PREFACE

Foreword to Module M 5 "Optics"

This book is the fifth module of the discipline "Physics". It helps to elucidate essential principles of wave optics.

As a result of studying this module, students must ***know*** the definitions of such concepts as light ray, coherence, basic regularities of light propagation and such phenomena as interference, diffraction, polarization, dispersion.

Students must get ***skills*** to research and apply theoretical and experimental methods of wave optics, plot graphs, estimate errors of physical measurements and use theoretical knowledge for solving practical problems.

It is necessary to ***understand***, that such phenomena as interference, diffraction, polarization, dispersion are based on the wave nature of light.

The differential and integral calculus is widely used in the module but for the first year students' level.

The module "Optics" consists of the following **Study Units** (**SU**):

SU 1 — Electromagnetic properties of light;
SU 2 — Interference of light;
SU 3 — Diffraction of light;
SU 4 — Polarization of light;
SU 5 — Dispersion of light;
SU 6 — Laboratory works;
SU 7 — Individual home tasks;
Supplementary SU — Key words, Help tables.

The Preliminary unit contains the basic concepts and laws of electricity and magnetism and oscillations and waves that are necessary to study efficiently this module and a glossary with explanations of mathematics and physics terminology.

"Study Units 1–5" include theoretical material, test questions, sample problems, as well as problems for work in class. "Study Unit 6" gives instructions on how to perform laboratory works. "Study unit 7" contains problems to be solved by students on their own. "Supplementary Units" are aimed at facilitating the module study.

For effectiveness, we advise using self-check questions. Each question is provided with information where to find an answer. Concepts, which are studied in the module, are basic for all engineering fields of study; they are used in aeronavigation, radiolocation, technical electrodynamics etc.

ELECTROMAGNETIC PROPERTIES OF LIGHT

Optics is a science about light phenomena. Historically it has two stages of development. The first stage corresponds to classical or *wave optics* (till 1900) and is based on the wave nature of light; the second one is connected with a discovery of photons — quanta of electromagnetic energy (so cold *quantum optics*).

In this manual we shall treat the wave (classical) optics that considers light as electromagnetic waves.

As it was pointed in Module 4 "Oscillation and Waves", Maxwell established that *light is an electromagnetic wave*. So, light phenomena must be described by the same equations that express the origin and propagation of electromagnetic waves including their interaction with substances.

According to the Maxwell's electromagnetic theory, we have to regard three characteristics of substance: permittivity ε, permeability μ and conductivity σ. Conductivity σ determines absorption of waves, and permittivity ε and permeability μ determine the phase velocity of the electromagnetic waves propagation in a medium

$$v = 1/\sqrt{\varepsilon\varepsilon_0\mu\mu_0}\ .$$

As the phase velocity of light in vacuum is:

$$c = 1/\sqrt{\varepsilon_0\mu_0}\ (\varepsilon = 1,\ \mu = 1),\ \text{so}\ v = c/\sqrt{\varepsilon\mu}\ .$$

The ratio of the speed of light in vacuum c to its phase velocity in a medium v is called the *absolute refractive index n*:

$$n = \frac{c}{v},$$

where $n = \sqrt{\varepsilon\mu}$.

For the majority of transparent substances $\mu = 1$, therefore $n = \sqrt{\varepsilon}$.

The refractive index characterizes the optical density of the medium: a medium with $n = \text{const}$ is called *optically homogeneous*, a medium with a greater n is called *optically denser*.

A line along which light energy propagates is called a *ray*. In optically homogeneous medium for a plane or spherical wave the rays are straight. In isotropic medium the rays are perpendicular to the wave surfaces, in anisotropic medium they are not.

1.1. Reflection and Refraction of Plane Electromagnetic Waves at Interface Between Two Dielectrics

Experiments show that if a light wave falls on the interface between two dielectrics, it is divided into two waves: one of them is reflected on the interface and is propagated in the first medium, and the second wave is refracted and propagated in the second medium. It may be shown that the *frequencies* of the reflected and refracted waves *coincide* with that of the falling (incident) wave.

1.1.1. Constancy of Wave Frequency at Reflection and Refraction

Let us regard a plane electromagnetic wave that falls on the infinite interface between two homogeneous isotropic dielectrics with the refractive indexes n_1 and n_2. Let us determine the direction of propagation by means of the wave vector $\vec{k}$ for the incident wave, the wave vector $\vec{k}'$ for the reflected wave and the wave vector $\vec{k}''$ for the refracted wave. The behavior of the wave at the interface where free charges and currents are absent is determined by the boundary conditions:

$$E_{\tau 1} = E_{\tau 2}\,, \quad H_{\tau 1} = H_{\tau 2}\,, \tag{1.1}$$

where $E_{\tau 1}$, $E_{\tau 2}$ and $H_{\tau 1}$, $H_{\tau 2}$ are the tangential components of electric and magnetic field intensities in the first and second media (see Module 3 “Electricity and Magnetism”, subsections 1.5 and 4.3).

The electric field intensity of the incidence wave that propagates in the direction of vector $\vec{k}$ may be presented in the form:

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k}\vec{r})}. \tag{1.2}$$

According to the superposition principle (see subsection. 2.1) the electric field intensity in the first medium is determined by the intensities of the incident and reflected waves:

$$\vec{E}_1 = \vec{E} + \vec{E}' = \vec{E}_0 e^{i(\omega t - \vec{k}\vec{r})} + \vec{E}'_0 e^{i(\omega' t - \vec{k}'\vec{r})}\,, \tag{1.3}$$

and in the second medium by the intensity of the refracted wave only:

$$\vec{E}_2 = \vec{E}_0'' e^{i(\omega'' t - \vec{k}''\vec{r})}. \qquad (1.4)$$

Here $E_0 = |E_0| e^{i\alpha}$, $E_0' = |E_0'| e^{i\alpha'}$, $E_0'' = |E_0''| e^{i\alpha''}$ are the complex amplitudes of the incident, reflected and refracted waves; α, α', α'' are the initial phases of these waves correspondingly; $\vec{r}$ is the position vector that starts arbitrary and ends at the wave falling point at the interface of dielectrics.

According to the equation (1.1), the tangential components at the interface must be the same:

$$E_{0\tau} e^{i(\omega t - \vec{k}\vec{r})} + E_{0\tau}' e^{i(\omega' t - \vec{k}'\vec{r})} = E_{0\tau}'' e^{i(\omega'' t - \vec{k}''\vec{r})}. \qquad (1.5)$$

For this equality at any time and at any point at the interface such conditions are necessary and sufficient:

$$\omega t = \omega' t = \omega'' t \Rightarrow \omega = \omega' = \omega''. \qquad (1.6)$$

$$\vec{k}\vec{r} = \vec{k}'\vec{r} = \vec{k}''\vec{r} \Rightarrow k_r r = k_r' r = k_r'' r \Rightarrow k_r = k_r' = k_r'', \qquad (1.7)$$

where k_r, k_r', k_r'' are the projections of the wave vectors onto the vector $\vec{r}$.

It follows from the equation (1.6), that the *frequency of the electromagnetic wave at reflection and refraction does not change*.

1.1.2. Relation between Angles of Incidence, Reflection and Refraction

Let us regard a plane electromagnetic wave that falls on the interface between two homogeneous isotropic dielectrics with the refractive indexes n_1 and n_2 (Fig. 1.1).

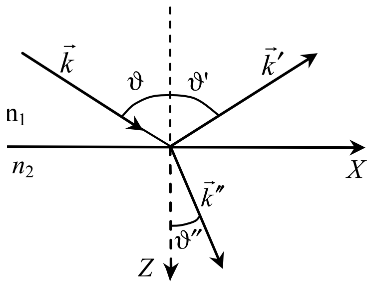


Fig. 1.1

Let us determine the direction of propagation with the aid of the wave vector $\vec{k}$ for the incident wave, the wave vector $\vec{k}'$ for the reflected wave and the wave vector $\vec{k}''$ for the refracted wave. The angles ϑ, ϑ' and ϑ'' that are counted from the normal *Z*, are called the *angle of incidence* (ϑ), the *angle of reflection* (ϑ') and the *angle of refraction* (ϑ'').

Law of reflection of light: the reflected and incident rays and the normal to the point of incidence lie in one plane; the angle of reflection equals the angle of incidence:

$$\vartheta = \vartheta' . \tag{1.8}$$

Law of refraction of light (Snell's law): the refracted and incident rays and the normal to the point of incidence lie in one plane; the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for given substances and is equal to the relative refractive index of these substances:

$$\frac{\sin\vartheta}{\sin\vartheta''} = n_{12} . \tag{1.9}$$

The relative refractive index of the second substance with respect to the first one equals the ratio of their absolute refractive indices $n_1 = c/v_1$ and $n_2 = c/v_2$. Therefore,

$$n_{12} = \frac{n_2}{n_1} = \frac{v_1}{v_2} . \tag{1.10}$$

In the general case a refractive index depends on a wave length, temperature and pressure.

Transforming the equations (1.9) and (1.10) as:

$$n_1 \sin\vartheta = n_2 \sin\vartheta'' ,$$

we may understand, that when light passes from an optically *less denser* medium to an optically *denser* one ($n_1 < n_2$), the angle of incidence is greater then the angle of refraction $\vartheta > \vartheta''$ (Fig. 1.1). On the contrary, if light passes from an optically *denser* medium to an optically *less denser* one ($n_1 > n_2$), the angle of refraction is greater then the angle of incidence $\vartheta < \vartheta''$ and the refracted ray moves away from a normal to the interface of the media (Fig. 1.2, *a*).

If the angle of incidence ϑ increases, the angle of refraction ϑ'' grows even more rapidly (Fig. 1.2, *b*) and, at so called the *critical (limit) angle* of incidence ϑ_{cr}, the angle of refraction becomes equal $\vartheta'' = \pi/2$ (Fig. 1.2, *c*). It is clear that

$$\sin\vartheta_{cr} = n_2 / n_1 = n_{21}, \quad (n_1 > n_2). \tag{1.11}$$

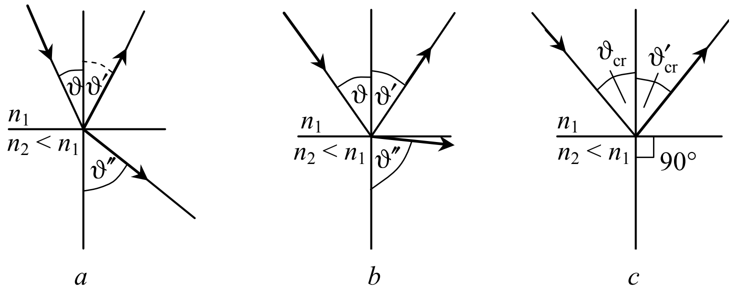


Fig. 1.2

For the angles of incidence $\vartheta > \vartheta_{cr}$ (i.e. from ϑ_{cr} to $\pi/2$), the light wave penetrates into the second medium to a distance of the order of a wavelength λ and then returns to the first medium. This phenomenon is called *total internal reflection*.

Phenomenon of total internal reflection is used, for example, in a right-angle prism to return the direction of rays on 90° and 180° or to overturn the image (Fig. 1.3 *a*, *b*, *c*).

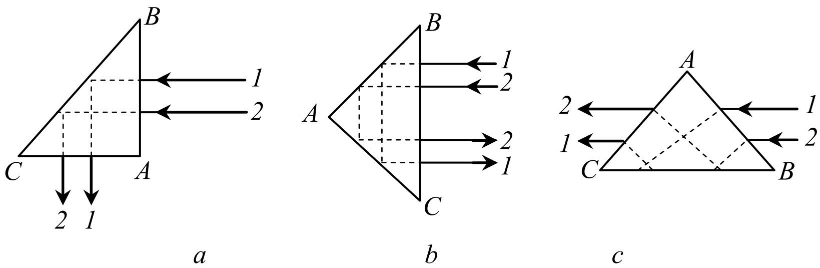


Fig. 1.3

1.1.3. Fresnel's Formulas

For a complete description of reflection and refraction of light it is necessary to know the relation between the amplitudes and phases of electromagnetic waves at the interface of two media. These relations were derived first by Fresnel (1823) and they are called *Fresnel's formulas*. Using Maxwell's electromagnetic theory it is possible to obtain these formulas.

We know that what oscillates in an electromagnetic wave are the vectors $\vec{E}$ and $\vec{H}$. But the most actions of light (photoelectrical, photochemical, physiological, etc.) are due to the oscillations of the electric field intensity vector $\vec{E}$. Therefore we shall regard in the following the behavior of this *electric vector* $\vec{E}$ (sometimes it is called the *light vector*) remembering, that the magnetic vector $\vec{H}$ is always perpendicular to it.

Assume that a plane electromagnetic wave falls on the interface of two transparent homogeneous and isotropic media. As in the general case light is natural, the vector $\vec{E}$ (and $\vec{H}$) oscillations occur in all planes and change with time. But at any moment of time each of these vectors may be resolved on two components, directed parallel and perpendicular to the plane of incidence. Therefore we shall consider two cases:

1) the electric vector lies in the plane of incidence (||) (and the magnetic vector is perpendicular to it);

2) the electric vector is perpendicular to the plane of incidence ($\perp$) (and the magnetic vector lies in it).

These two cases are shown in Fig. 1.4, *a* and *b*.

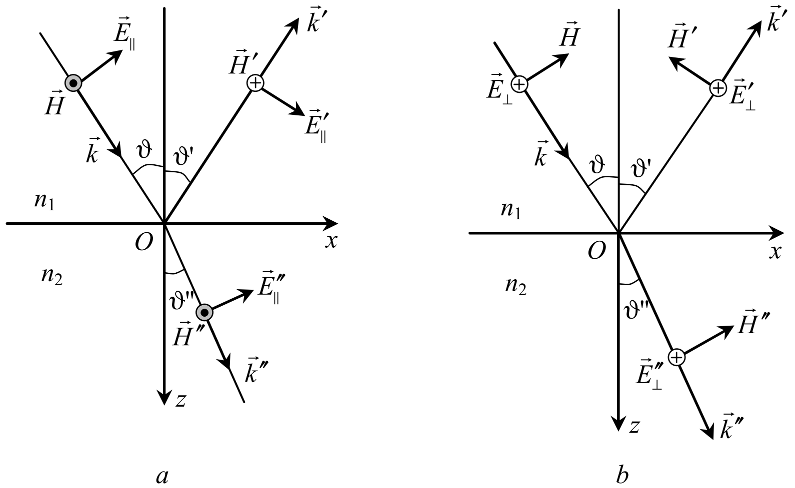


Fig. 1.4

As it follows from Maxwell's equations, there is a relation for the amplitudes of a plane electromagnetic wave: $|E_0|\sqrt{\varepsilon_0\varepsilon} = |H_0|\sqrt{\mu_0\mu}$ (see Module 4 "Oscillations and Waves", subsection 4.2). For transparent dielectrics $\mu \approx 1$ (in an optical region of spectrum), so the absolute refractive index is $n = \sqrt{\varepsilon}$, and the magnetic field intensity is:

$$|H_0| = |E_0|\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0}} = \frac{n|E_0|}{\sqrt{\mu_0/\varepsilon_0}} = \frac{n|E_0|}{Z}, \qquad (1.12)$$

where the constant value $Z = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$ is called the *wave resistance of vacuum*. As at a certain space point the vectors $\vec{E}$ and $\vec{H}$ oscillate in phase, the connection between the amplitudes [the equation (1.12)] holds also for the instantaneous values:

$$H = \frac{nE}{Z}. \qquad (1.12a)$$

For the case shown in Fig. 1.5, *a*, according to the boundary conditions the equation (1.1) and the equation (1.12a), we get:

$$E_{\parallel}\cos\vartheta + E'_{\parallel}\cos\vartheta = E''_{\parallel}\cos\vartheta''; \quad n_1E_{\parallel} - n_1E'_{\parallel} = n_2E''_{\parallel}. \qquad (1.13)$$

According to the equations (1.6) and (1.7), at the interface of two dielectrics the phase factor $\exp(\omega t - \vec{k}\vec{r}_\tau)$ is the same for the incident, reflected and refracted waves. Therefore the ratio of the instantaneous values of electric field intensities of these waves is equal to the ratio of their amplitudes. The ratio $E'_{\parallel}/E_{\parallel} = E'_{0\parallel}/E_{0\parallel}$ we denote as $r_{\parallel}$, and $E''_{\parallel}/E_{\parallel} = E''_{0\parallel}/E_{0\parallel}$ — as $t_{\parallel}$.

Here $E_{0\parallel}$, $E'_{0\parallel}$ and $E''_{0\parallel}$ are, in general, the complex amplitudes of the plane wave that is polarized parallel to the plane of incidence.

The values $r_{\parallel}$ and $t_{\parallel}$ are called the *amplitude coefficients of reflection and transmission* for a plane wave, polarized in the plane of incidence.

Now the equation (1.13) may be rewritten as:

$$\cos\vartheta + r_{\parallel}\cos\vartheta = t_{\parallel}\cos\vartheta''; \quad n_1 - n_1r_{\parallel} = n_2t_{\parallel}. \qquad (1.14)$$

Solving this system of equations using the law of refraction $\sin\vartheta/\sin\vartheta'' = n_2/n_1$, we obtain:

$$r_{\parallel} = \frac{E'_{\parallel}}{E_{\parallel}} = \frac{E'_{0\parallel}}{E_{0\parallel}} = -\frac{\sin 2\vartheta - \sin 2\vartheta''}{\sin 2\vartheta + \sin 2\vartheta''} = -\frac{\mathrm{tg}(\vartheta - \vartheta'')}{\mathrm{tg}(\vartheta + \vartheta'')}; \qquad (1.15)$$

$$t_{\parallel} = \frac{E''_{\parallel}}{E_{\parallel}} = \frac{E''_{0\parallel}}{E_{0\parallel}} = \frac{2\cos\vartheta \sin\vartheta''}{\sin(\vartheta + \vartheta'')\cos(\vartheta - \vartheta'')}. \qquad (1.15a)$$

For the case shown in Fig. 1.6, *b*, the boundary conditions (1.1) are:

$$E_{\perp} + E'_{\perp} = E''_{\perp}; \;\; n_1(E_{\perp} - E'_{\perp})\cos\vartheta = n_2 E''_{\perp}\cos\vartheta''. \qquad (1.16)$$

Performing calculations similar to the previous, we get:

$$r_{\perp} = \frac{E'_{\perp}}{E_{\perp}} = \frac{E'_{0\perp}}{E_{0\perp}} = -\frac{\sin(\vartheta - \vartheta'')}{\sin(\vartheta + \vartheta'')}; \qquad (1.17)$$

$$t_{\perp} = \frac{E''_{\perp}}{E_{\perp}} = \frac{E''_{0\perp}}{E_{0\perp}} = \frac{2\cos\vartheta \sin\vartheta''}{\sin(\vartheta + \vartheta'')}, \qquad (1.17a)$$

where $E_{0\perp}$, $E'_{0\perp}$ and $E''_{0\perp}$ are the complex amplitudes; $r_{\perp}$ and $t_{\perp}$ are the *amplitude coefficients of reflection and transmission* for a plane wave, polarized perpendicular to the plane of incidence.

Relations (1.15), (1.15a) and (1.17), (1.17a) between the complex amplitudes of the incident, reflected and refracted waves are called *Fresnel's formulas.*

Based on the Fresnel's formulas, we may obtain all the relations between the phases of the incident, reflected and refracted waves in all possible cases. For simplicity we assume that light falls normally on the interface ($\vartheta = \vartheta'' = 0$). At small angles of incidence, the sines and tangents in the equations (1.15) and (1.17) may be replaced by the angles themselves, so, the law of refraction (1.12) gets the form $\vartheta / \vartheta'' = n_2 / n_1$. Then,

$$r_{\parallel} = \frac{E'_{0\parallel}}{E_{0\parallel}} = r_{\perp} = \frac{E'_{0\perp}}{E_{0\perp}} = -\frac{(\vartheta - \vartheta'')}{(\vartheta + \vartheta'')} = -\frac{(\vartheta/\vartheta'' - 1)}{(\vartheta/\vartheta'' + 1)} = \frac{n_1 - n_2}{n_1 + n_2}. \qquad (1.18)$$

Taking into account that $\vec{E}_0 = \vec{E}_{0\parallel} + \vec{E}_{0\perp}$, and $\vec{E}'_0 = \vec{E}'_{0\parallel} + \vec{E}'_{0\perp}$, the equation (1.18) may be presented in a vector form:

$$\vec{E}'_0 = \frac{n_1 - n_2}{n_1 + n_2}\vec{E}_0 \Rightarrow \vec{E}' = \frac{n_1 - n_2}{n_1 + n_2}\vec{E}. \qquad (1.19)$$

Performing similar calculations with the equations (1.15a) and (1.17a), we get:

$$t_{\parallel} = \frac{E''_{0\parallel}}{E_{0\parallel}} = t_{\perp} = \frac{E''_{0\perp}}{E_{0\perp}} = \frac{2n_1}{n_1 + n_2}, \tag{1.20}$$

or, in a vector form,

$$\vec{E}'' = \frac{2n_1}{n_1 + n_2}\vec{E}. \tag{1.21}$$

As it follows from the equation (1.21), vectors $\vec{E}''$ and $\vec{E}$ have the same direction at any moment of time, i.e. their oscillations are in-phase. It means that at the interface the phases of the incident and refracted waves coincide. Namely, *the refracted wave in all cases conserves the phase of the incident wave without any change.*

For the reflected wave [the equation (1.19)] vectors $\vec{E}'$ and $\vec{E}$ also have the same direction, but only when $n_1 > n_2$. Namely, *if a wave reflects from the optically less dense medium, its phase does not change.* But when $n_1 < n_2$, the fraction in the equation (1.19) becomes negative, therefore the direction of vector $\vec{E}'$ is opposite to the direction of vector $\vec{E}$. It means, that *at the reflection of a wave from the optically denser medium its phase sharply changes by* π.

Indeed, for the complex amplitudes the equation (1.19) in the case $n_1 < n_2$ may be written as:

$$|E'_0| e^{i\alpha'} = -\left|\frac{n_1 - n_2}{n_1 + n_2} E_0\right| e^{i\alpha}.$$

Since $e^{\pm i\pi} = -1$, then

$$|E'_0| e^{i\alpha'} = e^{\pm i\pi}\left|\frac{n_1 - n_2}{n_1 + n_2} E_0\right| e^{i\alpha} = \left|\frac{n_1 - n_2}{n_1 + n_2} E_0\right| e^{i(\alpha\pm\pi)}.$$

Two complex magnitudes are equal if their modules $|E'_0| = \left|\frac{n_1 - n_2}{n_1 + n_2} E_0\right|$ and phases $\alpha' = \alpha \pm \pi$ are equal. The last equation means, that the phase changes sharply by π. This result is also true in the case of oblique incidence of light on the interface of two transparent isotropic dielectrics (see subsection 4.2.1).

To find the relationships between the coefficients of reflection and transmission we may recall, that the intensity of an electromagnetic wave I is a period average value of an energy flux density, i.e. an average value of Poynting vector S: $I = \langle S \rangle \sim |E_0||H_0|$ (see Module 4 “Oscillations and Waves”, subsection 4.3). According to the equation (1.12), $|H_0| \sim n|E_0|$, therefore $I \sim n|E_0|^2$. By the definition, the coefficient of reflection is: $R = I'/I = n_1|E_0'|^2 / n_1|E_0|^2$, where I', I are the intensities; $|E_0'|$, $|E_0|$ are the ordinary amplitudes of the reflected and incident waves. According to the equation (1.19) we get:

$$R = \frac{I'}{I} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 . \tag{1.22}$$

It is seen, that R does not depend on the direction of the wave incidence: from the first medium into the second, or vice versa.

Similarly, we find the transmission coefficient as the ratio $T = I''/I = n_2|E_0''|^2 / n_1|E_0|^2$, or, with regard to the equation (1.21),

$$T = \frac{I''}{I} = \frac{4n_1 n_2}{(n_1 + n_2)^2} . \tag{1.23}$$

Easy to see that in the absence of absorption $R + T = 1$ (the law of energy conservation). Intensities I, I' and I'' may be calculated as the sum of corresponding parallel (||) and perpendicular ($\perp$) components.

Let us estimate the reflection coefficient at normal (or nearly normal) incidence of light from air ($n_1 \approx 1$) into glass ($n_2 \approx 1{,}5$). In this case [see the equation (1.22)] $R = 0.04$, i.e. 4 % of the incident energy reflects and 96 % passes through the glass.

Taking into account the equations (1.15) and (1.17), the equations for the reflection coefficients for *polarized light* in the case of oblique incidence may be represented as:

$$\rho_{\parallel} = r_{\parallel}^2 = \left[\frac{\operatorname{tg}(\vartheta - \vartheta'')}{\operatorname{tg}(\vartheta + \vartheta'')}\right]^2, \quad \rho_{\perp} = r_{\perp}^2 = \left[\frac{\sin(\vartheta - \vartheta'')}{\sin(\vartheta + \vartheta'')}\right]^2 . \tag{1.24}$$

If *natural light* falls on the interface (all the directions of the electric vector oscillations are equally probable), the wave energy is divided equally between the parallel and perpendicular components. So the total coefficient of reflection is:

$$\rho = \frac{\rho_{\parallel} + \rho_{\perp}}{2} . \tag{1.25}$$

? Test Questions

1. What does optics study?
2. Give the definitions of the refractive index of a medium.
3. Does the frequency of electromagnetic wave change at reflection or refraction?
4. Formulate the law of reflection of light.
5. Formulate the law of refraction of light.
6. What is the phenomenon of total internal reflection?
7. What is the limit (critical) angle of total internal reflection equal to?
8. What do Fresnel's formulas describe?
9. How does the phase of a light wave that is reflected from an optically denser medium change?

Sample Problems

Problem 1. Show, that in the case of refraction in a prism with a small angle of refraction ϑ, a ray deviates from its initial direction on an angle $\alpha = (n-1)\,\vartheta$ regardless of the angle of incidence, if this angle is also small.

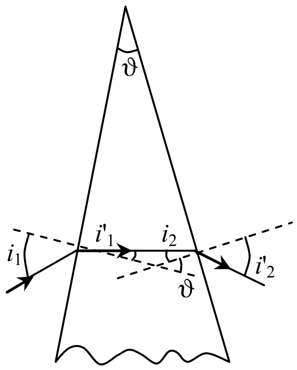


Fig. 1.5

Solution. As the angles of incidence are small, the sines in the law of refraction may be replaced by the angles themselves.

Using the law of refraction for the front and back surfaces of the prism (Fig. 1.5) we obtain the relations:

$$i_1 = ni_1', \; ni_2 = i_2', \; i_1' + i_2 = \vartheta. \qquad (1.26)$$

As it is shown in Fig. 1.1, the required angle is equal to

$$\alpha = (i_1 - i_1') + (i_2' - i_2). \qquad (1.27)$$

After substitution the angles from the equation (1.26) into the equation (1.27) we get:

$$\alpha = (n-1)\,\vartheta.$$

Problems

1. The angle of incidence of light on a glass plate (n = 1.5) is. 60°. After passing through the plate the ray shifts at 15 mm. What is the thickness of the plate? (≈ 28 mm)

2. The refractive index for glass equals 1.52, and for water — 1.33. Find the limit angle of total internal refraction for the interface: 1) glass — air; 2) water-air; 3) glass-water. (41.1°; 48.7°; 61°)

3. A light ray falls at an angle i on a body with the refractive index n. Find the relation between i and n if the reflected ray is perpendicular to the refracted one. ($\tan i = n$)

4. Find the speed of light in a substance if it is known, that at the angle of incidence 45°, the angle of refraction is 30°. ($2.13 \cdot 10^8$ m/s)

INTERFERENCE OF LIGHT

2.1. Superposition Principle

Interference of light is a vivid example of the demonstration of the wave properties of light.

Different optical experiments pointed, that the light beams propagate independently. Therefore, the resulting intensity at a given space point equals the vector sum of intensities of separate waves. This statement is called the *principle of superposition*. Superposition may be considered for waves of any nature (sound, electromagnetic, on the water surface, etc.).

For the light waves, that are electromagnetic waves, the superposition principle mathematically means that the electric field intensity vector (so called *light vector*) $\vec{E}_1$ of one wave simply is added to the electric field intensity vector $\vec{E}_2$ of another wave without any change:

$$\vec{E} = \vec{E}_1 + \vec{E}_2,$$

or for any number of light waves:

$$\vec{E} = \sum_i \vec{E}_i.$$

2.2. Conception of Coherence. Interference of Light Waves

Before considering conception of coherence let us recall addition of oscillations and waves.

Let there be two monochromatic waves of the same frequency that superpose and, at a certain point of space, they produce oscillations of the same direction x with the amplitudes A_1 and A_2:

$$x_1 = A_1 \cos(\omega t + \varphi_1) \text{ and } x_2 = A_2 \cos(\omega t + \varphi_2).$$

The resultant oscillation at the given point has the same frequency and its amplitude is:

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta\,, \qquad (2.1)$$

where δ is the phase difference $\delta = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$.
The initial phase determines by the equation:

$$\mathrm{tg}\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \qquad (2.2)$$

(see Module 4 "Oscillations and Waves", subsection 1.7).

We see, that the amplitude of the resultant oscillation depends on the phase difference δ. If this phase difference of oscillation set up by the waves varies chaotically in time, corresponding waves (and oscillations) are called *incoherent*. In this case δ varies continuously and takes on any values with an equal probability. Therefore, the time-averaged value of $\cos\delta$ equals zero. Thus,

$$< A^2 > = < A_1^2 > + < A_2^2 > .$$

As the intensity is proportional to the average value of the square of the wave amplitude $I \sim < A^2 >$, it may be written as:

$$I = I_1 + I_2\,. \qquad (2.3)$$

So in the case of the superposition of incoherent waves (or oscillations) the resultant intensity equals the sum of the intensities of separate waves (or oscillations).

If the phase difference δ remains constant in time, corresponding waves (and oscillations) are called *coherent*. In the case of superposition of coherent waves, $\cos\delta$ has a constant value and according to the equation (2.1) the resultant intensity is:

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta\,. \qquad (2.4)$$

We see, that in this case the resultant intensity depends on $\cos\delta$: if $\cos\delta > 0$, the resultant intensity $I > I_1 + I_2$ (amplification of waves); if $\cos\delta < 0$, the resultant intensity $I < I_1 + I_2$ (attenuation of waves). Thus, the resultant intensity differs from the sum of intensities of separate waves (or oscillations). Such superposition of coherent waves that results in redistribution of their intensities in space is called the *interference* of waves.

The points, where the waves amplify each other, are called the *maxima of interference*; the points, where the waves attenuate each other, are called the *minima of interference*;

The interference is seen the most clearly when the intensity of both waves is the same: $I_1 = I_2 = I_0$. Then, according to the equation (2.4), if the phase difference $\delta = \pm 2\pi m$, where $m = 0, 1, 2 \ldots$, the resultant intensity $I = 4I_0$ (maximum of interference); if the phase difference $\delta = \pm 2\pi m$, where $m = 0, 1, 2 \ldots$, the resultant intensity $I = 0$ (minimum of interference). Note, that for incoherent waves we get the same resultant intensity $I = 2I_0$ everywhere [the equation (2.3)].

Thus, two waves are called *coherent*, if their phase difference does not depend on time. The sources of such waves are called the *coherent sources* (for example, the Sun, the lamps, the identical radio antennas). The overlapping of coherent ways leads to the *interference* and corresponding *interference pattern* with places of maximum and minimum intensity is observed. For light monochromatic waves maximum and minimum intensity correspond to light and dark places. The region where coherent waves overlap is called the *interference field*.

Natural light sources (for example, lamps, even monochromatic) are not coherent. It is due to the fact that light is emitted by many individual atoms. Each atom emits a wave during 10^{-9} – 10^{-8} (so called a *wave train* of a length of about 3 m) and then a new emitted wave has another phase. So the phase difference of emitted waves changes over very short time intervals and therefore these waves are not coherent. When they overlap, we observe only an average uniform distribution of illumination, but not their interference.

To obtain coherent light waves we must split the wave emitted by a single source (atom) into two parts (by means of reflections or refractions). If these waves then overlap, interference in nature light is observed.

Let there be two coherent point sources S_1 and S_2 of monochromatic waves and the oscillations of vector $\vec{E}$ occur perpendicular to the plane of the drawing (Fig. 2.1).

The interference of these waves gives the interference pattern on screen *E*.

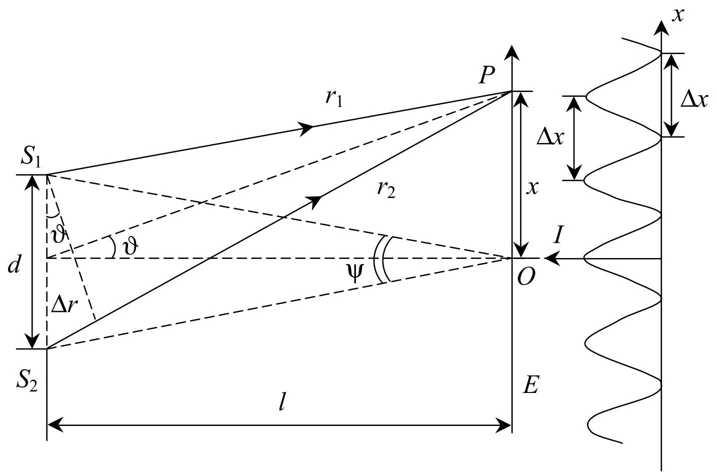


Fig. 2.1

The position of point P on the screen is characterized by the coordinate x (the origin is at point O relatively to which the sources S_1 and S_2 are arranged symmetrically). Also we count, that the distance between the sources d is considerably smaller than the distance l to the screen. The distance x to point P is also considerably smaller than l.

Let us assume that two coherent waves with a wavelength λ coincide at point P. As at the sources S_1 and S_2 oscillations of vector $\vec{E}$ are identically directed, we may use the equations of oscillations in scalar form. Then the first and the second waves excite the oscillations at point P as:

$$E_1 = A_1\cos(\omega t - kr_1 + \varphi_1) = A_1\cos(\omega t + \alpha_1);$$

$$E_2 = A_2\cos(\omega t - kr_2 + \varphi_2) = A_2\cos(\omega t + \alpha_2),$$

($\alpha_1 = \varphi_1 - kr_1$; $\alpha_2 = \varphi_2 - kr_2$), and their superposition is:

$$E = E_1 + E_2 = A\cos(\omega t + \alpha).$$

The amplitude A and the phase α of the resulting oscillation at the point P may be found according to the equations (2.1) and (2.2). But in this case the phase difference δ is determined not only by the difference of initial phases, but also by the difference of the paths r_1 and r_2, that is called the *geometrical path difference*.

Then the phase difference of two waves at the point *P* may be written as:

$$\delta = \alpha_2 - \alpha_1 = -\left[k(r_2 - r_1) - (\varphi_2 - \varphi_1)\right] = -\left[k\Delta r - (\varphi_2 - \varphi_1)\right],$$

where $\Delta r = r_2 - r_1$ is the geometrical path difference of two waves. According to the equation (2.4), the intensity at this point is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left[k\Delta r - (\varphi_2 - \varphi_1)\right] \qquad (2.5)$$

(sign "minus" disappears because cosine is an even function). As the waves are coherent, $\varphi_2 - \varphi_1 = \text{const}$, so the intensity distribution at different points of the screen depends on geometrical path difference Δr (Fig. 2.1).

For simplicity we may take $\varphi_2 - \varphi_1 = 0$. Then, the interference pattern is symmetrical about point *O* (Fig. 2.1) and the intensity is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(k\Delta r), \qquad (2.6)$$

where

$$\delta = k\Delta r = \frac{2\pi}{\lambda}\Delta r \qquad (2.7)$$

is *the phase difference that has appeared due to the path difference*; λ is the wavelength in the medium.

The equations (2.6) and (2.7) establish, that if $\Delta r = \pm m\lambda$ ($m = 0, 1, 2, ...$), i.e. when the geometrical path difference equals an *integral number of wavelengths*, the resultant intensity is maximum.

Corresponding phase difference is $\delta = \pm 2m\pi$, and the oscillations at these points occur in phase. If $\Delta r = \pm(2m+1)\lambda/2$ ($m = 0, 1, 2, ...$), i.e. when the geometrical path difference equals a *half- integral number of wavelengths*, the resultant intensity is minimum.

Corresponding phase difference is $\delta = \pm(2m+1)\pi$, and the oscillations at these points are in counterphase.

Thus, the equation

$$\Delta r = \pm m\lambda,\ \delta = k\Delta r = \pm 2m\pi,\ (m = 0,1,2,...) \qquad (2.8)$$

is the condition for an interference *maximum*, and the equation

$$\Delta r = \pm(2m+1)\lambda/2\,,\ \delta = k\Delta r = \pm(2m+1)\pi\,,\ (m = 0,1,2,...) \quad (2.9)$$

is the condition for an interference *minimum*.

2.3. Optical Path Difference

In general, the interfering waves can propagate in several media with different refractive indices. Assume that up to the observation point the first wave travels the path r_1 in a medium with the refractive index n_1. The second wave travels the path r_2 in a medium with the refractive index n_2. The wavelengths in the media are $\lambda_1 = \lambda_0 / n_1$ and $\lambda_2 = \lambda_0 / n_2$, where λ_0 is the wavelength in vacuum. Corresponding wave numbers are $k_1 = 2\pi/\lambda_1$ i $k_2 = 2\pi/\lambda_2$. Then the phase difference (2.7) has a form:

$$\delta = k_2 r_2 - k_1 r_1 = 2\pi\left(\frac{r_2}{\lambda_0 / n_2} - \frac{r_1}{\lambda_0 / n_1}\right) = \frac{2\pi}{\lambda_0}(n_2 r_2 - n_1 r_1) = \frac{2\pi}{\lambda_0}\Delta, \quad (2.10)$$

where a quantity

$$\Delta = n_2 r_2 - n_1 r_1 = L_2 - L_1 \quad (2.11)$$

is called the *optical path difference*.

If the waves propagate in the same medium with the refractive index $n = n_1 = n_2$, the optical path difference equals the product of the refractive index and the geometrical path difference:

$$\Delta = n(r_2 - r_1) = n\Delta r\,. \quad (2.12)$$

The *optical path* $L = nr$ points that the time spent by light in covering the distance r with the speed v in the medium is the same as in vacuum to cover the optical path L with the speed c.

In fact,

$$t = \frac{L}{c} = \frac{nr}{c} = \frac{r}{c/n} = \frac{r}{v}.$$

Using the optical path difference we may rewrite the equations (2.8) and (2.9) in the form of (2.8a) and (2.9a) respectively.

Eq. (2.8a) $\Delta = \pm m\lambda_0\,,\ \delta = k\Delta = \pm 2m\pi\,,\ (m = 0, 1, 2,...)$

for the condition for an interference *maximum*, and

Eq. (2.9a) $\Delta = \pm(2m+1)\lambda_0/2\,,\ \delta = k\Delta = \pm(2m+1)\pi\ (m = 0, 1, 2, ...)$

for the condition for an interference *minimum*.

It is clear, that if the waves propagate in vacuum ($n = 1$) or in air ($n \cong 1$), the optical path difference Δ coincides with the geometrical path difference $\Delta = \Delta r$.

2.4. Width of Interference Fringes

Let us return to Fig. 2.1 and consider the interference pattern that is a system of successive rectilinear light and dark fringes (bands). Let us find the width of the interference fringes and the distance between them.

To obtain a distinguishable interference pattern, the conditions $d << l$ and $x << l$ must be fulfilled. Under these conditions the right triangles with a small angle ϑ (in practice $\vartheta << 1^{\circ}$) may be considered as similar, and the average distance from the sources to the observation point as $(r_1 + r_2)/2 \approx l$. Thus, mathematically we get:

$$\frac{\Delta r}{d} = \frac{x}{l} \Rightarrow \Delta r = xd/l.$$

Multiplied by the refractive index n of the medium, we get the optical path difference:

$$\Delta = n \cdot \Delta r = nxd/l. \quad (2.13)$$

From the equations (2.8a) and (2.13) we find the positions of the interference maxima (light fringes) on the screen:

$$\Delta = nxd/l = \pm m\lambda_0 \Rightarrow$$

$$\Rightarrow x_{\max} = \pm ml\lambda/d, \quad (m = 0, 1, 2, ...), \quad (2.14)$$

and from the equations (2.9a) and (2.13) — the positions of the interference minima (dark fringes) on the screen:

$$\Delta = nxd/l = \pm(2m+1)\lambda_0/2 \Rightarrow$$

$$\Rightarrow x_{\min} = \pm(2m+1)l\lambda/2d, \quad (m = 0, 1, 2, ...), \quad (2.15)$$

where $\lambda = \lambda_0/n$ is the wavelength in the medium between the sources and the screen.

The number m is called the *order of interference*. The distance Δx between two adjacent maxima or minima that corresponds to a change of m by one is called the *width of an interference fringe*.

It follows from the equations (2.14) and (2.15) that the width of a fringe is:

$$\Delta x = l\lambda/d. \quad (2.16)$$

The width of the interference fringe may be expressed through an convergence angle of the rays ψ (Fig. 2.1). Since as usual this angle is small, it may be seen that $d = l\psi$, or $\psi = d/l$. Substituting this equation into Eq. (2.16), we get:

$$\Delta x = \lambda / \psi. \quad (2.17)$$

The *angular width* of the interference fringes is the angular distance between the adjacent maxima that is observed in the location of sources. Indeed, the position of maxima (light fringes) can be determined by means an angle $\vartheta = x/l$ (Fig. 2.1). The condition of maximum is $\Delta = ndx/l = \pm m\lambda_0$, thus $\vartheta = \pm m\lambda/d$ ($\lambda = \lambda_0/n$), and the angular width becomes equal to:

$$\Delta\vartheta = \frac{\lambda}{d} \quad \text{or} \quad \Delta\vartheta = \frac{\Delta x}{l}. \quad (2.18)$$

The interference pattern is the alternation of light and dark fringes (bands) only in monochromatic light $\lambda = \text{const}$. The interference pattern in white light consists of the alternation of colored fringes, as the positions of minima for one wavelength coincide with the positions of maxima for another wavelength.

By measuring the distances Δx between the adjacent maxima for a certain color and knowing l and d, the wavelength corresponding to this color may be found [see the equation (2.16)]. It is exactly from the experiments on the interference of light, the wavelengths for light rays of various colors were determined for the first time (Table 2.1)

Table 2.1

Correspondence colors and wavelengths for visible light

Color	λ, nm	Color	λ, nm
Red	760–630	Blue-green	500–450
Orange	630–600	Blue	450–430
Yellow	600–570	Violet	430–00
Green	570–500		

Recall, that the rays with wavelengths greater than 760 nm and less than 400 nm, the human eye does not perceive. The first of them are called *infrared*, the second — *ultraviolet*. Light of a certain wavelength ($\lambda = \text{const}$) is called *monochromatic*.

2.5. Intensity Distribution

Let us consider the case of a particularly clear interference, when the intensities of the coherent sources S_1 and S_2 are the same $I_1 = I_2 = I_0$. According to the equation (2.6) the resultant intensity at the points, where the phase difference equals δ [the equation (2.10)], has a form:

$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\delta/2. \tag{2.19}$$

The phase difference δ depends on the optical path difference $\delta = 2\pi\Delta/\lambda_0$, and as $\Delta = nxd/l$ [Eq. (2.13)], then $\delta = 2\pi xd/l\lambda$ ($\lambda = \lambda_0/n$). Finally, we obtain:

$$I = 4I_0\cos^2\frac{\pi xd}{l\lambda}. \tag{2.20}$$

The phase difference δ grows proportionally to *x*. Hence, the intensity varies along the screen in accordance with the law of cosine square. The right-hand part of Fig. 2.1 shows the dependence of I on x in monochromatic light. The intensity varies from 0 at points $x_{\min}$ to $4I_0$ at points $x_{\max}$.

On the whole it may be stated that the *interference is the phenomenon of superposition of coherent waves that leads to a steady in time redistribution of a wave energy flux in the form of interference maxima and minima.*

2.6. Coherence

Let us return to the conception of coherence and consider it in details. Two types of coherence are distinguished: time (temporal) and space (spatial) coherence.

2.6.1. Time Coherence. Coherence Length

An absolute monochromatic wave is an idealized notion. A real light wave is more or less nonmonochromatic. It may be considered as a set of monochromatic waves whose lengths vary in a finite interval from λ to $\lambda+\Delta\lambda$.

Let us assume, that the lengths of the monochromatic waves uniformly and continuously fill sufficiently narrow interval from λ to $\lambda+\Delta\lambda$ and intensity of the waves is approximately the same. Let us take from this interval two extreme waves with the wavelengths λ and $\lambda' = \lambda + \Delta\lambda$ and find the condition under which the interference maxima of this pair of waves coincide.

The distance of the *m*-th maximum from the center of the screen to the point *P* (Fig. 2.1) is directly proportional to $x_{max} \sim m\lambda$ [see Eq. (2.14)]. Hence, the *m*-th maximum of the greater wavelength $\lambda' > \lambda$ is placed farther away from the center of the screen then the *m*-th maximum of the wavelength λ. Between these maxima there is a shift $m(\lambda' - \lambda)$ and it will grow with the increasing of the interference order *m*. For a certain value of *m* this shift will reach the value of the wavelength λ, i.e. the path difference between these waves will be one wavelength $\lambda = m(\lambda' - \lambda)$. It means that the interference maxima of two waves λ and λ' will coincide in the same place of the screen:

$$m\lambda' = m\lambda + \lambda, \quad \text{or} \quad m(\lambda + \Delta\lambda) = (m+1)\lambda. \qquad (2.21)$$

The positions of maxima for the extreme wavelengths of the spectral interval λ and $\lambda + \Delta\lambda$ are given in Fig. 2.2, *a*. Solid lines correspond to the order $m+1$ for the wavelength λ, dotted lines —to the order *m* for the wavelength $\lambda + \Delta\lambda$. In the shaded areas maxima of the intermediate wavelengths are located.

The interference pattern is distinguished until the dip between adjacent maxima of the wavelength λ is completely filled with the intermediate maxima of the wavelengths from the interval (λ, $\lambda + \Delta\lambda$). If the maximum of the *m*-th order for $\lambda + \Delta\lambda$ coincides with the maximum ($m+1$)-st order for λ, then all this dip will be filled with maxima of intermediate wavelengths (Fig. 2.2, *a*, the dip between the 4-th and the 5-th orders of interference). It means that from this place the interference pattern becomes indistinguishable. The character of the smearing of the interference pattern (the disappearing of the interference fringes) is shown in Fig. 2.2, *b*.

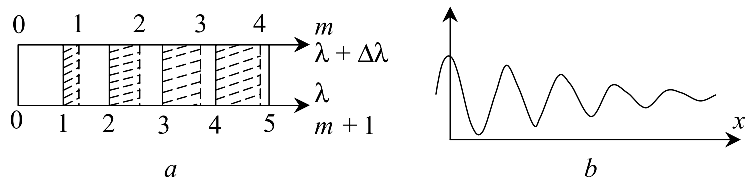


Fig. 2.2

Thus, the interference pattern becomes indistinguishable under the condition $m(\lambda + \Delta\lambda) = (m+1)\lambda$, where m is the limit interference order from which the fringes disappear. Hence we get

$$m \approx \lambda / \Delta\lambda . \quad (2.22)$$

The quantity $\lambda / \Delta\lambda$ characterizes the *degree of light monochromaticity*: the greater it is, the light is more monochromatic.

It can be seen from the equation (2.22) that the higher the interference order is, the narrower the spectral interval $\Delta\lambda$ must be, at which the interference pattern may still be observed.

For non-monochromatic light the path difference when the interference pattern disappears corresponds to the interference order m [the equation (2.22)]. Such path difference is called the *coherence length* for optical radiation

$$l_{\text{coh}} \approx m\lambda \Rightarrow l_{\text{coh}} \approx \lambda^2 / \Delta\lambda . \quad (2.23)$$

The coherence length is directly connected with the degree of light monochromaticity ($\lambda / \Delta\lambda$): the greater it is, the more the coherence length is.

Thus, to obtain an interference pattern by splitting a natural wave into two parts, the optical path difference Δ must be smaller than the coherence length:

$$\Delta < l_{\text{coh}} . \quad (2.24)$$

As it is known, the waves emitted by the atoms in a single act of radiation retain regular only during a *limited time interval*. During this time an amplitude and a phase of atom oscillations are approximately constant (then they significantly vary). Such sequence of regular oscillations is called a *train of waves* or *a wave train*. Time of radiation of the wave train (i.e. time of an atom transition from the exited state to the normal one — one act of radiation) is called the duration of a train or the *coherence time* τ_{coh}. The length of the train in space at a given moment of time may be presented as a small segment of sinusoidal oscillations and a wave may be considered here as monochromatic.

The train length L and the coherence time are connected by the relation $L = c\tau_{\text{coh}}$, where c is the speed of light. The average space length of the train is about 3 m, and the time duration is 10^{-8} s. Thus, on an average, through such time intervals the radiation of one wave train ends and the radiation of a new train starts. Besides the amplitudes, phases and polarizations of two successive trains are not related in any way. Now it becomes clear that to observe the interference pattern the train length must be equal to the coherence length $L = l_{coh}$. So it is essential that the optical path difference Δ is smaller (or equal) than the coherence length $\Delta \le c\tau_{coh} = l_{coh}$, otherwise the overlapping of different and independent trains takes place.

Experiments show that the coherence length does not exceed a few tens of centimeters.

Using the equation (2.23) the connection between the width of the spectral interval $\Delta\lambda$ and the coherence time τ_{coh} may be found:

$$|\Delta\lambda| \approx \frac{\lambda^2}{l_{coh}}, \text{ or } |\Delta\lambda| \approx \frac{\lambda^2}{c\tau_{coh}}. \tag{2.25}$$

As $\lambda = c/\nu$, differentiation of this equation yields $|\Delta\lambda| = c\Delta\nu/\nu^2$ and as a result we get:

$$\tau_{coh} \approx \frac{1}{\Delta\nu}, \tag{2.26}$$

where $\Delta\nu$ is the width of the spectral interval on a scale of frequencies.

From this equation it can be seen that the smaller the range of $\Delta\nu$ (or $\Delta\lambda$) is, the greater the coherence time is.

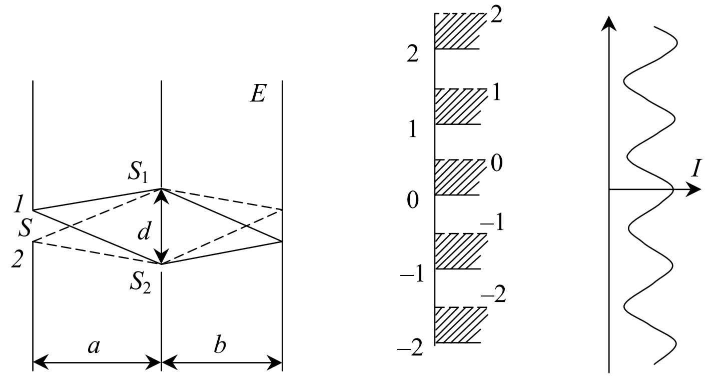
If $l_{coh}(\text{Cd}) = 30\ \text{cm}$, $l_{coh}(\text{laser}) = 3\ \text{km}$, then the corresponding coherence time equals: $\tau_{coh}(\text{Cd}) = 10^{-9}\ \text{s}$, $\tau_{coh}(\text{laser}) = 10^{-5}\ \text{s}$.

2.6.2. Space Coherence

Let us consider the effect of the size of a light source on an interference pattern using Young experiment (1802). The English scientist Thomas Young observed the interference of light waves for the first time in history.

The scheme of Young experiment is shown in Fig. 2.3, *a*. A sunlight beam falls on a narrow slit S and then on two narrow slits S_1 and S_2 behind which there is a screen E. The slits S_1 and S_2 become the sources of the coherent waves that overlap and form an interference pattern on the screen.

It is necessary to find out, at which the width of the slit S the observed interference pattern is still clear. We assume that radiation is monochromatic.



a *b* *c*

Fig. 2.3

If the primary slit S is narrow enough, the slits S_1 i S_2 may be considered as the sources of coherent cylindrical waves, formed by dividing a cylindrical wavefront from a common source S. Increasing of the width of the primary slit S leads to the scattering of the interference pattern and even to its complete disappearance. Thus, the dimensions of the source affect the formation of the interference pattern.

In Fig 2.3, *a* the path of the rays from the upper edge (a solid line) and the lower edge (a dotted line) of the slit S is shown. The interference maxima produced by the rays from upper edge *1* are shown in Fig. 2.3, *b* as the solid lines, and the interference minima produced by the rays from lower edge *2* — as the dotted lines. In the shaded areas the interference maxima produced by the slit points between edges *1* and *2* are located.

If the slit S is infinitely narrow, the interference maxima from edge points *1* and *2* will coincide. But if the width of the slit increases, the interference pattern will scatter and disappear completely (in this case the interval between adjacent maxima will be completely filled with the intermediate maxima of the wavelengths from another points of the slit).

The disappearance if the interference pattern, when the width of the slit S increases, signifies that the slits S_1 and S_2 are not coherent any more. We must find the greatest distance between the slits at which interference can still be observed.

Therefore we may introduce the notion of the *coherence width* h_{coh} of the wave that falls on the slits S_1 and S_2, and connect this notion with the space coherence of the light source *S*.

Under the width of the coherence h_{coh} we understand the characteristic distance between the points of the surface that is perpendicular to the direction of wave propagation. (In Young experiment such surface passes through the slits S_1 and S_2).

Let us find the equation for h_{coh}. For simplicity, assume that $a = b = l$ (Fig. 2.3). Then the secondary sources S_1 and S_2 cease to be coherent if the width *s* of the primary slit (*S*) becomes equal to the width of the interference fringe: $s \approx \Delta x$. As $\Delta x = l\lambda / d$ [Eq. (2.16)], these relations give:

$$ds \approx \lambda l\,. \tag{2.27}$$

The distance between two slits S_1 and S_2 that satisfies condition (2.27), is called the *space coherence length*

$$h_{\text{coh}} \approx d = \frac{\lambda l}{\Delta x} \approx \frac{\lambda l}{s} \approx \frac{\lambda}{s/l} \approx \frac{\lambda}{\varphi}, \tag{2.28}$$

where $\varphi = s/l$ is the angular dimension of the slit *S* relative to a diaphragm with two slits S_1 andi S_2 (Fig. 2.4). It follows from the equation (2.28), that for fixed values λ and *l* a space coherence length may be increased by reducing the width of the primary slit, or, equivalently, reducing its angular dimension φ. In Young experiment to observe clear (contrast) interference pattern the condition $s < \Delta x$ must take place, so

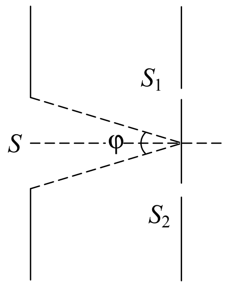


Fig. 2.4

$$ds < \lambda l \quad \text{or} \quad d < h_{\text{coh}} \approx \frac{\lambda}{\varphi}. \tag{2.29}$$

2.7. Methods of Producing of Coherent Light Beams

Let us consider the certain optical schemes of the experiments where interference may be observed. They are connected with two methods of obtaining the coherence rays — the method of wavefront division and the method of amplitude division.

2.7.1. Method of Wavefront Division

The first example is Young experiment that was regarded already in subsection 2.3.2. In this case a primary wavefront divides onto two parts by passing through two closely located very narrow slits. The same effect may be obtained when a primary light refracts (Fresnel's biprism).

Fresnel's Biprism. A light wavefront from a source *S* (a slit) divides (splits) with the aid of a biprism with a small refractive angle ϑ (Fig. 2.5). An interference pattern is seen on a screen *E*.

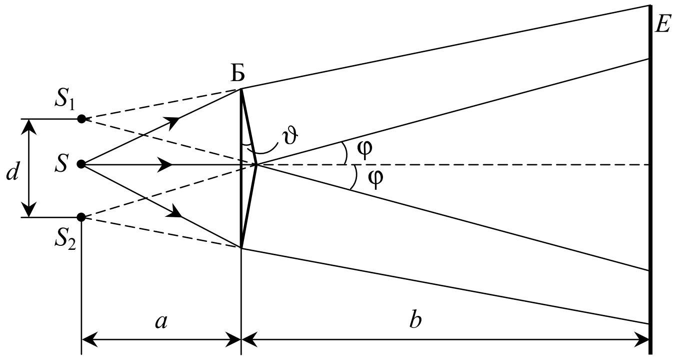


Fig. 2.5

The angle of incidence of the rays on the biprism is not great. Therefor, all the rays are deflected by each half of the biprism through the same angle φ. It looks like two coherent cylindrical waves emerge from virtual sources S_1 and S_2 in the same plane as *S*.

It may be shown (see laboratory work 5 of SU 6) that when the refractive angle ϑ of the biprism is very small and the angles of incidence of the rays on the face of the prism are not very great, all the rays are deflected through an identical small angle that is equal to:

$$\varphi = (n-1)\vartheta, \tag{2.30}$$

where *n* is the refractive index of the prism.

The distance between the virtual sources S_1 and S_2 of the source *S* equals:

$$d = 2a\sin\varphi \approx 2a\varphi. \tag{2.31}$$

The distance to the screen is $l = a + b$. The width of an interference fringe is found by the equation (2.16):

$$\Delta x = \frac{\lambda l}{d} = \frac{\lambda(a+b)}{2a\varphi} = \frac{\lambda}{2\varphi}\left(1+\frac{b}{a}\right). \qquad (2.32)$$

If a plane wave falls on the biprism ($a \to \infty$), the width of the interference fringes does not depend on the distance to the screen:

$$\Delta x = \frac{\lambda}{2\varphi}. \qquad (2.33)$$

The maximum number N of fringes observed may be found by taking into account the size $x \approx 2b\varphi$ of the coherent waves overlap, namely:

$$N \approx \frac{x}{\Delta x} = \frac{4\varphi^2}{\lambda}\frac{ab}{a+b}. \qquad (2.34)$$

2.7.2. Method of Amplitude Division

This method is suitable both for point and for linear sources and provides a much greater intensity of the interference fringes than the method of wavefront division. The coherent waves are formed under certain conditions due to the reflection of incident light on the top and the bottom surfaces of the transparent film (or plate). Interference of these rays results, for example, in the rainbow color of soap bubbles or thin films of oil or kerosene (gasoline), etc.

Plane-Parallel Plate (Film). Assume that a plane monochromatic wave falls on a transparent glass plate. This wave may be considered as a parallel beam of rays. As the result of reflection on both surfaces of the plate, the primary wave splits onto two waves with about the same amplitudes. Note, that besides these reflected waves ($1'$ and $2'$) a multiple reflection appears (Fig. 2.6), but intensity of these waves is rather small and we may take no account of them.

The pass of the rays that are formed due to the refraction and reflection of light in a plane-parallel film is shown in Fig. 2.6.

We assume that a plate with a refractive index n and a width d is placed into a medium with a refractive index n_0. Ray *1* falls on the surface of the film at the angle *i*. Rays $1'$ and $2'$ reflect on the top and bottom surfaces of the plate, and rays $1''$ and $2''$ pass through the plate. These rays, under certain conditions, may be coherent and interfere. Let us find these conditions.

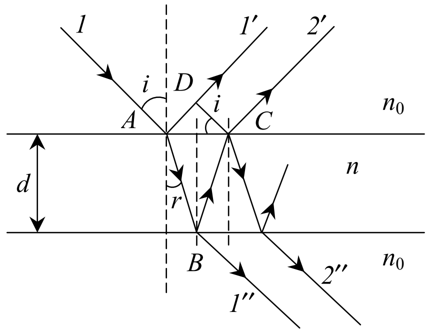


Fig. 2.6

The optical path difference of reflected rays $1'$ and $2'$ (Fig. 2.6) equals to:

$$\Delta = n(AB + BC) - n_0(AD \pm \lambda/2),$$

or

$$\Delta = n(AB + BC) - n_0 AD \pm \lambda_0/2, \qquad (2.35)$$

where λ_0 is the wavelength in vacuum, $\lambda = \lambda_0/n_0$ is the wavelength in a medium with a refractive index n_0. The term $\pm\lambda_0/2$ means that in the case of the wave reflection on an optically denser medium a wave phase sharply changes on π [see the equation (1.19)], i.e. half a wavelength is lost. If $n > n_0$, half a wavelength is lost at the point *A* and the term $\lambda_0/2$ will have sign "minus". If $n < n_0$, half a wavelength is lost at the point *B* and this term will have sign "plus".

A glance at Fig. 2.6 shows that $AB + BC = 2d/\cos r$ $(AB = BC)$ and $AD = 2AB\sin r\sin i = 2d\,\mathrm{tg}\,r\sin i$. As the angles of incidence *i* and refraction *r* are connected by the relation $\sin i/\sin r = n/n_0$, then $AD = 2nd\sin^2 r/n_0\cos r$. Thus,

$$n(AB + BC) - n_0 AD = 2nd\cos r = 2nd\sqrt{1-\sin^2 r} =$$

$$= 2nd\sqrt{1-\frac{n_0^2}{n^2}\sin^2 i} = 2d\sqrt{n^2 - n_0^2\sin^2 i}. \qquad (2.36)$$

Finally for the optical path difference we get

$$\Delta = 2d\sqrt{n^2 - n_0^2\sin^2 i} \pm \lambda_0/2. \qquad (2.37)$$

In practice, the medium surrounding the plate is, as usual, air ($n_0 \cong 1$, $n_0 < n$), therefore equation (2.37) is simplified to the form:

$$\Delta = 2d\sqrt{n^2 - \sin^2 i} - \lambda_0/2. \qquad (2.37a)$$

If reflected rays *1′* and *2′* are coherent, the condition of *maxima for reflection* is $\Delta = m\lambda_0$, namely:

$$2d\sqrt{n^2 - \sin^2 i} = (m + 1/2)\lambda_0, \qquad (2.38)$$

and of *minima* is $\Delta = (m + 1/2)\lambda_0$, namely

$$2d\sqrt{n^2 - \sin^2 i} = (m + 1)\lambda_0. \qquad (2.39)$$

Interference may be observed not only in reflected but also in transmitted light. As a wave, that passes from the optically denser medium to the optically less denser one, does not change its phase (the loss of half a wave is absent), the optical path difference of rays *1″* and *2″* that pass through the plate will be (Fig. 2.6):

$$\Delta = 2d\sqrt{n^2 - \sin^2 i}.$$

Thus, the maximum of interference in transmitted light will correspond to the minimum of interference in reflected light, and vice versa.

According to the equation (2.38) the position of maxima depends on the wavelength. Therefore, if sunlight falls on the plate (film), the interference pattern will form by the rays of different colors (wavelengths), and the plate acquires the coloring of a rainbow.

To obtain interference of rays *1′* and *2′*, the conditions of both time and space coherence must be performed. For the time coherence the path difference must not exceed the coherence length $l_{coh} \approx \lambda_0^2/\Delta\lambda_0$ [see the equation (2.23)].

So condition (2.24) must be fulfilled:

$$2d\sqrt{n^2 - \sin^2 i} - \lambda_0/2 < \lambda_0^2/\Delta\lambda_0,$$

or

$$d < \frac{\lambda_0(\lambda_0/\Delta\lambda_0 + 1/2)}{2\sqrt{n^2 - \sin^2 i}}.$$

As $\lambda_0 / \Delta\lambda_0 >> 1/2$, we may disregard $1/2$ in comparison with $\lambda_0 / \Delta\lambda_0$. The equation $\sqrt{n^2 - \sin^2 i}$ has a magnitude of the order of unity (for $n = 1,5$ the magnitude of this equation varies within the limits from 1.12 at $i = 90^\circ$, to 1,50 at $i = 0$). Therefore, we can assume, that

$$d < \lambda_0^2 / (2\Delta\lambda_0)\,, \qquad (2.40)$$

i.e. the double plate thickness must be less than the coherence length. For example, if $\lambda = 5000\ \text{Å}$ and $\Delta\lambda = 20\ \text{Å}$ (for air $\lambda \approx \lambda_0$), the extreme thickness equals, according to the equation (2.40),

$$d = \frac{5000^2}{2 \cdot 20} \approx 6 \cdot 10^5\ \text{Å} = 0,06\ \text{mm}\,. \qquad (2.41)$$

As to the space coherence: Fig. 2.6 shows that the distance *DC* between two reflected rays *1′* and *2′* is:

$$DC = 2d\,\text{tg}r \cos i = \frac{d \sin 2i}{\sqrt{n^2 - \sin^2 i}}\,.$$

If the same distance exists between the incident rays and does not exceed h_{coh} of the incident wave (the equation (2.28)), rays *1′* and *2′* will be coherent. Thus,

$$h_{\text{coh}} = \frac{d \sin 2i}{\sqrt{n^2 - \sin^2 i}}\,. \qquad (2.42)$$

If $n = 1,5$, then for $i = 45^\circ$ we get $h_{\text{coh}} = 0,8d$, and for $i = 10^\circ$ we get $h_{\text{coh}} = 0,1d$. For normal incidence ($i = 0$), $h_{\text{coh}} = 0$ for any *n*.

The coherence radius of sunlight $h_{\text{coh}} \sim 0,05$ mm. At an angle of incidence of $i = 45^\circ$, we may assume that $h_{\text{coh}} \approx d$. Hence, for interference to occur, the following condition must be fulfilled:

$$d < h_{\text{coh}}\,, \text{ or } \quad d < 0,05\ \text{mm}\,. \qquad (2.43)$$

So we arrive to the conclusion that owing to the restrictions imposed by time and space coherence, interference is observed only if the thickness of the plate (film) does not exceed a few hundredths of a millimeter.

In practice interference is observed by placing in the path of the reflected rays a lens that gathers the rays at one of the points of the screen (or the part of the lens and the screen can be played by an eye and a retina).

Plate (Film) of Varying Thickness (Wedge). Assume that a monochromatic light falls on a plate (film) in the form of a wedge with a small apex angle $\alpha << 1°$. Parallel rays *1* and *2* reflect on the top and the bottom surfaces of this wedge (Fig. 2.7)

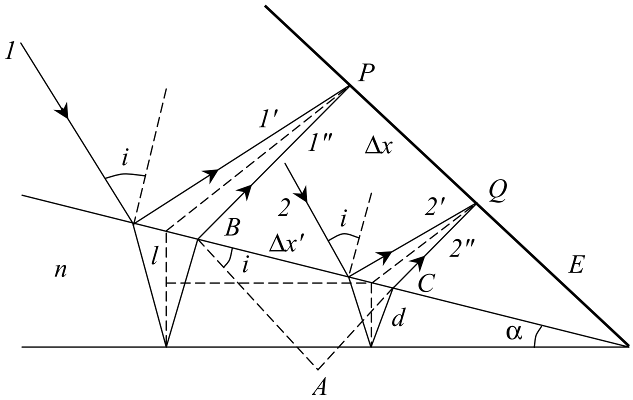


Fig. 2.7

Now the reflected rays *1′* and *1″* and, correspondingly, *2′* and *2″* will not be parallel. These rays will interfere at points *P* and *Q* on screen *E* and an interference pattern will be observed in the form of light and dark fringes parallel to the edge of the wedge. Usually for all points of the wedge with the same thickness *d* conditions of maxima or minima are equal. Thus, maxima and minima hold the points that correspond to the same thickness of the wedge; therefore they are called the *fringes of equal thickness*.

Let us find the distance between the adjacent fringes (width of the fringe) Δx on the screen *E* (Fig. 2.7). As the apex angle is small ($\alpha << 1°$), the optical path difference of rays *2′* and *2″* may be found according to the equation (2.37a):

$$\Delta = 2d\sqrt{n^2 - \sin i} - \lambda_0 / 2, \qquad (2.44)$$

where *d* is the thickness of the wedge at the place where ray *2* falls on it; *n* is the refractive index; *i* is the angle of incidence of ray *2* on the top surface of the wedge. Similarly the optical path difference of rays *1′* and *1″* is:

$$\Delta_1 = 2(d+l)\sqrt{n^2 - \sin i} - \lambda_0/2, \qquad (2.45)$$

where $d+l$ is the thickness of the wedge at the place where ray *1* falls on it *1*.

If $\Delta x'$ is the distance between rays *1* and *2* on the top surface of the wedge, then,

$$\Delta x' \sin\alpha = l. \qquad (2.46)$$

Taking into account, that $AB \approx \Delta x$, from the triangle ABC we get:

$$\Delta x' = \Delta x / \cos i. \qquad (2.47)$$

Since Δx is the distance between the adjacent fringes (width of the fringe), the condition $\Delta_1 - \Delta = \lambda_0$ is to be performed, or according to the equations (2.44) and (2.45) it is:

$$2l\sqrt{n^2 - \sin^2 i} = \lambda_0. \qquad (2.48)$$

Having the equations (2.46), (2.47) and $\sin\alpha \approx \alpha$, finally from the equation (2.48) we obtain:

$$\Delta x = \frac{\lambda_0 \cos i}{2\alpha\sqrt{n^2 - \sin^2 i}}. \qquad (2.49)$$

With normal incidence of the light ($i \approx 0$) from the equation (2.49) we get:

$$\alpha = \frac{\lambda_0}{2n \cdot \Delta x}, \qquad (2.50)$$

so very small angles ($0{,}1'$ and less) may be determined.

Newton's Rings. A classic example of fringes of equal thickness is Newton's rings. Usually they are observed in light reflected on an air wedge that is formed by a thick plane-parallel glass plate in contact with a plano-convex lens having a large radius of curvature (Fig. 2.8). With normal incidence of the light, fringes have the form of concentric rings centered at the point of contact of the lens with the plane-parallel plate. With inclined incidence, these rings transform into ellipses.

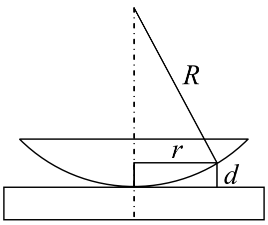


Fig. 2.8

We have to note, that owing to the grate thickness of the plate and the lens, no interference fringes appear as a result of reflection on other surfaces.

Let us find the radii of Newton's rings when the light falls along a normal to the plate ($i = 0$). It is assumed that the lens and the plate have the same refractive index *n*. The refractive index of the air wedge is $n_0 \cong 1 < n$. To obtain the equation of the optical path difference in the air wedge, it is enough to change *n* on n_0 in the equation (2.44) for a glass wedge:

$$\Delta = 2dn_0 + \lambda_0 / 2 .$$

Since $n_0 \cong 1$, then

$$\Delta = 2d + \lambda_0 / 2 . \qquad (2.51)$$

Sign "plus" appears before $\lambda_0 / 2$, as the loss of half a wave occurs at the bottom of the air wedge (Fig. 2.8). For condition of minima (dark rings) Δ must be equal to the odd number of a half of waves $\Delta = 2d + \lambda_0 / 2 = (2m+1)\lambda_0 / 2$, hence,

$$2d = m\lambda_0 \ (m = 0,1,2,...). \qquad (2.52)$$

By Pythagoras' theorem (Fig. 2.8) $r^2 = R^2 - (R-d)^2$. Since $d << R$, then

$$r^2 = 2dR . \qquad (2.53)$$

From the equation (2.52) and (2.53) it follows, that the radius of the *m*-th *dark ring* is:

$$r_m = \sqrt{m\lambda_0 R} , \quad (m = 0,1, 2, ...). \qquad (2.54)$$

(value $m = 0$ corresponds to a minimum of a dark spot).

For condition of maxima (bright rings) Δ must be equal to the integer number of the waves $\Delta = 2d + \lambda_0 / 2 = m\lambda_0$. Here the interference order starts with one ($m = 1, 2, 3...$), as $m = 0$ corresponds to the minimum. After similar calculations we obtain the formula for the radii of *bright rings*:

$$r_m = \sqrt{(m - 1/2)\lambda_0 R} , \quad (m = 1, 2, 3....). \qquad (2.55)$$

Formulas (2.54) and (2.55) can be written as follows:

$$r_m = \sqrt{m\lambda_0 R} = \sqrt{\lambda_0 R/2}\sqrt{2m}\,, \qquad (2.54a)$$

$$r_m = \sqrt{(m-1/2)\lambda_0 R} = \sqrt{\lambda_0 R/2}\sqrt{2m-1}\,. \qquad (2.55a)$$

In *transmitted light* the interference pattern is *reverse* (half a wave does not lose): the equations (2.55) and (2.55a) are used for the radii of *dark rings*, and the equations (2.54) and (2.54a) for the radii of *bright rings*.

2.8. Interferometers

Interferometers are the optical measuring instruments, which are based on the interference of light. Using these devices, we can very accurately measure linear and angular distances, refractive indexes, etc.

The most famous is the Michelson interferometer, which played a fundamental role in the development of science and technology. By means of this interferometer Michelson made the first comparison of the wavelength of the red line of cadmium with the length of the standard metre, also the famous experiment of detecting motion of the Earth relative to the hypothetic ether was conducted (so called Michelson-Morley experiment). It proved the independence of the speed of light from the Earth motion (the absence of "ether wind").

A schematic view of the **Michelson interferometer** is given in Fig. 2.9. It works by the method of amplitude division and consists of two flat mirrors M_1, M_2 and semitransparent plane-parallel plate P_1.

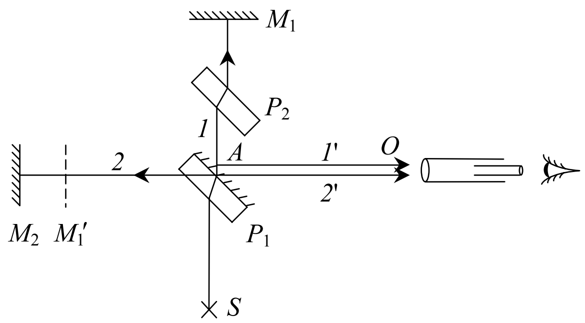


Fig. 2.9

A light beam from the source *S* falls at an angle 45° on the plate P_1. A back surface of this plate is coated with a thin layer of silver with the coefficient of reflection $R \approx 0,5$. The falling beam is splitting into two rays *1* and *2* with approximately equal intensities. The rays *1* and *2* reflect on the mirrors M_1 and M_2 and return to the plate P_1, where each of them again is splitting into two rays of equal intensity. As a result, the rays $1'$ and $2'$ of equal intensity are formed. If conditions of time and space coherence are fulfilled, these rays will interfere and give the interference pattern that may be observed for example, through the eyepiece (ocular).

The result of interference depends on the optical path difference from the plate P_1 to the mirrors M_1 and M_2 and back. The ray *2* passes through the plate three times, and the ray *1* only once. To compensate the additional optical path difference (especially for waves of different wavelengths), the plate P_2 is placed on the path of the ray *1*. This plate is identical to the plate P_1, except of the silver coating. This arrangement makes the paths of the rays *1* and *2 in glass* equal.

The observed interference pattern corresponds to the interference in an air layer formed by the mirror M_2 and a virtual image M_1' of the mirror M_1 in the plate P_1 (Fig. 2.9). The optical path difference of the rays $1'$ and $2'$ in the plane-parallel air layer is:

$$\Delta = 2n_0(l_1 - l_2) = 2n_0\Delta l = 2\Delta l, \qquad (2.56)$$

where $n_0 \cong 1$ is the refractive index of air; l_1 and l_2 are the interferometer arms, i.e. corresponding distances from the plate P_1 to the mirrors M_1 and M_2; Δl is the thickness of the air layer.

? Test Questions

1. Formulate the superposition principle for light waves.
2. What waves are called coherent?
3. What is the phenomenon of interference?
4. How are the coherent waves formed in optics?
5. What is the geometrical path difference, the optical path difference?
6. Point the conditions of maximum and minimum for the optical path difference.
7. What is the time coherence?
8. What is the space coherence?
9. Explain the notion of the coherence length.
10. Give the examples of the optical plants for light interference observation.
11. What are the instruments called interferometers? Give an example.

Sample Problems

Problem 1. A monochromatic light beam with λ = 0.6 μm normally falls onto a thin film (n = 1.4) that covers a thick glass plate (n = 1.5). Find the thickness of the film if the reflected light is maximally attenuated due to the interference.

Solution. Let us distinguish one ray *SA* from the light beam. The path of this ray for the general case, when $i \neq 0$, is given in Fig. 1. At the points *A* and *B* the incident ray partially reflects and partially refracts. The reflected rays AS_1 and BCS_2 fall on a lens *L* and interfere. It is important to note, that a refractive index of air (n_1 = 1.0) is less then a refractive index of a film (n_2 = 1.4) and, in its turn, is less then a refractive index of a plate (n_3 = 1.5). Hence, in both cases reflection occurs from optically denser media. Therefore, at the point *A* a phase of oscillations of the reflected ray AS_1 changes on π. Similarly, at the point *B* the phase of the reflected ray BCS_2 also changes on π. Thus, these rays will interfere as if there is no phase change.

As it is known, the optical path difference for minimum of interference equals the odd number of a half of wavelengths: $\Delta = (2m+1)\frac{\lambda}{2}$. As it is seen on Fig. 2.10, the optical path difference is $\Delta = (AB + BC)n_2 - ADn_1$. Thus, the condition when light is maximally attenuated is:

$$(AB + BC)n_2 - ADn_1 = (2m+1)\frac{\lambda}{2}.$$

If the angle of incidence i_1 decreases up to zero, then $AD \to 0$ and $AB + BC \to 2d$, where d is the thickness of the film.

In the limiting case, if i_1 = 0, we have $\Delta = 2dn_2 = (2m+1)\frac{\lambda}{2}$, and then the required thickness of the film equals $d = \frac{(2m+1)\frac{\lambda}{2}}{4n_2}$.

Counting m = 0, 1, 2, 3, ..., we obtain the possible values of the film thickness: 0.11 μm; 0.33 μ, etc.

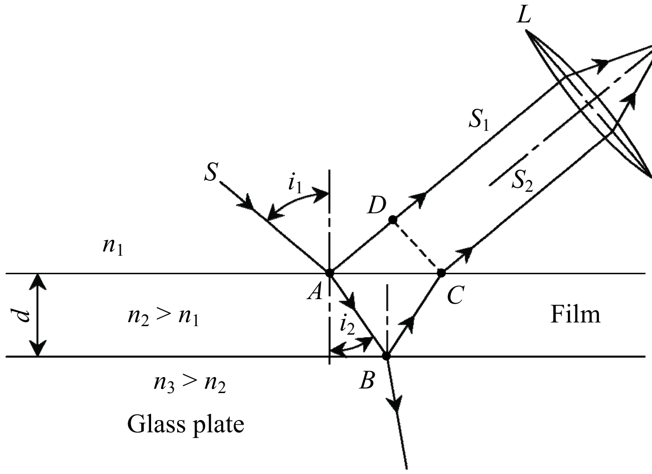


Fig. 2.10

Problem 2. A beam of light (λ = 0.6 μm) falls normally onto a glass wedge (Fig. 2.11). The number of interference fringes per 1 cm equals 10. Find the apex angle (the refractive angle) of the wedge.

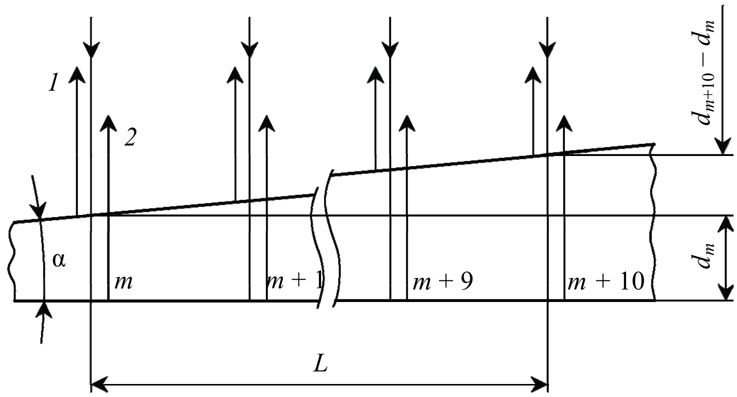


Fig. 2.11

Solution. A beam of parallel rays reflects both on the top and the bottom of the wedge. These rays are coherent, so the interference pattern is observed. We assume that the reflected rays *1* and *2* are almost parallel (Fig. 2.11).

For the dark fringes the optical path difference is:

$$\Delta = (2m+1)\frac{\lambda}{2},$$

where m = 0, 1, 2, ...

On the other hand, the optical path difference equals the difference of the optical paths of these rays $2dn\cos i_2$ and a half of the wavelength

Рис.1 $\frac{\lambda}{2}$ (because of the reflection on the optically denser medium). Therefore we get:

$$2d_m n\cos i_2 + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}, \qquad (2.57)$$

where n is the refractive index of glass (n = 1.5); d_m is the thickness of the wedge where the dark fringe, that corresponds the number m, is observed; i_2 is the refractive angle; λ is the wavelength.

Accordingly to the problem condition the angle of incidence equals zero, so the angle of refraction i_2 also equals zero and $\cos i_2 = 1$. The equation (2.57) transforms into:

$$2d_k n = k\lambda. \qquad (2.58)$$

Let us take an arbitrary dark fringe with the number m that corresponds to a certain thickness of the wedge d_m at this place, and a dark fringe with the number $m + 10$ that corresponds the thickness d_{m+10}. As there are ten fringes per l = 1 cm, the required angle will be (Fig. 2.11):

$$\alpha = \frac{d_{m+10} - d_m}{l}, \qquad (2.59)$$

where $\sin\alpha \approx \alpha$ (angle α is expressed in radians). Finding d_m and d_{m+10} from the equation (2.58) and substituting them into the equation (2.59), we have:

$$\alpha = \frac{\frac{m+10}{2n}\lambda - \frac{m}{2n}\lambda}{l} = \frac{5\lambda}{nl}.$$

Substituting the numerical data, we obtain:

$$\lambda = \frac{5\cdot 0,6\cdot 10^{-6}}{1,5\cdot 1\cdot 10^{-2}} = 2\cdot 10^{-4}\,\text{rad}.$$

Let us express α in degrees. For this we may use the relation between radians and seconds:

$$1 \text{ rad} = 206265'' = 2''.06 \cdot 10^5, \text{ namely } \alpha = 2 \cdot 10^{-4} \cdot 2''.06 \cdot 10^5 = 41''.2.$$

Or, according to the general rule of transition from radians to degrees,

$$\alpha_{\text{deg}} = \frac{180^{\circ}}{\pi} \alpha \text{ rad}, \ \alpha = \frac{180^{\circ}}{3.14} \cdot 2 \cdot 10^{-4} = 1^{\circ}.15 \cdot 10^{-2} = 0'.688 = 41''.2.$$

Problem 3. A thin glass plate is placed in the path of one of the interfering rays in Young's experiment. This causes the central light band to shift into the position which was initially occupied by the fifth light band (not considering the central one). The ray falls onto the plate perpendicularly. The refractive index of the plate is 1.5. The wavelength is $6 \cdot 10^{-7}$ m. What is the thickness of the plate?

Data:

$n = 1.5$

$k = 5$

$\lambda = 6 \cdot 10^{-7}$ m

$h = ?$

Solution

The glass plate causes the optical paths difference $\Delta = n \cdot h - h = h(n - 1)$. As a result of the introduction of a glass plate, the displacement of interferential bands by k took place. Hence, the additional difference in the paths due to the plate is $k\lambda$. Thus, condition of maximum is $h\,(n - 1) = k\lambda$. Eliminate h and obtain:

$$h = \frac{k\lambda}{n-1} = \frac{5 \cdot 6 \cdot 10^{-7}}{1.5-1} = 6 \cdot 10^{-6} \text{ m}.$$

Problem 4. Two slits are parallel and 0.5 mm apart. The interfringe distance obtained on a screen 2 m away from the slits is 2 mm. Calculate the wavelength of monochromatic radiation used.

Data:

$d = 0.5$ mm

$L = 2$ m

$\Delta y = 2$ mm

$\lambda = ?$

Solution

We know that the distance between the interference bands on a screen and the corresponding wavelength are linked by the equation $\lambda = \frac{\Delta y}{L}\lambda$. So the wavelength can be found as:

$$\lambda = \frac{\Delta y}{L} = \frac{2 \cdot 10^{-3} \cdot 0.5 \cdot 10^{-3}}{2} = 0.5 \cdot 10^{-6} \text{ m}.$$

Problem 5. A vertical soap film forms a wedge due to the liquid trickling down. By observing the interference bands in the reflected light of a mercury arc (λ = 5461 Å), we find that the distance between five bands is 2 cm. Find the wedge angle in seconds. The light falls at right angles to the film surface. The soapy water refractive index is 1.33.

Data:

$\lambda = 5461 \cdot 10^{-10}$ m
$k = 5$
$l = 0{,}02$ m
$n = 1.33$

α = ?

Solution

Let us denote thickness of the film corresponding to adjacent bands by h_1 and h_2. The optical paths differences for these bands differ by $\Delta = 2n\ (h_2 - h_1) = 2n \cdot \Delta h$. But adjacent bands always have optical paths differences changed by λ. Therefore $\lambda = 2n \cdot \Delta h$. Let us denote the distance between the adjacent bands by l. It may be assumed that $\Delta h = l \tan \alpha$, where α is a wedge angle. Hence

$$\tan\alpha = \frac{k\lambda}{2nl} = \frac{5 \cdot 5461 \cdot 10^{-10}}{2 \cdot 1.33 \cdot 0.02} = 5.13 \cdot 10^{-5}; \alpha = 11''.$$

Problem 6. Find the distance between the third and the sixteenth Newton's dark rings if the distance between the second and the twentieth dark ring is equal to 4.8 mm. The observation is made in reflected light.

Data:

$r_{20} - r_2 = 4.8$ mm

$r_{16} - r_3$ = ?

Solution

When Newton's rings are observed in reflected light of wavelength λ, the condition of the minimum for a ring is determined by the formula $r_k = \sqrt{kR\lambda}$, where k is the number of the ring and R is the radius of curvature of the lens. Let us use the formula for the twentieth dark ring and for the second one:

$$r_{20} - r_2 = \sqrt{20R\lambda} - \sqrt{2R\lambda} = 4.8\,\text{mm};$$

$$\sqrt{R\lambda} = 1.57\ \text{mm}.$$

Let's do the same for the sixteenth and the third Newton's dark rings:

$$r_{16} - r_3 = \sqrt{16R\lambda} - \sqrt{3R\lambda} =$$
$$= \sqrt{16 \cdot 1.57} - \sqrt{3 \cdot 1.57} = 2.83\ \text{mm}.$$

Problems

1. How many wavelengths with a frequency $5 \cdot 10^{14}$ Hz lay on a path of 1.2 mm length: 1) in vacuum; 2) in glass with $n = 1.5$? ($2 \cdot 10^3$; $3 \cdot 10^3$)

2. Find the optical path difference change of a ray that propagates in air, if a glass plate of 2 mm width is placed on its path. Perform the calculations for two cases: the ray falls normally and the ray falls at an angle of 60°. The refractive index for the glass is 1.5. (1 mm; 0.32 mm)

3. A light source with a diameter of 30 cm is 200 m apart from the point of observation. This source radiates the waves with wavelengths from 490 to 510 nm. Find a coherence time and a coherence length for this radiation. ($4 \cdot 10^{-14}$ s; 0.01 mm)

4. Two coherent light beams with $\lambda = 400$ nm cross. What will be observed in a cross point – maximum or minimum of interference, if a path difference is 0.5 mm? (Maximum)

5. White light falls at an angle of 45° onto a soap film ($n = 1.33$). At what minimum thickness of the film will the reflected rays be colored yellow with $\lambda = 600$ nm? (≈ 0.13 μm)

6. White light falls normally onto a soap film ($n = 1.33$) with a thickness of $5.92 \cdot 10^{-4}$ mm. This light reflects and produces an interference maximum at a wavelength of $\lambda_1 = 630$ nm. At what wavelength λ_2 the nearest interference minimum of the same order will be observed? (525 nm)

7. A plant for producing Newton's rings is illuminated by normally incident monochromatic light. The radius of the lens is 15 m. The distance between the fifth and the twenty-fifth Newton's bright rings is 9 mm. Find the wavelength of the incident light. The observation is made in the reflected light. (675 nm)

8. A plant used to observe Newton's rings is illuminated by normally incident monochromatic light with a wavelength of 600 nm. Find the thickness of the air layer between the lens and the glass plate where the fourth dark ring is observed in the reflected light. (1.2 μm)

9. A plant used to observe Newton's rings in reflected rays is illuminated by normally incident monochromatic light with a wavelength of 600 nm. The radius of the lens curvature is 15 m. The space between the lens and the glass plate is filled with liquid. Find the refractive index of the liquid if the radius of the third dark ring is 4.7 mm. (1.22)