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Mode-Matching Method Applied to the Sound Reception Problem Using Helmholtz Resonator

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The study explores the problem of forming an acoustic field in the Helmholtz resonator cavity using the mode-matching method. Acoustic boundary value problems for the description of acoustic fields in resonator cavities under traditional boundary conditions and boundary conditions at the edge (and in its absence), which is characterized by the known acoustic properties, are set and solved. For certain dimensions of an air-filled resonator, the basic field characteristics in the resonator are calculated, analyzed and compared with experimental data. The influence on the results of field formation of the involved condition at the edge with acoustically rigid boundaries-surfaces is estimated. In view of the foregoing, the research aims to formulate and solve the wave problem of acoustic field formation in the cubic Helmholtz resonator taking into account classical boundary conditions, conjugation conditions, and conditions at angular points of structural elements formed by the mode-matching and applying the eponymous method.

Keywords: Resonator, Acoustic field, Mode-matching method, Frequency characteristic, Interference.

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1. INTRODUCTION

Traditionally, the study of acoustic properties of the Helmholtz resonator as an oscillatory system with concentrated parameters was carried out using the methods of acousto-mechanical analogies (see, for example, [1]). However, further advancement and development of the methodology of numerical and experimental research of acoustic field formation in cavities or outer space of resonators and systems based on them required enrichment and refinement of research approaches to the use of resonator models as distributed parameter systems (e.g., tube-shaped sets), which is shown in [2]. Calculations of the acoustic fields of single receivers more accurately presented the picture of acoustic field formation in the resonator.

The next step in the development should be an approach to involving in the formulation of wave problems not only the canonical idealized surfaces and formulated for them classical Dirichlet and Neumann conditions, their combinations, conjugation of force and kinematic type conditions, but also additional surface conditions that will include angular points and lines.

The computational situation can be supported by such a numerical method as the finite domain method, which involves dividing the oscillatory system into a certain number of constituent elements, an arbitrary choice of an approximating function [3] followed by integration of individual solutions. The disadvantages of this situation were an arbitrary approach to the choice of this function, as well as the accuracy of the solution associated with such a choice. However, this method in wave acoustics problems was later developed as the mode-matching method. The method was researched and implemented in acoustic practice by scientific schools of the Institute of Hydromechanics of the National Academy of Sciences of Ukraine and the Department of Acoustic and Multimedia Electronic Systems of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute". The main provisions for its application are introduced in [4-7].

This method gained widespread attention due to its capability to divide the studied acoustic object into the canonical in their shape areas and the use of appropriate solutions for the fields' constituents. The combination of individual elements of areas into a single whole, which forms and describes the studied structure, requires not only traditional boundary conditions such as Dirichlet and Neumann conditions, kinematic and force conjugation conditions for all selected rectangular (cubic) areas, but also angular conditions for points (edges), which are formed at the intersections of the transition from one area to another [4].

Thus, the approximation of the computational situation to the real physical one determines the relevance of the work – as a new scientific approach to solving the problems of receiving wave acoustics reception for resonators of the Helmholtz type.

In connection with the above, this study aims to formulate and solve the wave problem of acoustic field formation in the cubic Helmholtz resonator taking into account classical boundary conditions, conjugation conditions and conditions at angular points of structural elements formed by the mode-matching method.

2. FORMULATION OF THE PROBLEM

Let us consider a cubic resonator (Fig. 1) with elements represented by three areas: two main areas I and II (area I – neck, area II – resonating cavity) and

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one additional (III), which covers the area around the top of the rectangular wedge point (Fig. 2). The size of area III is much smaller than the wavelength of external influence: $\rho_0 << \lambda$.

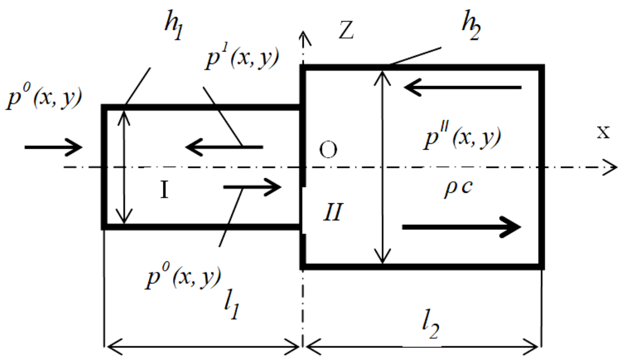


Fig. 1 – Schematic representation and basic dimensions of the cubic resonator

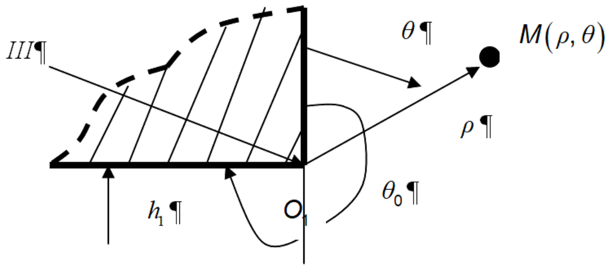


Fig. 2 – Area III and the angular point of a rectangular wedge

Suppose that the resonator is presented by a flat waveguide of the variable cross section with acoustically rigid surfaces (Fig. 1). The waveguide is filled with an ideal medium with the density and velocity of sound and is oriented along the axis of the rectangular coordinate system xOy. The dimensions of the sections are h_1, l_1 for the neck and h_2, l_2 for the resonating cavity, where h_1 is the vertical cross-sectional neck size, l_1 is the neck length, h_2 is the vertical section of the cavity, l_2 is the length of the resonator cavity. Areas I and II are separated by a segment $-h_1 \le y \le h_1$ of the intersection $x = 0$, and the bottom of the resonator acts as a reflective surface and is represented by a segment $-h_2 \le y \le h_2$ in the intersection $x = l_2$. The condition of a narrow tube is fulfilled for all resonator sizes and the size of area III.

The solutions to the problem in areas I and II must correspond to the known [7, 8] solutions of the Helmholtz equations in rectangular coordinates obtained for regular waveguides and mode-matching components. Regarding area III, we note that when the environment bypasses the surfaces areas that include angular points, the oscillation velocity field is characterized by the presence of local features i.e., the effect of increasing to infinity of the oscillating velocity values when approaching the vertex.

It is known that for a small (point) source the smallness of the area allows us to consider the medium in it as uncompressed, and the motion of the medium in area III as described by the Laplace equation for the wave potential $\Phi(\rho, \theta, t)$ with the appropriate boundary conditions:

$$\begin{gathered}\Delta\Phi^{III}(\rho,\theta,t)=0,\ \rho\in[0,0\pm\rho],\ \theta\in[0,3\pi/2],\\ \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\Phi^{III}(\rho,\theta,t)\right)=\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\Phi^{III}(\rho,\theta_0,t)\right)=0,\\ \rho\in[0,0\pm\rho],\ \theta_0\in[0,3\pi/2].\end{gathered} \tag{1}$$

In the future, one can go from potential terms to pressure terms without a multiplier $e^{-i\omega t}$:

$$\begin{gathered}\Delta p^{III}(\rho,\theta)=0,\ \rho\in[0,0\pm\rho],\ \theta\in[0,3\pi/2],\\ \frac{1}{i\omega\rho_m}\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p^{III}(\rho,\theta)\right)=\frac{1}{i\omega\rho_m}\frac{1}{\rho}\left(p^{III}(\rho,\theta_0)\right)=0,\\ \rho\in[0,0\pm\rho],\theta_0=3\pi/2,\end{gathered} \tag{2}$$

where Δ is the Laplace operator in rectangular coordinates, ω is the circular frequency.

We set the conjugation conditions of force and kinematic type, and the boundary conditions in the form of functional equations:

$$\begin{gathered}p_s^I(x,y)=p^{II}(x,y),\ x=0,\ y\in[0,0\pm h_1],\\ v_n=\frac{1}{i\omega\rho_m}\frac{\partial}{\partial x}\left(p_s^I(x,y)\right)=\frac{1}{i\omega\rho_m}\frac{\partial}{\partial x}\left(p^{iI}(x,y)\right),\\ x=0,\ y\in[0,0\pm h_1],\\ v_n=\frac{1}{i\omega\rho_m}\frac{\partial}{\partial x}\left(p^{iI}(x,y)\right)=0,\ x=l_2,\ y\in[0,0\pm h_2],\\ p^{III}(\rho,\theta)=p_s^I(x,y),\ x=0-\rho_0,\ y\in[h_1,h_1\pm\rho_0],\\ \theta_0\ge\theta\ge 0,\ \theta_0=\frac{3\pi}{2};\\ p^{III}(\rho,\theta)=p_s^{II}(x,y),\ x=0+\rho_0,\ y\in[h_1,h_1\pm\rho_0],\\ 0\le\theta\le\theta_0,\theta_0=\frac{3\pi}{2};\end{gathered} \tag{3}$$

where the first two functional equations are the conditions of conjugation of force and kinematic type, respectively. The third equation demonstrates the boundary conditions on an acoustically rigid surface and, in fact, is a Neumann problem.

The group of the last two functional equations represents the conditions on the surface of area III, the size and boundary of which is determined by the value ρ_0. This last pair of functional equations formally contains the conjugation conditions of the force type of area III and areas I and II when approaching the angular point from the left and right.

3. SOLUTION OF THE PROBLEM

Let a plane wave propagates from the far field of the specified system $p_0(x, y, t)$, penetrating through the section $x = l_1$ to the neck. The value of the cross section of the neck is chosen so that the inequality $2h_1 \le \lambda/2$ is fulfilled, where λ is the wavelength perturbed by the resonator. This will meet the condition of the above-mentioned "narrow tube", at which there can be only the lowest mode of the waveguide with rigid boundaries $n = 0$ in the resonator neck. If the condition of the "narrow tube" extends to the intersection $x = l_2$, inequalities $2h_1 \le \lambda/2$, $2h_2 \le \lambda/2$ do not hold, the number of waveguide modes increases, which further leads to the solution of an infinite system of linear algebraic equations with respect to unknown coefficients of series expansions.

Consider two consecutive situations:

– solution of the problem of the field formation in the resonator without taking into account the conditions at the edge with the angular point O_1;

– solution of the problem of the field formation in the resonator taking into account the conditions at the edge with the angular point O_1.

3.1 Problem 1. Solution of the Problem of the Field Formation in a Cubic Resonator without Taking into Account the Conditions at the Edge with the Angular Point O_1

Let the following waves propagate in the neck of the resonator:

● harmonic direct plane wave

$$p^0(x,y,t) = -i\omega\rho_m a_0 e^{-i(\omega t-kx)} \rightarrow p^0(x,y) = A_0 e^{ikx}, \quad (4)$$

where $k = \omega/c$ is the wave number, $A_0 = i\omega\rho_m a_0$, A_0 is the amplitude of the incident wave (hereinafter we believe that $A_0 = 1$, and the amplitudes of the direct and reflected waves in the respective areas A_n, B_n, C_n, D_n will be written down hiding the multiplier $i\omega\rho_m$;

● reflected wave $p^{I}(x, y)$, which is formed as a result of falling to the limit $x = 0$ of the flat wave (4);

● wave $p^{II}(x, y)$, which has passed into the volume of the cavity (to the right of the boundary $x = 0$, area II).

Let us consider these waves.

In area I, the acoustic field is formed as a superposition of normal waves with unknown amplitudes and incident waves. It can be written as a series:

$$p^I(x,y) = \sum_{n=0}^{\infty}\left(B_n cos(\alpha_n y)e^{-i\beta_n x}\right), \; n = 0, 1, 2, 3, \dots, \quad (5)$$

where $\alpha_n = n\frac{\pi}{h_1}$; $\beta_n = \sqrt{k^2 - (\alpha_n)^2}, k \geq \alpha_n$ contain wave numbers, B_n is an unknown coefficient.

Thus, the full field $p^{I}(x, y)$ in area I can be written as:

$$p_s^I(x,y) = p^0(x,y) + p^I(x,y) = A_0 e^{ikx} + \sum_{n=0}^{\infty}\left(B_n cos(\alpha_n y)e^{-i\beta_n x}\right), \; n = 0, 1, 2, 3, \dots \quad (6)$$

Let us turn to area II, which is a closed volume with acoustically rigid walls (boundary condition for the system of functional equations (3)). We will assume that the sound field in area II is formed only due to fluctuations in the original cross section of the neck $x = 0$ and repeated reflection of sound waves mainly from the bottom of the cavity $x = l_2$. Conditions at the edge are disregarded.

We present the field $p^{II}(x, y)$ as a superposition of normal waves of a plane waveguide limited in cross section $x = l_2$ by the acoustically rigid bottom, i.e.,

$$p^{II}(x,y) = \sum_{n=0}^{\infty}\left(C_n cos(\xi_n y)e^{i\gamma_n x}\right) + \sum_{n=0}^{\infty}\left(D_n cos(\xi_n y)e^{-i\gamma_n x}\right), \quad (7)$$
$$n = 0, 1, 2, 3, \dots$$

where $\xi_n = n\frac{\pi}{h_2}$; $\gamma_n = \sqrt{k^2 - (\xi_n)^2}$, $k \geq \xi_n$ contain wave numbers, C_n, D_n are unknown coefficients.

Due to the determination of unknown coefficients B_n, C_n, it is possible to fulfill the conjugation conditions (the first and second equations of system (1)) taking into account equations (5)-(7) and the properties of orthogonality of functions $\cos\left(n\frac{\pi}{h_1}y\right), \cos\left(n\frac{\pi}{h_2}y\right)$.

Substituting the equations for fields (6) and (7) into the functional equations (1), after a series of transformations, we obtain a system of the form:

$$\sum_{n=0}^{\infty} C_n \cos(n\frac{\pi}{h_2}y) + \sum_{n=0}^{\infty} D_n \cos(n\frac{\pi}{h_2}y) = 1 + \sum_{n=0}^{\infty} B_n \cos(n\frac{\pi}{h_1}y), \quad (8)$$

$$\sum_{n=0}^{\infty} \gamma_n C_n cos(n\frac{\pi}{h_2}y) + \sum_{n=0}^{\infty} \gamma_n D_n cos(n\frac{\pi}{h_2}y) = k + \sum_{n=0}^{\infty} \beta_n B_n cos(n\frac{\pi}{h_1}y), \quad (9)$$

where equation (8) is written for the force conjugation conditions, and (9) is for kinematic ones.

The transition from a system of functional equations to algebraic ones must take place in the traditional way. To do this, we multiply equation (8) by $cos\left(n\frac{\pi}{h_1}y\right)$ and integrate the left and right parts of (8) along the segment $[0, h_1]$. After integration we obtain

$$\sum_{n=0}^{\infty}\left(C_n + D_n\right)Sa(n) = \sum_{n=0}^{\infty} B_n h_1(1+\delta_n) + 2h_1\delta_n \;, \quad (10)$$

where $\delta_n = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$ is the Kronecker symbol,

$$Sa(n) = \left[2(-1)^n \frac{sin\left(\frac{n\pi h_2}{h_1}\right)}{\frac{n\pi h_2}{h_1}} \frac{h_2^2}{\left(h_2 - \frac{h_1^2}{h_2}\right)} \right].$$

Next, we multiply equation (9) by $cos\left(n\frac{\pi}{h_2}y\right)$ and integrate the left and right parts of (9) along the segment $[0, h_2]$. After integration we have:

$$\sum_{n=0}^{\infty}\gamma_n\left(C_n - D_n\right)h_2(1+\delta_n) = kh_2\delta_n + \sum_{n=0}^{\infty}\beta_n B_n \; Sa(n). \quad (11)$$

Thus, we obtain two algebraic equations (10) and (11) with three unknown coefficients B_n, C_n, D_n. To find the third coefficient (for example, D_n), we apply the condition of oscillating velocity $\upsilon_n(x, y)$ on an acoustically rigid boundary $x = l_2$ from the system of functional equations (1):

$$\text{v}_n(x,y)\big|_{x=l_2} = \frac{1}{i\omega\rho_m}\frac{\partial}{\partial x}\left(p^{iI}(x,y)\right) = 0$$

and the equation for the field in the cavity (7).

In narrow tubes with acoustically rigid boundaries, only the lower mode is present in areas I and II. The same, after substituting equation (10) into (11) and performing a series of transformations, we obtain for the required coefficients:

$$D_n = \frac{kh_2\delta_n + B_n\beta_n Sa(n)}{\gamma_n(cos(2\gamma_n l_2) - 1)h_2(1+\delta_n)}, \qquad (12)$$

$$C_n = D_n\ cos(2\gamma_n l_2), \qquad (13)$$

$$B_n = D_n a_n - \frac{2h_1\delta_n}{h_2(1+\delta_n)} = D_n b_n - \frac{kh_2\delta_n}{\beta_n Sa(n)}\ , \qquad (14)$$

where

$$a_n = \frac{\gamma_n(cos(2\gamma_n l_2)+1)Sa(n)}{h_2(1+\delta_n)}\ , b_n = \frac{\gamma_n((cos(2\gamma_n l_2) - 1))h_2(1+\delta_n)}{\beta_n Sa(n)}.$$

And finally,

$$D_n = \frac{\dfrac{2h_1\delta_n}{h_2(1+\delta_n)} - \dfrac{kh_2\delta_n}{\beta_n Sa(n)}}{a_n - b_n}. \qquad (15)$$

At the same time, we understand that in further transformations and calculations the index $n = 0$.

Thus, expressions (12)-(15) allow us to calculate the unknown coefficients of expansions (6) and (7) with their subsequent use to determine the field in the resonator.

3.2 Problem 2. Solution of the Problem of the Field Formation in a Cubic Resonator Taking into Account the Conditions at the Edge with the Angular Point O_1

In accordance with [5] for area III in polar coordinates (ρ, θ), the Laplace equation, written with respect to the potential, has a partial solution of the form:

$$\Phi_\nu(\rho,\theta) = \rho^\nu\left(M\ cos(\nu\theta) + N\ sin(\nu\theta)\right), \qquad (16)$$

where ν is a positive real number.

In the case of acoustically rigid boundaries (bounds) of the edge (see condition (2)), the coefficients of decomposition for the potential $\Phi_\nu(\rho, \theta)$ from expression (16) can be defined as follows. From the condition on the bounds $\theta = 0$ it follows that $N = 0$, and for the bound $\theta = \theta_0$ we obtain:

$$\begin{aligned} &\Phi_\nu(\rho,\theta) = \rho^{\nu_n} M_n cos(\nu_n\theta); \\ &\Phi(\rho,\theta) = \sum_{n=0}^{\infty}\rho^{\nu_n} M_n cos(\nu_n\theta),\ \nu_n = n\pi/\theta_0. \end{aligned} \qquad (17)$$

Thus [6], on the boundary $\rho = \rho_0$, series (17) is a complete Fourier series and the conjugation conditions with areas I and II (i.e., for points belonging to space $\rho \geq \rho_0$) should be performed for area III when approaching the angular point from the left (neck) and right (resonator) side.

Similar ideas lead to the situation of the acoustic field representation in terms of pressure.

Next, based on equation (17), we determine the components of the oscillating velocity for area III:

$$\begin{aligned} &\mathrm{v}_\theta = -\frac{\partial\Phi(\rho,\theta)}{\rho\partial\theta} = -\sum_{n=0}^{\infty}\rho^{\nu_n-1}\mathrm{v}_n\ M_n sin(\nu_n\theta); \\ &\mathrm{v}_\rho = -\frac{\partial\Phi(\rho,\theta)}{\partial\rho} = -\sum_{n=0}^{\infty}\rho^{\nu_n-1}\mathrm{v}_n\ M_n cos(\nu_n\theta). \end{aligned} \qquad (18)$$

Therefore, when bending the angular point of the wedge edge, the velocity field must have local features of the type $1/\rho$ for $\rho \to 0$. The wedge acoustic properties and the wedge angle determine the encroaching velocity of the components of the oscillating velocity to the infinity and their angular distribution.

Thus, taking into account the conditions of growth to infinity of the oscillating velocity when approaching O_1, it is necessary to adjust the value of the oscillating velocity, which is obtained without taking into account the boundaries of the wedge and area III (Problem 1). If the angle θ_0 varies in the interval $0 < \theta_0 \leq \pi$, then all terms of series (18) are finite, and for small values of the additional variable $n = 1$ a situation arises $\nu_n = \left.\dfrac{n\pi}{\theta_0}\right|_{n=1} > 1$. If $\theta_0 > \pi$, then for small values of the additional variable $n = 1$, $\nu_n = \left.\dfrac{n\pi}{\theta_0}\right|_{n=1} > 1$. Thus, the power $\nu_n - 1$ of the multiplier ρ^{ν_n-1} is negative, which corresponds to the tendency of the oscillatory velocity to infinity. So, for $\theta_0 = \dfrac{3}{2}\pi$, an increase in the velocity is determined by the multiplier $\rho^{-1/3}$. For a narrow tube ($n = 0$), the regularity connected with the considered case $1/\rho$ is also preserved for $\rho \to 0$v.

Understanding that ρ is the distance from the top of the right angle to a point in the field and that we can have $\rho \leq \rho_0$, we need to find not only the change in the amplitude of the velocity, but also its angular distribution, using the relationship of polar and rectangular coordinates. In this case, the change in velocity at the boundaries of mode-matching areas should be determined by two directions of angular point approach i.e., from area I and from area II. This situation, according to [6], when $n = 1$ determines the velocity at the boundaries with areas I and II, the amplitude of which is obtained after taking the derivatives (18) and substituting the resulting value of the angle $\theta_0 = \dfrac{3}{2}\pi$.

Then for the edge: $\mathrm{v}_x(0,z) = \dfrac{1}{3}\mathrm{v}_0\rho^{-1/3},\ z \to h_1 - 0$, when moving from the cavity to the angular point (approaching from the right), and:

$$\mathrm{v}_x(0,z) = -\frac{2}{3}\mathrm{v}_0\rho^{-1/3},\ z \to h_1 + 0, \qquad (19)$$

(approaching from the left).

As can be seen, when approaching from the cavity to the angular point, the velocity is twice less than that when approaching from the left along the acoustically rigid boundary to the angular point. This result is confirmed by [6]. That is, at the intersection $x = 0$, the velocity around the corner point can be given by some addition to the regular function υ_{reg} and written as

$$\mathrm{v}_x(0,z) = \mathrm{v}_{reg} + \frac{1}{3}\mathrm{v}_0\left(h_2^2 - z^2\right)^{-1/3},\ z \le h_1,$$

when moving from the cavity to the angular point (approaching from the right), and

$$\mathrm{v}_x(0,z) = \mathrm{v}_{reg} - \frac{2}{3}\mathrm{v}_0\left(-h_2^2 + z^2\right)^{-1/3},\quad h_2 < z < h_1 \qquad (20)$$

(approaching from the left).

In case of a narrow tube and selected boundary conditions, equations (18)-(20) are simplified, because $n = 0$. So, let us take this into account:

$$v_n = 0, \Rightarrow \mathrm{v}_\theta = 0, \qquad (21)$$
$$\mathrm{v}_\rho = \rho^{-1}\mathrm{v}_0 M_0$$

or $\mathrm{v}_x(0,z) = \mathrm{v}_{reg} + \mathrm{v}_0\left(h_2^2 - z^2\right)^{-1},\ z \le h_1,$ when moving from the cavity to the angular point (approaching from the right), and

$$\mathrm{v}_x(0,z) = \mathrm{v}_{reg} - \mathrm{v}_0\left(-h_2^2 + z^2\right)^{-1},\quad h_2 < z < h_1 \qquad (22)$$

(approaching from the left).

As can be seen, due to the choice of a narrow tube, the oscillating velocity circumference O_1 is characterized by a symmetric angular velocity distribution, contains only a radial component, and when approaching the angular point decreases more slowly than in case of soft boundaries and excitation in the waveguide of only the first mode.

The final step of the solution is the choice of a regular function υ_{reg} and unspecified value υ_0. Function υ_{reg} does not contradict the problem statement, it is an unambiguous solution at angular point circumference and can have derivatives in other points of the neck and resonator fields. Therefore, as a regular function we choose the value of the oscillation velocity calculated at the boundaries of areas I and II, in the absence of conditions at the edge. We assume that the unspecified value υ_0 is conditionally equal to one.

Thus, the content of **Problem 2** (taking into account the conditions at the edge) is formally reduced to determining the additive $\upsilon_x(0, z)$ to the calculated velocities obtained for the kinematic conditions of **Problem 1**.

Therefore, only the decomposition coefficients M_n for the field in area III remain unknown. They are found by using expressions (12)-(15), conditions (17)-(20) and fulfilling the conditions of conjugation of kinematic type at the boundaries of area III when approaching the point O_1:

$$A_0 k + \beta_n B_n cos(\alpha_n y) + v_0\rho^{-1}M_n = -(C_n - D_n)\gamma_n cos(\xi_n y) - -v_0\rho^{-1}M_n,$$

where

$$A_0 k + \beta_n B_n cos(\alpha_n y) + v_0\rho^{-1}M_n = -(C_n - D_n)\gamma_n cos(\xi_n y) - -v_0\rho^{-1}M_n, \qquad (23)$$
$$M_n = \frac{A_0 k + \beta_n B_n cos(\alpha_n y) + (C_n - D_n)\gamma_n cos(\xi_n y)}{(-2)v_0\rho^{-1}},$$

or, after the transition to rectangular coordinates and intersections $x = 0$, from (23) we have:

$$M_n = \frac{A_0 k + \beta_n B_n cos(\alpha_n y) + (C_n - D_n)\gamma_n cos(\xi_n y)}{(-2)v_0\left(h_1^2 - y^2\right)^{-1}}. \qquad (24)$$

Thus, we have all the necessary expressions for the expansion coefficients (12)-(15) to find the field in the cubic Helmholtz resonator represented by narrow tubes, taking into account the condition at the edge.

4. CALCULATION RESULTS

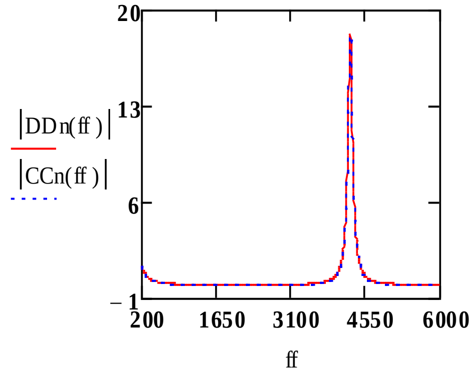
An air-filled cubic (reduced to a flat rectangular) Helmholtz resonator with the following dimensions was chosen for the calculations (Fig. 1, Fig. 2): neck length $l_1 = 0.01$ m, height of the neck section $h_1 = 0.01$ m, resonator cavity length $l_2 = 0.04$ m, height of the cross section of the resonator cavity $h_2 = 0.03$ m, studied frequency range $\Delta f = 200$-5000 Hz, wedge-shaped edge with angle $\theta_0 = 3\pi/2$.

The following were calculated:

- frequency characteristic coefficients of series expansions recorded for potential or pressure (Fig. 3);
- distribution of pressure along the acoustic axis of the neck (Fig. 4a) and the resonator within its longitudinal size $x \in [0; 0.04]$;
- coordinate dependences of the resonator gain in the section h_2 in a given frequency range Δf (Fig. 5);
- the amplitude of the oscillating velocity without taking into account the conditions at the edge (intersection $x = 0$);

The calculated data show that the moduli of the frequency characteristics of the field expansion coefficients $B_n = BBn$, $C_n = CCn$, $D_n = DDn$ (or $Cn = (CCn(ff))$, $Dn = (DDn(ff)$ (Fig. 3a), $Bn = (BBn(ff)$ (Fig. 3b)) for the mode $n = 0$ illustrate the resonant frequency dependence in the neck and cavity of the resonator. The result of the interference of the forward and reverse waves gives us an extreme region, which, according to the selected initial data, is determined by the frequency value of 4270 Hz (Fig. 2b).

The coordinate dependence of the field in the neck and cavity for the frequencies of 1000, 2000 and 4270 Hz is shown in Fig. 4. The complication of the wave pattern with increasing frequency is obvious, at which the pressure is practically constant in the initial section of the neck ($x - 0$) and variable in the cavity. This seems correct, given the invariance of acoustic pressure and the low frequency dependence in the selected computational situation of the resulting field in the neck of a narrow tube. Therefore, the behavior dynamics of a medium, which the resonator is filled with, corresponds to injection-rarefaction cycles, taking into account differences in local pressure and its elastic and inertial properties. In this case (Fig. 5), the field in the



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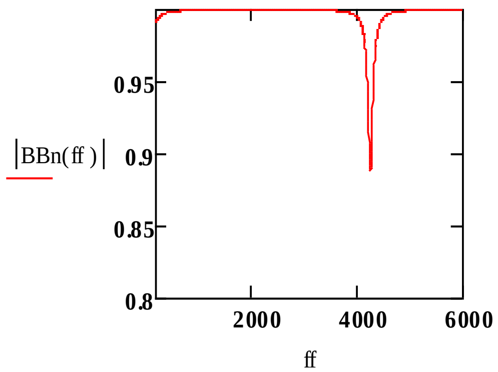
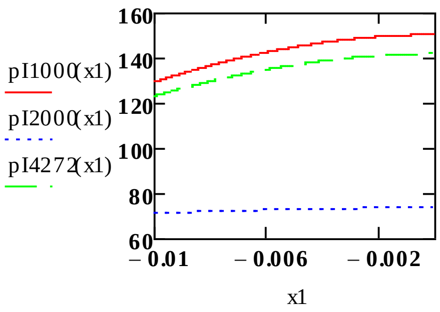
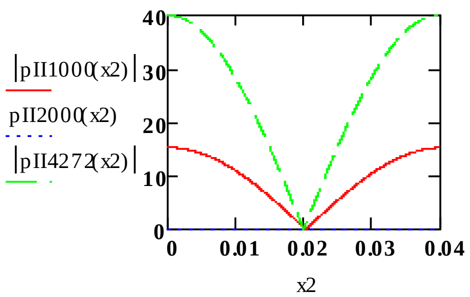


b

Fig. 3 – Frequency characteristic coefficients of series expansions: (a) *Cn* = (*CCn*(*ff*)), *Dn* = (*DDn*(*ff*)), (b) *Bn* = (*BBn*(*ff*))



a



b

Fig. 4 – Pressure distribution along (a) the acoustic axis of the neck and (b) the longitudinal resonator size $x \in [0; 0.04]$

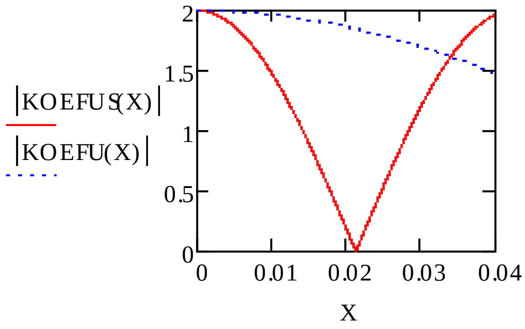
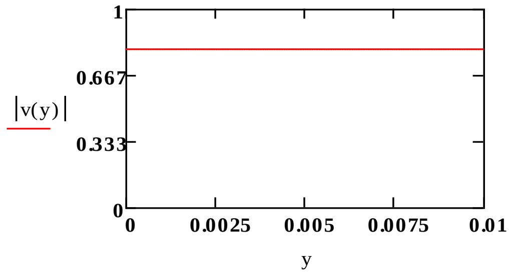
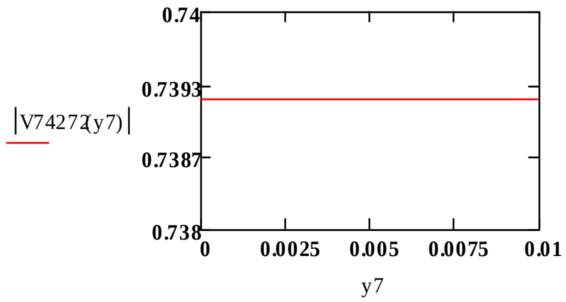


Fig. 5 – Coordinate dependences of the coefficient in the resonator



a



b

Fig. 6 – Velocity at the boundary of the cavity and the neck (angular point O_1): (a) taking into account conditions at the edge, (b) without taking into account the boundary conditions at the edge

resonator also has local extreme regions, the number and magnitude of which increase with increasing frequency. So, along the *Ox*-axis, the pressure can drop to 3-6 dB. These results coincide with the calculated data on the pressure field levels in the Helmholtz resonator cavity, which are presented in [8].

5. CONCLUSIONS

The determination of the resonator gain (Fig. 5), which was found in [1] for a system with concentrated parameters, should be clarified. This is connected with the essence of the applied mode-matching method, which helps to accurately characterize the formation of acoustic fields not only at the bottom, but also in any part of the mode area, considering the resonator as a system with the distributed parameters. Thus, the calculation results presented in Fig. 5 show the dependence of the gain on the coordinates. They show that in the computational case, there are cavity areas where reinforcement is completely impossible (for instance, the coordinate region near the circumference point $x = 0.02$ m and at a frequency of about 2000 Hz. That is, the coordinate and the corresponding frequency dependences of the gain (Fig. 5) show the variability of pressure values not only at the bottom of the cavity, but also in the entire resonator. The results also show that taking into account the condition at the edge leads to a certain adjustment of the amplitude values of the oscillating velocity in the zero intersection of the cavity. This is due to the choice of boundary conditions. In the above case, the general nature of the oscillating velocity is determined only by the radial component. The angular velocity distribution around the point O_1 is symmetrical, and the introduction, in fact, of the conditions at the edge, leads to an increase in the velocity amplitude by 10-11 %. Fig. 6 illustrates the above considerations, which shows the dependence of the velocity in the section $(x - 0)$ on the coordinate at the resonance frequency. Thus, the fulfillment of the conditions at the edge is a sufficiently influential factor in the formation of acoustic fields in closed volumes such as Helmholtz resonators, and the choice of boundary conditions on the elements of its cavities allows to perform amplitude-phase adjustment of oscillations, potentials and pressures.

REFERENCES

1. Leo L. Beranek, J.Mellow Tim. Acoustics – Sound Fields and Transducers (San Diego: Elsevier – Academic Press: 2012) ISBN: 9780123914217.
2. R.L. Panton, J.M. Miller, *J. Acoust. Soc. Am.* **57** No 6, 1533 (1975).
3. Olek C. Zienkiewicz, Robert L. Taylor, J.Z. Zhu. The Finite Element Method: Its Basis and Fundamentals (Elsevier Butterworth-Heinemann: 2005) ISBN 978-0-7506-6320-5.
4. V.T. Grinchenko, I.V Vovk, V.T. Matsypura. *Fundamentals of acoustics* (Kyiv: Naukova Dumka: 2007).
5. V.T. Grinchenko, I.V. Vovk, V.T. Matsypura, *Acoustic wave problems* (Begell House: 2018).
6. O. Korzhyk, S. Naida, S. Kurdiuk, V. Nizhynska, M Korzhyk, A. Naida, *EUREKA Phys. Eng.* **5** (2021).
7. O. Korzhyk, S. Naida, T. Zheliaskova, O. Chaika, N. Naida, *East. Eur. J. Enterprise Technol.* **5** No 5, 113 (2021).
8. E. Webster, C. Davies, *Sensors (Basel)* **10** No 12, 10663 (2010).

Метод часткових областей в задачі прийому звуку резонатором Гельмгольця

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У роботі розглянуто задачу формування акустичного поля в порожнинах резонатора Гельмгольця із застосуванням метода часткових областей. Поставлено і розв'язано краєві задачі акустики щодо описання акустичних полів в порожнинах резонатора із залученням традиційних граничних умов та граничних умов на ребрі (і за його відсутністю), яке характеризується відомими акустичними властивостями. Для певних розмірів резонатора в умовах повітряного наповнювача обчислено, проаналізовано та порівняно з експериментальними даними основні характеристики поля в резонаторі. Оцінено вплив на результати формування поля залученої умови на ребрі з акустичо жорсткими гранями-поверхнями. У зв'язку з вищенаведеним, метою роботи є постановка та розв'язок хвильової задачі формування акустичного поля в кубічному резонаторі Гельмгольця з врахуванням класичних граничних умов, умов спряження та умов на кутових точках елементів конструкції, які утворені при обранні часткових областей відповідно до однойменного методу.

Ключові слова: Резонатор, Акустичне поле, Метод часткових областей, Частотна характеристика, Інтерференція.