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Velocity of a falling object with air resistance

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We consider the problem of determining the velocity **v** of a falling object with air resistance.

The differential equation for the falling speed of an object can be written using Newton's second law, which states that the force acting on an object is equal to the object's mass multiplied by its acceleration. In the case of an object falling in the zone of air resistance, it can be assumed that the force of resistance is proportional to the square of the object's speed. Then the differential equation of the falling speed of the object will have the form:

$$\frac{dv}{dt} = g - \frac{k}{m}v^2$$

where m is the mass of the object, g is the acceleration of free fall, k is the coefficient of air resistance, v is the falling speed of the object, and t is time.

This equation can be solved by numerical methods, if the initial conditions of the speed and position of the object at the initial moment of time are known. we assume that the object was falling from rest so that v = 0 when t = 0, i.e. v(0) = 0.

To solve the equation let v_∞ will be the terminal volacity given by $v_\infty = \sqrt{g/k}$, the ODE can be written as follows:

$$\frac{dv}{dt} = k(v_\infty - v^2)$$

It is a separable equation with two trivial solutions $\pm v_\infty$ but the negative one is not acceptable as it indicates a constant upward velocity. In fact, the ODE is incorrect for v < 0,
because the motion and air resistance cannot be in the same direction.

To solve the new ODE divide through by $(v_\infty - v)(v_\infty + v)$ to get the separate equation:

$$\frac{1}{(v_\infty - v)(v_\infty + v)}dv = kdt$$

Solving using the partial fraction decomposition we get

$$\ln\left|\frac{v_\infty - v}{v_\infty + v}\right| = 2kv_\infty t$$

To solve for v, first exponentiate and choose the plus sign since it is in accord with the initial condition. Therefore,

$$v = v_\infty \tanh(kv_\infty t)$$