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ANALYSIS OF THE ACCURACY'S CRITERIA OF THE FLIGHT CONTROL AND GUIDANCE SYSTEM

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Abstract—*It is shown that in the practice of designing Flight Control and Guidance System it is necessary to solve not only single-objective optimization problems related to the field of deterministic processes, but also to optimize random variables and functions. It is to such tasks that the analysis of the criterion of accuracy of the designed Flight Control and Guidance System belongs. In the paper, considering the navigation process as a stationary random process, a probabilistic criterion for the quality of aircraft navigation is formulated in the form of a quantitative assessment of the accuracy of the Flight Control and Guidance System that ensures that the aircraft does not leave the specified corridor for a specified time. Formulas are obtained for calculating the accuracy criterion of Flight Control and Guidance System in across track separation. It is shown that in order to calculate the accuracy criterion of the Flight Control and Guidance System, it is sufficient to know the correlation function of the lateral deviation errors, which can be obtained either by the calculation method, or by modeling, or by a flight experiment.*

Index Terms—Accuracy; efficiency; navigation task; flight along the route; corridor; separation; probability; relative number of aircraft.

I. Introduction

The design of a Flight Control and Guidance System (FMGS) is a complex task that is part of the overall task of designing aircraft. Due to the complexity of such design objects, as well as the impossibility of a mathematical description of a number of processes of their functioning, there is currently no single formalized methodology for their synthesis. Therefore, in the synthesis process, informal methods are used in combination with the solution of a number of separate formalized problems and a comparison of competing options.

One of these tasks is the development of certain criteria for evaluating the effectiveness of FMGS [2], among which the accuracy criteria can significantly affect the structure of FMGS and information processing methods.

II. Problem Statement

The correct formulation of the FMGS accuracy criterion can be obtained by considering the navigation process as a stationary random process. Appeal to the theory of stationary random processes is based on the processing of a significant number of flights performed by a large number of aircraft. Studies have confirmed the stationary nature of the piloting process. However, the dead reckoning process (the main method of determining the position of an aircraft in modern FMGS) is a non-stationary process, which is characterized by an increase in the dispersion of errors in determining the position in the dead reckoning segment.

If we formulate the requirement for the navigation process as ensuring flight within a given corridor, then the crew, fulfilling this requirement, must perform aircraft position correction at those moments of time when possible dead reckoning errors do not exceed the width of the corridor [1], [3]. Such a correction, performed at random times, leads to the fact that the root-mean-square errors in determining the position of the aircraft along the route will be close to each other in magnitude. The non-stationary process of dead reckoning will be transformed into a stationary process of aircraft navigation.

Thus, the formulation of the problem is reduced to the formulation of the accuracy efficiency of the FMGS.

III. Problem Solution

When formulating the accuracy efficiency criterion, the following factors should be taken into account:

- accuracy efficiency is a conditional probability of fulfilling the main navigation task, determined by the FMGS errors;

- accuracy efficiency characterizes to what extent the FMGS accuracy satisfies the needs of lateral and vertical separation and depends on the value of the given boundaries;
- the navigation process is a differentiated normal random process, so its description can be quite correctly performed using the correlation theory of random functions;
- in the process of navigation, the airway is a separate section of the reckoning of coordinates and their correction;
- counting sections are characterized by the accumulation of errors, which causes the non-stationary nature of this process.

When determining the accuracy efficiency, changes in the errors in the calculation of coordinates are considered as a non-stationary normal random process, characterized by an increase in time of the dispersion of coordinate errors. This approach to the navigation process is confirmed by the results of flight tests of a number of navigation systems.

Since the measurement errors of the main navigation parameters are independent, a correct description of the random navigation process can be performed within the framework of the correlation theory of random processes, in particular, its emission theory [5].

The theory of outliers of random processes, suitable for differentiated random processes, makes it possible to obtain a solution to this problem by determining the average number of outliers of a random function over a given time interval. Given the significantly rare and independent nature of outliers (crossings of given boundaries), we can accept the distribution of the number of outliers as subject to the Poisson law.

The simplest Markov process described by a differential equation of the form

$$\dot{x} + \alpha x = \sigma\sqrt{2\alpha}\xi(t),$$

where ξ is white noise.

The correlation function of the stationary solution of this equation has the form:

$$k_x(\tau) = \sigma^2 e^{-\alpha\tau}.$$

When evaluating the efficiency of complexes for several given boundaries, for example, a corridor, half an echelon and an adjacent corridor, it is necessary to determine the accuracy's efficiency for the same boundaries for the entire working subset of FMGS states.

Considering that the lateral, longitudinal and vertical separation norms are independent, the criteria of the accuracy's efficiency for the lateral and longitudinal channels are also evaluated independently.

The quality of the solution of the main navigation task, which is to ensure flight along a given route, will be characterized by the probability that the aircraft will never leave the corridor in a given area.

- during $(0, T)$;
- during $\Delta t = t_2 - t_1$.

This probability, in turn, can be represented as the ratio of the number of aircraft that have never left the given corridor to the total number of aircraft flying along the given route.

Consider a sufficiently large flow of aircraft N, equipped with the same FMGS and flying at the same speed along the same desired track (DT). All aircraft at the beginning of the DT ($t = 0$) completed the correction.

Of the total number N of aircraft, only n_1 aircraft after the correction ended up inside the corridor, and $N - n_1$ aircraft outside the corridor.

During the flight during the time t $(0, T)$ out of this number of planes (n_1) some number of planes (n_3) left the corridor. During the same time, out of the number of planes $N - n_1$, a certain number of planes n_4 entered the corridor.

Thus, by the time T, there were n_2 planes inside the corridor, and $N - n_2$ outside the corridor. It is clear that $n_1 + n_4 = n_3 + n_2$.

The navigation accuracy for a stream of aircraft N can be characterized by the number of aircraft that never left the given corridor during the time $t(0, T)$. The relative number of such aircraft will be equal to $\frac{n_1}{N}\left(1 - \frac{n_3}{n_1}\right)$. The probability of such a process can be represented as a criterion for the accuracy of the FMGS efficiency:

$$\Phi = P_{\text{T.E}} = P_1 - P_3,$$

where $P_1 = n_1/N$ and $P_3 = n_3/N$.

As a consequence, the criterion $P_{\text{T.E.}}$ characterizes the probability of the navigation process, in which the aircraft will never leave the specified corridor in the specified flight segment.

The use of the criterion $P_2 = n_2/N$ gives overestimated values of the accuracy's efficiency of the FMGS:

$$P_2 = P_{\text{T.E.}} + P_4.$$

The use of criterion 1 – P_3 (probability of not crossing the corridor level from the inside) also gives an overestimation of the accuracy of the efficiency of the FMGS:

$$1 - P_3 = P_{\text{T.E}} + (1 - P_1).$$

Thus, as a criterion for exactly the same efficiency when flying along the route, we will take the ratio of the number of aircraft that have never left the given corridor to the total number of aircraft flying along the given section of the route. Let us give this relation the meaning of the probability of completing the navigation task [4].

The probability is determined by two ratios n_1/N and n_3/N. Their physical meaning is clear:

- $P_1 = n_1/N$ is the relative number of aircraft inside the corridor by the start of a given route segment;
- $P_3 = n_3/N$ is the relative number of aircraft that left the corridor on a given section of the route.

Considering the navigation process as a random process, we accept P_3 in accordance with the theory of random process outliers as the average number of random function outliers in the time interval under study.

It should be noted that the criterion $\Phi = P_{\text{T.E}}$ satisfies the basic requirements for the efficiency's criteria of large systems, the most important of which is the ability to measure the quantitative assessment of efficiency.

The criterion of accuracy's efficiency also satisfies other requirements for efficiency criteria. It is characterized by efficiency in the statistical sense, small dispersion and, as a result, sufficient accuracy, completeness of the assessment, simplicity, the presence of a physical meaning, the possibility of normalizing values from zero (the worst quality) to one (the case of an ideal characteristic).

To derive accuracy's efficiency equations, we consider a sufficiently large flow of aircraft flying independently of each other and following along the DT with the same speed W. All aircraft, the number N of which is sufficiently large, are equipped with the same FMGS.

Let us determine the probabilistic characteristics of the aircraft being inside the corridor (– c, + c). Let us denote: n_1 is the number of aircraft that passed the section inside the corridor $\pm c$; n_2 is the number of aircraft remaining inside the corridor $\pm c$; n_3 is the number of aircraft that flew out of the corridor $\pm c$ between sections; n_4 is the number of aircraft that flew into the corridor $\pm c$ between sections; n_3+ and n_4+ are the number of aircraft that crossed the border $+c$; n_3^-, n_4^- are the number of aircraft that crossed the border $-c$.

Naturally, with the same number of aircraft, the following equalities are correct:

$$n_1 + n_4 = n_2 + n_3,$$

$$n_1 + n_4^+ + n_4^- = n_2 + n_3^+ + n_3^-.$$

Dividing these equalities by the total flow of aircraft N, we obtain expressions for the probabilities:

$$P_1 + P_4 = P_2 + P_3,$$

$$P_1 + P_4^+ + P_4^- = P_2 + P_3^+ + P_3^-.$$

where the probabilities of the flight the intersections indicated above for n are expressed through P with the same indices.

For a normal random navigation process, the one-dimensional and two-dimensional probability densities for the moment t are [4]:

$$w(Z/t) = \frac{1}{\sigma_z\sqrt{2\pi}} e^{-\frac{(Z-m_z)^2}{2\sigma_z^2}},$$

$$w(Z,\dot{Z}/t) = \frac{1}{2\pi\sigma_z\sigma_{\dot{z}}\sqrt{1-r^2}} \times \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(Z-m_z)^2}{\sigma_z^2} - 2r\frac{(Z-m_z)(\dot{Z}-m_{\dot{z}})}{\sigma_z\sigma_{\dot{z}}} + \frac{(Z-m_z)}{\sigma_{\dot{z}}^2}\right]\right\},$$

Herewith

$$m_z = m_{\dot{z}}(t) = \frac{dm_z(t)}{dt},$$

$$\sigma_z^2 = \sigma_z^2(t) = k_z(t_1,t_2)\Big|_{t_1=t_2=t},$$

$$\sigma_{\dot{z}}^2 = \sigma_{\dot{z}}^2(t) = k_{\dot{z}}(t_1,t_2)\Big|_{t_1=t_2=t} = \frac{\partial^2 k_z(t_1,t_2)}{\partial t_1 \partial t_2}\Big|_{t_1=t_2=t},$$

$$r = r(t) = \frac{k_{z\dot{z}}(t_1,t_2)\big|_{t_1=t_2=t}}{\sigma_z\sigma_{\dot{z}}} = \frac{1}{\sigma_z\sigma_{\dot{z}}}\frac{\partial k_z(t_1,t_2)}{\partial t_2}\Big|_{t_1=t_2=t},$$

where σ_Z, m_Z are the standard deviation and the mathematical expectation of the lateral deviation error at time t; $\sigma_{\dot{Z}}$, $m_{\dot{Z}}$ are the standard deviation and the mathematical expectation of the error in the rate of change of the lateral deviation at time t; $k_z(t_1, t_2)$ is the correlation function of lateral deviation errors; $k_{\dot{z}}(t_1, t_2)$ is the correlation function of errors in the rate of change of lateral deviation; $k_{z\dot{z}}(t_1, t_2)$ are the mutual correlation function of errors Z and $\dot{Z}$; r is the normalized cross-correlation function between errors $\dot{Z}$ and Z at times t.

First of all, let us determine the probabilities P_1 and P_2 of being inside the corridor $\pm c$ at moments t and $t + \Delta t$:

$$P_1 = \int_{-c}^{+c} dZ \int_{-\infty}^{+\infty} w(Z, \dot{Z}/t) d\dot{Z} = \Phi\left[\frac{c}{\sigma_z(t)}\right],$$

where $\Phi_x = \sqrt{\frac{2}{\pi}} \int_0^x e^{-\frac{\tau^2}{2}} d\tau,$

$$P_3(c/t) = \left(\frac{\sigma_z\sqrt{1-r^2}}{2\pi\sigma_z} e^{-\frac{(c-m_z)^2}{2\sigma_z^2}}\right) \times \left\{\exp\left(-\frac{1}{2}\mathrm{M}^2 + \sqrt{\frac{\pi}{2}}\mathrm{M}\Phi(M) + \sqrt{\frac{\pi}{2}}\mathrm{M}\right)\right\}.$$

Let us define the temporal probability density

$$P_4(c/t) = -\int_{-\infty}^{0} w(c, Z/t)\dot{Z} d\dot{Z}$$

It's obvious that

$$\int_{-\infty}^{\infty} w(c, \dot{Z}/t)\dot{Z} d\dot{Z} = \int_{\infty}^{0} w(c, \dot{Z}/t)\dot{Z} d\dot{Z} + \int_{0}^{\infty} w(c, \dot{Z}/t)\dot{Z} d\dot{Z}.$$

Then

$$P_4(c/t) = \int_0^{\infty} w(c, \dot{Z}/t)\dot{Z} d\dot{Z} - \int_{-\infty}^{\infty} w(c, \dot{Z}/t)\dot{Z} d\dot{Z} = P_3(c/t) - \int_{-\infty}^{\infty} w(c, \dot{Z}/t)\dot{Z} d\dot{Z}$$

or $P_4(c/t) = I_1 - I_2$

where the integral I_2 differs from I_1 only by the lower integration limit.

It is known that

$$I_2 = \int_{-\infty}^{\infty} x e^{-px^2 + 2qx} dx = \frac{q}{p}\sqrt{\frac{\pi}{p}} \exp\left(\frac{q^2}{p}\right),$$

then

$$I_1 - I_2 = \frac{1}{2p} + \frac{q}{2p}\sqrt{\frac{\pi}{p}} \exp\left(\frac{q^2}{p}\right) \times \left[1 + \Phi\left(\frac{\sqrt{2}q}{\sqrt{p}}\right)\right] - \frac{q}{p}\sqrt{\frac{\pi}{p}} \exp\left(\frac{q^2}{p}\right) = \frac{1}{2p} - \frac{q}{2p}\sqrt{\frac{\pi}{p}} \exp\left(\frac{q^2}{p}\right)\left[1 - \Phi\left(\frac{\sqrt{2}q}{\sqrt{p}}\right)\right].$$

Substituting the value of $I_1 - I_2$, we get

$$P_4(c/t) = \left(\frac{\sigma_{\dot{z}}\sqrt{1-r^2}}{2\pi\sigma_z} e^{-\frac{(c-m_z)^2}{2\sigma_z^2}}\right) \times \left\{\exp\left(-\frac{1}{2}M^2\right) + \sqrt{\frac{\pi}{2}} M\Phi(M) - \sqrt{\frac{\pi}{2}} M\right\}.$$

Since $m_z = 0$ and $m_{\dot{z}} = 0$, then

$$M = \frac{r}{\sqrt{1-r^2}} \frac{c}{\sigma_z}.$$

Let us now determine the time's density of probability the level crossing $-c$:

$$P_3(-c/t) \text{ and } P_4(-c/t).$$

Obviously, the probability $P_3(-c/t)$ can be obtained from $P_4(c/t)$ by substituting $-c$ for c. The relations between $P_4(-c/t)$ and $P_3(c/t)$ will be similar.

As a result, we get

$$P_3(-c/t) = P_3(c/t),$$

$$P_4(-c/t) = P_4(c/t).$$

Using the relations for $P_3(c/t)$, $P_3(-c/t)$, $P_4(c/t)$, $P_4(-c/t)$ and integrating over time t, we obtain the probabilities $P_3^+, P_4^+, P_3^-, P_4^-$.

Note that the time densities of the level crossing from bottom to top and from top to bottom $P_3(\pm c/t)$ and $P_4(\pm c/t)$ can be determined as follows:

$$P_3(\pm c/t) = 2P_3(c/t), \quad P_4(\pm c/t) = 2P_4(c/t).$$

And the probabilities P_3 and P_4 of crossing the level from bottom to top and from top to bottom during time T can be determined using the relations:

$$P_3 = 2P_3^+ = 2\int_0^T P_3(c/t)dt, \quad P_4 = 2P_4^+ = 2\int_0^T P_4(c/t)dt.$$

Thus, the final formulas for calculating the accuracy efficiency will have the form:

$$\Phi = P_1 - P_3,$$

$$P_1 = \Phi\left[\frac{c}{\sigma_z(0)}\right], \qquad P_3 = 2\int_0^T P_3(c/t)dt,$$

$$P_3(c/t) = \left(\frac{\sigma_{\dot z}\sqrt{1-r^2}}{2\pi\sigma_z} e^{-\frac{c^2}{2\sigma_z^2}}\right)$$

$$\times\left\{\exp\left(-\frac{1}{2}M^2\right) + \sqrt{\frac{\pi}{2}}M + \sqrt{\frac{\pi}{2}}M\Phi(M)\right\},$$

where $M = \frac{r}{\sqrt{1-r^2}}\frac{c}{\sigma_z} = \frac{1}{\sigma_z\sigma_{\dot z}}k_{z\dot z}(t_1,t_2)\Big|_{t_1=t_2=t},$

$$\sigma_z^2 = k_z(t_1,t_2)\Big|_{t_1=t_2=t},$$

$$\sigma_{\dot z}^2 = \frac{\partial^2 k_z(t_1,t_2)}{\partial t_1 \partial t_2}\Big|_{t_1=t_2=t},$$

$$r = \frac{1}{\sigma_z\sigma_{\dot z}}\frac{\partial k_z(t_1,t_2)}{\partial t_2}\Big|_{t_1=t_2=t},$$

To calculate the accuracy's efficiency for a given corridor ± c, it is sufficient to know the correlation function of the lateral deviation errors – $k_z(t_1, t_2)$.

An effective solution to the problem of estimating the accuracy of aircraft navigation under the conditions of meeting modern requirements can only be achieved with the help of a comprehensive accuracy assessment technique [1], [3], which can be obtained by a calculation method, simulation, and flight experiment.

IV. Conclusions

The technique for estimating the accuracy's efficiency of the FMGS, which is based on the consideration of the navigation process as a stationary random process, makes it possible to estimate the conditional probability of completing the main navigation task, which is determined by the errors of the FMGS. To calculate the FMGS accuracy criterion for a given corridor with lateral separation, it is sufficient to know the correlation function of lateral deviation errors, which can be obtained either by a calculation method, or by modeling, or by a flight experiment.

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М. К. Філяшкін Аналіз критерію точності пілотажно-навігаційних комплексів

Показано, що у практиці проектування пілотажно-навігаційних комплексів (ПНК) доводиться вирішувати не тыльки завдання однокритеріальної оптимізації, які стосуються області детермінованих процесів, але й оптимізувати випадкові величини і функції. Саме до таких завдань відноситься аналіз критерію точності проектованого ПНК. У роботі, розглядаючи процес навігації як стаціонарний випадковий процес, сформульований ймовірнісний критерій якості літаководіння у вигляді кількісної оцінки точності ПНК, що забезпечує не вихід літака за межі заданого коридору протягом заданого часу. Одержано формули для розрахунку критерію точності ПНК при бічному ешелонуванні. Показано, що для розрахунку критерію точності ПНК достатньо знати кореляційну функцію помилок бічного відхилення, яку можна отримати або розрахунковим методом, або моделюванням або льотним експериментом.

Ключові слова: точність; ефективність; навігаційне завдання; політ за маршрутом; коридор; ешелонування; ймовірність; відносна кількість літаків.

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Напрям наукової діяльності: комплексна обробка інформації в пілотажно-навігаційних комплексах, автоматизація та оптимізація керування повітряними суднами на різних етапах польоту.
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Н. К. Филяшкин. Анализ критерия точности пилотажно-навигационных комплексов

Показано, что в практике проектирования пилотажно-навигационных комплексов приходится решать не только задачи однокритериальной оптимизации, относящиеся к области детерминированных процессов, но оптимизировать случайные величины и функции. Именно к таким задачам относится анализ критерия точности проектированного ПНК. В работе, рассматривая процесс навигации как стационарный случайный процесс, сформулирован вероятностный критерий качества самолетовождения в виде количественной оценки точности ПНК обеспечивающий не выход самолета за пределы заданного коридора в течение заданного времени. Получены формулы для расчета критерия точности ПНК при боковом эшелонировании. Показано что для расчета критерия точности ПНК достаточно знать корреляционную функцию ошибок бокового отклонения, которую можно получить либо расчетным методом, либо моделированием либо летным экспериментом.

Ключевые слова: точность; эффективность; навигационная задача; полет по маршруту; коридор; эшелонирование; вероятность; относительное число самолетов.

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