**MINISTRY OF EDUCATION AND SCIENCE
OF UKRAINE
NATIONAL AVIATION UNIVERSITY**

MECHANICS

Method Guide to Doing Homework Assignments

**for students of major
6.051103 “Avionics”**

KYIV 2013

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Методичні рекомендації включають завдання, рекомендації для виконання домашнього завдання з дисципліни "Механіка" та приклади розрахунків і проектування одноступінчатих редукторів.
Методичні рекомендації призначені для студентів напряму 6.051103 "Авіоніка"

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The method guide includes tasks, recommendations for carrying out the homework assignments on the subject “Mechanics” and examples of analysis and design of one stage speed reducers.
It is intended for students of major 6.051103 “Avionics”.

Навчальне видання

МЕХАНІКА

МЕТОДИЧНІ РЕКОМЕНДАЦІЇ
для виконання домашнього завдання

для студентів напряму
6.051103 "Авіоніка"

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GENERAL GUIDELINES

Mechanics is one of the hardest disciplines for non-mechanical specialities, technical colleges and the first engineering discipline that combines theory and methodology of engineering designing.

Discipline "Mechanics" consists of two parts: "Strength of Materials" and "The calculation and designing of mechanisms and their parts."

Study of the subject "Mechanics" as a discipline of applied nature is combined with work in the laboratory, where theoretically sound calculation formulas are checked according to the design and experimental data. At preparing and doing laboratory works the recommended literature should be used.

The discipline includes lectures, laboratory classes, consultations, homework assignment, two module tests on the sections of the lecture course and the exam.

Students must do homework assignments on their own, and present them to the teacher for reviewing and then defend them before the first and second modular control work. Laboratory work must be performed under the guidance of a teacher in laboratory studies. After defending of homework, performance and protection of laboratory work, and writing both modular examinations, students pass the exam.

While studying the subject "Mechanics" it is recommended to use these guidelines, and other training manuals, guidelines and reference books.

METHOD GUIDE TO CALCULATION AND GRAPHIC WORK

Calculation and graphic work is done by students in accordance with the curriculum and the program of discipline "Mechanics".

To solve the problem, choose the number of the work and the variant that corresponds to the last two digits of the student's record book. Number of the work corresponds to the penultimate digit number of the student’s record book, and number of variant - the last digit. If the last number is zero, the student must perform the tenth variant. If the penultimate digit of a record book is zero, the student must fulfill the tenth number or the work. For example, a student whose record book number is 830865, should perform the fifth task of the sixth variant.

Kinematic drives are shown in Fig. A, B, C, D, and the initial data are given in Table A.

Calculation and graphic work should contain explanatory notes and the graphic part.

The explanatory note is to be done in ink legibly on one (right) side of A4 paper, leaving 20 mm on the left margin for binding, 30 mm on the right for writing down the final results of calculations and notes of the reviewer. The distance from the first (last) line of the sheet to the top (or bottom) edge of the sheet must be not less than 10 mm. All pages must be numerated.

The first page of the explanatory note is the title page, the second - the problem task's kinematic scheme and initial data for them. Then goes the explanatory note proper. On the last page there is a list of references which must be referred to in the calculations.

On the title page of the explanatory note, in block letters, should be presented:

- university;
- department;
- discipline;
- number of homework assignment, task number and variant;
- surname, name of the student;
- course, faculty, record book number;
- date the assignment was done.

Calculation part of the work should be performed in accordance with the problem task. The text should have a clear category structure (sections, paragraphs and items with clear and concise headings). Contraction of the words in the text and captions are not allowed.

Formula, the empirical coefficients and other reference data should always be accompanied by references to the literature which must specify the numbers in square brackets according to the serial number in bibliography. When using the standards, make reference to them, for example choosing of asynchronous motors by means of Table B by standard 19523-81.

The used calculation formulas must have a name, and symbols with appropriate explanation.

To simplify checking the work check by the author or reviser and to avoid errors, it is recommended to do calculations in the following way: firstly you write the formula in symbols then without any algebraic changes substitute numerical values in formula, and after that the result of the calculation. For example, at determining the pitch diameter of the gear, the calculation is written as: $d_2 = m\ z_2 = 20 * 3 = 60$ mm where m it’s modulus, z_2 - number of teeth on the wheel.

Stick to this rule otherwise it would be difficult to check and verify the calculation and, more over, it may result in error.

Missing data in the work should be selected at you own making reference to the relevant sources.

Calculations must be accompanied by illustrations (diagrams, sketches) done in pencil with exceptional clarity and completeness using a ruler and compass with indication of symbols and calculated values. Pictures may be placed either in the explanatory note or in the end of it, as an appendix. All illustrations in the explanatory notes should be numbered in Arabic numerals through the text (e.g. Fig. 1, Fig. 2) and accompanied by the notes exactly matching the content of the image.

Start drawing sketches as soon as all preliminary calculations provide sufficient data for the drawing. Drawings and calculations must be performed almost simultaneously, so that the calculations slightly precede the drawing, otherwise the errors which are inevitable can be revealed later and their correction will take time and effort. You must stick to the rule: all calculated data are checked immediately by marking them in the drawing.

Graphical part of each calculation task and graphic work should be done in pencil on drawing paper A2 in accordance with the standards for engineering drawings. In the right lower corner of the sheet the corner stamp of the title block in the drawing and diagrams (55x185) must be filled.

Drawing of the gearing unit should be performed in two projections according to the scale and indication of sizes defined by calculations (see Annex A).

The sheets of drawings should be folded and filed at the end of the explanatory notes after references within one cover.

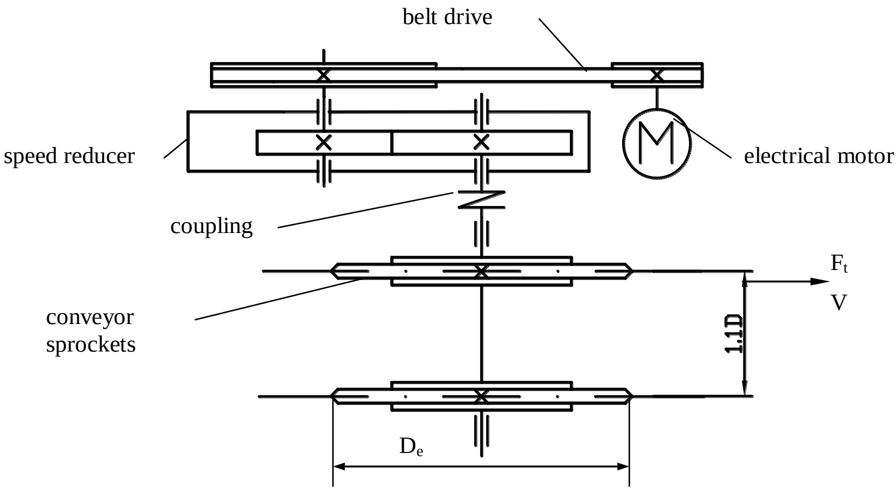


Fig A. Design a belt conveyor mechanical drive. If turning force Ft, peripheral speed V at well as diameter of sprockets D are given (Table A).

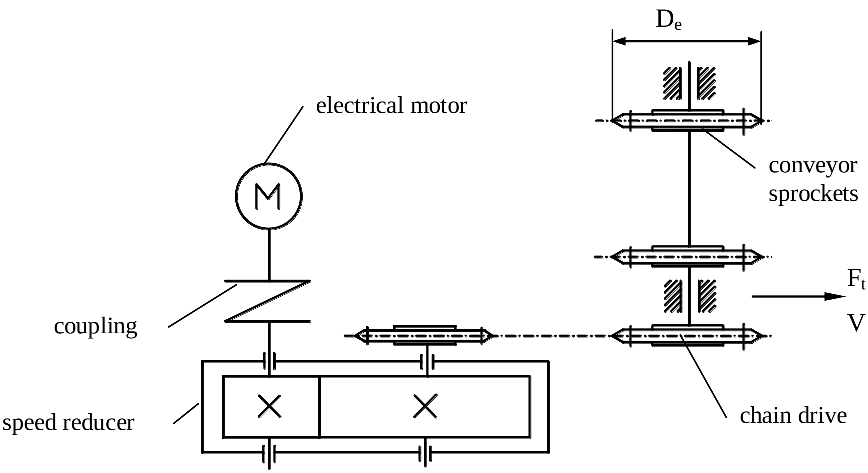


Fig B. Design a chain conveyor mechanical drive. If turning force Ft, peripheral speed V at well as diameter of sprockets D are given (Table A).

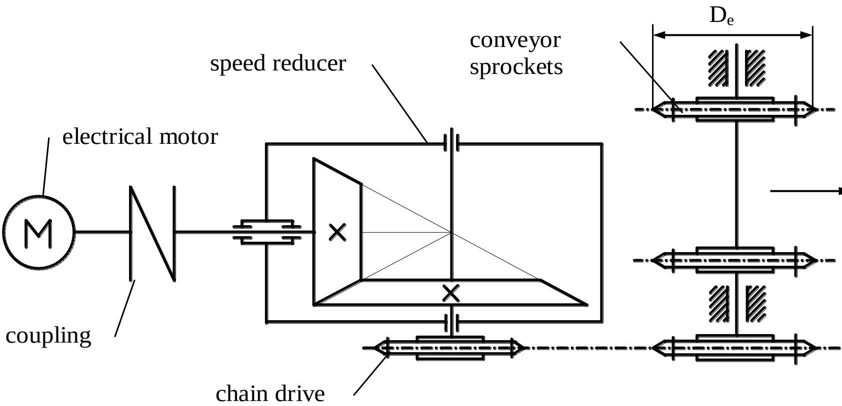


Fig. C. Design a chain conveyor mechanical drive. If turning force Ft, peripheral speed V at well as diameter of sprockets D are given (Table A).

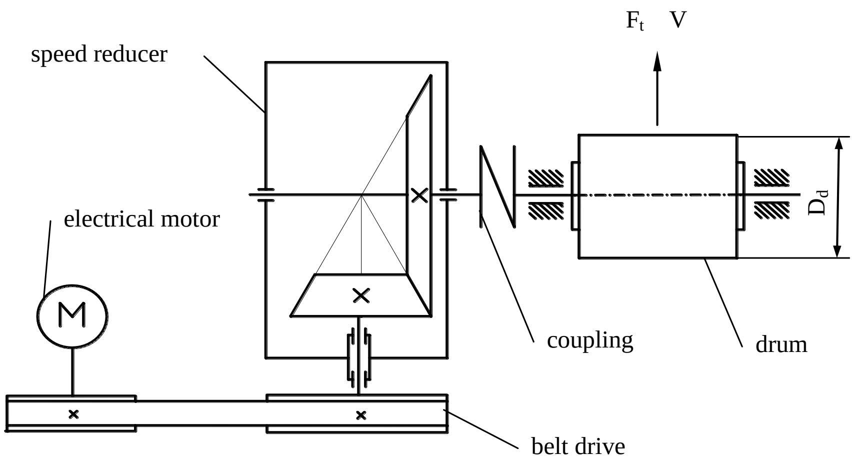


Fig D. Design a belt conveyor mechanical drive. If turning force Ft, peripheral speed V at well as diameter of sprockets D_d are given (Table A).

Table A

Initial data

Number of task	Number of figure	Type of gearing		Variant number									
		Engagement	Belt drive or Chain drive	1	2	3	4	5	6	7	8	9	10
1	A	Spur gears	Belt drive	$\frac{1{,}0}{2{,}0}$	$\frac{1{,}5}{0{,}9}$	$\frac{2{,}0}{0{,}7}$	$\frac{2{,}5}{1{,}1}$	$\frac{3{,}0}{0{,}8}$	$\frac{3{,}5}{1{,}1}$	$\frac{4{,}0}{0{,}5}$	$\frac{4{,}5}{1{,}2}$	$\frac{5{,}0}{1{,}0}$	$\frac{5{,}5}{1{,}3}$
2	B	Spur gears	Chain drive	$\frac{5{,}0}{1{,}0}$	$\frac{4{,}0}{1{,}2}$	$\frac{6{,}0}{1{,}5}$	$\frac{4{,}0}{1{,}7}$	$\frac{3{,}0}{1{,}9}$	$\frac{5{,}0}{2{,}0}$	$\frac{3{,}0}{2{,}1}$	$\frac{2{,}0}{2{,}2}$	$\frac{3{,}0}{2{,}3}$	$\frac{2{,}8}{2{,}5}$
3	C	Bevel gears	Chain drive	$\frac{1{,}8}{2{,}0}$	$\frac{2{,}0}{2{,}5}$	$\frac{2{,}4}{3{,}0}$	$\frac{2{,}5}{4{,}0}$	$\frac{2{,}0}{3{,}5}$	$\frac{1{,}8}{3{,}8}$	$\frac{2{,}0}{2{,}7}$	$\frac{3{,}0}{3{,}2}$	$\frac{2{,}2}{4{,}2}$	$\frac{2{,}6}{2{,}8}$
4	D	Bevel gears	Belt drive	$\frac{4{,}2}{1{,}5}$	$\frac{2{,}0}{1{,}8}$	$\frac{1{,}4}{2{,}0}$	$\frac{2{,}0}{1{,}0}$	$\frac{3{,}0}{1{,}7}$	$\frac{4{,}5}{2{,}2}$	$\frac{3{,}0}{2{,}4}$	$\frac{2{,}4}{3{,}0}$	$\frac{2{,}0}{2{,}7}$	$\frac{1{,}0}{2{,}8}$
5	A	Helical gear	Belt drive	$\frac{5{,}2}{1{,}3}$	$\frac{4{,}3}{1{,}2}$	$\frac{3{,}3}{0{,}6}$	$\frac{4{,}8}{2{,}2}$	$\frac{3{,}8}{0{,}7}$	$\frac{2{,}6}{1{,}2}$	$\frac{2{,}4}{1{,}4}$	$\frac{1{,}8}{1{,}9}$	$\frac{1{,}4}{1{,}2}$	$\frac{1{,}2}{1{,}6}$
6	B	Helical gear	Chain drive	$\frac{2{,}7}{2{,}5}$	$\frac{2{,}1}{2{,}5}$	$\frac{3{,}2}{1{,}4}$	$\frac{3{,}6}{1{,}3}$	$\frac{5{,}2}{1{,}4}$	$\frac{5{,}4}{2{,}0}$	$\frac{4{,}2}{1{,}7}$	$\frac{4{,}4}{1{,}2}$	$\frac{1{,}7}{2{,}3}$	$\frac{1{,}9}{2{,}5}$
7	C	Bevel gears	Chain drive	$\frac{5{,}3}{1{,}2}$	$\frac{4{,}2}{1{,}1}$	$\frac{3{,}2}{0{,}8}$	$\frac{4{,}7}{2{,}1}$	$\frac{3{,}7}{0{,}8}$	$\frac{2{,}7}{1{,}3}$	$\frac{2{,}3}{1{,}5}$	$\frac{1{,}7}{2{,}0}$	$\frac{1{,}3}{1{,}4}$	$\frac{1{,}6}{1{,}2}$
8	D	Bevel gears	Belt drive	$\frac{2{,}1}{2{,}5}$	$\frac{2{,}7}{2{,}6}$	$\frac{3{,}4}{1{,}3}$	$\frac{3{,}8}{1{,}4}$	$\frac{5{,}4}{2{,}0}$	$\frac{5{,}2}{1{,}4}$	$\frac{4{,}4}{1{,}2}$	$\frac{4{,}2}{1{,}7}$	$\frac{1{,}9}{2{,}5}$	$\frac{1{,}7}{2{,}3}$
9	C	Bevel gears	Chain drive	$\frac{5{,}0}{1{,}0}$	$\frac{4{,}4}{1{,}2}$	$\frac{3{,}0}{0{,}9}$	$\frac{4{,}6}{2{,}0}$	$\frac{3{,}6}{2{,}0}$	$\frac{2{,}6}{1{,}4}$	$\frac{2{,}2}{1{,}6}$	$\frac{1{,}6}{2{,}1}$	$\frac{1{,}2}{1{,}6}$	$\frac{1{,}7}{2{,}1}$
10	A	Helical gear	Belt drive	$\frac{1{,}9}{2{,}0}$	$\frac{2{,}1}{2{,}6}$	$\frac{2{,}3}{3{,}0}$	$\frac{2{,}6}{4{,}0}$	$\frac{2{,}0}{2{,}5}$	$\frac{1{,}7}{3{,}7}$	$\frac{2{,}0}{3{,}5}$	$\frac{3{,}2}{3{,}0}$	$\frac{3{,}4}{2{,}8}$	$\frac{2{,}9}{3{,}4}$

Note: In the numerator circumferential force on the drum or sprocket F_t is given, in denominator - the speed of the belt or chain V. The diameter of drum or sprocket should be selected within the range of 300 - 500 mm.

Table B

Asynchronous squirrel cage induction motors series 4A by standard 19523-81

Power, kW	Motor type	Rotational speed, rpm	Motor type	Rotational speed, rpm	Motor type	Rotational speed, rpm	Motor type	Rotational speed, rpm
0,25	-	-	-	-	-	-	71B6	680
0,55	-	-	71A4	1390	71A6	910	80A8	675
0,75	71A2	2840	71B4	1390	71B6	900	80B8	700
1,1	71B2	2810	80A4	1420	80A6	915	90 A8	700
1,5	80A2	2850	80B4	1415	80B6	920	90 B8	700
2,2	80B2	2850	90 4	1425	90 6	935	100 8	700
3,0	90 2	2840	100 4	1435	100 6	950	112MA8	700
4,0	100M2	2880	100 4	1430	112MA6	955	112MB8	700
5,5	100 2	2880	112M4	1445	112MB6	950	132 8	720
7,5	112M2	2900	132 4	1455	132 6	965	132M8	720
11,0	132M2	2900	132M4	1450	132M6	970	160 8	730
15,0	160 2	2940	160M4	1465	160 6	975	160M8	730
18,5	160M2	2940	160M4	1465	160M6	975	180M8	730
22,0	180 2	2945	180 4	1470	180M6	975	-	-
30,0	180M2	2945	180M4	1470	-	-	-	-

Execution order of calculation and graphics work is the following:

1. Choose the initial data for calculation and the kinematic scheme of the drive from Table 1.

2. Define the purpose, principle and condition of the drive according to kinematic scheme.

3. Make kinematic calculation of the drive to:

- determine the power requirements of the motor and the shaft rotation frequency;
- find the standard motor using the catalogue;
- determine the overall gear ratio of the drive and distribute it between each transmission.

4. Calculate the strength of the gear and:

- select materials for gears;
- determine the allowable stress;
- calculate distance between gear centers;
- determine all necessary dimensions of gears.

5. To determine and calculated the set diameter of a shaft taking into account the condition of torsional strength.

6. Knowing the diameter of the shaft for gears installation, choose the key parameters from the standard and calculate key strength.

7. Write the explanatory note with the complete calculation of the drive.

8. Draw the gear unit with the main geometric dimensions of gears on drawing sheet A2 (examples of drawing see in Annex A).

1. Kinematic and force analysis of a mechanical drive

Initial data:
Output power P_{out} = 5.5 kW;
Rotational speed n_{out}=100 rpm;

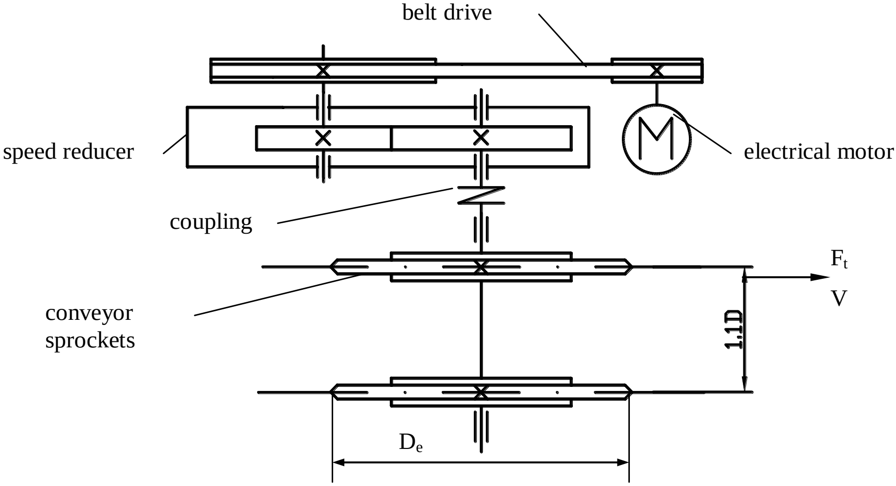


Fig A. Design a belt conveyor mechanical drive. If turning force Ft, peripheral speed V at well as diameter of sprockets D are given (Table A).

The mechanism consists of: asynchronous 4A series motor; belt drive with flat belt; helical spur gear speed reducer; rolling contact bearings; coupling with rubber bushed studs.

1.1. Determine the total drive efficiency.

In general, the efficiency of the drive is determined as a product of efficiencies of all kinematic pairs and links where the input power is lost.

$$\eta=\eta_1\cdot\eta_2\cdot\ldots\cdot\eta_n,$$

In this case

$$\eta = \eta_{hsg}\cdot\eta_{cd}\cdot\eta_c\cdot\eta_b^2,$$

where η_{cd} is the efficiency of the chain drive;
η_{hsg} is the efficiency of the helical spur gears;
η_c is the efficiency of the coupling;

η_b takes into account losses in one pair of bearings.
The magnitudes of all efficiencies are given in table 1.1.

Let us assume that η_{cd} = 0.96, η_{hsg}= 0.97, η_c = 0,996, η_b= 0.99. Then

$$\eta = 0.96 \cdot 0.97 \cdot 0.996 \cdot 0.99^2 = 0.909.$$

Mind that the magnitude of the total efficiency must be rounded off to thousands.

Table 1.1

Name	Efficiency	
	Closed drive	Opened drive
Gearings:		
- straight spur gears	0,98 - 0,99	0,94 - 0,96
- helical spur gears	0,97 - 0,98	0,94 - 0,95
- bevel gears	0,96 - 0,98	0,92 - 0,94
Worm gearing:		
-one thread worm	0,7 - 0,75	
-two thread worm	0,75 - 0,82	
-four thread worm	0,82 - 0,92	
Belt drives:		
- flat belt drive		0,96 - 0,98
- V-belt drive		0,95 - 0,97
- toothed belt drive		0,94 - 0,97
Chain drives:		
- roller chain		0,94 - 0,96
- toothed chain		0,96 - 0,97
Couplings:		
- with rubber bushed studs	0,996	
- flexible coupling	0,985 - 0,995	
- rigid coupling	1	
Bearings:		
- rolling bearings	0,99 - 0,995	
- sliding bearings	0,98 - 0,985	

1.2. Determine the input power.

Taking into account the fact that the efficiency is determined as ratio of the output power to the input one

$$\eta = \frac{P_{out}}{P_{inp}}$$

we can find the needed power of the electrical motor by the formula

$$P_{inp} = \frac{P_{out}}{\eta} = \frac{5.5}{0.909} = 6.05\,kW\,.$$

1.3. Select the electrical motor type.

For the given mechanical drives we use asynchronous electrical motor. It is explained by the fact that in comparison with the other types of motors, asynchronous electrical motors are simpler in design and maintenance, more reliable and less expensive.

Asynchronous motors are chosen according to table 1.2 the choice depends on the input power P_{inp} of a mechanical drive and the synchronous rotational speed n_s (rotational speed of a magnetic field that characterizes motor operation without load). It is recommended to take asynchronous motors of either synchronous rotational speed n_s= 1500 rpm or n_s= 1000 rpm for the given mechanical drives.

In our case we select 4A132S4 Induction Motor (P_r = 7.5 kW, n_s = 1500 rpm).

Table 1.2

Rated Power P_r, kW	Synchronous rotational speed n_s, rpm					
	3000		1500		1000	
	Type designation	*S*,%	Type designation	*S*,%	Type designation	*S*,%
0,55	63B2	8,5	71A4	7,3	71B6	10
0,75	71A2	5,9	71B4	7,5	80A6	8,4
1,1	71B2	6,3	80A4	5,4	80B6	8,0
1,5	80A2	4,2	80B4	5,8	90L6	6,4
2,2	80B2	4,3	90L4	5,1	100L6	5,1
3,0	90L2	4,3	100S4	4,4	112MA6	4,7
4,0	100S2	3,3	100L4	4,7	112MB6	5,1
5,5	100L2	3,4	112M4	3,7	132S2	3,3
7,5	112M2	2,5	132S4	3,0	132M6	3,2
11,0	132M2	2,3	132M4	2,8	160S6	2,7
15	160S2	2,1	160S4	2,3	160M6	2,6
18,5	160M2	2,1	160M4	2,2	180M6	2,7
22	180S2	2,0	180S4	2,0	200M6	2,8
30	180M2	1,9	180M4	1,9	200L6	2,1

1.4. Determine the motor rated rotational speed n_r.

$$n_r = n_s(1 - \frac{S}{100}),$$

where S is relative speed loss that is determined according to table 1.2. In our case S = 3 %. After substituting the corresponding magnitudes we obtain

$$n_r = 1500 \cdot (1 - \frac{3}{100}) = 1455 \text{ rpm.}$$

1.5. Determine the output rotational speed.

$$n_{out} = \frac{60 \cdot V}{\pi \cdot D} = \frac{60 \cdot 0.7}{3.14 \cdot 0.4} = 33.44 \text{ rpm.}$$

1.6. Determine the total velocity ratio of the mechanical drive

$$u = \frac{n_{inp}}{n_{out}} = \frac{1455}{17.37} = 83.75 .=1455/100=14{,}55$$

1.7. Distribute the total velocity ratio between mechanical drive steps.

The total velocity ratio can be found by the formula

$$u = u_{b.d.} \cdot u_{red},$$

where $u_{bd.}$ is the belt drive velocity ratio; u_{red} is the helical spur gear speed reducer velocity ratio.

First, determine the velocity ratio of speed reducer.
It should correspond to the following standard series*.

* - for spur and bevel gear speed reducers:
1.25; 1.4; 1.6; 1.8; 2.0; 2.24; 2.5; 2.8; 3.15; 3.55; 4.0; 4.5; 5.0; 5.6

Let us take u_{red} =4.
Then the velocity ratio u_{bd} =u/u_{red}=14.55/4=3.637 (for belt drives the obtained value of u_{bd} should range from 2 to 4; for chain drives u_{cd} = from 1.5 to 4).

1.8. Determine the rotational speed of all shafts.

$n_1 = n_r$ = 1455 rpm;

$$n_2 = \frac{n_1}{u_{bd}} = \frac{1455}{3{,}637} = 400 \; rpm;$$

$$n_3 = \frac{n_2}{u_{hsg}} = \frac{491}{4} = 100 \; rpm;$$

The obtained value of n_3 must be equal to n_{out} according to the initial data. Error ε must be not more than 1%. In our case ε=0.

1.9. Determine the angular velocity of all mechanical drive shafts:

$$\omega_1 = \frac{\pi n_1}{30} = \frac{3.14 \cdot 1455}{30} = 152.29\ \sec^{-1};$$

$$\omega_2 = \frac{\omega_1}{u_{bd}} = \frac{152.29}{3{,}637} = 41{,}87\ \sec^{-1};$$

$$\omega_3 = \frac{\omega_2}{u_{ssg}} = \frac{41{,}87}{4} = 10.45\ \sec^{-1};$$

1.10. Determine the power at mechanical drive shafts.
Calculation is carried out with respect to P_{inp}, determined in point 1.3.
$P_1 = P_{inp} = 6.05$ kW;
$P_2 = P_1 \cdot \eta_{bd} \cdot \eta_b = 6.05 \cdot 0.96 \cdot 0.99 \cdot 0.996 = 5.727$ kW;
$P_3 = P_2 \cdot \eta_{hsg} \cdot \eta^2_b \cdot \cdot \eta_c = 5.767 \cdot 0.97 \cdot 0.99 = 5.499$ kW;
The obtained magnitude of P_3 must be equal to P_{out} according to the initial data. Error should not be more than 1%. In our case ε=0.02.
1.12. Determine the torques at all shafts.

$$T_1 = \frac{P_1}{\omega_1} = \frac{6.05 \cdot 10^3}{152.29} = 39.845\ N \cdot m;$$

$$T_2 = \frac{P_2}{\omega_2} = \frac{5.727 \cdot 10^3}{41{,}87} = 136{,}78\ N \cdot m;$$

$$T_3 = \frac{P_3}{\omega_3} = \frac{5.5 \cdot 10^3}{10{,}45} = 511{,}62\ N \cdot m;$$

All obtained values should be listed in a summary table.

2. Calculation of allowable stresses

2.1. Select the material of toothed wheels.

The main material of toothed wheels is carbon and alloy steels. Depending on material hardness, toothed wheels are subdivided into two groups:

- toothed wheels with surface hardness $H \le 350\,\mathrm{BHN}$:
- toothed wheels with surface hardness H >350 BHN Brinell hardness number.

For general purpose speed reducers, the following alternatives are possible:

1. A pinion and a gear are made of identical carbon or alloy steel, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni). Heat treatment of both, the gear and the pinion is martempering. The pinion hardness is ranged from 269 to 302 BHN and the gear hardness is ranged from 235 to 262 BHN.
2. A pinion and a gear are made of identical alloy steel, such as 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 35XM (0.35C-Cr-Mo). Heat treatment of the gear is martempering to hardness ranged from 269 to 302 BHN. Heat treatment of the pinion is martempering and surface (induction) hardening to hardness ranged from 45 to 50 HRC.

Toothed wheels of the straight and helical spur gears are recommended to produce according to the 1st alternative. If we deal with bevel gearing, the 2nd alternative is more preferable.

2.2. Determine the mean values of the gear and the pinion hardness:

- for the pinion $$H_m^p = \frac{H_{min}^p + H_{max}^p}{2};$$
- for the gear $$H_m^g = \frac{H_{min}^g + H_{max}^g}{2}.$$

2.3. Determine the allowable contact stress for the pinion and gear.

Determine the limit of contact endurance for the pinion $\sigma_{H\,lim}^p$ and for the gear $\sigma_{H\,lim}^g$ according to table 2.1.

Table 2.1

Contact and bending endurance limits

Heat treatment	Tooth hardness: case	Tooth hardness: core and root	Gear material	$\sigma_{H\ lim}$, MPa	$\sigma_{b\ lim}$, MPa
Normalizing, martempering	Brinell 180 to 350		Carbon and alloy steels, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 50XH (0.50C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$2H_m + 70$	$1.8H_m$
Full hardening	Rockwell C, 40 to 55		Carbon and alloy steels, such as 45 (0.45C), 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$18H_m + 150$	500
Surface hardening	Rockwell C, 40 to 58	Rockwell C, 25 to 35	Alloy steels, such as 40X (0.40C-Cr), 40XH (0.40C-Cr-Ni), 50XH (0.50C-Cr-Ni), and 35XM (0.35C-Cr-Mo)	$17H_m + 200$	650
Case hardening	Rockwell C, 54 to 64	Rockwell C, 30 to 45	Alloy steels, such as 20XH2M (0.20C-Cr-2Ni-Mo)	$23H_m$	950
Nitriding	Rockwell C, 50 to 60	Rockwell C, 24 to 40	Alloy steels, such as 40XH2MA (0.40C-Cr-2Ni-Mo, quality)	1050	$300 + 1.2H_m$ (of tooth core)

Table 2.2

Base number of stress cycles

BHN_m	up to 200	250	300	350	400	450	500	550	600
HRC_m	-	25	32	38	43	47	52	56	60
$N_{H0} \cdot 10^6$	10	16.5	25	36.4	50	68	87	114	143

$$\sigma_{H\lim}^{p} = 17 \cdot H_{m}^{p} + 200 = 17 \cdot 47.5 + 200 = 1007.5\text{MPa}$$
$$\sigma_{H\lim}^{g} = 2 \cdot H_{m}^{g} + 70 = 2 \cdot 285.5 + 70 = 641\text{MPa}$$

Determine the base number of stress cycles for the pinion N_{H0}^{p} and gear N_{H0}^{g}. For this purpose we use table 2.2.

$N_{H0}^{p} = 68.9 \cdot 10^{6}$ stress cycles;
$N_{H0}^{g} = 22.5 \cdot 10^{6}$ stress cycles.

2.3.3. The gearing service life in hours is:

t =4000…5000 hours

2.3.4. Let factor K_{HE} that reduces variable load conditions to the constant load equivalence be:

K_{HE}=1

2.3.5. Determine the equivalent number of cycles for the pinion and the gear.

$$N_{HE}^{p} = 60 \cdot n^{p} \cdot t \cdot K_{HE};$$
$$N_{HE}^{g} = 60 \cdot n^{g} \cdot t \cdot K_{HE}$$

where n^p and n^g are rotational speeds of the pinion and the gear correspondingly.

2.3.6. Determine the durability factor for the pinion and gear if:

$$N_{HE} \geq N_{HO} \text{ then } K_{HL}=1,$$
$$N_{HE} < N_{HO} \text{ then } K_{HL} = \sqrt[6]{\frac{N_{H0}}{N_{HE}}}.$$

$$N_{HE}^{p} = 60 \cdot 400 \cdot 5000 \cdot 1 = 120 \cdot 10^{6};\ N_{H0}^{p} = 68.9 \cdot 10^{6},\ N_{HE}^{p} > N_{H0}^{p},$$
$$\text{then } K_{HL}^{p} = 1;$$
$$N_{HE}^{g} = 60 \cdot 100 \cdot 1 \cdot 5000 \cdot 1 = 30 \cdot 10^{6};\ N_{H0}^{g} = 22.5 \cdot 10^{6},\ N_{HE}^{g} > N_{H0}^{g},$$
$$\text{then } K_{HL}^{g} = 1.$$

2.3.7. Determine the safety factor S_H for the pinion and gear.

- for homogeneous structure of the material (heat treatment is normalizing, martempering and full hardening) $S_H = 1.1$;

- for heterogeneous structure of the material (heat treatment is surface hardening, case hardening, nitriding) S_H=1.2.

2.3.8. Determine the contact allowable stresses for the gear and for the pinion

$$\left[\sigma_H^p\right] = \frac{\sigma_{H\lim}^p \cdot K_{HL}}{S_H^p}, \qquad \left[\sigma_H^g\right] = \frac{\sigma_{H\lim}^g \cdot K_{HL}}{S_H^g}.$$

In our case: $S_H^p = 1.2;\ S_H^g = 1.1;$

$$\left[\sigma_H^p\right] = \frac{1007.5 \cdot 1}{1.2} = 839.58\text{MPa}; \qquad \left[\sigma_H^g\right] = \frac{641 \cdot 1}{1.1} = 582.73\text{MPa}.$$

If H^p-$H^g \leq 70$BHN, we assume that the design allowable contact stress is less value of above calculated stresses, where H^p and H^g are hardness of the pinion and gear materials correspondingly.

Otherwise, the design allowable contact stress is determined by the following formula:

$$\left[\sigma_H\right] = 0.45 \cdot \left(\left[\sigma_H^p\right] + \left[\sigma_H^g\right]\right) \leq 1.23 \cdot \left[\sigma_H^g\right].$$

Thus, for further calculations we assume as the design allowable contact stress $\left[\sigma_H\right] = 640.04\text{MPa}$.

2.4. Determine the allowable bending stresses for the pinion and for the gear.

2.4.1 . Determine the limits of the bending endurance for the pinion $\sigma_{b\lim}^p$ and for the gear $\sigma_{b\lim}^g$. For this purpose we use table 2.1.

In our case

$$\sigma_{B\lim}^p = 650\text{MPa}$$

$$\sigma_{H\lim}^g = 1.8 \cdot H_m^g = 1.8 \cdot 285.5 = 513.9\text{MPa}$$

2.4.2. Determine the base number of stress cycles N_{b0}.

For steels $N_{b0} = 4 \cdot 10^6$.

2.4.3. Let factor K_{bE} that reduces variable load conditions to the constant load equivalence be

$$K_{bE} = 1$$

2.4.4. Determine the equivalent number of cycles for the pinion and the gear.

$$N_{bE}^{p} = 60 \cdot n_p \cdot t \cdot K_{bE};$$

$$N_{bE}^{g} = 60 \cdot n_g \cdot t \cdot K_{bE};$$

$$N_{bE}^{p} = 60 \cdot 400 \cdot 5000 \cdot 1 = 120 \cdot 10^6;$$

$$N_{bE}^{g} = 60 \cdot 100 \cdot 5000 \cdot 1 = 30 \cdot 10^6.$$

2.4.5. Determine the durability factor for the pinion the gear if:

$$N_{bE} \geq N_{bO} \text{ then } K_{bL}=1,$$

$$N_{bE} < N_{bO} \text{ then } K_{bL} = \sqrt[m]{\frac{N_{b0}}{N_{bE}}},$$

where m=3 for toothed wheels with hardness $H \leq 350$ BHN and m=9 if $H > 350$ BHN.

In our case: $N_{bE}^{p} > N_{b0}^{p}$, then $K_{bL}^{p} = 1$;

$N_{bE}^{g} > N_{b0}^{g}$, then $K_{bL}^{g} = 1$.

2.4.6. Determine safety factor S_b for the pinion and for the gear.

- for wheels made of forged blanks (our case) $S_b = 1.75$;
- for wheels made of cast blanks $S_b = 2.3$.

2.4.7. Determine the bending allowable stresses for the gear and the pinion

$$\left[\sigma_b^p\right] = \frac{\sigma_{b\lim}^p \cdot K_{bL}}{S_b^p}, \qquad \left[\sigma_b^g\right] = \frac{\sigma_{b\lim}^g \cdot K_{bL}}{S_b^g}.$$

In our case: $S_b^p = S_b^g = 1.75$;

$$\left[\sigma_b^p\right] = \frac{650 \cdot 1}{1.75} = 371.43\text{MPa}; \quad \left[\sigma_b^g\right] = \frac{513.9 \cdot 1}{1.75} = 293.657\text{MPa}$$

For further calculations we assume that the design allowable bending stress has less value of above calculated stresses $\left[\sigma_b\right] = 293.657\text{MPa}$.

3. Strength calculation of the straight spur gears

Initial data: torque at the pinion shaft T^p =74 N·m; torque at the gear shaft T^g =370 N·m; velocity ratio of the gearing u=5; allowable contact stress $[\sigma_H]$=515 MPa; allowable bending stress $[\sigma_b]$=255 MPa; hardness of the gear material H^g=285 BHN, angular velocity of the gear shaft ω^g = 40 rad/sec.

3.1. Determine the centre distance of the straight spur gears

$$a_w = 0.85 \cdot (u+1) \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}},$$

where the sign ("+") is used for gears with external toothing as in our case; **u** is the velocity ratio of the spur gears; T^g is the torque at the gear shaft in N·mm; $[\sigma_H]$ is the allowable contact stress in MPa; E_{tr} is the transformed modulus of elasticity in MPa; $K_{H\beta}$ is the load concentration factor; $\psi_{ba}= b^g/a_w$ is the gear face width factor.

Transformed modulus of elasticity E_{tr} is determined as

$$E_{tr} = \frac{2 \cdot E^p \cdot E^g}{E^p + E^g},$$

where E^p and E^g are modulus of elasticity of pinion and gear materials respectively. Since the pinion and the gear are made of steel we can make the conclusion that $E_{tr} = E^p = E^g = 2.1 \cdot 10^5$ MPa.

Load concentration factor $K_{H\beta}$ is determined by means of table 3.2 depending upon disposition of toothed wheels with respect to bearings and factor $\psi_{bd}= b^g/d^p$. Since b^g and d^p are not determined we find this factor by the following formula

$$\psi_{bd} = \frac{b^g}{d^p} = \frac{0.5 \cdot b^g}{a_w} \cdot (u+1) = 0.5 \cdot \psi_{ba} \cdot (u+1),$$

where gear face width factor ψ_{ba} is taken from table 3.1 depending upon the position of the gear relative to bearings, remembering that the value of ψ_{ba} should correspond to the standard. The greater ψ_{ba} the less overall dimensions of the gearing. That is why we select the greater value of ψ_{ba}.

In our case the gear is located symmetrically relative to support. That is why we take ψ_{ba} = 0.5, $\psi_{bd} = 0.5 \cdot 0.4 \cdot (5+1) = 1.2$, $K_{H\beta}$=1.05.

$$a_w = 0.85\cdot(5+1)\cdot\sqrt[3]{\frac{511.62\cdot10^3\cdot1.19\cdot2.1\cdot10^5}{640^2\cdot5^2\cdot0.4}} = 163\text{mm}$$

Obtained magnitude of a_w is rounded up according to the series given in table 3.3. We assume a_w=180 mm.

Table 3.1

Recommended values of the gear face width factor ψ_{ba}

Gear arrangement with respect to bearings	Tooth hardness	ψ_{ba}
Symmetrical	Any	0.315; 0.4; 0.5
Non-symmetrical	Brinell BHN, up to 350 Rockwell C, 40 upwards	0.315; 0.4 0.25; 0.315
On shaft cantilevers	Brinell BHN, up to 350 Rockwell C, 40 upwards	0.25 0.2
For herringbone gears	Any	0.4 ; 0.5; 0.63
For internal gears	Any	0.2

Table 3.2

Approximate values of $K_{H\beta}$

Gear arrangement with respect to bearings	Tooth surface hardness, BHN	$\psi_{bd} = \frac{b^g}{d^p}$					
		0.2	0.4	0.6	0.8	1.2	1.6
On cantilevers, ball bearings	up to 350 over 350	1.08 1.22	1.17 1.44	1.28 -	- -	- -	- -
On cantilevers, roller bearings	up to 350 over 350	1.06 1.11	1.12 1.25	1.19 1.45	1.27 -	- -	- -
Symmetrical	up to 350 over 350	1.01 1.01	1.02 1.02	1.03 1.04	1.04 1.07	1.07 1.16	1.11 1.26
Non-symmetrical	up to 350 over 350	1.03 1.06	1.05 1.12	1.07 1.20	1.12 1.29	1.19 1.48	1.28 -

Table 3.3

Standard values of the centre distance a_w

Series 1	63	80	100	125	160	200	250	315	400	500
Series 2	71	90	112	140	180	224	280	355	450	560

3.2. Determine the nominal pitch circle diameter of the gear

$$d^g = \frac{2\cdot a_w\cdot u}{u\pm1} = \frac{2\cdot180\cdot5}{5+1} = 300\text{mm}.$$

3.3. Determine the face width of the gear

$$b^g = \psi_{ba} \cdot a_w = 0.4 \cdot 180 = 72 \text{ mm}.$$

3.4. Determine the module according to the strength condition for bending

$$m \geq \frac{2 \cdot K_m \cdot T^g}{d^g \cdot b^g \cdot [\sigma_b]} = \frac{2 \cdot 6.8 \cdot 370 \cdot 10^3}{300 \cdot 72 \cdot 255} = 0.91\text{mm},$$

where K_m is taken as 6.8 for straight spur gears.

Obtained value of the module should be rounded up according to the standard series given in table 3.4. It is necessary to note that for general-purpose speed reducers the minimum value of the module is m_{min} = 2 mm.

For our further calculations we assume m = 2 mm.

Table 3.4

Standard values of m_n

Series 1	1.0	1.25	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	12.0
Series 2	1.125	1.375	1.75	2.25	2.75	3.5	4.5	5.5	7.0	9.0	11.0	14.0

Note: Series 1 is preferable to Series 2

3.5. Determine the total number of teeth

$$z_\Sigma = \frac{2 \cdot a_w}{m} = \frac{2 \cdot 180}{2} = 180.$$

Obtained value of z_Σ we should be rounded off to the nearest integer number.

3.6. Determine the number of pinion teeth

In our case $z^p = \dfrac{z_\Sigma}{u \pm 1} \geq z_{min}$,

where z_{min}=17 for straight spur gears.

Obtained value of z^p should round off to the nearest integer number. If z^p <17 it is necessary to decrease the module or to use nonstandard toothed wheels

$$z^p = \frac{z_\Sigma}{u \pm 1} = \frac{180}{6} = 30 \geq z_{min} = 17.$$

3.7. Determine the number of teeth of the gear

$$z^g = z_\Sigma \mp z^p = 180 - 30 = 150.$$

3.8. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p} = \frac{150}{30} = 5.$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less then or equal to 4%. Otherwise the number of teeth z^p, z^g and z_Σ must be rounded down.

3.9. Determine the nominal pitch circles diameters for the pinion and the gear

$$d^p = m \cdot z^p = 2 \cdot 30 = 60 \text{ mm},$$
$$d^g = 2 \cdot a_w \mp d^p = 2 \cdot 180 - 60 = 300.$$

3.10. Determine the addendum circles diameters for the pinion and the gear

$$d_a^p = d^p + 2 \cdot m = 60 + 2 \cdot 2 = 64 \text{ mm},$$
$$d_a^g = d^g \pm 2 \cdot m = 300 + 2 \cdot 2 = 304 \text{ mm}.$$

3.11. Determine the dedendum circles diameters for the pinion and the gear

$$d_f^p = d^p - 2.5 \cdot m = 60 - 2.5 \cdot 2 = 55 \text{ mm},$$
$$d_f^g = d^g \mp 2.5 \cdot m = 300 - 2.5 \cdot 2 = 295 \text{ mm}.$$

3.12. Determine forces that act in the engagement of the straight spur gears:

- turning force $F_t = \frac{2 \cdot T^g}{d^g} = \frac{2 \cdot 370}{0.3} = 2467\text{N}$;
- radial force $F_r = F_t \cdot tg\alpha_w = 2467 \cdot tg20° = 898\text{N}$,

where $\alpha_w = 20°$ is the pressure angle for the pitch circle.

3.13. Determine the maximum contact stress that develops in the contact zone of teeth

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T^p \cdot K_H \cdot E_{tr}}{\left(d^p\right)^2 \cdot b^g \cdot \sin 2\alpha_w} \cdot \left(\frac{u_{act} \pm 1}{u_{act}}\right)} =$$
$$= 1.18 \cdot \sqrt{\frac{74 \cdot 10^3 \cdot 1.19 \cdot 1.24 \cdot 2.1 \cdot 10^5}{60^2 \cdot 72 \cdot \sin 40°} \cdot \left(\frac{5+1}{5}\right)} = 465\text{MPa},$$

where T^p is the torque at the pinion shaft in N·mm; K_H is the design load factor that is determined as $K_H = K_{H\beta} \cdot K_{HV}$ where $K_{H\beta}$ is the load concentration factor; K_{HV} is the dynamic load factor.

The load concentration factor $K_{H\beta}$ is specified in table 3.2. It depends upon $\psi_{bd} = \frac{b^g}{d^p} = \frac{72}{60} = 1.2$.

In order to determine K_{HV} it is necessary to find the peripheral gear speed V^g

$$V^g = \frac{\omega^g \cdot d^g}{2} = \frac{40 \cdot 0.3}{2} = 6 \text{ m/sec},$$

and the gearing accuracy of manufacturing (table 3.5), where ω^g is the angular velocity of the gear.

The dynamic load factor K_{HV} is determined according to table 3.6.

Table 3.5

Gearing accuracy of manufacturing

Types of gear drives	Peripheral speed V, m/sec			
	under 5	5 - 8	8 – 12.5	over 12.5
Straight spur gear	9	8	7	6
Helical spur gear	9	9	8	7
Straight bevel gear	8	7	-	-
Spiral bevel gear	9	9	8	7

Table 3.6

Dynamic load factor K_{HV}

Gear drive accuracy	Tooth surface hardness, BHN	Peripheral speed V, m/sec					
		1	2	4	6	8	10
7	up to 350	1.04/1.02	1.07/1.03	1.14/1.05	1.21/1.06	1.29/1.07	1.36/1.08
	over 350	1.03/1.00	1.05/1.01	1.09/1.02	1.14/1.03	1.19/1.03	1.24/1.04
8	up to 350	1.04/1.01	1.08/1.02	1.16/1.04	1.24/1.06	1.32/1.07	1.40/1.08
	over 350	1.03/1.01	1.06/1.01	1.10/1.02	1.16/1.03	1.22/1.04	1.26/1.05
9	up to 350	1.05/1.01	1.10/1.03	1.20/1.05	1.30/1.07	1.40/1.09	1.50/1.12
	over 350	1.04/1.01	1.07/1.01	1.13/1.02	1.20/1.03	1.26/1.04	1.32/1.05

Note: The figures in the numerators refer to straight spur gears and those in the denominators - to helical spur gears.

Obtained value of σ_H should correspond to the following condition:

$$\sigma_H = (0.8\ldots1.1)\cdot[\sigma_H].$$

Otherwise it is necessary to change the center distance a_w and make calculations once more.

3.14. Determine the maximum bending stress

$$\sigma_b = \frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Y_b}{m \cdot b^g} = \frac{2467 \cdot 1.42 \cdot 1.58 \cdot 3.6}{2 \cdot 72} = 138.4\text{MPa} \le [\sigma_b] = 255\text{MPa},$$

where $K_{b\beta}$ is the load concentration factor that is determined according to table 3.7; K_{bv} is the dynamic load factor determined according to table 3.8; Y_b is the tooth shape factor that is determined by means of table 3.9 depending upon the number of gear teeth when the offset factor x=0.

If the obtained value of $\sigma_b > [\sigma_b]$ it is necessary to increase the module.

Table 3.7

Approximate values of $K_{b\beta}$

Gear arrangement with respect to bearings	Tooth surface hardness, BHN	$\psi_{bd} = \frac{b^g}{d^p}$					
		0.2	0.4	0.6	0.8	1.2	1.6
On cantilevers, ball bearings	up to 350	1.16	1.37	1.64	-	-	-
	over 350	1.33	1.70	-	-	-	-
On cantilevers, roller bearings	up to 350	1.10	1.22	1.38	1.57	-	-
	over 350	1.20	1.44	1.71	-	-	-
Symmetrical	up to 350	1.01	1.03	1.05	1.07	1.14	1.26
	over 350	1.02	1.04	1.08	1.14	1.30	-
Non-symmetrical	up to 350	1.05	1.10	1.17	1.25	1.42	1.61
	over 350	1.09	1.18	1.30	1.43	1.73	-

Table 3.8

Dynamic load factor K_{bV}

Gear drive accuracy	Tooth surface hardness, BHN	Peripheral speed V, m/sec					
		1	2	4	6	8	10
7	up to 350	1.08/1.03	1.16/1.06	1.33/1.11	1.50/1.16	1.62/1.22	1.80/1.27
	over 350	1.03/1.01	1.05/1.02	1.09/1.03	1.13/1.05	1.17/1.07	1.22/1.08
8	up to 350	1.10/1.03	1.20/1.06	1.38/1.11	1.58/1.17	1.78/1.23	1.96/1.29
	over 350	1.04/1.01	1.06/1.02	1.12/1.03	1.16/1.05	1.21/1.05	1.26/1.08
9	up to 350	1.13/1.04	1.28/1.07	1.50/1.14	1.72/1.21	1.98/1.28	1.25/1.35
	over 350	1.04/1.01	1.07/1.02	1.14/1.04	1.21/1.06	1.27/1.08	1.34/1.09

Note: The figures in the numerators refer to straight spur gears and those in the denominators - to helical spur gears.

Table 3.9

Tooth form factor Y_b

z or z_v	17	20	22	24	26	28	30	35	40	45	50	65	80	100
Y_b	4.27	4.07	3.98	3.92	3.88	3.81	3.8	3.75	3.7	3.66	3.65	3.62	3.61	3.6

4. Strength calculation of the helical spur gears for

4.1. Determine the center distance of the helical spur gears

$$a_w = 0.75 \cdot (u+1) \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}},$$

where u is the velocity ratio of the gearing; T^g is the torque at the gear shaft in N·mm; $[\sigma_H]$ is the allowable contact stress in MPa; E_{tr} is the transformed modulus of elasticity in MPa; $K_{H\beta}$ is the load concentration factor; $\psi_{ba} = b^g/a_w$ is the gear face width factor.

Transformed modulus of elasticity E_{tr} is determined as

$$E_{tr} = \frac{2 \cdot E^p \cdot E^g}{E^p + E^g},$$

where E^p and E^g are modulus of elasticity of pinion and gear materials respectively. Since the pinion and the gear are made of steel we may make the conclusion that $E_{tr} = E^p = E^g = 2.1 \cdot 10^5$ MPa.

Load concentration factor $K_{H\beta}$ is determined according to table 3.2. This factor depends upon disposition of tooth wheels with respect to bearings and factor $\psi_{bd} = b_g/d_p$. Since b_g and d_p were not determined, we find this factor by the following formula:

$$\psi_{bd} = \frac{b^g}{d^p} = \frac{0.5 \cdot b^g}{a_w} \cdot (u \pm 1) = 0.5 \cdot \psi_{ba} \cdot (u \pm 1),$$

where gear face width factor ψ_{ba} is determined according to table 3.1 depending on the position of the gear relative to bearings taking into account that the value of this factor should correspond to the standard. The greater ψ_{ba} the less overall dimensions of the gearing. That is why we select the greater magnitude of ψ_{ba}.

Obtained value of a_w we round up according to the series given in table 3.3.

In this case: T^g = 464300 N·mm; T^p = 161120 N·mm; u = 4; $[\sigma_H]$ = 640 MPa; $E_{tr} = 2.1 \cdot 10^5$ MPa**;** $[\sigma_b]$ = 293.657 MPa.

From table 3.1 we take ψ_{ba} = 0.5; $\psi_{bd} = 0.5 \cdot 0.5 \cdot (4+1) = 1.25$, and $K_{H\beta}$ = 1.073 (for symmetrical gear arrangement and tooth surface hardness up to 350MPa).

Thus $a_w = 0.75 \cdot (4+1) \cdot \sqrt[3]{\dfrac{464300 \cdot 1.073 \cdot 2.1 \cdot 10^5}{640^2 \cdot 4^2 \cdot 0.5}} = 118.965\text{mm}$

according to table 3.3 we take a_w = 125 mm for the further calculations.

4.2. Determine the nominal pitch circle diameter of the gear

$$d^g = \frac{2 \cdot a_w \cdot u}{u+1}. \qquad d^g = \frac{2 \cdot 125 \cdot 4}{4+1} = 200\text{mm}.$$

4.3. Determine the face width of the gear

$$b^g = \psi_{ba} \cdot a_w. \qquad b^g = 0.5 \cdot 125 = 62.5 \text{ mm}.$$

4.4. Determine the normal module according to the strength condition for bending

$$m_n \ge \frac{2 \cdot K_m \cdot T^g}{d^g \cdot b^g \cdot [\sigma_b]},$$

where K_m is 5.8 for helical spur gears.

The obtained value of the module should be rounded up according to the standard series given in table 3.4. It is necessary to note that for general-purpose speed reducers, the minimum value of the module is m_{min} = 2 mm.

$m_n = \dfrac{2 \cdot 5.8 \cdot 464300}{200 \cdot 62.5 \cdot 293.657} = 1.467\text{mm}$, round off to m_n = 2 mm

4.5. Determine the helix angle

$$\beta = \arcsin\left(\frac{3.5 \cdot m_n}{b^g}\right) = \arcsin\left(\frac{3.5 \cdot 2}{62.5}\right) = 6.43 = 6^\circ 25'.$$

For helical spur gears this angle should be ranged from 8 to 18°. Otherwise, it is necessary to change the normal module m_n and in our case this condition is not satisfied.

That's why we take m_n=2.5 mm, then $\beta = \arcsin\left(\dfrac{3.5 \cdot 2.5}{62.5}\right) = 8.048 = 8^\circ 2'.$

4.6. Determine the total number of teeth

$$z_\Sigma = \frac{2 \cdot a_w \cdot \cos\beta}{m_n}.$$

The obtained value of z_Σ should be rounded off to the nearest integer number.

4.7. Specify the helix angle according to the integer number of z_Σ

$$\beta = \arccos\left(\frac{m_n \cdot z_\Sigma}{2 \cdot a_w}\right).$$

The value of this angle must be ranged from 8 to 18°.

4.8. Determine the number of teeth of the pinion

$$z^p = \frac{z_\Sigma}{u \pm 1} \ge z_{min},$$

where $z_{min}=17 \cdot \cos^3\beta$ for helical spur gears.

The obtained value of z^p should be rounded off to the nearest integer number. If $z^p < 17 \cdot \cos^3\beta$ it is necessary to decrease the module or to use nonstandard toothed wheels.

In our case

$$z_\Sigma = \frac{2 \cdot 125 \cdot \cos 8^\circ 2'}{2.5} = 99, \ \beta = \arccos\left(\frac{2.5 \cdot 99}{2 \cdot 125}\right) = 8.11^\circ = 8^\circ 6',$$

$$z^p = \frac{99}{4 \pm 1} = 19.8 \Rightarrow z^p = 20 > z_{min} = 17 \cdot \cos^3 8^\circ 6' = 16.5.$$

4.9. Determine the number of teeth of the gear

$$z^g = z_\Sigma - z^p, \qquad z^g = 99 - 20 = 79.$$

4.10. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p}.$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less then or equal to 4%. Otherwise the number of teeth z^p, z^g and z_Σ must be rounded down.

In our case the condition is satisfied, as

$$u_{act} = \frac{79}{20} = 3.95; \qquad \varepsilon = \left|\frac{3.95-4}{4}\right| \cdot 100\% = 1.25 < 4\%$$

4.11. Determine the nominal pitch circles diameters for the pinion and the gear

$$d^p = \frac{m_n}{\cos\beta} \cdot z^p = \frac{2.5}{\cos 8^\circ 6'} \cdot 20 = 50.5\text{mm},$$

$$d^g = 2 \cdot a_w - d^p = 2 \cdot 125 - 50.5 = 199.5 \text{ mm}.$$

4.12. Determine the addendum circles diameters for the pinion and the gear

$$d_a^p = d^p + 2m_n = 50.5 + 2 \cdot 2.5 = 55.5\text{mm},$$

$$d_a^g = d^g + 2m_n = 199.5 + 2 \cdot 2.5 = 204.5\text{mm}.$$

4.13. Determine the dedendum circles diameters for the pinion and the gear

$$d_f^p = d^p - 2.5 \cdot m_n = 50.5 - 2.5 \cdot 2.5 = 44.25\text{mm},$$

$$d_f^g = d^g - 2.5 \cdot m_n = 199.5 - 2.5 \cdot 2.5 = 193.25\text{mm}.$$

4.14. Determine forces that act in the engagement of the helical spur gears:

- turning force $F_t = \frac{2 \cdot T^g}{d^g} = \frac{2 \cdot 464300}{199.5} = 4654.64\text{N}$;
- radial force $F_r = \frac{F_t}{\cos\beta} \cdot tg\alpha_w = \frac{4654.64}{\cos 8^\circ 6'} \cdot tg20^\circ = 1711.22\text{N}$;
- axial force $F_a = F_t \cdot tg\beta = 4654.64 \cdot tg8^\circ 6' = 662.45\text{N}$,

where $\alpha_w = 20^\circ$ is the pressure angle for the pitch circle.

4.15. Determine the maximum contact stress developed in the contact zone of teeth

$$\sigma_H = 1.18 \cdot Z_{H\beta} \cdot \sqrt{\frac{T^p \cdot K_H \cdot E_{tr}}{\left(d^p\right)^2 \cdot b^g \cdot \sin 2\alpha_w} \cdot \left(\frac{u_{act} \pm 1}{u_{act}}\right)},$$

where $Z_{H\beta}$ takes into account rising contact strength of the helical spur gears in comparison with the straight spur gears; T^p is the torque at the pinion shaft in N·mm; K_H is the design load factor that is determined as

$$K_H = K_{H\beta} \cdot K_{HV},$$

where $K_{H\beta}$ is the load concentration factor; K_{HV} is the dynamic load factor.

The load concentration factor $K_{H\beta}$ is specified in table 3.2 and depends upon $\psi_{bd} = \frac{b^g}{d^p}$.

In order to determine K_{HV} it is necessary to find the peripheral speed V^g of the gear

$$V^g = \frac{\omega^g \cdot d^g}{2}$$

and the accuracy of the gearing (table 3.5), where ω^g is the angular velocity of the gear.

The dynamic load factor K_{HV} is specified in table 3.6.

Factor $Z_{H\beta}$ is determined in the following way

$$Z_{H\beta} = \sqrt{\frac{K_{H\alpha} \cdot \cos^2 \beta}{\varepsilon_\alpha}},$$

where $K_{H\alpha}$ takes into account non-uniform load distribution between several pairs of teeth; ε_α is the contact ratio.

$K_{H\alpha}$ depends upon the accuracy of manufacturing and the peripheral speed and is determined according to table 4.1.

Table 4.1

Factors $K_{H\alpha}$, $K_{b\alpha}$ that take into account non-uniform load distribution between some pairs of teeth

Peripheral speed V, m/sec	Accuracy degree	$K_{H\alpha}$	$K_{b\alpha}$
To 5	7	1.03	1.07
	8	1.07	1.22
	9	1.13	1.35
From 5 to 10	7	1.05	1.2
	8	1.10	1.3
From 10 to 15	7	1.08	1.25
	8	1.15	1.40

Contact ratio ε_α is found by the following formula

$$\varepsilon_\alpha = \left[1.88 - 3.2 \cdot \left(\frac{1}{z^p} + \frac{1}{z^g}\right)\right] \cdot \cos\beta \,.$$

The obtained value of σ_H should meet to the following condition:

$$\sigma_H = (0.8 \ldots 1.1) \cdot [\sigma_H].$$

Otherwise it is necessary to change the center distance a_w and recalculate the gearing.

In our case: $\psi_{bd} = \frac{62.5}{50.5} = 1.238$; $K_{H\beta} = 1.072$;

$V^g = \frac{19.19 \cdot 0.1995}{2} = 1.914 \mathrm{m/sec} \Rightarrow K_{HV} = 1.01$;

Gear drive accuracy of manufacturing is 9; $K_H = 1.072 \cdot 1.01 = 1.083$;

$\varepsilon_\alpha = \left[1.88 - 3.2 \cdot \left(\frac{1}{20} + \frac{1}{79}\right)\right] \cdot \cos 8^\circ 6' = 1.663$ $K_{H\alpha} = 1.13$;

$Z_{H\beta} = \sqrt{\frac{1.13 \cdot \cos^2 8^\circ 6'}{1.663}} = 0.816$;

$\sigma_H = 1.18 \cdot 0.816 \cdot \sqrt{\frac{161120 \cdot 1.083 \cdot 210000}{50.5^2 \cdot 62.5 \cdot \sin 2 \cdot 20^\circ} \cdot \left(\frac{3.95 + 1}{3.95}\right)} = 644.63 \mathrm{MPa}$;

$\sigma_H < 1.1[\sigma_H]$ so the strength condition is satisfied.

4.16. Determine the maximum bending stress

$$\sigma_b = \frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Z_{b\beta} \cdot Y_b}{m_n \cdot b^g} \le [\sigma_b],$$

where $K_{b\beta}$ is the load concentration factor that is determined according to table 3.7; K_{bv} is the dynamic load factor specified in table 3.8; Y_b is the tooth shape factor that is determined in table 3.9; it depends on the number of teeth of the equivalent straight spur gear $z_v^g = \frac{z^g}{\cos^3\beta}$ for the case when the shift factor x=0.

Factor $Z_{b\beta}$ is the analogy of $Z_{H\beta}$ and is determined as

$$Z_{b\beta} = \frac{K_{b\alpha} \cdot Y_\beta}{\varepsilon_\alpha},$$

where $K_{b\alpha}$ is chosen from table 4.1; $Y_\beta = 1 - \frac{\beta^\circ}{140}$ is the correction factor.

If obtained value of $\sigma_b > [\sigma_b]$ it is necessary to increase the module.

In our case: $K_{b\beta} = 1.155$; $K_{bv} = 1.02$; $z_v^g = \frac{79}{\cos^3 8^\circ 6'} = 81.42 \Rightarrow 81$;

$Y_b = 3.61$; $K_{b\alpha} = 1.35$; $Y_\beta = 1 - \frac{8^\circ 40'}{140} = 0.939$; $Z_{b\beta} = \frac{1.35 \cdot 0.939}{1.663} = 0.762$;

$$\sigma_b = \frac{4654.64 \cdot 1.155 \cdot 1.02 \cdot 0.762 \cdot 3.61}{2.5 \cdot 62.5} = 96.54\text{MPa} < [\sigma_b] = 293.657\text{MPa}.$$

Strength condition is satisfied.

5. Strength calculation of the bevel gear

Initial data: torque at the gear shaft T^g =460 N·m; velocity ratio of the gearing u=3; allowable contact stress $[\sigma_H]$=620 MPa; allowable bending stress $[\sigma_b]$=168 MPa, hardness of the gear material H^g=285 BHN.

5.1. Determine the external pitch diameter of the gear

$$d_e^g = 1.7 \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr} \cdot u}{\nu_H \cdot [\sigma_H]^2 \cdot \psi_{bR} \cdot (1-\psi_{bR})}},$$

where T^g is the torque at the gear shaft in N·mm; E_{tr} is the transformed modulus of elasticity; $K_{H\beta}$ is the load concentration factor; u is the velocity ratio; ν_H = 0.85 is the correction factor that takes into account reducing bevel gears strength in comparison with the spur gears; $[\sigma_H]$ is the allowable contact stress; $\psi_{bR}=b^g/R_e$ is the gear face width factor that determines proportions of the face width of the gear with respect to the external cone distance. Factor ψ_{bR} must be less than 0.3. Recommended value of ψ_{bR} = 0.285.

Since both pinion and gear are made of steel, the transformed modulus of elasticity E_{tr} =$2.1 \cdot 10^5$ MPa.

Load concentration factor $K_{H\beta}$ depends upon the hardness of the gear material. If $H^g \le 350$ BHN, $K_{H\beta}$ is ranged from1.23 to 1.35. Otherwise (H^g>350 BHN) $K_{H\beta}$ is ranges from1.25 to 1.45. It is necessary to note that greater values of $K_{H\beta}$ are intended for the case when one of tooth wheels is on the cantilever shaft. Let us take $K_{H\beta}$ = 1.3

$$d_e^g = 1.7 \cdot \sqrt[3]{\frac{T^g \cdot K_{H\beta} \cdot E_{tr} \cdot u}{\nu_H \cdot [\sigma_H]^2 \cdot \psi_{bR} \cdot (1-\psi_{bR})}} = 1{,}7 \cdot \sqrt[3]{\frac{460 \cdot 10^3 \cdot 1.3 \cdot 2.1 \cdot 10^5 \cdot 3}{0.85 \cdot 620^2 \cdot 0.285 \cdot (1-0.285)}} = 302.9\text{mm}.$$

The obtained value of d_e^g should be rounded up according to standard series given in table 5.1.
In our case we assume d_e^g=315 mm.

Table 5.1

Standard values of the external pitch diameter d_e^g

Series 1	40	50	63	80	100	125	160	200	250	315	400	500
Series 2	-	-	71	90	112	140	180	224	280	355	450	560

5.2. Determine pitch angles for the pinion and the gear.

δ_2 = arctg u= arctg 3=71°36', δ_1 = 90°- δ_2=90-71.6=18°24'.

5.3. Determine the external cone distance

$$R_e = \frac{d_e^g}{2 \cdot \sin \delta_2} = \frac{315}{2 \cdot \sin 71°36'} = 165.98\text{mm} .$$

5.4. Determine the face width of the gear

$$b^g = \psi_{bR} \cdot R_e = 0.285 \cdot 165.98 = 47.3 \text{ mm}.$$

5.5. Determine the external module

$$m_e = \frac{14 \cdot T^g \cdot K_{b\beta}}{\nu_b \cdot d_e^g \cdot b^g \cdot [\sigma_b]} = \frac{14 \cdot 460 \cdot 10^3 \cdot 1{,}32}{0.85 \cdot 315 \cdot 47.3 \cdot 168} = 3{,}99\text{mm} ,$$

where ν_b = 0.85 is the correction factor; $K_{b\beta}$ is the load concentration factor that is determined according to table 3.7 and depends upon ψ_{bd} factor, where the latter is found as

$$\psi_{bd} = \frac{b^g}{d_m^p} = 0.166 \cdot \sqrt{u^2 + 1} = 0.166 \cdot \sqrt{3^2 + 1} = 0.53 .$$

Let us take $K_{B\beta}$ = 1.32 (for gear arrangement on cantilevers, mounted on roller bearings).

5.6. Determine the number of the gear teeth

$$z^g = \frac{d_e^g}{m_e} = \frac{315}{3.99} = 78{,}9$$

and round off z^g to the integer number. In our case $z^g = 79$.

5.7. Determine the number of the pinion teeth

$$z^p = \frac{z^g}{u} = \frac{79}{3} = 26.3$$

and round off z^p to the integer number too. In our case $z^p = 26$.

5.8. Specify the velocity ratio of the gearing

$$u_{act} = \frac{z^g}{z^p} = \frac{78}{26} = 3.04 .$$

The error $\varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\%$ should be less then or equal to 4%. Otherwise, we should round down values of z^p and z^g .

$$\text{In this case } \varepsilon = \left|\frac{u_{act} - u}{u}\right| \cdot 100\% = \left|\frac{3.04 - 3}{3}\right| \cdot 100\% = 1.33 < 4\%.$$

5.9. Specify pitch angles for the pinion and the gear

$$\delta_2 = \text{arctg } u_{act} = \text{arctg } 3.04 = 71°48', \qquad \delta_1 = 90° - \delta_2 = 18°12'$$

5.10. Determine external pitch diameters of the pinion and the gear.

$$d_e^p = m_e \cdot z^p = 3.99 \cdot 26 = 103.74\text{mm},$$
$$d_e^g = m_e \cdot z^g = 3.99 \cdot 79 = 315.21\text{mm}.$$

5.11. Determine diameters of addendum circles at the outer section for the pinion and the gear

$$d_{ae}^p = d_e^p + 2 \cdot m_e \cdot \cos\delta_1 = 103.74 + 2 \cdot 3.99 \cdot \cos 18°12' = 111.32 \text{ mm},$$
$$d_{ae}^g = d_e^g + 2 \cdot m_e \cdot \cos\delta_2 = 315.21 + 2 \cdot 3.99 \cdot \cos 71°48' = 317.70 \text{ mm}.$$

5.12. Determine diameters of dedendum circles in the outer section for the pinion and the gear.

$$d_{fe}^p = d_e^p - 2.4 \cdot m_e \cdot \cos\delta_1 = 103.74 - 2.4 \cdot 3.99 \cdot \cos 18°12' = 94.64 \text{ mm},$$
$$d_{fe}^g = d_e^g - 2.4 \cdot m_e \cdot \cos\delta_2 = 315.21 - 2.4 \cdot 3.99 \cdot \cos 71°48' = 312.22 \text{ mm}.$$

5.13. Specify the external cone distance

$$R_e = 0.5 \cdot m_e \cdot \sqrt{\left(z^p\right)^2 + \left(z^g\right)^2} = 0.5 \cdot 3.99 \cdot \sqrt{26^2 + 79^2} = 165.92\text{mm}.$$

5.14. Specify the face width of the gear

$$b^g = \psi_{bR} \cdot R_e = 0.285 \cdot 165.92 = 47.23 \text{ mm}.$$

5.15. Determine mean pitch diameters for the pinion and for the gear

$$d_m^p=\frac{d_e^p\cdot(R_e-0.5\cdot b^g)}{R_e}=d_e^p\cdot\left(1-0.5\cdot\psi_{bR}\right)=103.74\cdot(1-0.5\cdot 0.285)=88.96\text{mm}$$

$$d_m^g=\frac{d_e^g\cdot(R_e-0.5\cdot b^g)}{R_e}=d_e^g\cdot\left(1-0.5\cdot\psi_{bR}\right)=315.21\cdot(1-0.5\cdot 0.285)=270.29\text{mm}$$

5.16. Determine forces that acts in the engagement of the bevel gears

- turning force $\quad F_t=\frac{2\cdot T^g}{d_m^g}=\frac{2\cdot 420\cdot 10^3}{270.29}=3108\ \text{N};$

- radial force at the gear

$$F_r^g=F_t\cdot\text{tg}\,\alpha_w\cdot\cos\delta_2=3108\cdot\text{tg}20°\cdot\cos71°48'=353.3\ \text{N};$$

- axial force at the gear

$$F_a^g=F_t\cdot\text{tg}\,\alpha_w\cdot\sin\delta_2=3108\cdot\text{tg}20°\cdot\sin71°48'=1074.4\ \text{N}\ .$$

5.17. Determine the maximum contact stress that develops in the contact zone of teeth:

$$\sigma_H=1.18\cdot\sqrt{\frac{T^p\cdot K_H\cdot E_{tr}}{\nu_H\cdot\left(d_m^p\right)^2\cdot b^g\cdot\sin 2\alpha_w}\cdot\left(\frac{\sqrt{u_{act}^2+1}}{u_{act}}\right)}=$$

$$=1.18\cdot\sqrt{\frac{153\cdot 10^3\cdot 1.29\cdot 2.1\cdot 10^5}{0.85\cdot 88.96^2\cdot 47.23\cdot\sin 40°}\cdot\left(\frac{\sqrt{3.04^2+1}}{3.04}\right)}=545.3\text{MPa},$$

where T^p is measured in N·mm; K_H is the design load factor determind as

$$K_H=K_{H\beta}\cdot K_{HV}.$$

Load concentration factor $K_{H\beta}$ is specified according to table 3.2 which depends upon factor $\psi_{bd}=\frac{b^g}{d_m^p}$.

Dynamic load factor K_{HV} is determined according to table 3.6 and depends upon the peripheral speed of the gear ($V^g=\frac{\omega^g\cdot d_m^g}{2}$) and the accuracy of manufacturing (table 3.5). If we using table 3.6 for bevel gears we should reduce the accuracy of manufacturing by 1.

In this case $\psi_{bd}=\frac{b^g}{d_m^p}=\frac{47.23}{88.96}=0.53$, $V^g=\frac{\omega^g \cdot d_m^g}{2}=\frac{25 \cdot 0.27}{2}=3.4$ m/sec.
$K_H=K_{H\beta} \cdot K_{HV}=1.16 \cdot 1.11=1.29$.

Obtained value of σ_H should correspond to the following condition

$$\sigma_H=(0.8\ldots1.1)\cdot[\sigma_H]=(0.8\ldots1.1)\cdot620=496\ldots682 \text{ MPa}.$$

Otherwise it is necessary to change the external pitch diameter and make calculation once more.

5.18. Determine the maximum bending stress

$$\sigma_b=\frac{F_t \cdot K_{b\beta} \cdot K_{bV} \cdot Y_b}{\nu_b \cdot m_m \cdot b^g}=\frac{3108 \cdot 1.32 \cdot 1.27 \cdot 3.6}{0.85 \cdot 3.42 \cdot 47.23}=136.6\text{MPa} \le [\sigma_b]=168\text{MPa},$$

where $K_{b\beta}$ is the load concentration factor defined according to table 3.7; K_{bv} is the dynamic load factor given in table 3.8 (for bevel gears we should reduce the degree of accuracy by 1); Y_b is the tooth shape factor that is defined according to table 3.9 depends upon the number of teeth of the equivalent straight spur gear $z_v^g=\frac{z^g}{\cos\delta_2}=\frac{79}{\cos 71°48'}=253$ for the case when the offset factor x=0;

ν_b = 0.85 is the correction factor; $m_m=\frac{d_m^g}{z^g}=\frac{270.29}{79}=3.42\text{mm}$ is the mean module.

6. **Analysis and design of shafts**

6.1 Find the minimum diameter of speed reducer shafts

$$d_{min} = \sqrt[3]{\frac{T}{0.2 \cdot [\tau]}} ,$$

where T is the torque at the shaft is measured in N·mm; [τ] is the allowable torsion stress in MPa.

In order to compensate action of bending stresses, the allowable tangential stress is considered as down rated. For steels [τ] = 15…20 MPa.

The obtained value of d_{min} is rounded up according to the following standard series: 20, 21, 22, 23, 24, 25, 26, 28, 30, 32, 34, 36, 38, 40, 42, 45, 48, 50, 52, 55, 58, 60, 65, 70, 75, 80, 85, 90, 95, 100,105, 110, 115, 120, 130, 140, 150.

In general-purpose speed reducers the stepped shafts with solid cross-section are used as a rule.

For the input shaft d_{min} is the diameter of the shaft cantilever portion where such elements as a half coupling, a pulley, a sprocket or a pinion may be mounted (Fig. 6.1). In order to fix the above mentioned elements in the axial direction we use a shoulder which height t_1 may be ranged from 2 to 5 mm depending on the shaft diameter. Recommended values of t_1 are given in table 6.1.

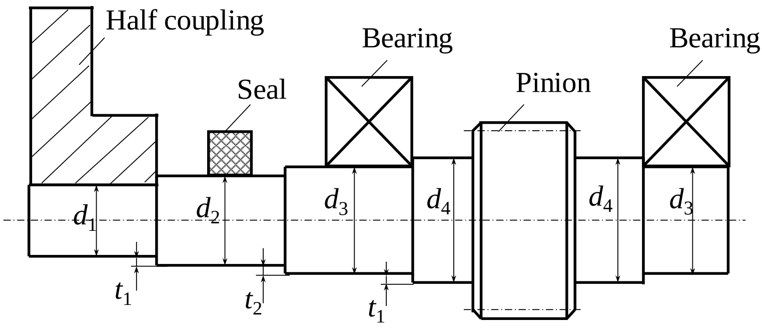


Fig.6.1. Input shaft

The next shaft section of diameter $d_2=d_1+2\cdot t_1$ (the value of d_2 must correspond to standard series) is for seal installation. Seals are used to prevent bearing assemblies from dust and dirt accumulation and lubrication leakage from bearings. For general-purpose speed reducer commercial seals are used more frequently used.

In order to reduce friction at the point of seal contact with the shaft, the corresponding section should be polished. For this purpose this section is additionally surface hardened to 45-50 HRC.

Table 6.1

Recommended values of t_1 and t_2

d, mm	20 - 50	55 - 120
t_1, mm	2; 2.5	5
t_2, mm	1; 1.5	2.5

The next shaft section is used for bearing mount. The diameter of this section is determined as

$$d_3 = d_2 + 2\cdot t_2,$$

where t_2 is the height of the shoulder that is used for differentiation of shaft surfaces by hardness and roughness. Recommended values of t_2 are given in table 9.2. It is necessary to note that t_2 should be chosen to obtain shaft diameter d_3 ended by 0 or 5. It is explained by the fact that bearings are standard elements with the inner ring diameter value should be multiple of 5.

Bearings must be fixed in the axial direction. That is why the diameter of the next section of the shaft, where a pinion or gear is installed, is determined as

$$d_4 = d_3 + 2\cdot t_1.$$

The obtained value of d_4 should correspond to standard series.

A pinion may be made either as integral with the shaft or as a separate part. In order to increase shaft strength and rigidity it is recommended to use pinion shafts.

The last section of the shaft is for installing the second bearing. The diameter of this section should be the same as the diameter of the first bearing. In our case it is d_3.

The output shaft has the same design as the input one. But in contrast to the latter a gear is mounted on the shaft section of diameter

d_4 (Fig. 6.2). In order to fix the gear in the axial direction we should provide the shoulder height t_1. That is why the diameter of the next section of the shaft is $d_5 = d_4 + 2 \cdot t_1$.

For our case we should design the output shaft where a helical spur gear is mounted. We will have the following diameters:

$d_{min} = \sqrt[3]{\dfrac{400 \cdot 10^3}{0.2 \cdot [20]}} = 46{,}4\ \text{mm}$, that's why d_1 = 48 mm (according to the standard series);

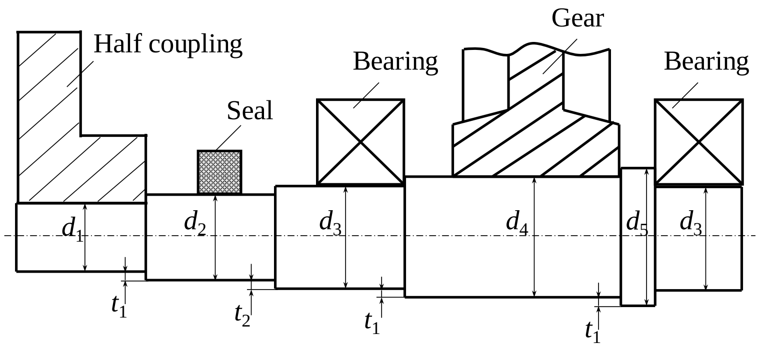


Fig. 6.2. Output shaft

$$d_2 = d_1 + 2 \cdot t_1 = 48 + 2 \cdot 2.5 = 53\ \text{mm},\ d_2 = 55\ \text{mm};$$
$$d_3 = d_2 + 2 \cdot t_2 = 55 + 2 \cdot 2.5 = 60\ \text{mm};$$
$$d_4 = d_3 + 2 \cdot t_1 = 60 + 2 \cdot 5 = 70\ \text{mm};$$
$$d_5 = \text{d}_4 + 2 \cdot t_1 = 70 + 2 \cdot 5 = 80\ \text{mm}.$$

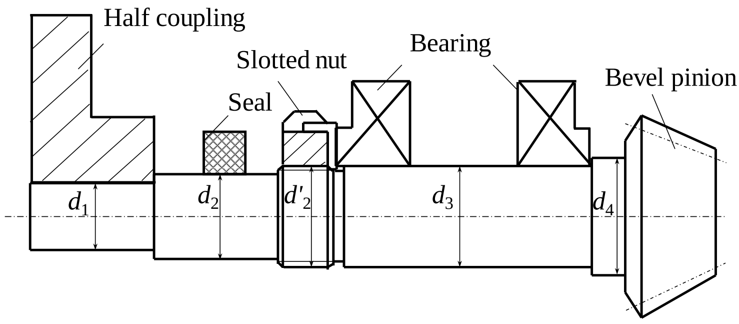


Fig. 6.3. Bevel pinion shaft construction

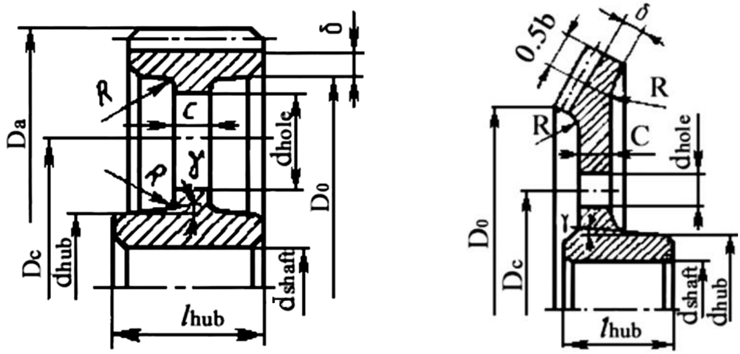
6.2. Determine sizes of elements that are installed on the shaft.

6.2.1. Pinion.

Face width of the pinion $b^p = b^g + 5$.

6.2.2. Spur and bevel gears (Fig.6.4, *a, b*)

- thickness of the rim $\delta = (3\ldots4)\cdot m$;
- thickness of the web $C = (0.2\ldots0.3)\cdot b^g$;
- diameter of the hub $d_{hub}=(1.5\ldots1.7)\cdot d_{shaft}$;
- length of the hub $l_{hub}=(1.2\ldots1.5)\cdot d_{shaft}$;
- diameter of the hole $d_{hole} = \dfrac{D_0 - d_{hub}}{4}$;
- diameter of the hole centre line $D_c = \dfrac{D_0 + d_{hub}}{2}$;
- fillet radii $R \geq 6$ mm and angle $\gamma \geq 7^o$.



a *b*

Fig.6.4. Spur gear (*a*), bevel gear (*b*)

7. Calculation of keyed joints.

Dimensions of keys are chosen according to table 7.1 which depends upon the shaft diameter. The length of the key should be less than the hub length by 5…10 mm and correspond to the standard series.

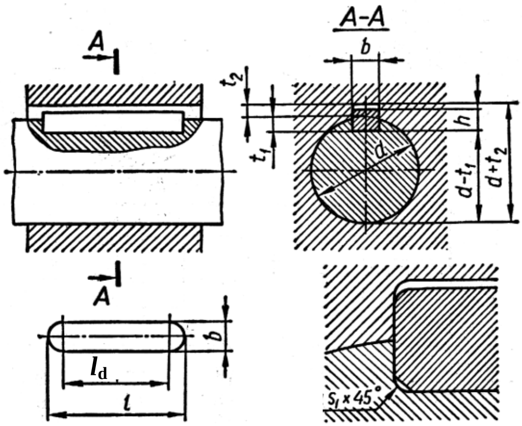
In general-purpose speed reducer, keyed joints are usually analyzed to prevent bearing stresses.

$$\sigma_{bear} = \frac{2\cdot T}{d\cdot(h-t_1)\cdot l_d} \leq [\sigma_{bear}],$$

where T is the torque in N·mm; d is the diameter of the shaft in mm; h is the height of the key in mm; t_1 is the depth of the slot in the shaft; l_d is the design length of the key in mm (for keys with round sides $l_d = l - b$; for keys with square sides $l_d = l$, where l is the length of the key; b is the width of the key); [σ_{bear}] is the allowable bearing stress (for cast-iron hubs [σ_{bear}]=60…80 MPa; for steel hubs [σ_{bear}]=100…120 MPa).

Table 7.1

Standard Sunk Keys



Shaft diameter d	Key cross section		Keyseat depth		Length l
	b	h	shaft, t_1	hub, t_2	
Over 17 to 22	6	6	3.5	2.8	Over 14 to 70
Over 22 to 30	8	7	4	3.3	Over 18 to 90
Over 30 to 38	10	8	5	3.3	Over 22 to 110
Over 38 to 44	12	8	5	3.3	Over 28 to 140
Over 44 to 50	14	9	5.5	3.8	Over 36 to 160
Over 50 to 58	16	10	6	4.3	Over 45 to 180
Over 58 to 65	18	11	7	4.4	Over 50 to 200
Over 65 to 75	20	12	7.5	4.9	Over 56 to 220
Over 75 to 85	22	14	9	5.4	Over 63 to 250
Over 85 to 95	25	14	9	5.4	Over 70 to 280
Over 95 to 110	28	16	10	6.4	Over 80 to 320
Over 110 to 130	32	18	11	7.4	Over 90 to 360

Note: The length of the key is chosen according to the following series: 6; 8; 10; 12; 14; 16; 18; 20; 25; 28; 32; 35; 40; 45; 50; 56; 63; 70; 80; 90; 100; 110; 125; 140; 160; 180; 200.

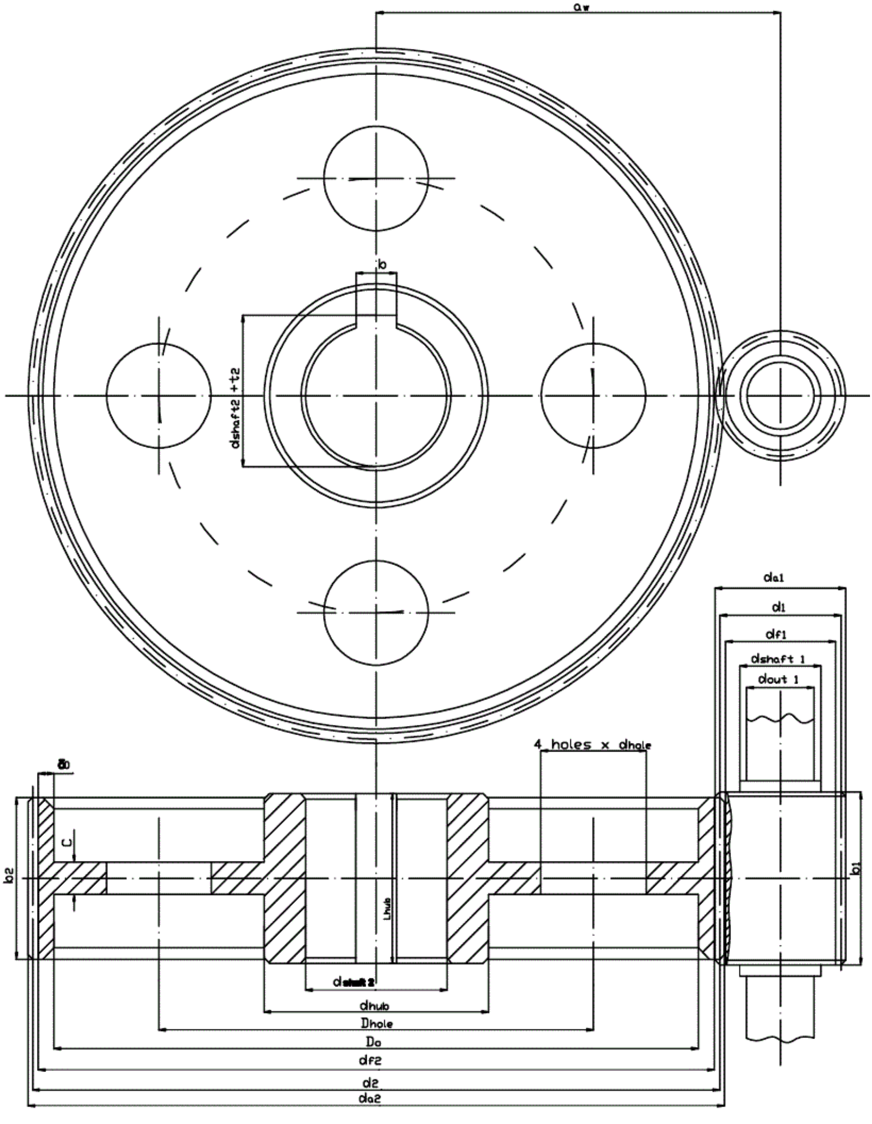


Fig. 1. Helical-Spur Gearing

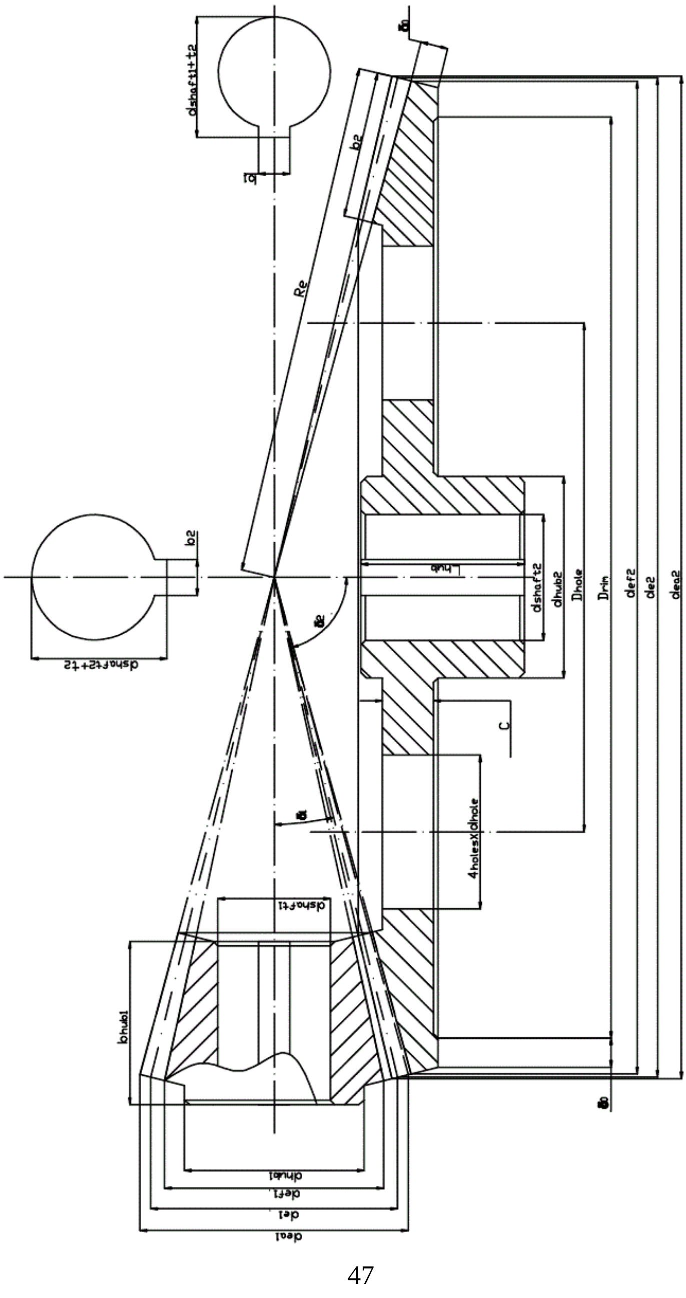
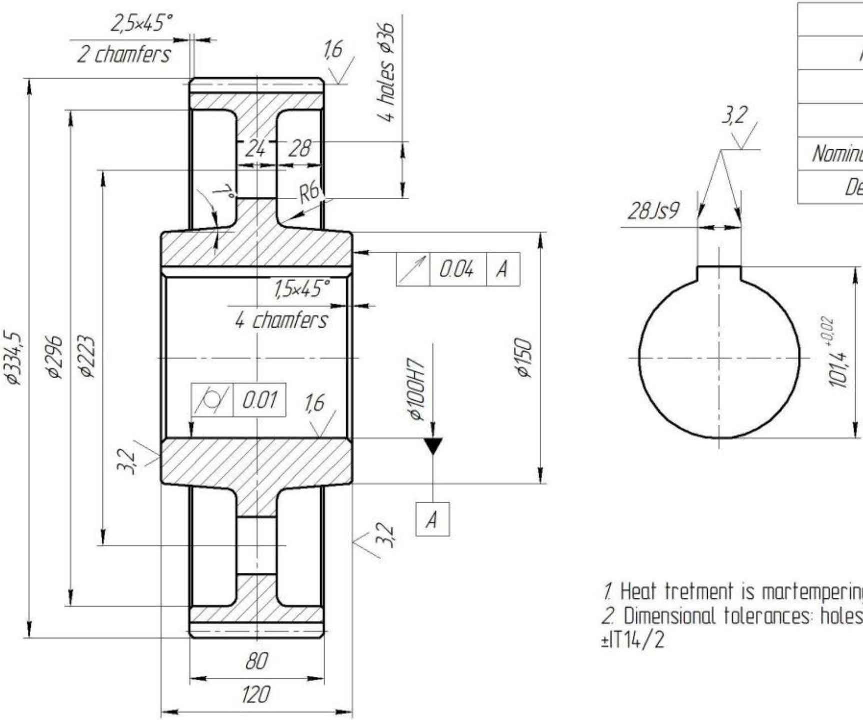


Fig. 2. Bevel gearing

6,3

Module	m	4
Number of teeth	z	98
Helix angle	β	11.48°
Offset factor	x	0
Nominal pith circle diameter	d	326.5
Degree of accuracy		9



1. Heat tretment is martempering to hardness 269–302 BHN
2. Dimensional tolerances: holes – H14, shafts – h14, other elements ±IT14/2

Fig. 3. Spur gear construction

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