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The application of linear differential equations of the second order for finding the function of price dependence on time in economic problems

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Modern mathematics is used in various fields of science, enterprises, spheres of life, various fields of knowledge through the construction and analysis of a model of the phenomenon being studied. Mathematical models of a real process or object can be presented in the form of formulas, equations, graphs, etc. In practice, differential equations are widely used for the description of various processes.

A characteristic property of differential equations is the an infinite number of solutions. Therefore, having solved the differential equations described the course of a certain process, it is impossible to unambiguously find the relationship between the values characterizing this process. In order to find a specific solution of the equation that corresponds to a specific problem, it is necessary to have additional information that characterizes the initial conditions, that is, to solve the Cauchy problem. Recall that a differential equation is a non-identical relationship between the independent variable, the desired function, and its derivatives with respect to the independent variable up to and including a certain order.

Wide use of mathematical methods is an important direction of improvement of economic analysis, increases the efficiency of analysis of the activities of enterprises and their divisions. This is achieved by shortening the analysis time, more comprehensive coverage of the influence of factors on the results of commercial activity, replacing approximate or simplified calculations with accurate calculations, setting and solving new multidimensional analysis problems.

The differential equation is used in models of economic dynamics, which reflect not only the dependence of variables on time, but also their interrelationship over time.

Let's take some goods. At a given price p per unit of the product, we denote by $s(p)$ the number of units of the product that sellers on the market offer for sale. The function $s = s(p)$ is called the product supply function. By $q(p)$ denote the number of units of the product that buyers want to buy at a given price p . The function $q = q(p)$ is called the product demand function. For economic reasons, the supply function *s = s(p)* is increasing, and the demand function *q=q(p)* is decreasing. The price at which the demand for a certain product is equal to the supply of this product in the market is called the equilibrium price. That is, at the equilibrium price *p**, equality is fulfilled *s(p*)=q(p*)* .

Formulation of the problem. Let the demand and supply of the product be determined by the ratios, respectively $q = 2p'' - p' - p + 15,\ S = 3p'' + p' + p + 5,$
where *p* is the price of the product, *p'* is the trend of price formation, *p''* is the rate of price change. Let also at the initial moment of time *p(0) = 6, q(0) = S(0) = 10.* Based on the demand-supply matching requirement, find the dependence of the price on time.

Solution. Based on the demand-supply matching requirement, the requirement q = S must be met. We add the equation $2p'' - p' - p + 15 = 3p'' + p' + p + 5$, from which we obtain a linear inhomogeneous differential equation of the second order with constant coefficients:

$$p'' + 2p' + 2p = 10.$$

The general solution of which is: $p(t) = e^{-t}(C_1 cost + C_2 sint) + 5C2$.
Given the initial condition *p(0) = 6*, отримаємо $C_1 = 1$.
Then $p(t) = e^{-t}(cos\,t + C_2\,sin\,t) + 5;$
$p'(t) = -e^{-t}(\cos t + C_2 \sin t) + e^{-t}(-\sin t + C_2 \cos t) = e^{-t}[(C_2 - 1)cos\,t - (C_2 + 1)sin\,t];$
$p''(t) = -e^{-t}[(C_2 - 1)cos\,t - (C_2 + 1)sin\,t] + e^{-t}[-(C_2 - 1 * sin\,t - (C_2 + 1)cos\,t] = e^{-t}(2C_2\,cos\,t + 2sin\,t).$
From here $p'(0) = C_2 - 1,\ p''(0) = -2C_2$.
Taking into account that $q = 2p'' - p' - p + 15$ and $q(0) = 10,$ we find $C_2 = 0$.

Finally, the searched function acquires the form $p(t) = 5 + e^{-t}cos\,t.$

So, a mathematical model for finding the function of the price dependence on time has been constructed, based on which it is possible to make forecasts of pricing. Summarizing the above, we can make the conclusion that differential equations can be used to define a huge number of processes, including economic ones. Mathematical modeling and precise numerical methods of research are the key to scientific and technological progress and a better understanding of all processes from the simplest to the most complex.

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