inverse matrix correspond to the changes of elements of initial matrix. It is necessary for providing of stability of inverse matrix, that the determinant of matrix was not too small. At all events, his size must not be the size of the second order of trifle as compared to the known Hadamard's estimate for the value of determinant:

$$\Delta \leq \sqrt{\prod_{i=1}^{n} \sum_{j=1}^{n} \left|a_{ij}\right|^2} \quad (11)$$

In [8, p.27] it is shown that the change da_{kl} of every element a_{kl} of inverse matrix, caused by the change da_{ij} of other element a_{ij} of this matrix, is equal to this change increased on work of some two elements of matrix: $da_{kl} = -\sum_{i,j} a_{ki} a_{lj} da_{ij}$. If the elements of inverse matrix are great (that at small determinant always takes place) enough, an insignificant error in the elements of initial matrix entails the considerable changes in the elements of inverse matrix.

Further there is a question: at the set eigenvector and all matrices, which is he got from, is the risk of transition great from one of them on any other at presence of small indignations in elements? In particular, is transition possible from the matrix of attitudes toward any other matrix?

By other question-at consideration of two eigenvectors being small indignations of each, are there small indignations which translate one class of the proper matrices in other?

In [1] for these questions is not brought an answer over. An attempt to fill in this blank is here done. Thus we will not impose restraints (certainly, within reasonable limits) on the absolute values of indignations.

Often also there is a question, as far as the priorities set by the components of eigenvector are sensible, to the small changes in the sizes of agreement [1, p.20-25]. Desirably, that priorities did not hesitate in wide limits at the small changes in judgement. There are three methods of verification of this sensitiveness: 1) finding of mathematical estimation of oscillation; 2) receipt of the answers, based on the large number of the computer calculations built properly for verification of sensitiveness; 3) combination of previous two methods, special at impossibility of conducting of complete argumentation analytically.

We will consider dependence of spectral number of conditionality (1) of the matrix got as a result of decision of task on the eigenvalues of initial matrix **A**. As a result of transformation of similarity of matrix **A** with the use of matrices of right eigenvectors we get the diagonal matrix of kind

$$\mathbf{H}^{-1}\mathbf{AH} = \mathbf{diag}(\lambda_i),\ i = \overline{1,n} \tag{12}$$

Lets from casual indignations of elements a_{ij} within the limits of some ε- neighbourhood it is brought a matrix **A** over to the kind, the eigenvalues of which are μ_i, $i = \overline{1,n}$. Then, consequently,

$$\mathbf{AI}(1+\varepsilon-\mu_i) \tag{13}$$

– special matrix.

We will execute transformation of similarity of matrix (6):

$$\mathbf{H}^{-1}\left[\mathbf{AI}(1+\varepsilon-\mu_i)\right]\mathbf{H} = \mathbf{diag}(\lambda_i-\mu_i)+\varepsilon\mathbf{H}^{-1}\mathbf{AH} \tag{14}$$

Right part of matrix equalization (14) also is the special matrix.

We will assume that $\mu_i \neq \lambda_i$ for all i. Such supposition, as well as supposition that matrices **A** do not have the multiple eigenvalues, is based on that from the random errors of task of elements of matrix, methodical errors and errors of calculations always casual deviations of results from the truth values take place. It admits, the elements of matrix **A** are the independent Gauss distributed sizes with the mean values a_{ij} and identical dispersion of errors of presentation. In probabilistic sense the number of conditionality gives the relation to most semiaxis to the least semiaxis for the ellipsoid of dispersion of vector, the components of which there are the errors of presentation of elements [4, p. 145]. Therefore probabilities of appearance of multiple eigenvalues μ_i,λ_i or coincidence μ_i and λ_i practically equal to the zero.

Then

$$\mathbf{diag}(\lambda_i-\mu_i)+\varepsilon\mathbf{H}^{-1}\mathbf{AH} = \mathbf{diag}(\lambda_i-\mu_i)\left[\mathbf{I}+\varepsilon\mathbf{diag}(\lambda_i-\mu_i)^{-1}\mathbf{H}^{-1}\mathbf{AH}\right] \tag{15}$$

where a matrix in square brackets in right part of expression (15) is also special. In this case, if a matrix (I+M) is special, $\|\mathbf{M}\| \geq 1$ for the norm of any kind. Otherwise at $\|\mathbf{M}\| \leq 1$ none of eigenvalues (**I+M**) can be zeroing. Consequently, $\left\|\varepsilon\mathbf{diag}(\lambda_i-\mu_i)^{-1}\mathbf{H}^{-1}\mathbf{AH}\right\| \geq 1$ and, accordingly,

$$\varepsilon \max\left|(\lambda_i-\mu_i)^{-1}\right|\left\|\mathbf{H}^{-1}\right\|\left\|\mathbf{A}\right\|\left\|\mathbf{H}\right\| \geq 1 \tag{16a}$$

or

$$\varepsilon\left\|\mathbf{H}^{-1}\right\|\left\|\mathbf{A}\right\|\left\|\mathbf{H}\right\| \geq \min\left|(\lambda_i-\mu_i)\right| \tag{16б}$$

From expressions (16a) and (16б) follows, that in any case

$$|\lambda_i-\mu_i| \leq \varepsilon k(\mathbf{H})\|\mathbf{A}\| \tag{17}$$

at least, at one value *i*. Here $k(\mathbf{H}) = \left\|\mathbf{H}^{-1}\right\|\left\|\mathbf{H}\right\|$ is spectral number of conditionality.

Thus, the general sensibility of eigenvalues of matrix **A** directly depends on $k(\mathbf{H})$, so it is possible to interpret $k(\mathbf{H})$ as number of conditionality for the problem of eigenvalues.

Taking into account Hadamard relaxed condition of the regularity $|a_{ii}| - \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| \geq 0$ and in accordance with a Gershgorin circle theorem [8, p.415] it is possible to apply expression (17) for localization of roots of matrix (13).

IV. Remarks and conclusions

We will mark important conditions and results of conducted analysis.

1. The problem of singularity of matrices has different specifics for the calculation of eigenvalues and eigenvectors, from one side, and for matrix inversion and solution of system of equations, from another side.
2. In practice, for reduction of sensibility of calculation of eigenvalues sometimes is useful to apply some "noisiness" of non-diagonal elements of priorities matrix. Actually we eliminate the opposite symmetric character of matrix and observe stability of solution.
3. The got results are valid for any norm, for which $\left\| \mathbf{diag}(\lambda_i - \mu_i)^{-1} \right\| = \max |\lambda_i - \mu_i|^{-1}$. It is therefore possible to use any norm of matrix, following considering of comfort and concrete maintenance of the examined task. For example, the Euclid norm is often used for practical aims, because it is easily to count up her.
4. At the conclusion of expressions (16a), (16б) and (17) it has not required that region ε be the small neighbouring of points a_{ij}.

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