UDC 621.3

SOME ASPECTS OF COMPLEX NUMBERS APPLICATION IN ELECTRICAL ENGINEERING

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Introduction. The description of electromagnetic processes in alternating current circuits is reduced to solving many complicated integrals. Calculating simple circuits which contain a fairly small number of sources, circuits, and inductive connections, the trigonometric solution method is most often used, but if the electrical circuit becomes more complex, this form of calculation gets very difficult to find the result. In this situation, complex numbers come to the rescue.

Let us consider sinusoidal current (harmonic) [1]:

(1) $U(t) = U_m \cdot \sin(\omega \cdot t + \varphi_1)$, where:

$U(t) -$ AC voltage; $U_m -$maximum voltage value; $\varphi_1 -$initial phase $(t = 0)$;

$\omega -$ radial frequency; $T -$period ; $f -$ frequency.

$\omega = 2\pi \cdot f;\ f = \frac{1}{T}.$

(2) $I(t) = I_m \cdot \sin(\omega \cdot t + \varphi_2)$, where:

$I(t) -$ AC amperage; $I_m -$ maximum amperage value; $\varphi_2 -$ initial phase $(t = 0)$.

According to Ohm's law we have:

(3) $Z = \frac{U}{I}$,

$Z -$ resistance.

Let us replace $U(t)$ and $I(t)$ with the corresponding complex forms [2]:

$U(t) = U_m \cdot e^{j(\omega t + \varphi_1)}; I(t) = I_m \cdot e^{j(\omega t + \varphi_2)}$, where $j -$ imaginary unit, $(j^2 = -1)$.

Hence $U(t) = U_m \cdot (\cos(\omega \cdot t + \varphi_1) + j \cdot \sin(\omega \cdot t + \varphi_1))$ and

$I(t) = I_m \cdot (\cos(\omega \cdot t + \varphi_2) + j \cdot \sin(\omega \cdot t + \varphi_2))$.

Results

1) Let $U(t) = 8 \cdot \sin(\omega \cdot t + \varphi_1); I(t) = 4 \cdot \sin(\omega \cdot t + \varphi_2)$;

$\varphi_1 = \frac{\pi}{3}; \varphi_2 = \frac{\pi}{6}.$

Find active and reactive resistance.

$U(t) = 8 \cdot e^{j(\omega t + \varphi_1)};\ I(t) = 4 \cdot e^{j(\omega t + \varphi_2)}.$

$$Z=\frac{8\cdot e^{j(\omega t+\ \ _{1})}}{4\cdot e^{j(\omega t+\varphi_2)}}=2\cdot e^{j(\varphi_1-\varphi_2)}.$$

$$Z=2\cdot(\cos(\varphi_1-\varphi_2)+j\cdot\sin(\varphi_1-\varphi_2)),$$

where:

$$\varphi_1-\varphi_2=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}.$$

$$Z=2\cdot(\cos(\varphi_1-\varphi_2)+j\cdot\sin(\varphi_1-\varphi_2))=2\cdot\left(\cos\left(\frac{\pi}{6}\right)+j\cdot\sin\left(\frac{\pi}{6}\right)\right)=2\cdot\left(\frac{\sqrt{3}}{2}+j\cdot\frac{1}{2}\right);$$

$$Z=\sqrt{3}+j.$$

Thus $R=Re(Z)=\sqrt{3}$; $X=Im(Z)=1,$

where: $R-$ active resistance; $X-$ reactive resistance.

2) Let we have the parallel connection.

$I_1=8\cdot\sin(\omega\cdot t+\varphi_1); I_2=4\cdot\sin(\omega\cdot t+\varphi_2);$

$$\varphi_1=\frac{\pi}{6};\ \varphi_2=\frac{7\pi}{6}.$$

Find total amperage I.

$$I_1=8\cdot\left(\cos\left(\omega\cdot t+\frac{\pi}{6}\right)+j\cdot\sin\left(\omega\cdot t+\frac{\pi}{6}\right)\right)=8\cdot\left(\frac{\sqrt{3}}{2}+j\cdot\frac{1}{2}\right)=4\sqrt{3}+4j;$$

$$I_2=4\cdot\left(\cos\left(\omega\cdot t+\frac{7\pi}{6}\right)+j\cdot\sin\left(\omega\cdot t+\frac{7\pi}{6}\right)\right)=4\cdot\left(-\frac{\sqrt{3}}{2}-j\cdot\frac{1}{2}\right)=-2\sqrt{3}-2j.$$

$$I=I_1+I_2=; I=2\sqrt{3}+2j;\ I=4\cdot\sin\left(\omega\cdot t+\frac{\pi}{6}\right).$$

Conclusion

The described approach is effectively used to analyze the linear circuits for which the AC current and voltage are presented in the sinusoidal forms with the same frequency.

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