УДК 625.8 / 539.3 DOI: 10.33744/0365-8171-2023-113.1-012-020
UDC 625.8 / 539.3

CONSIDERATION THE RHEOLOGICAL PROPERTIES WHEN INVESTIGATING THE HETEROGENEOUS BEHAVIOR OF A ROCK MASS IN TIME

ВРАХУВАННЯ РЕОЛОГІЧНИХ ВЛАСТИВОСТЕЙ ПРИ ДОСЛІДЖЕННІ ПОВЕДІНКИ НЕОДНОРІДНОГО ПОРОДНОГО МАСИВУ У ЧАСІ

Bondarenko Liudmyla P., *Candidate of Engineering Science (Ph.D.), Associate Professor, National Transport University, Associate Professor of Department of Transport Construction and Property Management, e-mail: luda_bond@ukr.net, тел.+380442803942,*

https://orcid.org/0000-0002-8239-065X

Liashenko Yana G., *PhD in Physics and Mathematics, National Aviation University, Associate Professor of the Department of Higher Mathematics, e-mail: Ya_Lyashnko@ukr.net,*
тел.+380444067323

https://orcid.org/0000-0002-7772-7879

Balashova Yuliia Borisovna, *candidate of science (technical), associate professor, associate professor of the department of highways, geodesy and land management at the Prydniprovska State Academy of Civil Engineering and Architecture. e-mail: balashova.yuliia@pdaba.edu.ua, tel. +380507865446, Ukraine, 49600, Dnipro, 24-a, Architect Oleg Petrov St., of. 421.*

https://orcid.org/0000-0002-2286-9263

Abstract. The paper is devoted to the solution of nowadays relevant issue regarding the scientific substantiation of the most effective methods of mining rocks for various needs of the national economy, including for the construction of highways. The research was carried out on the basis of mathematical modeling methods, taking into account the rheological properties of rocks, heterogeneity of their structure, microdamage and behavior of the rock massif over time. As part of the work, geological material consisting of an isotropic viscoelastic matrix with stochastically placed inclusions in different directions was considered. The change in the stress-strain state of rocks with viscoelastic properties and containing randomly placed inclusions is determined. Provided that the size of the body is much greater than the size of the microinhomogeneities, the area containing the environment is considered infinite. The mathematical model is constructed on the basis of the fact that when homogeneous loads interact on a statistically homogeneous body, the random fields of stresses and strains that arise are also statistically homogeneous, and therefore, volume averaging can be performed as statistical averaging. The derivation of the calculation formulas is connected with the setting of an explicit form of density distribution of inclusions by direction. Based on the constructed mathematical model, microstructural stresses were investigated, effective parameters were calculated, and their dependence on the shape, orientation, and volume concentration of inclusions was determined. In addition, as a particular case, a fractured environment is considered. Taking into account such a significant heterogeneity of the geological rock and the presence of microdamages, the dependence of viscoelastic deformations on time and degree of damage was obtained. The obtained results make it possible to further evaluate the geomechanical situation, as well as to obtain the parameters of development systems for underground or open mining operations, which in turn will allow efficient mining of useful material.

Key words: rocks, rheological problem, stress-strain state, viscoelasticity operator, creep parameters.

Introduction. At nowadays, stone materials are one of the main building materials that are quite widely used in industrial and residential construction, hydraulic structures, and in the construction of highways and railways. So, for example, more than 13 thousand cubic meters of stone materials need to be spent to build 1 km of the II category road. At the same time, effective extraction and use of stone materials is not possible without scientific conclusions and justifications, which determines the relevance of corresponding scientific research. A comprehensive investigation of the mechanism and patterns of rock behavior will provide an opportunity to obtain knowledge that will allow, in specific mining and geological conditions, based on the initial data of geological exploration, to assess the geomechanical situation, as well as the parameters of development systems in underground or open mining operations, which in turn will allow effectively extract a useful component.

The development of the study of deformation processes of geological structures [1] is ensured, in particular, by the methods of mechanics of continuous heterogeneous environment [2]. Researching the mechanisms of accumulation of tectonic stresses, creep under constant loads, etc., are extremely important tasks. Indeed, under the influence of stresses, the formation of micro- and macro-cracks occurs, the processes of delamination, faults, and, ultimately, the destruction of the rock structure develop. Mechanical properties of rocks can be into deformation properties, which characterize the ability of rocks to be deformed under load; strength, which characterizes the resistance of rocks to various force influences; rheological, which characterize the change in strength and deformation properties in time.

Real geological massifs mainly have a random heterogeneous structure, which is determined by both the arbitrary shape of the filling particles and their random placement. The solution of this class of problems is connected with the application of mechanical models that take into account the heterogeneity of structures [3]. Application of methods of the theory of random functions allows to determine almost all real structures of heterogeneous environments [4]. Rocks with an irregular structure can be represented as a environment with effective mechanical characteristics that are random functions of coordinates.

Purpose and methods. An important task is to predict the behavior of the rock mass, taking into account the heterogeneity of its structure and microdamage. Indeed, even at the stage of designing a mining enterprise, an engineer needs to know what the load will be on the fastening of the mine workings, whether the soil of the workings, the sides of the pits and the slopes of the dumps will be stable, and about many other geomechanical indicators that would allow safe and economical development of the earth's subsoil in the future . For this, it is necessary to construct, to define mathematically and to analyze the appropriate geomechanical models.

The purpose of the work is to find viscoelastic operators and solve the problem of changing the stress-strain state of the geological rock in time, considering the heterogeneity of the material and the presence of microdamages.

The object of the research is geological materials of random heterogeneous structure with viscoelastic properties.

Research methods are mathematical and mechanical modeling of the stress-strain state of a heterogeneous material.

Results and explanations. A geological material consisting of an isotropic viscoelastic matrix with stochastically placed inclusions in different directions is considered. That is, this environment has the same mechanical properties in all directions. The problem of rheology the purpose of which is to define the stress-strain state of rocks and other materials, considering their tendency to creep and relaxation takes place.

This problem is reduced to compiling the so-called equations of state, i.e. such equations that connect the components of stresses, strains and their time derivatives into a single relationship. In all available investigations, it is considered that the equations of state sufficiently accurately determine changes in the stress-strain state of materials if the external conditions are such that deformations and stresses cannot be considered constant. For clarity of presentation of rheological processes, the method of structural models is usually used. This method consists in the fact that the properties of the body are defined using a specially selected mechanical model. The model should consist of elements that perfectly reflect the fundamental qualities of the initial material. So, for example, at a certain level of loads and their application quickly enough, all solid and cohesive rocks behave as elastic bodies obeying Hooke's law. Special investigations have established that grain boundaries in polycrystalline materials, which include rocks, behave like a viscous material. This circumstance leads to the fact that the temperature in a certain way changes the internal friction in such bodies, their deformation and strength characteristics. Under conditions of sufficiently long external loads, polycrystalline materials generally behave like a very viscous material.

Equations of equilibrium in stresses for arbitrary continuous environment [1], which are valid at every point of the volume V are following

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho F_i = 0 . \tag{1}$$

Equations of compatibility of deformations are

$$\varepsilon_{mlj} \varepsilon_{nik} \varepsilon_{ij,kl} = 0 \tag{2}$$

The generalized Hooke's law in the case of an isotropic linear elastic environment under the influence of only mechanical factors has the form

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu} \delta_{ij} \varepsilon_{kk} \right). \tag{3}$$

Here δ_{ij} is the Kronecker's symbol $\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}, \; i, j = 1, 2, 3 .$ (4)

Relationship between strains and stresses for such an environment has the form

$$\varepsilon_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \delta_{ij} \sigma_{kk} . \tag{5}$$

The form of Hooke's law is the most widely used through the Lamé coefficients λ and μ , which are related to Young's modulus E and Poisson's ratio ν by the formulas

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \; \mu = \frac{E}{2(1+\nu)} \tag{6}$$

namely

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{7}$$

In the rheological problem, Hooke's generalized law has the form [5, 6]:

$$\sigma_{ij} = \tilde{\lambda}_{ijkl} \varepsilon_{kl} , \tag{8}$$

where $$\tilde{\lambda}_{ijkl} = \lambda_{ijkl}\left[f - \int_0^t K_{ijkl} f(\tau)\, d\tau \right]. \quad (9)$$

Here $\tilde{\lambda}_{ijkl}$ is tensor of linear operators of viscoelasticity [7], λ_{ijkl} is tensor of instantaneous elastic moduli, K_{ijkl} are kernels of viscoelastic operators. It is known to be, for environments with randomly located inhomogeneities, tensors λ_{ijkl} and K_{ijkl} depend on space coordinate randomly. Since the time integration procedure is commutative with space integration and statistical averaging then the solution of the elastic problem can be used to solve the viscoelastic problem. Namely, in the final result, elastic moduli are replaced by viscoelastic operators.

The tensor of the macroscopic modulus of elasticity $\overset{*}{\lambda}_{ijkl}$ of a micro-inhomogeneous body is defined by the relations

$$\bar{\sigma}_{ij} = \overset{*}{\lambda}_{ijkl}\bar{\varepsilon}_{kl}. \quad (10)$$

Here $$\bar{\sigma}_{ij} = \frac{1}{v}\int_V \sigma_{ij} dv; \qquad \bar{\varepsilon}_{ij} = \frac{1}{v}\int_V \varepsilon_{ij} dv. \quad (11)$$

Equilibrium equations must be satisfied at all points of the studied body. Upon reaching the border of the area, the stress components must be such that they are in equilibrium with the external forces applied to the border. Because of this, external forces can be considered as a continuation of internal tensions.

Relations (1) are written on the basis of equilibrium equations

$$\sigma_{ij,j} = 0 \quad (12)$$

Hooke's law connecting stresses σ_{ij} and strains e_{ij} at an arbitrary point for a micro-inhomogeneous body has the following form

$$\sigma_{ij} = \lambda_{ijkl} e_{kl}, \quad (13)$$

where e_{ij} is unit antisymmetric tensor, λ_{ijkl} is a tensor of elastic moduli, which is a statistically homogeneous random function of coordinates. Substituting (13) into (12), we get the problem in displacements

$$(\lambda_{ijkl} u_{kl})_{,j} = 0 \quad (14)$$

Determining the stresses from (14) as a function of average deformations $\bar{\varepsilon}_{kl}$ and averaging them over the volume, we obtain (10).

We assume that any volume of the environment is continuously filled with its substance, that is, it forms a so-called material continuum. Such an assumption makes it possible to consider that an arbitrarily small volume of the area of space occupied by the body around an arbitrary point contains the given substance. If the size of the body is much larger than the size of the micro-inhomogeneities, then the area occupied by the environment can be considered infinite.

Interacting of homogeneous loads on a statistically homogeneous body, the arisen random fields of stresses σ_{ij} and strains ε_{ij} are also statistically homogeneous. Therefore, volume averaging coincides with statistical averaging

$$\bar{\sigma}_{ij} = \langle \sigma_{ij} \rangle; \quad (15) \qquad \bar{\varepsilon}_{ij} = \langle \varepsilon_{ij} \rangle, \quad (16)$$

where
$$\langle \sigma_{ij} \rangle = \int \sigma_{ij} f_2(\sigma_{ij}) d\sigma_{ij} \quad (17)$$

$$\langle \varepsilon_{ij} \rangle = \int \varepsilon_{ij} f_3(\varepsilon_{ij}) d\varepsilon_{ij} \, . \quad (18)$$

In this case, Hooke's law aquires the form

$$\langle \sigma_{ij} \rangle = \langle \lambda_{ijkl} \varepsilon_{kl} \rangle = \langle \lambda_{ijkl} \rangle \langle \varepsilon_{kl} \rangle + \langle \lambda^0_{ijkl} \varepsilon^0_{kl} \rangle . \quad (19)$$

For statistically homogeneous deformations, displacements are presented in the form

$$u_i = \langle \varepsilon_{ij} \rangle x_i + u_i^0 , \quad (20)$$

where u_i^0 is fluctuation of displacements. Then the equation of equilibrium in displacements follows from equation (4).

$$\langle \lambda_{ijkl} \rangle u^0_{k,l\,j} + \lambda^0_{ijkl,j} \langle \varepsilon_{kl} \rangle + \left(\lambda^0_{ijkl} u^0_{k,l} \right)_{,j} = 0 . \quad (21)$$

Since the regular part of the displacements $\langle \varepsilon_{ij} \rangle x_i$ at infinity increases indefinitely, the fluctuation u_i^0 at infinity is equal to zero

$$u_i^0 \Big|_{\infty} = 0 . \quad (22)$$

Therefore, this problem is reduced to the solution of equation (21) with the boundary condition (22).

Let's construct an equation for moments $\left\langle u_k^{0(1)} \lambda^{0(2)}_{pqmn} \right\rangle$. To do this, we will carry out [8, 9] certain transformations and lead statistical averaging of the equilibrium equation (21). As a result, we get

$$\langle \lambda_{ijkl} \rangle \left\langle u^{0(1)}_{kj} \lambda^{0(2)}_{pqmn} \right\rangle_{,l\,i} = -\left\langle \lambda^{0(1)}_{ijkl} \lambda^{0(2)}_{pqmn} \right\rangle \langle \varepsilon_{kl} \rangle_{,j} - \left\langle \lambda^{0(1)}_{ijkl} \, u^{0(1)}_{k,l} \, \lambda^{0(2)}_{pqmn} \right\rangle_{,j} . \quad (23)$$

The equilibrium equation (23) with respect to moments can be written [10] in following form

$$\left\langle \lambda_{ijkl} \right\rangle Q^{k}_{pqmn},_{li}(y_i) = -F^{ijkl}_{pqmn},_{j} \left\langle \varepsilon_{kl} \right\rangle . \qquad (24)$$

This equation is written in the correlation approximation. Moreover

$$Q^{k}_{pqmn}(y_i) = \left\langle u^{0(1)}_{k} \lambda^{i0(2)}_{pqmn} \right\rangle ; \qquad F^{ijkl}_{pqmn}(y_i) = \left\langle \lambda^{i0(1)}_{ijkl} \lambda^{i0(2)}_{pqmn} \right\rangle ; \qquad y_i = x_i^{(1)} - x_i^{(2)} . \qquad (25)$$

Then Hooke's law (19) for averaged stresses aquires the form [11, 12]

$$\left\langle \sigma_{ij} \right\rangle = \left\langle \lambda_{ijkl} \right\rangle \left\langle \varepsilon_{kl} \right\rangle + Q^{k}_{ijkl},_{l}(0) . \qquad (26)$$

After application of Green's function

$$\Gamma * f = \int G_{\alpha(i,j)\beta}(x-x) f(x) \partial V + \int G_{\alpha(i,j)}(x-x) f(x) N_{\beta} \partial S , \qquad (27)$$

we obtain

$$Q^{k}_{pqmn}(x_i^{(1)}) = \int_V G_{kl},_{h}(x_i^{(1)} - x_i^{(2)}) F^{lhpq}_{ijmn},_{h}(x_i^{(2)}) dV^{(2)} \left\langle \varepsilon_{pq} \right\rangle . \qquad (28)$$

Taking into account that $F^{lkpq}_{ijmn}(\infty) = 0,$ and having integrated the last expression we get

$$Q^{k}_{ijkl},_{l}(0) = \int_V G_{mr},_{hn}(-x_i) F^{rhkl}_{ijmn}(x_i) dV \left\langle \varepsilon_{pq} \right\rangle . \qquad (29)$$

The tensor of macroscopic elastic moduli is written in the form

$$\lambda^{*}_{ijkl} = \left\langle \lambda_{ijkl} \right\rangle + \int_V G_{mr},_{hn}(-x_i) F^{rhkl}_{ijmn}(x_i) dV . \qquad (30)$$

For geological environments with granular stochastic structure

$$Q_{jk},_{n}(0) = -\frac{1}{\left\langle \mu \right\rangle \left\langle \lambda + 2\lambda \right\rangle} [(5\left\langle \mu \right\rangle F_{ik}(0) - 2\left\langle \lambda + \mu \right\rangle F_{ik}(0)) \left\langle \varepsilon_{pp} \right\rangle \delta_{jn} +$$

$$+2\left\langle \lambda + 8\mu \right\rangle F_{2k}(0) \left\langle \varepsilon_{jn} \right\rangle] . \qquad (31)$$

Finally, we find the relationship between the average stresses and strains

$$\left\langle \sigma_{jk} \right\rangle = \lambda^{*} \left\langle \varepsilon_{pp} \right\rangle \delta_{jk} + 2\mu^{*} \left\langle \varepsilon_{jk} \right\rangle . \qquad (32)$$

This expression contains macroscopic Lame's constants

$$\lambda^{*} = \left\langle \lambda \right\rangle - \frac{1}{3\left\langle \lambda + 2\mu \right\rangle} \left[3F_{11}(0) + 4F_{12}(0) - \frac{4\left\langle \lambda + \mu \right\rangle}{5\left\langle \mu \right\rangle} F_{22}(0) \right] ; \qquad (33)$$

$$\mu^{*}=\langle\mu\rangle-\frac{2\langle 3\lambda+8\mu\rangle F_{22}(0)}{15\langle\mu\rangle\langle\lambda+2\mu\rangle} \tag{34}$$

Equations (32)-(34) are valid for arbitrary types of spatial orientation of inclusions. In addition, these equations allow taking into account damage to the geological environment. The derivation of the calculation formulas is related to the assignment of the explicit form of the density of the included distribution by direction. So, for example, in a partial case, the values of the effective creep parameters of an isotropic environment with cracks are calculated according to (33), (34) from the ratios that are expressed in terms of the corresponding creep parameters of the environment without cracks and the damage parameters, which depend on the radius of the cracks and their number on volume [7]

$$\lambda^{*}=\alpha\lambda, \tag{35}$$ $$\mu^{*}=\beta\mu; \tag{36}$$

where

$$\alpha=\frac{1}{1+\omega(1-\nu^{2})(1-2\nu)^{-1}}, \tag{37}$$

$$\beta=\frac{1}{1+\omega(1-\nu)(1-\nu/5)(1-\nu/2)^{-1}}. \tag{38}$$

The parameter ω characterizes the degree of rock damage

$$\omega=\frac{8}{3}\frac{na^{3}}{V}, \tag{39}$$

where a is the radius of the cracks, n is the number of cracks in the volume V. It should be noted that cracking leads to a significant decrease in the strength characteristics of the rock massif. Cracks are called breaks in rocks, along which displacements are completely absent or insignificant. A set of cracks that dismember this or that part of the earth's crust is called fracturing. Cracking, as an element of the structure, is also one of the characteristic features of the rock massif. Cracks traced in rocks are usually divided by genetic characteristics into natural (congenital, primary) cracks that arise during the formation of geological bodies; tectonic, formed as a result of orogenic processes; artificial ones that appear in the process of preparatory and cleaning works in mines, drilling and blasting, as well as due to other reasons. Effective viscoelastic operators cannot always serve as a reliable damage indicator. However, they are quite often used to model the quasi-brittle process of microdestruction of materials.

Conclusions and recommendations

The change in the stress-strain state of rocks having viscoelastic properties and containing randomly placed inclusions has been examined in paper. Microstructural stresses were studied, effective parameters were calculated, and their dependence on the shape, orientation, and volume concentration of inclusions was determined. In addition, as a partial case, a fractured medium is considered. Taking into account such a significant heterogeneity of the geological rock and the presence of microdamages, the dependence of viscoelastic deformations on time and degree of damage was obtained.

References

1. Lavrenyuk M. Modeli mexaniky` deformivnogo tverdogo tila neodnoridny`x seredovy`shh.: Navchal`ny`j posibny`k. – Ky`yiv: KNU im. Tarasa Shevchenka, 2012. – 86 s.
2. Shashenko O.M. Mexanika girs`ky`x porid: Navch. Posibny`k. – Dnipropetrovs`k: Nacional`na girny`cha akademiya Ukrayiny`, 2002. – 302 s.
3. Vy`zhva S.A., Maslov B.P., Prodajvoda G.T. Effektivnye uprugie svojstva nelinejnyx mnogokomponentnyx geologi`cheski`x sred. // Geofy`zy`chesky`j zhurnal. 2005. - N6. – S.86-96.
4. Maslov B. P., Prodajvoda G. T., Vyzhva S. A. Novyj metod matematy`cheskogo modely`rovany`ya processov razrusheny`ya v ly`tosfere // Geoy`nformaty`ka. — 2006. — N3. — S. 53–61.
5. Maslov B.P. Thermal-stress concentration near inclusions in viscoelastic random composites. // Journal of Engineering Mathematics, 2008. – N61. – P.339-355.
6. Maslov B.P., Lyashenko Ya.G.Nelinijna povzuchist` trishhy`nuvaty`x geologichny`x seredovy`shh. // Visny`k Ky`yivs`kogo universy`tetu. Seriya: geologiya. 2002. - N23-24. – S. 52-54.
7. Maslov B., Lyashenko Ya., Maksy`menko O. Prognozuvannya dovgotry`valoyi micznosti girs`kogo masy`vu u geologichny`x seredovy`shhax skladnoyi struktury`// Visny`k KNU im. Tarasa Shevchenka, Geologiya, - 2009- Vy`p.2 - S. 9-13.
8. Lyashenko, Ya.G. Stress concentration in microstructural elements of viscoelastic composite materials// Strength of Materials, 2005, 37(5), p. 541–550.
9. Jänicke, R., Quintal, B., Larsson, F. *et al.* Identification of viscoelastic properties from numerical model reduction of pressure diffusion in fluid-saturated porous rock with fractures. Comput. Mech **63**, 49–67 (2019). https://doi.org/10.1007/s00466-018-1584-7
10. Mexany`ka kompozy`tnых matery`alov y` эlementov konstrukcy`j. V 3-x t. T. 1. Mexany`ka matery`alov / Guz` A.N., Xoroshun L.P., Vany`n G.A. y` dr. – Ky`ev: Nauk. dumka, 1982. – 368 s.
11. Maslov B.P. Stress concentration in nonlinear viscoelastic composites/Mechanics and Advanced Technologies – 2017 – №1 – p.5-10. – Rezhy`m dostupu: http://nbuv.gov.ua/UJRN/madt_2017_1_3.

ВРАХУВАННЯ РЕОЛОГІЧНИХ ВЛАСТИВОСТЕЙ ПРИ ДОСЛІДЖЕННІ ПОВЕДІНКИ НЕОДНОРІДНОГО ПОРОДНОГО МАСИВУ У ЧАСІ

Бондаренко Людмила Петрівна, кандидат технічних наук, Національний транспортний університет, доцент кафедри транспортного будівництва та управління майном, e-mail: luda_bond@ukr.net, тел.+380442803942, https://orcid.org/0000-0002-8239-065X
Ляшенко Яна Григорівна, кандидат фізико-математичних наук, Національний авіаційний університет, доцент кафедри вищої математики, e-mail: Ya_Lyashnko@ukr.net, тел.+380444067323, https://orcid.org/0000-0002-7772-7879
Балашова Юлія Борисівна, кандидат технічних наук, доцент, доцент кафедри автомобільних доріг, геодезії та землеустрою Придніпровської державної академії будівництва та архітектури. e-mail: balashova.yuliia@pdaba.edu.ua, тел. +380507865446, Україна, 49600, м. Дніпро, вул. Архітектора Олега Петрова, 24-а, к. 421, https://orcid.org/0000-0002-2286-9263

Анотація. Роботу присвячено вирішенню актуального на сьогоднішній день питання щодо наукового обґрунтування найбільш ефективних способів видобування гірських порід для різних потреб народного господарства, у тому числі і для будівництва автомобільних доріг. Дослідження проведено на основі методів математичного моделювання з урахуванням реологічних властивостей гірських порід, неоднорідності їх структури, мікропошкодженості та поведінки породного масиву у часі. В рамках роботи розглянуто геологічний матеріал, що складається з ізотропної в'язко-пружної матриці із стохастично розміщеними включеннями у різних напрямках. Описано зміну напружено-деформованого стану гірських порід, що мають властивості в'язкопружності, та які містять випадково розміщені включення. При умові, що розміри тіла набагато більші розмірів мікронеоднорідностей, то область, що займається середовищем, розглядається як нескінченна. Математичну модель побудовано на основі того, що при взаємодії однорідних навантажень на статистично однорідне тіло випадкові поля напружень та деформацій, які виникають, також є статистично однорідними, а отже, усереднення по об'єму можна здійснювати, як статистичне усереднення. Виведення розрахункових формул пов'язано із заданням явного виду щільності розподілу включень за напрямками. На основі побудованої математичної моделі досліджено мікроструктурні напруження, обчислено ефективні параметри та визначено їх залежність від форми, орієнтації і об'ємної концентрації включень. До того ж, як частинний випадок, розглянуто тріщинувате середовище. Враховуючи таку суттєву неоднорідність геологічної породи та наявність мікропошкоджень отримано залежність в'язко-пружних деформацій від часу та міри пошкодженості. Отримані результати дають можливість в подальшому оцінити геомеханічну ситуацію, а також отримати параметри систем розробки при підземних чи відкритих гірських роботах, що в свою чергу дозволить ефективно видобувати корисний матеріал.

Ключові слова: гірські породи, реологічна задача, напружено-деформований стан, оператор в'язкопружності, параметри повзучості.

Перелік посилань

1. Лавренюк М. Моделі механіки деформівного твердого тіла неоднорідних середовищ.: Навчальний посібник. – Київ: КНУ ім. Тараса Шевченка, 2012. – 86 с.

2. Шашенко О.М. Механіка гірських порід: Навч. Посібник. – Дніпропетровськ: Національна гірнича академія України, 2002. – 302 с.

3. Вижва С.А., Маслов Б.П., Продайвода Г.Т. Эффективные упругие свойства нелинейных многокомпонентных геологических сред. // Геофизический журнал. 2005. - №6. – С.86-96.

4. Маслов Б. П., Продайвода Г. Т., Выжва С. А. Новый метод математического моделирования процессов разрушения в литосфере // Геоинформатика. — 2006. — № 3. — С. 53–61.

5. Maslov B.P. Thermal-stress concentration near inclusions in viscoelastic random composites. // Journal of Engineering Mathematics, 2008. – №61. – P.339-355.

6. Маслов Б.П., Ляшенко Я.Г.Нелінійна повзучість тріщинуватих геологічних середовищ. // Вісник Київського університету. Серія: геологія. 2002. - №23-24. – С. 52-54.

7. Маслов Б., Ляшенко Я., Максименко О. Прогнозування довготривалої міцності гірського масиву у геологічних середовищах складної структури// Вісник КНУ ім. Тараса Шевченка, Геологія, - 2009- Вип.2 - С. 9-13.

8. Lyashenko, Ya.G. Stress concentration in microstructural elements of viscoelastic composite materials// Strength of Materials, 2005, 37(5), p. 541–550.

9. Jänicke, R., Quintal, B., Larsson, F. *et al.* Identification of viscoelastic properties from numerical model reduction of pressure diffusion in fluid-saturated porous rock with fractures. Comput. Mech 63, 49–67 (2019). https://doi.org/10.1007/s00466-018-1584-7

10. Механика композитных материалов и элементов конструкций. В 3-х т. Т. 1. Механика материалов / Гузь А.Н., Хорошун Л.П., Ванин Г.А. и др. – Киев: Наук. думка, 1982. – 368 с.

Maslov B.P. Stress concentration in nonlinear viscoelastic composites/Mechanics and Advanced Technologies – 2017 – №1 – p.5-10. – Режим доступу: http://nbuv.gov.ua/UJRN/madt_2017_1_3.

Надійшла до редакції 10.04.2023.