Exponential Functions as Mathematical Models

Kushnir O.V.
National Aviation University, Kyiv
Supervisor-Klyus I.S.,assistant professor,PhD

Key words: **mathematical models, exponential functions,**

Exponential functions are useful in modeling many physical phenomena, such as populations, interest rates, radioactive decay, and the amount of medicine in the bloodstream. An exponential model is of the form $A =$ [redacted]$_0(b)^{t/c}$, where we have A_0 is the initial amount of whatever is being modelled, t is elapsed time.

Often problems dealing quantities that grow or shrink with respect to time can be modeled by exponential growth functions or exponential decay functions. Suppose $Q(t)$is a quantity whose rate of growth is directly proportional to the present quantity $Q(t)$.. Furthermore, suppose the initial quantity $Q(0)$ is denoted as.The mathematical formulation of these conditions can be written as $Q^{`}(t) = kQ(t)$ and $Q(0) = Q_0$.

1.

If the quantity increases as time increases and k is a positive number then we say that the exponential model is a growth model.

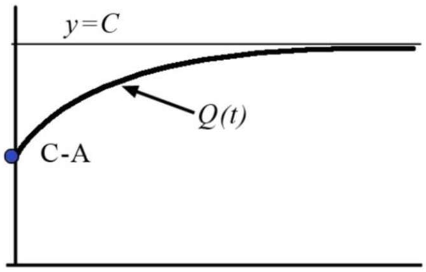
If the quantity decreases as time increases and k is a negative number we say $Q(t) = Q_0e^{kt}$is a decay model.

If k is positive, it is called a growth constant and if$k$ is negative, it is called a decay constant.

2.

A function of the type $Q(t) = C - Ae^{-kt}$, with all the constants positive, can be used to model the way people and animals learn new topics. When$t = 0$, we see the value of$Q(T)$is$C - A$.Notice that as$t$tends to increase without bound, e^{-kt} tends toward0.Thus, a horizontal asymptote for $Q(t)$ is $y = c$.

The graph for$Q(t)$is shown below:



Let's illustrate these:

3.

The half-life of a radioactive substance is4.3days. This means that the quantity of the substance decays by half after 4.3 days. Thus, if we begin with a 100-gram sample, only 50 grams will remain after 4.3 days. After another 4.3 days, only 25 grams will remain, and so forth. To the determine the decay constant$k$, we solve the equation 50=100$e^{k*4.3}$ for the constant$k$. Simple arithmetic leads to$k \simeq -.1612$We can now use this constant to write the general equation for the quantity present,Q=100$e^{-.1612*t}$.After 12 days, our 100-gram sample has decayed to$100e^{-.1612*12}$, which approximates to 14.45 grams.Then we would solve the equation$5 = 100e^{-.1612*t}$for the variable t.And the answer is approximately 18.58 days.

This function$Q(t)$ and its associated graph has been shown to closely model the way people and animals learn. Knowledge is gained quickly at first, but the rate of learning, the derivative of$Q(t)$, decreases as time increases. In the long run, the rate of learning tends to slow to the point where new material is difficult to learn.

References:

1. Higher mathematics: manual / V. P. Denisiuk, V. M. Bobkov, L. I. Grishina and others. –K. : NAU, 2006. – Part 1. – 268 p.
2. Larson R. College Algebra / R. Larson, R. Hosteller. – Houghton Mifflin, 1997. – 545 p.
3. Mizrahi A. Calculus and Analytic Geometry / A. Mizrahi, M. Sullivan. – California: Wadsmonth Publishing Company, 1987. – 1083 p.