METHODS OF FINDING THE SOLUTION OF THE EQUATIONS

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Solving equations is given great importance. There are many different ways of solving algebraic equations that require a lot of different knowledge and are widely used. This is due to the fact that many geometric problems, problems in Physics, Chemistry, Biology and other sciences are solved with their help. In most practical and scientific problems, where any quantity cannot be directly measured or calculated using a ready-made formula, there are relations that it satisfies. Comparing different ways of solving one problem allows students to find the most rational among them and leads them to believe that it is necessary to conduct a deeper analysis of the condition. Therefore, the question of applying various methods for solving equations to other types of problems is quite relevant. In my work I want to consider algebraic equations and methods for solving them. The expression $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$, where $a_s, s = 0,1, \dots, n$, are constants $(a \neq 0)$ and n is a positive integer, is called a *polynomial* in x of degree n. The polynomial $f(x) = 0$ is called an *algebraic equation of degree n.* If $f(x)$ contains some other functions such as trigonometric, logarithmic, exponential etc.; then $f(x) = 0$ is called a *transcendental equation.* The value of x which satisfies $f(x) = 0$, is called its root. Geometrically, a root of this equation is the value for which the graph of $y = f(x)$ crosses the *x*-axis. We often come across problems in deflection of beams, electrical circuits and mechanical vibrations which depend upon the solution of equations.

Let's consider the general properties of the solution of equations:

1. *If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $x - a$ and conversely.*

For instance, 3 is a root of the equation $x^4 - 6x^2 - 8x - 3 = 0$, because $x = 3$ satisfies this equation. In the other words, $x - 3$ divides $x^4 - 6x^2 - 8x - 3$ completely, i.e., $x - 3$ is its factor.

2. *Every equation of the nth degree has n roots (real or imaginary).*

Conversely, if $a_1, a_2, \dots, a_n$ be the roots of the *n*th degree equation $f(x) = 0$, then $f(x) = A(x - a_1)(x - a_2) \dots (x - a_n)$, where A is a constant.

If a polynomial of degree n vanishes for more than n value of x, it must be identically zero.

Solve the equation $2x^3 + x^2 - 13x + 6 = 0$.

By inspection, we find $x = 2$ satisfies the given equation. 2 is its root, i.e. $x - 2$ is a factor of $2x^3 + x^2 - 13x + 6$. Dividing this polynomial by $x - 2$, we get the quotient $2x^2 + 5x - 3$ and remainder 0. Equating the quotient to zero, we get $2x^2 + 5x - 3 = 0$. Hence, the roots of the given equation are $2, -3, 1/2$.

We can represent this process by using the synthetic division. The labor of dividing the polynomial by $x - 2$ can be saved considerably by the following simple device called **synthetic division.**

2	1	-13	6	2
	4		-6	
2	5	-3	0	

To find the solution of this equation we must do the following: (a) write down the coefficient of the powers of x in order (supplying the missing powers of x by zero coefficients and write 2 on extreme right; (b) put 2 as the first term of 3rd row and multiply it by 2, write 4 under 1 and add, giving 5; (*c*) multiply 5 by 2, write 10 under -13 and add, giving – 3; (*d*) multiply -3 by 2, write -6 under 6 and add given zero. Thus the quotient is $2x^2 + 5x - 3$ and remainder is zero.

3. **Intermediate value property.** If $f(a)$ and $f(b)$ have different sings, then the equation $f(x) = 0$ has at least one root between $x = a$ and $x = b$.

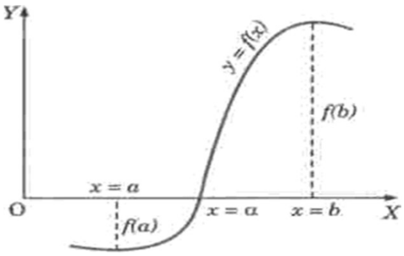


Fig. 1

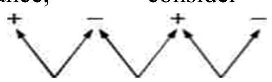
The polynomial $f(x)$ is a continuous function of x (Fig. 1). So while x changes from a to b, $f(x)$ must pass through all the values from $f(a)$ or $f(b)$. But since one of these quantities $f(a)$ or $f(b)$ is positive and the other negative, it follows that at least for one value of x (say α) lying between a and b, $f(x)$ must be zero. Then α is the required root. *In an equation with real coefficients, imaginary roots occur in conjugate pairs, i.e.,* if $\alpha + i\beta$ is a root of the equation $f(x) = 0$, then $\alpha - i\beta$ must also be its root. Every equation of the odd degree has at least one real root. This follows from the fact that imaginary roots occur in conjugate pairs.

Solve the equation $3x^3 - 4x^2 + x + 88 = 0$*, one root being* $2 + \sqrt{7}i$.

Since one root is $2 + \sqrt{7}i$, the other root must be $2 - \sqrt{7}i$. The factors corresponding to these roots are $(x - 2 - \sqrt{7i})$ and $(x - 2 + \sqrt{7i})$ or $(x - 2 - \sqrt{7i})(x - 2 + \sqrt{7i}) = (x-2)^2 + 7 = x^2 - 4x + 11$, which is a divisor of $3x^3 - 4x^2 + x + 88$. Division of this solution by $x^2 - 4x + 11$ gives $3x + 8$ as the quotient. Thus, the depressed equation is $3x + 8 = 0$. Its root is $-8/3$. Hence the roots of the given equation are $2 \pm \sqrt{7i}, -8/3$.

4. **Descarte's rule of signs.** *The equation* $f(x) = 0$ *cannot have more positive roots than the changes of signs in* $f(x)$*; and more negative roots than the changes of signs in* $f(-x)$.

For instance, consider the equation $f(x) = 2x^7 - x^5 + 4x^3 - 5 = 0$. Signs of $f(x)$ are



Clearly, $f(x)$ has 3 changes of signs (from + to – or – to +). Thus, cannot have more than 3 positive roots.

Also $f(-x) = 2(-x)^7 - (-x)^5 + 4(-x)^3 - 5 = -2x^7 + x^5 - 4x^3 - 5$. This shows that $f(x)$ has 2 changes of signs. Thus, this solution cannot have more than 2 negative roots.

The concept of an equation is one of the fundamental concepts of algebra, because an equation is a kind of form of analytical method of thinking. That is, the equation can be considered as a symbolic record for knowing real reality. It is by solving equations that we develop the ability to reason, compare equations with those solved earlier, and find common and different things in them. Solving algebraic equations makes it possible to apply in a complex all the methods familiar to us from polynomial transformations, as well as the need to expand the set of real numbers and form a field of complex numbers.

List of used sources:

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