УДК 532.526

UNIVERSAL MULTILEVEL APPROACH FOR MATHEMATICAL CONSTRUCTING THE MODELS OF NEAR-WALL TURBULENCE

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Turbulence modeling is one of very actual practical problems, having a wide range of different technical applications. At the same time, even now this is one of the most difficult scientific problem due to complexity of physical mechanism of turbulence phenomenon. The goal of this research is to build a universal as far as possible approach for constructing the mathematical description of turbulent viscosity and illustrate its applicability on several different levels of near-wall flow complexity. The following basic universal model form of turbulent viscosity coefficient ν_t, proposed by V. Movchan [1]:

$$\nu_t = \nu_{tout} \tanh \frac{\nu_{tin}}{\nu_{tout}}, \quad (1)$$

and different possibilities of description of ν_{tout}, ν_{tin} (values of ν_t in external and internal regions respectively) are the subjects of this research. According to traditional classification, the model of turbulence has algebraic or differential type depending on the applied methods of mathematical description of governing turbulence parameters. Structure of (1) allows to construct algebraic, differential and hybrid turbulence model by means of different forms of ν_{tout} and ν_{tin} presentation. The simplest **basic algebraic form** of ν_t can be effectively constructed as the following slightly modified form of (1)

$$\nu_t = \gamma \nu_{tout} \tanh \frac{\nu_{tin}}{\nu_{tout}},$$

where γ – intermittency factor. Internal region has the following structure of its mathematical description: $\nu_{t\,in} = lD_m$, $l = \kappa y \sqrt{\tau^+} \upsilon_*$,

$$D_m = \tanh \frac{\sinh^2 [\kappa_0 y^+ (1 + \kappa_3 |y^+ - 30|) \sqrt{\tau^+}] \ \tanh[\sinh^2 (\kappa_2 y^+ \sqrt{\tau^+})]}{\kappa y^+ \sqrt{\tau^+}}.$$

Linearization of the argument of the function $\sinh^2 [\kappa_0 y^+ (1 + \kappa_3 |y^+ - 30|) \sqrt{\tau^+}]$ transforms the damping factor D_m to the following simplified form

$$D_m = \tanh \frac{\sinh^2 (\kappa_1 y^+ \sqrt{\tau^+}) \ \tanh[\sinh^2 (\kappa_2 y^+ \sqrt{\tau^+})]}{\kappa y^+ \sqrt{\tau^+}}$$

that allows to obtain the approximate-analytical solutions for velocity profile within viscous and buffer sublayers of boundary layer.

External region can in frames of algebraic level of modeling be presented by the formula $\nu_{tout} = \chi U_H \delta^*$, $\chi = 0.0168$.
Here and above $\kappa_0, \kappa_1, \kappa_2, \kappa_3, \kappa, \chi$ – model coefficients; $y^+ = y\upsilon_* / \nu$ – normal to wall coordinate y, dimensionless according to the wall law; ν – kinematic viscosity coefficient; $\upsilon_* = \sqrt{\tau_w / \rho}$ – shear velocity; $\tau^+ = \tau/\tau_w$ – dimensionless shear stress by its value on a wall τ_w, that depends on pressure gradient parameter $p^+ = \frac{\nu}{\rho \upsilon_*^3} \frac{dp}{dx}$ as follows: $\tau^+ = 1 + p^+ y^+$ for $p^+ \geq 0$ and $\tau^+ = (1 - p^+ y^+)^{-1}$ for $p^+ < 0$; p – averaged pressure; ρ – density; U_H – velocity on the external free boundary of a wall shear flow; δ^* – displacement thickness, using as a linear scale in external region of boundary layer.

Modeling of near-wall effects of control. In case of necessity to account the influences of micro-riffling and/or polymeric additives we apply the fact that these effects, despite on their different physical mechanism, have similar effect of velocity profile shifting in semilogarithmic coordinates. Effective results have been found on the base of implementation and generalization of I. Rotta approach for roughness accounting. This approach requires modifying the normal coordinate y^+ into y_1^+ in the following way: $y_1^+ = 0$ for $s \leq 0$ and $y_1^+ = s$ for $s > 0$, where $s = y^+ + \Delta y^+_{rough} - \Delta y^+_{pol}$; Δy^+_{rough} – generalized parameter, accounting both irregular and regular roughness influence; Δy^+_{pol} – additional parameter that similarly to previous one describes the influence of polymeric additives injecting in near-wall region of a water boundary layer.

Modified algebraic model of turbulent viscosity. This modification effectively combines the described above set of dependencies for internal region ν_{tin} together with well-known and popular in turbomachinery Baldwin-Lomax approach for external region ν_{tout}, namely:

$$\nu_{t\,out} = \chi C_{cp} F_{wake}; C_{cp} = 1.6;\ F_{wake} = \min\left(y_{\max} F_{\max}; C_{wk} y_{\max} \Delta U^2 / F_{\max}\right);$$

$$C_{wk} = 0.25;\ \Delta U = |U_{\max}| - |U_{\min}|; \gamma = F_{Kleb} = \left[1 + 5.5(C_{Kleb} y / y_{\max})^6\right]^{-1}; C_{Kleb} = 0.3;$$

$$F(y) = y\left|\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\right| D_m;\ F_{\max} = \max[F(y)];\ y_{\max} = y(F_{\max}).$$

Hybrid algebraic – one-parametric differential model of turbulence. This is the next step of turbulence models elaboration, joining both algebraic and differential approaches for internal and external regions respectively. The turbulent viscosity coefficient is presented as a function of kinetic energy of turbulence k: $\nu_{t\,out} = \chi_k \Delta \sqrt{k}$, where $\Delta = U_H \delta^* / \upsilon_*$. This approach requires solving the corresponding transport partial differential equation for determination of k, that makes it more universal and exact for wider range of modeled types of boundary layers.

Hybrid algebraic – two-parametric differential model of turbulence. Here we demonstrate one more modification of basic turbulent viscosity mathematical description (1) for external region – two-parametric $k-\varepsilon$ approach: $\nu_{t\,out} = C_\mu k^2/\varepsilon$, $C_\mu = 0{,}09$, where ε – dissipation rate of k. Simplification and adopting of the constructed model for internal region allows to determine the profiles $k(y)$, $\varepsilon(y)$ and Reynolds stresses $\tau(y)$. For viscous and buffer sublayers these distributions in dimensionless form have the following structures:

$$\bar{k} = k/(\upsilon_*^2\tau^+) = \tanh(\kappa_1 y^+\sqrt{\tau^+})\sqrt{\tanh[\sinh^2(\kappa_2 y^+\sqrt{\tau^+})]/C_0};$$

$$C_0 = 0.16[1+\tanh(0.13y^+\sqrt{\tau^+})];\ k^+ = y\sqrt{k}/\nu;$$

$$\bar{\varepsilon} = \frac{\varepsilon\nu}{\upsilon_*^4\tau^{+2}} = \frac{\tanh^2(\kappa_{11}\sqrt{k^+})}{\cosh^2(\kappa_{11}\sqrt{k^+})}\tanh[\sinh^2(\kappa_{22}\sqrt{k^+})] + D_\varepsilon;$$

$$D_\varepsilon = 0.0316\,\bar{k}D_0/k^+;\ D_0 = 1-\tanh[0.5\sqrt{k^+}(\sqrt{k^+}-4.3)];$$

$$\bar{\tau} = -\overline{u'v'}/(\upsilon_*^2\tau^+) = \tanh^2(\kappa_{11}\sqrt{k^+})\tanh[\sinh^2(\kappa_{21}\sqrt{k^+})].$$

In logarithmic zone the corresponding dependencies have simpler structures: $\bar{k} \cong 1/C_0$; $\bar{\varepsilon} = 1/(\kappa_{01}k^+)$; $\bar{\tau} \cong 1$, where κ_{01}, κ_{11}, κ_{21}, κ_{22} – model coefficients.

The advantages of the last approach have been applied by Ye. Shkvar for elaborating the family of models of flow control factors, directed both on small-scale and large scale vortical structures of near-wall turbulence [2]. Nevertheless, even clearly algebraic turbulence models can be quite effective for modeling of some special kinds of practically interesting shear flows. As it has been demonstrated by T. Kozlova & Ye. Shkvar the algebraic description of turbulent viscosity gives reliable results in predictions of properties of gravity film flows developing along inclined surfaces of airstrips with strong artificial roughness [3].

Conclusions. The proposed multilevel family of approaches for semiempirical turbulence modeling demonstrates the correctness of the applied hypotheses and flexibility for wide range of wall shear flows types. In particular, these models have been effectively applied for constructing the models for accounting the basic methods of turbulent flow control (riblets, polymeric additives, Large Eddy Breakup Devices, wall jets) both in case of their single application and in different useful combinations.

References

1. Математичні моделі пристінної турбулентності: монографія/ М.С. Кулик, В.Т. Мовчан В.Т., Є.О. Шквар. – К.: НАУ, 2012. – 356 с.
2. Шквар Є.О. Математичне моделювання ефектів комбінованого управління турбулентними примежовими шарами / Є.О. Шквар // Вісник Черкаського унів.: сер. Прикл. матем. Інформ.– 2011. – Вип.194. – С.5–15.
3. Шквар Є.О. Математична модель шару стоку дощових опадів зі злітно-посадкової смуги за наявності бокового вітру / Є.О. Шквар, Т.В. Козлова // Системи озброєння і військова техніка – 2012. – №1(29). – С. 143–151.