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# DEPENDENCE OF THE DEFORMATION OF FLEXIBLE CYLINDRICAL SHELLS ON THEIR GEOMETRY AND THE ORTHOTROPY AND NONLINEAR PROPERTIES OF THE COMPOSITE MATERIALS

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**A study is made of geometrically and physically nonlinear inverse problems concerning the axisymmetric deformation of cylindrical shells into conical shells. Results obtained from the numerical solution of the problems are used to determine the laws of distribution of the surface loads, stresses, strains, and displacements in relation to the initial parameters and nonlinearities of the shells.**

Studies of the linear and nonlinear deformation of thin-walled structural elements in the form of shells and plates made of isotropic (metallic) and anisotropic (composite) materials have provided information on the shaping and deformation of these structures [1, 2, 4–6]. Inverse problems were formulated and solved and specific numerical results were presented in [1, 2, 4] in studies of the deformation of shells of revolution of nonlinearly elastic composites. Most of the results were obtained on the basis of geometrically and physically nonlinear theories of thin shells and were used to examine the axisymmetric deformation of cylindrical shells inside a shell of another form (conical, parabolic) with allowance for nonlinear factors.

It is interesting to study the effect of geometric and physicomechanical parameters, the character of the nonlinear deformation process, and the type of boundary conditions on the deformation of shells of a specific initial form: determine the parameters of their loading and the stress–strain state of the new form of the shells. As a outgrowth of previous studies [2], below we present specific results which are obtained from the numerical solution of nonlinear inverse problems and are used to determine the distributions of the surface loads, stresses, strains, and displacements in relation to the above-mentioned initial parameters of the shells and their nonlinear characteristics.

**1. Effect of the Geometric Parameters on the Deformation of the Shells.** It is assumed that a flexible cylindrical shell of constant thickness ($h$ = const) in which the meridian of the middle surface is determined by the equation

$$z=\alpha, \quad r=\sqrt{x^2+y^2}=R, \quad \alpha\in[0,l] \tag{1.1}$$

is made of a nonlinearly elastic orthotropic composite and is transformed into a conical shell by a system of surface and edge loads that are to be determined (Fig. 1)

$$\varphi\alpha-\gamma+w_0=0. \tag{1.2}$$

The kinematic condition which must be satisfied for such a transformation to occur has the form

$$w=\varphi(\alpha+u)+w_0, \tag{1.3}$$

where $\varphi$ and $w_0$ are parameters of the deformation process (the apex angle and the initial displacement).

To describe the stress–strain state of nonshallow thin shells referred to a conjugate curvilinear coordinate system $(\alpha, \beta, \gamma)$, we used the geometrically nonlinear theory of thin shells in a quadratic approximation and the nonlinear theory of elasticity and plasticity of anisotropic media. The main nonlinear relations and resolvent equations were presented in [4]

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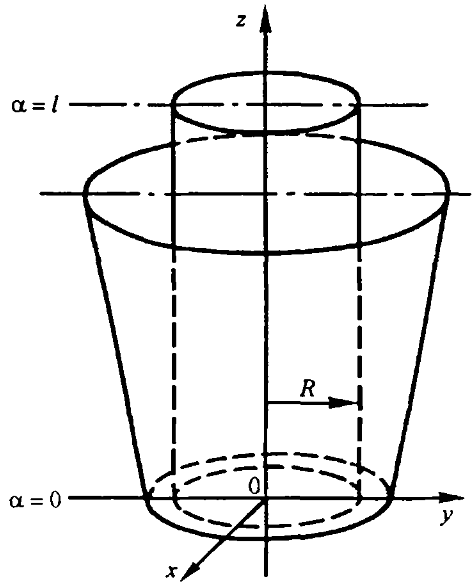


Fig. 1

along with a method for their numerical solution. The algorithm and application programs that were developed make it possible to obtain tables of values of the loading parameters, the components of the displacement vector, and the components of the strain and stress tensors for assigned values of the initial parameters ($\varphi$, $w_0$), the geometry of the shells ($\eta = R/h$ ; $\tilde{l} = l/h$ ), and the properties of their material ($E_i$, $\nu_i$, $i = \alpha, \beta$ ).

The values of surface pressure [ $\tilde{P}_\gamma = (P_\gamma / E_\alpha)\, 10^3$ ], the internal forces and moments ( $\tilde{T}_\rho = T_\rho / E_\rho h$ ; $\tilde{M}_\rho = M_\rho / E_\rho h^3$, $\rho = \alpha, \beta$ ), and the displacements ($u$, $w$), strains ( $e_{\rho\rho}$ ), and stresses ( $\sigma_{\rho\rho}$ ) were calculated at a series of nodal points ( $N$ ) subdividing the meridian of the shell and its thickness ( $\xi = \gamma/h$ ; $-0.5 \le \xi \le 0.5$ ).

The following equations [2, 7] were used in performing the indicated calculations

$$P_\gamma = T_\alpha \kappa_{\alpha\alpha} + T_\beta (k + \kappa_{\beta\beta}) - M_\alpha'' + (w' T_\alpha)' ; \tag{1.4}$$

$$e_{\alpha\alpha} = \varepsilon_{\alpha\alpha} + \gamma \kappa_{\alpha\alpha} ;$$

$$e_{\alpha\alpha} = u' + 1/2\, (w')^2 ; \quad \varepsilon_{\beta\beta} = k w ;$$

$$\kappa_{\alpha\alpha} = - w'' ; \quad \kappa_{\beta\beta} = - k^2 w ; \tag{1.5}$$

$$e_{\alpha\alpha} = \frac{1}{E_\alpha} (\sigma_{\alpha\alpha} - \nu_\alpha \sigma_{\beta\beta}) + \psi (q_{\alpha\alpha} \sigma_{\alpha\alpha} + q_{\alpha\beta} \sigma_{\beta\beta}), \quad \alpha \leftrightarrow \beta, \tag{1.6}$$

where $k$, $\varepsilon_{\rho\rho}$, and $\kappa_{\rho\rho}$ ( $\rho = \alpha, \beta$ ) are the principle curvature, strain, and change in curvature of the reference surface of the initial shell; $\psi$ and $q_{ij}$ ( $i, j = \alpha, \beta$ ) are the function and parameters that account for the anisotropy of the nonlinear properties of composites [3]; the "prime" used as a superscript denotes differentiation with respect to the argument $\alpha$.

The study was conducted for shells made of nonlinearly elastic glass-plastic with the characteristics

$$E_\alpha = 15 \text{ GPa} ; \quad E_\beta = 12 \text{ GPa} ; \quad \nu_\alpha = 0.12 ;$$

$$(q_{\alpha\alpha} = 2 ; \quad q_{\beta\beta} = 3{,}14 ; \quad q_{\alpha\beta} = -0.24) \tag{1.7}$$

TABLE 1

| $N$ | $-e_{\alpha\alpha}\cdot 10^2$ | $e_{\beta\beta}\cdot 10$ | $\tilde{\sigma}_{\alpha\alpha}$ | $\tilde{\sigma}_{\beta\beta}$ |
|---|---|---|---|---|
| 1 | 0.1504 | 0.1667 | –0.089 | 1405 |
| 3 | 0.2883 | 0.3327 | –0.011 | 2083 |
| 5 | 0.4229 | 0.4986 | –0.255 | 2542 |
| 7 | 0.5560 | 0.6643 | –0.615 | 2900 |
| 9 | 0.6884 | 0.8297 | –1.201 | 3198 |
| 11 | 0.8207 | 0.9949 | –2.023 | 3454 |

TABLE 2

| $N$ | $\tilde{l} = 50$ | | $\tilde{l} = 75$ | | $\tilde{l} = 100$ | |
|---|---|---|---|---|---|---|
| | $\tilde{w}$ | $\tilde{\sigma}_{\beta\beta}$ | $\tilde{w}$ | $\tilde{\sigma}_{\beta\beta}$ | $\tilde{w}$ | $\tilde{\sigma}_{\beta\beta}$ |
| 1 | 0.0000 | 0.09 | 0.0000 | 0.11 | 0.0000 | 0.13 |
| 3 | 0.4990 | 1403 | 0.7482 | 1784 | 0.9973 | 2082 |
| 5 | 0.9972 | 2082 | 1.4950 | 2542 | 1.9918 | 2900 |
| 7 | 1.4949 | 2542 | 2.2400 | 3054 | 2.9837 | 3454 |
| 9 | 1.9918 | 2900 | 2.9837 | 3454 | 3.9729 | 3887 |
| 11 | 2.4881 | 3197 | 3.7259 | 3787 | 4.9595 | 4248 |

in the case where the following boundary conditions (variant A) existed on the edges of the shell (Fig. 1):

$$\tilde{\alpha} = 0;\ u = 0 \quad \text{(fixed edge)};$$

$$\tilde{\alpha} = 1;\ T_k = 0 \quad \text{(free edge)}, \tag{1.8}$$

where $T_k$ is the force applied to the contour ( $\tilde{\alpha} = \alpha/l$ ).

The effect of the geometric parameters on the deformation of the shells was studied using two computational schemes: (1) $\eta = 30$ (fixed value); $\tilde{w}_0 = w_0/h = 0 \ldots 0.5$ ; $\tilde{l} = 50 \ldots 100$ ; $\varphi = 0.01 \ldots 0.2$ (variable parameters); (2) $\eta = 20 \ldots 100$ (variable quantity) and the other parameters fixed. We will present the results calculated in accordance with the solutions of the corresponding inverse problems in linear ($n = 1$) and nonlinear ($n \geq 10$) formulations for the indicated schemes of variation of the geometric parameters ($n$ is the number of iterations and $\Delta = 10^{-3}$ is the error of the calculations).

Table 1 shows data on the distribution of the components of the strain tensor ($e_{\rho\rho}$) and stress tensor $\tilde{\sigma}_{\rho\rho}$ ($\sigma_{\rho\rho} = \tilde{\sigma}_{\rho\rho}\, 10^5$ Pa) along the meridian of the shell when calculated in accordance with scheme (1) allowing for physical and geometric nonlinearities on its inside ( $\xi = -0.5$ ) and the following values: $\tilde{w}_0 = 0.5$ ( $\tilde{l} = 50$ ; $\varphi = 0.05$); the results obtained in the case $\tilde{w}_0 = 0$ were presented in [2] ($\eta = 30$; $\tilde{l} = 50$ ; $\varphi = 0.05$, this is the main variant of the calculation).

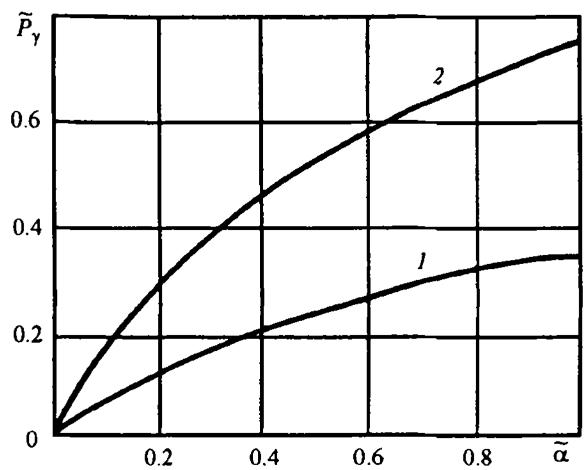


Fig. 2

TABLE 3

| $N$ | $\eta = 25$ | | | $\eta = 100$ | | |
|---|---|---|---|---|---|---|
| | $\tilde{w}$ | $e_{\beta\beta}$ | $\tilde{\sigma}_{\beta\beta}$ | $\tilde{w}$ | $e_{\beta\beta}$ | $\tilde{\sigma}_{\beta\beta}$ |
| 1 | 0.0000 | 0.0000 | 0.04 | 0.0000 | 0.0000 | 0.005 |
| 3 | 0.4989 | 0.1996 | 1569 | 0.4993 | 0.0499 | 565 |
| 5 | 0.9970 | 0.3988 | 2282 | 0.9983 | 0.0998 | 990 |
| 7 | 1.4943 | 0.5977 | 2765 | 1.4971 | 0.1497 | 1311 |
| 9 | 1.9907 | 0.7963 | 3141 | 1.9957 | 0.1996 | 1569 |
| 11 | 2.4864 | 0.9946 | 3454 | 2.4941 | 0.2494 | 1784 |

Figure 2 shows the change in the acting load $\tilde{P}_\gamma$ (normal pressure) along the meridian ( $\tilde{\alpha}$ ) for two values of the apex angle $\varphi$; the numbers of curves *1*, *2* correspond to the values $\varphi = 0.01$ and $\varphi = 0.05$ ( $\tilde{w}_0 = 0{,}5$; the other parameters correspond to the main variant). The results calculated with a change in the parameter $\tilde{l}$ are shown in Table 2 ( $\tilde{l} = 50$ , 75 , 100 ) .

The other components of the stress–strain state of the shells also agree within the stated computational error for different values $\alpha_1 \neq \alpha_2$ when the condition $w(\alpha_1) = w(\alpha_2)$ is satisfied. Such an agreement is obtained when any of the parameters ( $\tilde{w}_0$ , $\tilde{\alpha}$ , $\varphi$ ) are changed alone or in combination.

Calculations were performed with allowance for the nonlinear factors in accordance with scheme (2), with $\eta$ having different values and the other parameters corresponding to the main variant. Some of the results are shown in Table 3, which gives values of the displacements ( $\tilde{w}$ ), strains ( $e_{\beta\beta}$ ), and stresses ( $\tilde{\sigma}_{\beta\beta}$ ) calculated at a series of points of the meridian ($N$) on the inside.

It follows from these results that the above-noted agreement between the components of the stress–strain state for different sections is obtained in this case as well when the condition $w(\alpha_1)/R_1 = w(\alpha_2)/R_2$ is satisfied. Such an agreement is obtained for the values of normal pressure when the following condition is met: $P_\gamma(\alpha_1)/R_1 = P_\gamma(\alpha_2)/R_2$ .

TABLE 4

| Variants of problems | $E^*$ | $-\tilde{u}$ | $\tilde{w}$ | $\tilde{\sigma}_{\beta\beta}$ |
|---|---|---|---|---|
| LP | 1.25 | 0.199 | 2.49 | 9960 |
| PNP | 1.25 | 0.176 | 2.49 | 3199 |
| GNP | 1.25 | 0.261 | 2.49 | 9948 |
| PGNP | 1.25 | 0.239 | 2.49 | 3197 |
| LP | 0.80 | 0.249 | 2.49 | 12440 |
| PGNP | 0.80 | 0.311 | 2.48 | 4273 |

TABLE 5

| $N$ | $-\tilde{u}$ | $\tilde{w}$ | $-e_{\alpha\alpha}\cdot 10^2$ | $-e_{\beta\beta}\cdot 10^2$ | $\tilde{\sigma}_{\alpha\alpha}$ | $\tilde{\sigma}_{\beta\beta}$ |
|---|---|---|---|---|---|---|
| 1 | 0.000 | 0.500 | 0.200 | 1.667 | 0.025 | 1832 |
| 3 | 0.042 | 0.998 | 0.399 | 3.326 | 0.018 | 2754 |
| 5 | 0.105 | 1.495 | 0.598 | 4.982 | –0.014 | 3382 |
| 7 | 0.186 | 1.991 | 0.796 | 6.635 | –0.077 | 3869 |
| 9 | 0.288 | 2.486 | 0.994 | 8.285 | –0.173 | 4274 |
| 11 | 0.410 | 2.979 | 1.193 | 9.932 | –0.301 | 4623 |

**2. Effect of the Orthotropy and Nonlinear Properties of Composites.** Above, we presented numerical results for a cylindrical shell made of material (1.7) and having the parameters $E^* = E_\alpha/E_\beta = 1.25$, when the orthotropy axis $E_\alpha$ coincides with the axis of symmetry of the cylinder. The effect of the nonlinear properties of orthotropic composites on the deformation of the shells for other values of $E^*$ was studied with initial data corresponding to the main variant (Sec. 1). We will also present some of the results of these calculations, taking $E^* = 0.8$ ($E_\alpha = 12$ GPa; $E_\beta = 15$ GPa; $\nu_\alpha = 0.096$; $q_{\alpha\alpha} = 3.14$; $q_{\beta\beta} = 2$ ).

Some of the calculated data relating to the solution of the linear and nonlinear inverse problems is presented in Table 4 in the form of values of the displacements $\tilde{u}$, $\tilde{w}$ and the maximum hoop stresses $\tilde{\sigma}_{\beta\beta}$ calculated on the edge $\tilde{\alpha} = 1$ as a function of the degree of orthotropy of the material and the number of nodal points ($K = 21$). The other parameters corresponded to the main variant. The notation LP, PNP, GNP, and PGNP denotes the results obtained by solving the linear, physically nonlinear, geometrically nonlinear, and physically and geometrically nonlinear problems.

Table 5 shows the distribution of the displacements, strains, and stresses at a series of points of the meridian of the shells ( $\xi = 0$ ) with $\tilde{w}_0 = 0.5$, $E^* = 0.8$, and the other parameters corresponding to the main variant. The results and their analysis show that ignoring the nonlinearities results in overestimation of the maximum hoop stresses by 192% in the given example ($E^* = 0.8$); the effect of geometric nonlinearity on the values of $\tilde{\sigma}_{\beta\beta}$ in the given problems is negligible. The orthotropy of the material has a substantial effect on the values of $\tilde{\sigma}^{\max}_{\beta\beta}$, particularly in the nonlinear problems. For

TABLE 6

| No. of variant | $\tilde{\alpha}$ | $\tilde{u}$ | $\tilde{\sigma}_{\alpha\alpha}$ | $\tilde{\sigma}_{\beta\beta}$ |
|---|---|---|---|---|
| *A* | 0 / 0 | 0.0 / 0.0 | 0.0242 / 0.9700 | 0.0023 / 0.0931 |
| | 1 / 1 | −0.1995 / −0.2380 | 0.0241 / −0.9789 | 9960 / 3197 |
| *B* | 0 / 0 | 0.2011 / 0.2396 | 0.0242 / 0.9342 | 40.22 / 48.05 |
| | 1 / 1 | 0.0 / 0.0 | 0.0241 / −0.9600 | 10000 / 3204 |
| *C* | 0 / 0 | 0.0 / 0.0 | 607.8 / 385.4 | 58.35 / 37.00 |
| | 1 / 1 | 0.0 / 0.0 | 607.8 / 382.8 | 10060 / 3226 |

example, a small change in the parameter $E^*$ from 1.25 to 0.8 increases the stresses for the conical shell by 25 % in the linear problem and by 34 % in the nonlinear problems.

**3. Effect of the Type of Boundary Conditions on the Deformation of the Shells.** In contrast to the above-examined variant of boundary conditions A (1.8), here we present the results of studies performed with the following conditions on the edges of the shell ( $\tilde{\alpha} \in [0, 1]$ )

$$T_k = 0 \ (\tilde{\alpha} = 0), \quad u = 0 \ (\tilde{\alpha} = 1) \text{ (variant B)}; \tag{3.1}$$

$$u = 0 \ (\tilde{\alpha} = 0), \quad u = 0 \ (\tilde{\alpha} = 1) \text{ (variant C)} \tag{3.2}$$

Table 6 shows some of the values of the displacements $\tilde{u}$ and stresses $\tilde{\sigma}_{\alpha\beta}$, $\tilde{\sigma}_{\beta\beta}$ for the three variants of boundary conditions. The calculated data corresponds to the solution of the inverse problems in the linear (numerator) and doubly nonlinear (denominator) formulations ( $\xi = 0$ ).

It follows from the results that in the case of fixed edges (variant C) axial stresses are formed in the middle surface of the shell, as indicated by the presence of longitudinal tensile forces.

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