MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL AVIATION UNIVERSITY
FACULTY OF AIR NAVIGATION, ELECTRONICS AND TELECOMMUNICATIONS
DEPARTMENT OF AVIONICS

APPROVED
Head of the department
________Yu.V.Hryshchenko
‘___’___________2024

# QUALIFICATION PAPER

**(EXPLANATORY NOTES)**

**FOR THE DEGREE OF «BACHELOR»**

**SPECIALITY 173 ‘AVIONICS’**

## Theme: ‘Mathematical model of  small aircraft’

Done by: ___________Adeola Prince Oluwajuwon.____
(student, group, surname, name, patronymic)

Supervisor: _________DSc (Eng), prof._Sushchenko O.A._______
(scientific degree, academic rank, surname, name, patronymic)

Standard controller: ___________ Levkivskyi V.V.
(Signature) (surname, name, patronymic)

Kyiv 2024

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ
ФАКУЛЬТЕТ АЕРОНАВІГАЦІЇ, ЕЛЕКТРОНІКИ ТА ТЕЛЕКОМУНІКАЦІЙ
КАФЕДРА АВІОНІКИ

ДОПУСТИТИ ДО ЗАХИСТУ
Завідувач кафедри
_________Ю.В. Грищенко
«___» ___________2024

# КВАЛІФІКАЦІЙНА РОБОТА

**(ПОЯСНЮВАЛЬНА ЗАПИСКА)**

ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ «БАКАЛАВР»

ЗА СПЕЦІАЛЬНІСТЮ 173 «АВІОНІКА»

## Тема: «Математична модель малого безпілотного літального апарата»

Виконав: Адеола Принц Олуваджуон
(студент, група, прізвище, ім'я, по-батьвові)

Керівник: д.т.н., проф., Сущенко О.А.
(науковаступінь, вченезвання, прізвище, ім'я, по-батькові)

Нормоконтролер: ___________ В.В. Левківський
(підпис) (прізвище, ім'я, по-батькові)

Київ 2024

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Department of avionics

Specialty 173 ‘Avionics’

APPROVED

Head of the department
_________ Yu.V.Hryshchenko
‘___’___________2024

**TASK**

**for qualification paper**

Adeola Prince

1. Theme: ‘Mathematical Model of Small Aircraft’, approved by order 385/ст of the Rector of the National Aviation University of 14 March 2024.

2. Duration of which is from 13 May 2024 to 16 June 2024.

3. Input data of qualification paper: UAV Design Parameters, Performance Specifications, Specifications of Propulsion and Power Systems, Control and Navigation Systems, Load Capacity of UAV, Environmental Conditions.

4. Content of explanatory notes: List of conditional terms and abbreviations, Introduction, Chapter 1, Chapter 2, Chapter 3, References, Conclusions.

5. The list of mandatory graphic materials: Figures, charts, and graphs.

6. Planned schedule

| № | Task | Duration | Signature of supervisor |
|---|---|---|---|
| 1. | Validate the rationale of the qualification paper theme | 13.05.2024 | |
| 2. | Carry out a literature review | 15.05.2024 | |
| 3. | Develop the first chapter of the qualification paper | 22.05.2024 | |
| 4. | Develop the second chapter of the qualification paper | 29.05.2024 | |
| 5. | Develop the third chapter of the qualification paper | 05.06.2024 | |
| 6. | Tested for anti-plagiarism and obtained a review of the qualification paper | 12.06.2024 | |
| 7. | Preparation of presentation and report | 16.06.2024 | |

8. Date of assignment: ‘___ ‘ ____________ 2024

Supervisor ______________ O.A. Sushchenko

The task took to perform ______________ A. Prince

(Signature) (surname, name, patronymic)

# ABSTRACT

Explanatory notes to qualification paper ‘Mathematical Model of Small Aircraft' contains 61 pages, 23 figures, 10 graphs, and 27 references.

**Keywords:** AIRCRAFT, MATHEMATICAL MODEL, DIFFERENTIAL EQUATIONS, STATE SPACE REPRESENTATION, SIMULATION.

**The object of the research -** the process of creation mathematical model of a small unmanned aerial vehicle.

**The subject of the research -** A mathematical model of a small unmanned aerial vehicle.

**Purpose of qualification paper –** creation of the mathematical model and simulation of the aircraft flight along the given trajectory.

**Research Method** – Methods of flight control theory, methods of modern control theory, methods of mathematical modelling and simulation.

**Scientific novelty** – creation of the simplified mathematical model of small unmanned aerial vehicles with keeping sufficient accuracy of flight control.

**CONTENTS**

# LIST OF ABBREVIATIONS

| | |
|---|---|
| SUAV | Small Umanned Aerial Vehicle. |
| RPAs | Remotely Piloted Aircrafts |
| VTOL | Vertical Take off and Landing |
| GPS | Global Positioning System |

# INTRODUCTION

The rapid expansion of unmanned aerial vehicle (UAV) technology in both civilian and military applications underscore the urgent need for sophisticated mathematical models that accurately predict and enhance UAV performance. This qualification paper, "Mathematical Model of Small Unmanned Aerial Vehicle", is positioned at the forefront of this technological evolution, offering significant contributions to the field of aerospace engineering. The objective of this work is to develop and validate a comprehensive mathematical model that encapsulates the dynamics and operational characteristics of small UAVs, thereby facilitating better design, control, and application strategies.

Unmanned aerial vehicles, especially small UAVs, have become instrumental across various sectors including agriculture, surveillance, environmental monitoring, and logistics due to their versatility, cost-effectiveness, and capability to operate in environments that are either inaccessible or hazardous for humans. However, the effective operation of UAVs in these diverse roles depends critically on the reliability of their mathematical models. These models are essential for simulating flight dynamics, predicting behavior under different environmental conditions, and integrating new technologies that enhance UAV capabilities.

The introduction of this study starts with a detailed review of the current state of UAV technologies, emphasizing the gaps in existing mathematical modeling approaches particularly for small UAVs. It discusses the typical challenges encountered in the accurate modeling of aerodynamic forces, control system dynamics, and the interaction of these systems with variable operational environments. By highlighting these challenges, the paper sets the stage for the novel contributions of the proposed mathematical model, which aims to address these specific shortcomings.

This work also meticulously details the methodology employed in developing the mathematical model, incorporating elements from fluid dynamics, control theory, and simulation technologies. It discusses the theoretical underpinnings necessary for constructing a robust model that accurately reflects the UAV's behavior. The models validity is tested through a series of simulations and, where possible, experimental flights that provide empirical data to compare against model predictions. These activities underscore the practical implications of the research, demonstrating its relevance not only to UAV designers and manufacturers but also to operators seeking to deploy UAVs in complex environments.

The significance of this qualification paper extends beyond its academic merits, contributing to a broader understanding of UAV operation in real-world scenarios. By enhancing the mathematical modeling of UAVs, the research supports advancements in UAV design that lead to improved safety, efficiency, and performance. The potential applications of this research are vast, indicating its importance for ongoing and future projects in UAV technology development. In conclusion, the introduction of this qualification paper outlines the necessity and urgency of advancing UAV technology through refined mathematical modeling. It provides a clear overview of the aims, methodology, and significance of the research, setting a solid foundation for the detailed investigations that follow. This paper not only aims to enrich the academic discourse surrounding UAV development but also seeks to make a tangible impact on the practical aspects of UAV deployment in various sectors.

**Actuality.**

The qualification paper titled Mathematical Model of Small Unmanned Aerial Vehicle (UAV) is situated at a critical juncture in the evolution of aerospace technology, where unmanned systems are becoming increasingly prevalent across a variety of applications, from commercial deliveries and environmental monitoring to surveillance and disaster management. This research is deeply relevant and timely, given the expanding use of UAVs in both civilian and military contexts, and the consequent need for robust, reliable mathematical models to enhance their design, stability, and operational efficiency. As UAVs operate in increasingly complex environments, the precision of their mathematical models becomes crucial. These models are fundamental for simulating and predicting UAV behavior under various conditions, thus ensuring safety and effectiveness in their deployment. The focus of this study on developing and refining a mathematical model for a small UAV addresses the pressing challenge of achieving high accuracy and reliability in UAV flight dynamics, control systems, and mission planning.

This qualification paper not only contributes to theoretical knowledge in aerospace engineering and applied mathematics but also has significant practical implications. By advancing mathematical models, the study supports enhancements in UAV design that can lead to improved flight stability, energy efficiency, and payload capacity. Furthermore, the paper discusses the integration of new sensors and actuators within the UAV systems, facilitated by advanced modeling, which could lead to broader functionalities and applications of these aerial vehicles. The research methodology employed involves a detailed analysis of aerodynamic principles, control theory, and system dynamics, all of which are essential for the development of a comprehensive mathematical model. The work meticulously examines various factors such as air resistance, propulsion dynamics, and gyroscopic effects, which are critical for the UAV's performance. The use of simulation tools and experimental data to validate the models ensures that the research is grounded in empirical evidence, enhancing its credibility and applicability.

From a professional standpoint, the findings of this study have the potential to influence future UAV development projects significantly. The improved mathematical models can help in the design of UAVs that are not only safer and more efficient but also capable of performing more complex tasks. This is particularly important in an era where the demand for UAVs is increasing in sectors that require high precision and reliability.

Academically, this study enriches the researchers knowledge and skills in both the theoretical and application aspects of UAV technology. It provides a solid foundation for anyone looking to pursue a career in aerospace engineering, particularly in the burgeoning field of unmanned systems. The analytical skills, advanced mathematical techniques, and problem-solving abilities developed through this research are valuable assets that can be applied in various engineering challenges.

In conclusion, the qualification work on the mathematical model of a small unmanned aerial vehicle is of considerable importance both academically and professionally. It addresses a topical issue in aerospace technology with broad implications for the future of unmanned flight. By bridging theoretical research and practical application, the paper contributes significantly to the field, providing insights that could lead to more sophisticated, robust, and versatile UAV systems.

This study not only underscores the relevance of mathematical modeling in enhancing UAV technology but also highlights the dynamic interplay between engineering innovation and practical application, essential for advancing the capabilities of unmanned aerial systems.

**The purpose of the work** is to accurately depict the mathematical model of a small unmanned aerial vehicle and its environment.

**The following tasks should be done to achieve this purpose:**

1. Establish the meaning of a Small Unmanned Aerial Vehicle and highlights its differences from a Manned Aircraft.

2. To create the mathematical modeling of a SUAV

3. To carry out simulation of a SUAV motion.

**The object of the research** – small unmanned aerial vehicle.

**The subject of the research -** mathematical modeling of a small unmanned aerial vehicle.

**Research Method** – theory of flight control, method of mathematical modelling, simulation methods.

**Scientific novelty** – proposed model of modern SUAV’s and its simulation in different environmental conditions.

**Validation of obtained results** –simulation results using MatLab (Simulink Toolbox).

# CHAPTER 1
# GENERAL CHARACTERISTIC OF SMALL UNMANNED AERIAL VEHICLE

## 1.1 History of Small Unmanned Aerial Vehicle

The word “Drone” got its etymology from the German word Drohnen, which means male bee. But the technical name for this flying object is Unmanned Ariel Vehicle (UAV). Drones are also referred to as Unmanned Ariel System (UAS) or Remotely Piloted Aircraft (RPA).

Drones have an extensive history. On August 22, 1849, Austrians were among the first to use drones. They launched 200 pilotless balloons with bombs at Venice. In 1862, balloons were used for reconnaissance and bombing by both Confederate and Union soldiers during the United States Civil War. In 1898, during the Spanish-American War, the US military mounted a camera on a kite to capture the first airborne surveillance images. The contemporary drone evolved from "target drones" in the early 20th century. These "dumb" drones were employed to test and train combat pilots and missile operators.

This technology initially appeared during the initially World War. The Dayton-Wright Aeroplane Company invented the "grandfather" of drones, the rail-launched Kettering Aerial Torpedo "Bug".

Multicopters, or multi-propeller helicopters, arose during the development of the first helicopters, but their implementation was greatly complicated by the task of developing a transmission capable of transmitting torque from one motor to several rotors. The development of such structures proved to be more difficult compared to conventional helicopters, as it required a differential transmission of torque to the rotors or the use of control rudders under the rotors. With the invention of helicopter tail rotors and the yaw machine, this concept was dismissed as unpromising.

The first working prototype of a quadcopter (as well as all existing helicopters) was the Botezatu helicopter, which made its first flight in 1923. However, the story of the development of multicopters did not end then, and now these aircraft are widely used due to their compact size and maneuverability. Modern technology overcomes the difficulties faced by developers in the past and makes multicopters popular in many industries, including aerial photography, scientific research and a number of other applications.

In the 2000s, the idea of a quadcopter received a new flourish among aircraft modelers, thanks to the widespread access to electronics and radio equipment at that time. The initial success of home-made tricopters was achieved due to the simplicity of the implementation of this scheme, which did not require the use of any specific controller. It was enough to limit ourselves to the use of model brushless engines, speed controllers, helicopter gyroscopes and service machines for the implementation of a rotary control unit. This simplified and accelerated the process of creating tricopters with your own hands, making them accessible to modeling enthusiasts without deep knowledge in the field of specialized programming and electronics.

This approach opened up new opportunities for creativity in the field of aircraft modeling and experiments with self-made tricopters, using simple and affordable components.

Fig. 1.1. Tricopter (one of the first multicopters)

With the appearance of the flight controller in 2006, the era of multicopter popularity began. The micro-copter became affordable and easy to manufacture, and the fixation of light cameras, video cameras and even telephones on them opened up new possibilities for aerial photography.

Micro-copter released a flight controller that revolutionized the capabilities of multi-copters. With the help of this device, the machines could maintain their position using GPS, perform autonomous landing and auto-return when the signal was lost. These controllers opened up new perspectives for professional aerial photography, which began to develop actively.

However, controllers from micro-copter had their drawbacks, including high cost and complexity of assembly. The next step in this field was DJI-Innovations, which released the DJI Wookong-m controller. This controller was aimed at a wide audience and did not require special skills and knowledge in radio electronics. DJI has successfully established itself in the flight controller market thanks to its simplicity and reliability.

The next important step was the release by DJI-Innovations of the widely available Naza controller, which opened the possibilities of multi-copters to a wider range of users. This step developed the use of multicopters and contributed to the active development of the field of aerial photography.

Unmanned aircraft, which can cover long distances, performing complex aerial surveying tasks in almost any weather conditions, have impressive efficiency at a distance of no more than 70 km from the control point. Maintaining a high speed of up to 400 km/h, these planes can be in flight from 30 minutes to 8 hours. Despite their performance, their launch and landing require special devices, such as catapults or parachutes, designed to ensure launch speed and precise landing, which adds to their operational complexity and increases the risk of damage.

Unmanned helicopters, in turn, do not require special means of take-off or runways. With easier adaptation to different weather conditions, their flight time ranges from 30 minutes to 3 hours. They are characterized by less structural complexity compared to airplanes or multirotor machines, although they require the introduction of complex machines to control the inclination of the blades. However, an important advantage is the possibility of self-rotation of the main propeller (autorotation), which contributes to a significant reduction of losses in the event of engine failure.

Multi-motor unmanned aerial vehicles, or multi-copters, are characterized by the presence of several propellers (rotors). These drones typically have 3, 4, 6, 8, or 12 propellers, giving them the ability to take off vertically and fly at zero speed. Multi-copters are less structurally complex than airplanes or helicopters, reducing maintenance costs and increasing reliability. Their high maneuverability and availability make them an advantageous choice for a wide range of applications, including aerial surveying, surveying and vegetation monitoring.

A serious element of risk in such types of drones is the almost complete loss of controllability when one of the propellers fails. However, in modern times, intensive research and experiments are conducted aimed at ensuring safe landing or continued movement of six- and eight-rotor aircraft in similar emergency situations.

The most common design was the multi-rotor type with four valve-motor groups, where each group consists of a motor and a propeller with a constant pitch (the angle of inclination of the propellers). Such drones are distinguished by their compactness, ease of assembly and adjustment, as well as relatively low cost and energy consumption compared to multicopters with more propellers. Their characteristics make this type of drone effective and affordable for a wide range of tasks in different industries, including aerial photography, surveying and environmental monitoring.

In parallel with scientific research in the field of materials and the use of micro- and nanoelectromechanical systems in various devices, there has been a significant reduction in the size and weight of aircraft. Modernity allows us to use miniature aerial vehicles (MAUs) that cost less than $70 to build. Quadcopters included in this class of MLAs have become quite common.

Additional classes of unmanned aerial vehicles have been developed. Micro- and mini-UAVs are light and compact devices with a take-off weight of up to 5 kg and a flight range of up to 25-40 km, used mainly for observation.

Light UAVs weighing from 5 to 50 kg have a shorter range, but are distinguished by a flight range of 10 to 120 km.

Average UAVs weigh from 100 to 300 kg and have a flight range of 150 to 1000 km.

Scientific research and the use of the latest technologies have made it possible to have access to various classes of light and compact UAVs.

Heavy UAVs of medium range have a mass of more than 500 kg and a flight range of 70 to 300 km.

Long-duration heavy UAVs such as Predator, Reaper, Global Hawk (USA) and Heron (Israel) have their advantages and disadvantages.

It is also important to note the quadcopter, which is a type of aircraft with four rotors rotating diagonally in opposite directions. Compared to heavy UAVs with a long flight duration, it has its own unique features and applications.

Modern achievements in the field of microelectromechanical systems (MEMS) allow using components based on them to significantly reduce the size and weight of aircraft. Crystal MEMS microcircuits with sizes from 20 μm to 1 mm and micromechanical devices from 1 to 100 μm have become an important part of miniature aircraft.

The mass of the aircraft is important in its design, because it affects the ratio of the total mass to the lifting force, which directly determines the aerodynamic characteristics of the aircraft.

Quadcopters are distinguished by their great maneuverability, the ability to change the direction of flight without additional rotations. They do not require mechanical connections to change the angle of inclination of the rotor blades during rotation.

One of the key characteristics of a quadcopter is the propeller pitch, which determines the distance the propeller travels in one complete revolution. A larger propeller pitch means more engine load and a lower propeller speed, simplifying the design but reducing flight speed.

Quadcopters use four rotors, which allows you to reduce the diameter of the blades, saving kinetic energy and increasing the flight range. However, they have limited flight range and payload capacity.

Features of the quadcopter also include the absence of auxiliary mechanisms for flight stabilization, which can be important in the conditions of nonlinear dynamics of this aircraft. Abandoning the yaw machine may simplify the design, but requires attention to control and flight path planning.

Recent advances in the development of unmanned aerial vehicles (UAVs) have led to a rapid growth in their variety and applications. The ranking of the best quadcopter models with a camera for 2023 includes such models as Parrot Bebop Drone 2, Xiro XPLORER V, Autel Robotics X-Star Premium, Xiaomi Mi Drone 4K, YUNEEC Typhoon Q500 4K, DJI Phantom 3 SE, DJI Spark, DJI Mavic Air, DJI Mavic Pro and YUNEEC Typhoon H.

Below is a brief description of these UAV models

Parrot Bebop Drone 2:

Description: The Parrot Bebop Drone 2 is a popular quadcopter known for its portability and high-quality camera.

Characteristics: High video resolution, stable flight, maneuverability, possibility of use in various conditions.

Applications: Provides high-quality aerial photos and videos for entertainment and creative projects.

Xiro XPLORER V:

Description: Xiro XPLORER V is a quadcopter designed for high-quality photo and video shooting.

Features: GPS navigation, autopilot, high flight stability, high resolution camera.

Application: Used for aerial photo and video shooting in various conditions.

Autel Robotics X-Star Premium:

Description: Autel Robotics X-Star Premium is an aerial vehicle focused on high-quality images and ease of use.

Features: 4K Ultra HD camera, 12 MP photo, GPS navigation, long flight time.

Applications: Excellent for aerial photography and videography, especially for those looking for high quality in an affordable package.

Xiaomi Mi Drone 4K:

Description: Xiaomi Mi Drone 4K is an affordable and functional quadcopter from a well-known Chinese electronics manufacturer.

Features: 4K high resolution, stable flight, autopilot, long flight time.

Application: Excellent for high-quality photo and video shooting.

YUNEEC Typhoon Q500 4K:

Description: YUNEEC Typhoon Q500 4K is a high-quality quadcopter from a manufacturer known for its aerial photography products.

Features: 4K Ultra HD camera, 12 MP photo, GPS, Smart Mode for beginners.

## 1.2 **Types and Characteristics of Small Unmanned Aerial Vehicles**

A small unmanned Ariel vehicle or UAV as they are popularly called are UAVs are aircrafts without a dedicated pilot, a flying machine without a human pilot or passengers on board. As such, "unmanned" implies the complete absence of a human being in charge and actively in control plane. Control functions of unmanned aerial vehicles can be both on-board and external (remote control). UAVs are also called RPAs (Remotely Piloted Aircrafts), and Drones. sUAV’s used for UAVs weighting less than 25Kg. The letter “s” is especially written in lowercase, to accentuate the small size of these aircraft. An unmanned aerial vehicle contains cameras, sensors, means of communication, as well as other payload devices . It was created for military use and civilian use for border protection.

There are are primarily two types of sUAVs namely:

- Fixed wing sUAV;
- Rotatory wing sUAV.

Fixed Wing UAVs may transport a variety of payloads and data-link equipment. The fixed-wing has the greatest range of any of the other categories. The downside of this type of aircraft is that it requires a runway for landing or takeoff. UAV designers are actively working to tackle this issue by building a fixed-wing aircraft capable of taking off vertically and transitioning to horizontal flight. This type of fixed-wing aircraft uses Vertical Takeoff and Landing (VTOL) technology.

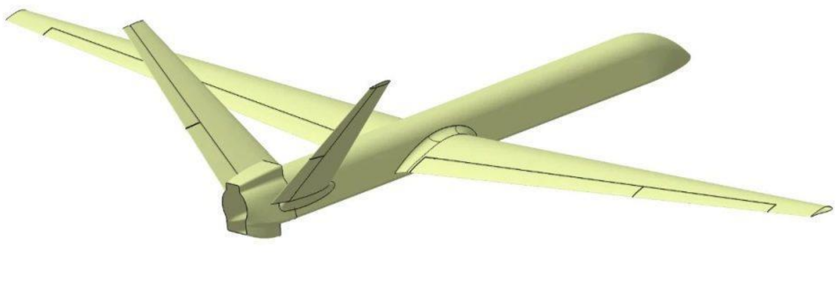


Fig. 1.2. A Fixed wing UAV.

Fig. 1.3. A Fixed wing UAV

Fixed-wing UAVs can travel greater distances, map considerably wider regions, and stay in place for extended periods of time to monitor their target. The average flying time is a few hours. However, with a higher energy density of fuel (gas engine powered), several fixed-wing UAVs can remain aloft for 16 hours or longer. This drone type can fly higher, carry more weight, and be more forgiving in the air than other drone types.

Fixed-wing UAVs are typically flown after receiving training. The first time you launch a fixed-wing drone, you must be confident in your ability to maintain control during the flight and return to a soft landing. A fixed-wing drone is continually moving forward and moves much faster than a multi-rotor, thus you may not be able to put it into a hover. In most circumstances, a launcher is required to lift a fixed-wing drone into the air. Flight is only the first step in fixed-wing aircraft. The hundreds of thousands of collected photos must be analysed and stitched into a single large tiled image. There is a lot more work to be done after that, including data analysis such as stockpile volume calculations, tree counts, overlaying other data into maps, and so on

*Application***:** Aerial mapping, Drone surveying for forestry and environmental purposes, Pipeline UAV surveys, and coastal UAV surveys; Agriculture; Inspection; Building; Security.

Fixed-Wing Hybrid-Alos known as Vertical Take-off and Landing (VTOL):
VTOL UAVs that combine the advantages of rotor- and fixed-wing systems are known as hybrid drones. This kind of drone can hover, take off, and land vertically since rotors are fastened to the fixed wings. There aren't many hybrids in this new category yet, but as technology develops, this choice may become considerably more common in the years to come.

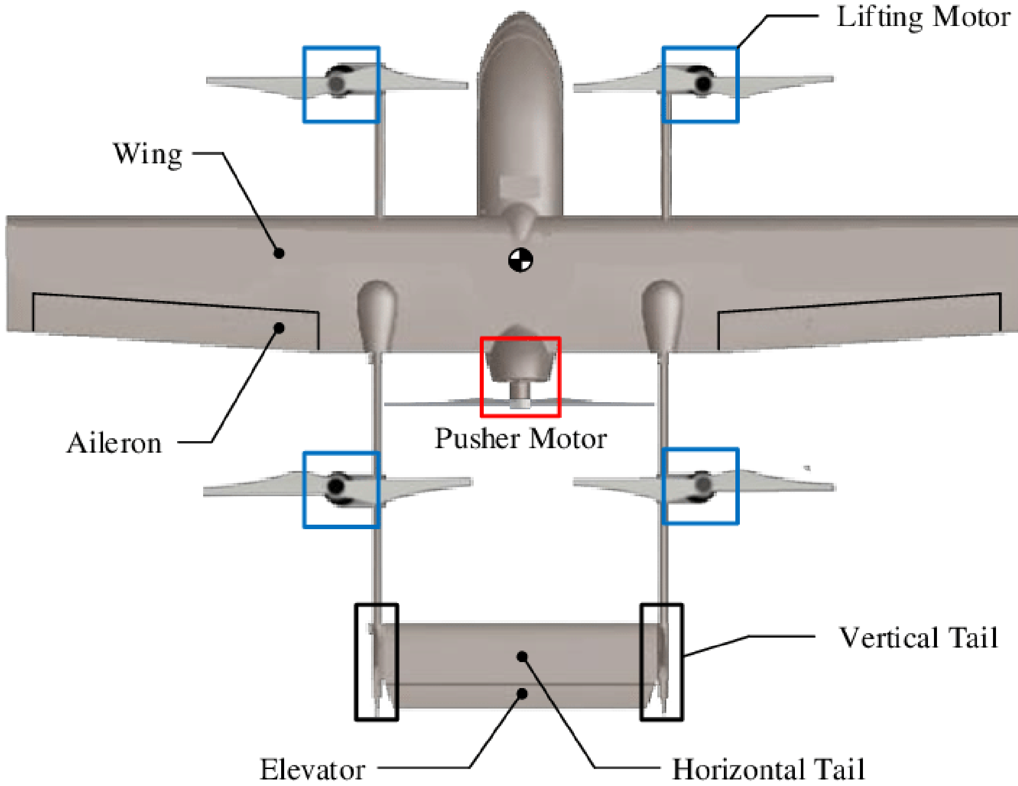



Fig. 1.4. VTOL UAV of the fist type

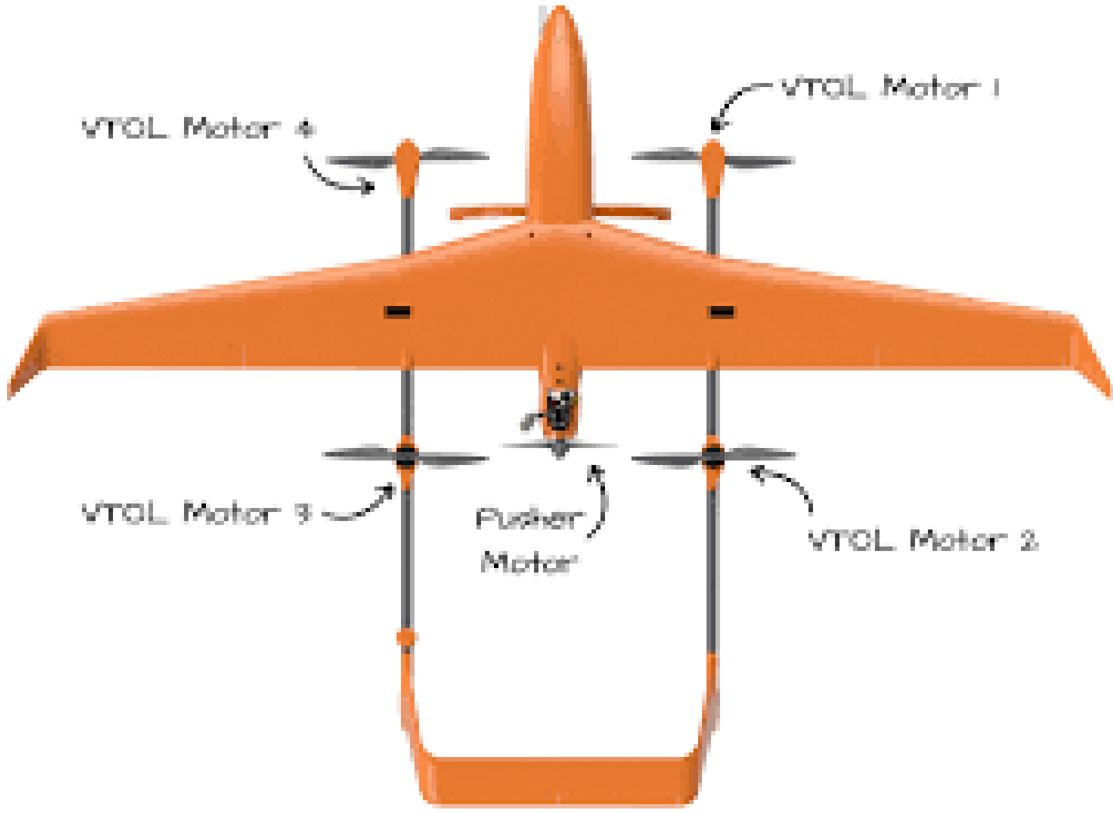



Fig. 1.5. VTOL UAV of the second type

Advantages: the UAVs autopilot can handle all of the difficult maintenance, leaving the human pilot to focus on the simpler task of navigating the vehicle through the air.

Fixed-wing and rotor-based designs combined provide you with the best of both worlds with hybrid VTOL drones. They are excellent at flying forward or hovering.

Drawbacks: there are currently just a few fixed-wing hybrid VTOLs available for purchase. The technology used in these drone types is still in the fledgling stage.

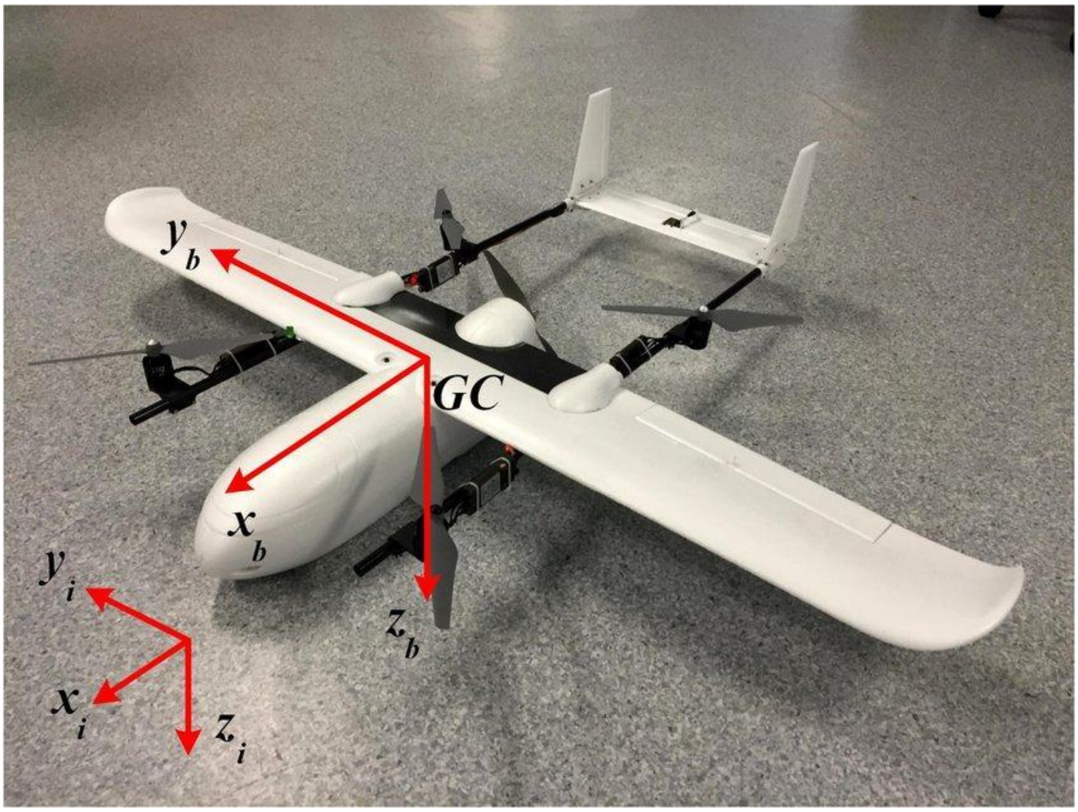



Fig. 1.6. Definition of Axis of Motion/inertial coordinate frame

Multi-rotor UAVs are the most convenient and cost-effective way to keep a watch on the sky. They also provide better control over location and framing, making them ideal for aerial photography and surveillance. They are known as multi-rotors because they feature more than one motor, including tricopters (3 rotors), quadcopters (4 rotors), hexacopters (6 rotors), and octocopters (8 rotors), among others. Quadcopters are by far the most popular type of multirotor UAV.

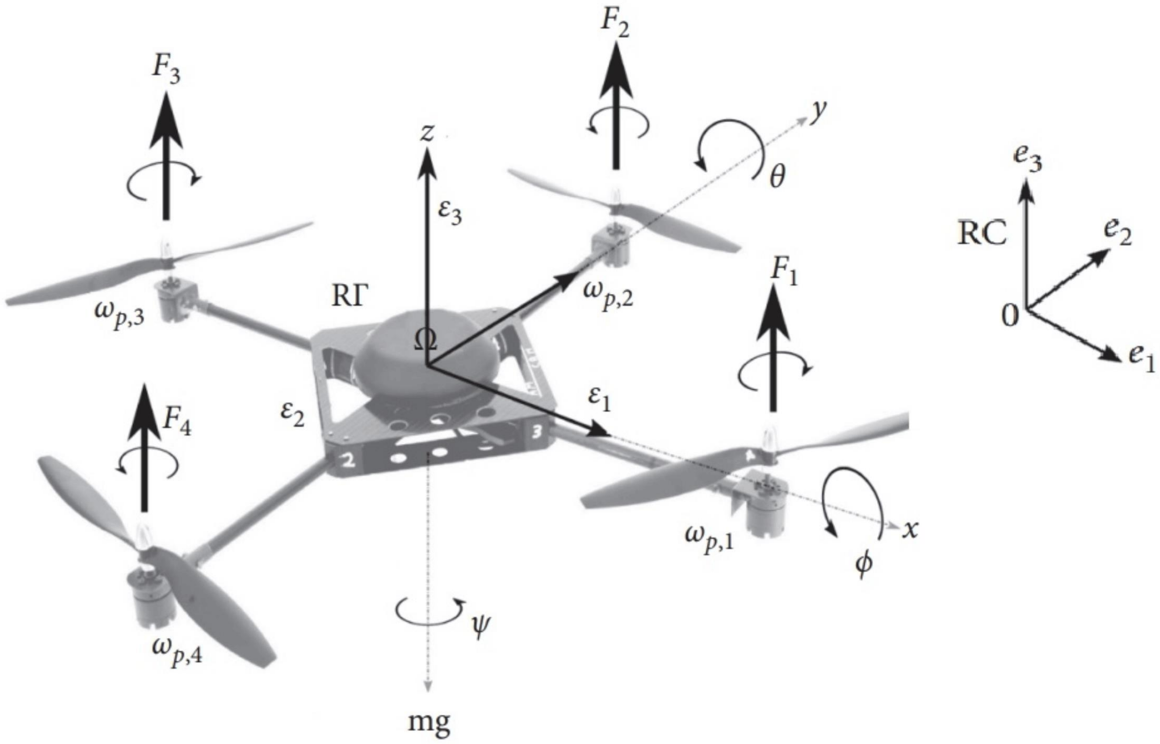



Fig. 1.7. Quadrotor orientation using Euler angles.

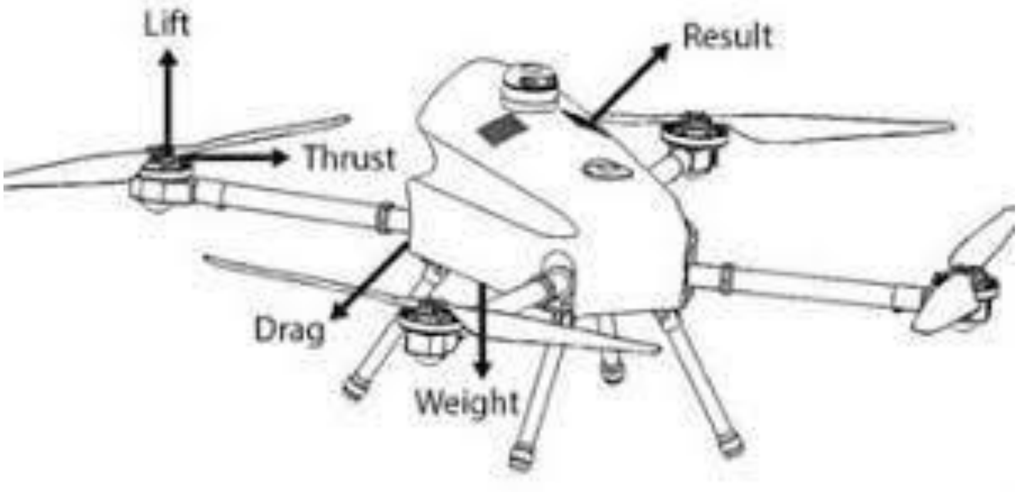



Fig. 1.8. Forces acting on the quadrotor

Advantages: It improves control of the aircraft during flight.

Because of its improved maneuverability, it can travel up and down the same vertical line, back and forth, side to side, and rotate in its own direction. It can fly significantly closer to structures and buildings. The capacity to carry numerous payloads every flight improves operating efficiency and lowers inspection time.

Disadvantages: Multi-rotor drones have limited endurance and speed, making them unsuitable for large-scale aerial mapping, long-duration monitoring, and long-distance inspections of pipelines, highways, and power lines. They are essentially inefficient, requiring a large amount of energy just to battle gravity and stay in the air. When carrying a lightweight camera payload, current battery technology limits them to approximately 20-30 minutes. Heavy-lift multi-rotors can carry more weight, but they have much shorter flight durations. Because multi-rotors require fast and precise throttle changes to maintain stability, using a petrol engine to power them is impractical, thus electric motors are the only option. So, until a new power source emerges, we can only expect tiny increases in flight time.

*Application:* Visual inspections, Thermal reports, Photography & videography 3D scans.

Fig. 1.9. Multi-rotor UAV

Single-rotor drones are sturdy and durable. In terms of structure and design, they resemble actual helicopters. A single-rotor UAV has only one rotor, which functions similarly to a large spinning wing, as well as a tail rotor for direction and stability control.

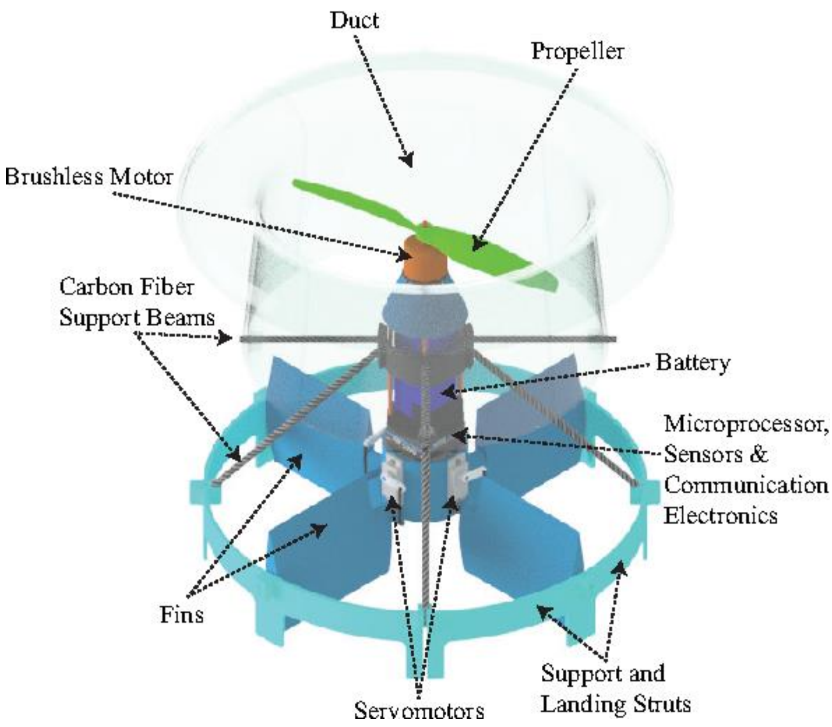



Fig.1.10. Instrumentation of UAV

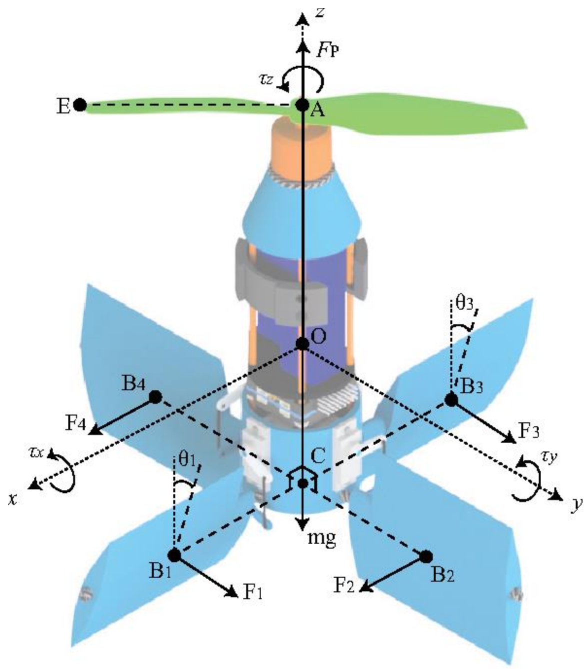



Fig. 1.11. Reference frames of UAV

A single-rotor helicopter is far more efficient than a multi-rotor, which improves if the drone is fueled by gas for even longer endurance. A single-rotor helicopter allows for extremely long blades that function more like a spinning wing than a propeller, resulting in high efficiency. If you need to hover with a substantial payload (e.g., an aerial LIDAR laser scanner) or do a combination of hovering with extended endurance and fast forward flight, a single-rotor helicopter is your best option. They're designed to be tough and long-lasting.

Disadvantages**:** Single-rotor uavs are complex and costly. They tremble and are less sturdy or forgiving in the case of a poor landing. Because of their mechanical intricacy, they require a great deal of maintenance and attention. The large, heavy rotating blades of a single rotor might be hazardous.

Application: Aerial laser scan. Drone Surveying. Carrying big workloads.

Fig. 1.12. A Single Rotor UAV

# CHAPTER 2

# MATHEMATICAL DESCRIPTION AND SIMULATION

## 2.1. Design of Small Unmanned Aerial Vehicle.

Aircraft-based system design typically involves three phases: **conceptual, preliminary, and detail design.**

**Conceptual research** is defined as a process in which research is carried out by watching and analysing existing knowledge on a certain issue. Conceptual research does not entail any real tests. It refers to abstract concepts or ideas. To ensure commercial viability, it's important to assess falls from an early stage. An initial design will be created to anticipate product performance and costs during development, manufacture, and operation.

a) Is it what the customer needs (rather than what he believes he wants)?

b) What is the market's expected size, in terms of units?

c) Will the consumer consider the unit manufacturing costs plus markup to be good value?

d) Will the customer be satisfied with the operating expenses and system reliability?

e) Will the program's nonrecurring costs be covered by sales returns within a reasonable time frame?

d) Are there any political or regulatory barriers to system sales?

To answer these problems, apply operational analysis, cost-benefit, and economic research techniques.

*The preliminary design.* After deciding to proceed, the overall system design will be explored in further detail. Optimisation trade-offs will be made to maximise the system's performance across all operational roles and conditions. To better understand component placement, maintenance accessibility, and operator ergonomics, a'mock-up' of the aircraft and control station areas can be built in wood or other easily workable materials. While 3D computer design programmes have reduced the need for this feature, the physical value of'real' hardware should not be overlooked. he system's components will be manufactured in-house or procured from external providers, with an estimated cost. The phase finishes with a detailed system design, including interfaces and specifications.

**Detail Design.** At this time, the scope of the task increases, and a larger number of people will be hired for the programme. A thorough analysis will be conducted on the aircraft's aerodynamics, dynamics, structures, and auxiliary systems, as well as the layout, mechanical, electronic, and environmental systems of the control station and any other sub-systems like launch and recovery systems. The system's pieces, including ground support and test equipment (unless purchased), will be designed and analysed for their worth. Tenders will be solicited for 'bought-out' items based on specifications.

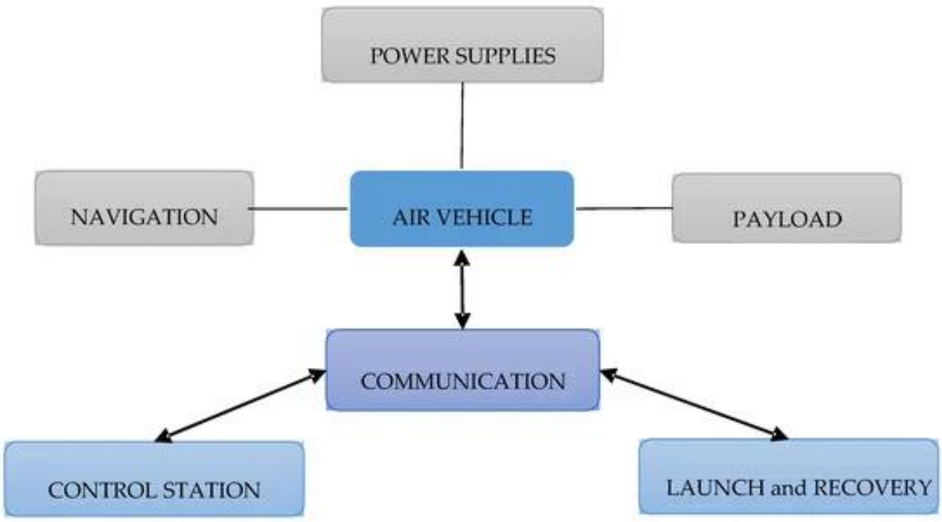



Fig. 2.1. Stages of UAV design

When dealing with the design of a UAV there are some key components that have to be considered when designing the UAV, these include: Air Vehicle-Payload, Air Vehicle-Speed Range, Air Vehicle- Endurance, Air Vehicle- Radius, Air Vehicle- Launch and systems, Environmental conditions, Maintenance, System Selection as Categories.

The payload of an air vehicle is the entire weight of equipment, cargo, or instruments that it can transport during a flight mission. Payload capacity is an important design factor for a wide range of air vehicles, including manned aircraft, unmanned aerial vehicles (UAVs), drones, and spacecraft.


| Multirotor | Fixed-Wing | Helicopter/VTOL |
|---|---|---|
| Wide selection | Ideal for corridor mapping | Spot takeoff/landing |
| Ease of use and maintenance | Highly stable | High altitude performance |
| Affordability and reliability | Long range and flight endurance | Highly stable |
| Greater maneuverability | Greater area coverage | High payload capacity |
| Unstable in windy conditions | Throw-launch complexity | Advanced piloting skills required |
| Limited flight endurance (battery) | Limited payload capacity | Maintenance difficulties |
| Small space for payload | Takeoff/Landing runway required | High cost |

Payloads can vary greatly based on the mission needs. They could comprise cameras and sensors for spying and surveillance, scientific instruments for research missions, communication equipment, freight for delivery, or special machinery for activities like crop spraying or search and rescue.  An air vehicle's payload capacity is defined by its design, which includes its frame strength, propulsion system power, and cargo space. It is critical to strike a balance between payload capacity and other performance metrics like as range, endurance, and control.

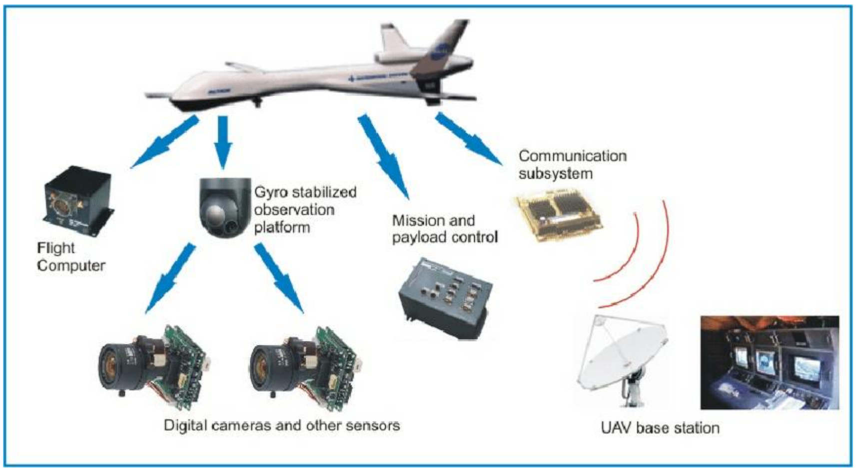



Fig. 2.2. UAV payload

*Payload Integration:* To ensure compatibility and optimal performance, it is necessary to carefully integrate mounting locations, power supply systems, data interfaces, and structural modifications when designing an air vehicle with payloads.

*Weight Distribution:* Accurate weight distribution is critical to ensuring the control and balance of the air vehicle in flight. Engineers must evaluate the payload's centre of gravity and aerodynamic effects to ensure flight is secure and efficient.

The payload-to-weight ratio measures an air vehicle's efficiency in conveying payloads relative to its own weight. Higher payload-to-weight ratios suggest improved payload conveyance or mission capability.

**UAV- Speed Range**

A UAVs speed range is the range of velocity it can reach while operating. For air vehicles, speed is an essential performance metric since it affects things like mission duration, range, payload capacity, and manoeuvrability.

The airplane's configuration and propelling power will be mostly determined by the needed speed range.

**UAV endurance**

UAV endurance, or the amount of time an aircraft can stay in the air on a single load of fuel or battery charge, is a crucial performance statistic. It is especially crucial for operations like long-haul transportation, search and rescue, surveillance, and reconnaissance that call for lengthy flights.

Things that affec the endurance of  UAV include:

Fuel Efficiency: One major factor influencing endurance is the propulsion system's efficiency. Air vehicles with engines or motors that use less fuel or energy per unit of time are able to stay in the air for longer.

Fuel/Battery Capacity: Endurance is directly impacted by fuel or battery capacity. Longer flight periods are made possible by larger fuel tanks or larger capacity batteries.

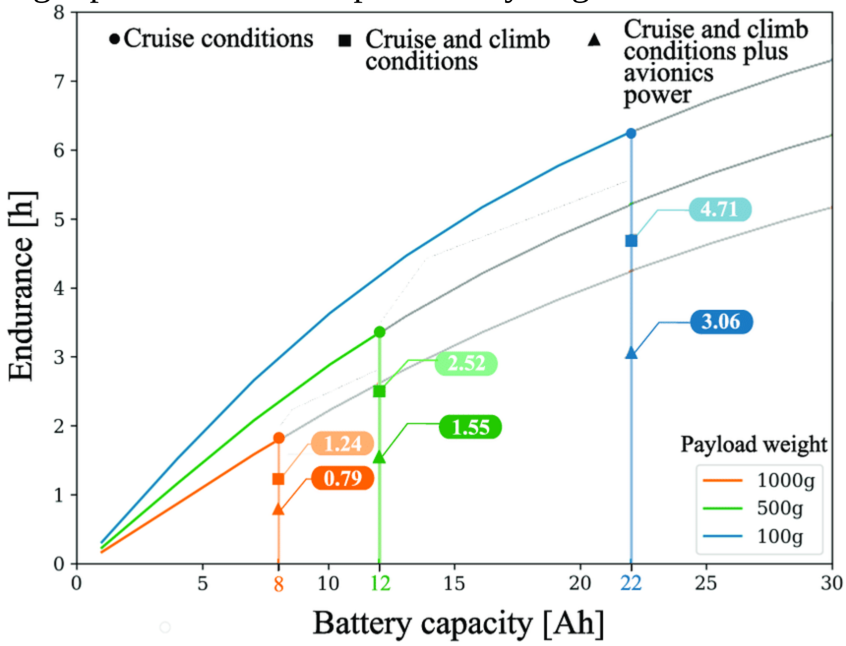



Fig. 2.3.

An Endurance Chart depicting the air vehicles endurance with varying payload weights.

Flight Profile: The air vehicle's speed, altitude, and manoeuvrability all have an impact on endurance. Maximising endurance can be achieved by optimising the flight profile for energy efficiency, such as cruising at an ideal speed and altitude.

Environmental Factors: Weather, altitude, and constraints imposed by air traffic control are examples of external factors that can affect endurance.

**UAV- Radius Range**

An air vehicle's maximum safe return distance from its base of operations without the need for fuel or recharging is known as its radius range. This measurement is essential for comprehending the vehicle's operational range and is particularly significant for missions involving long-range strikes, search and rescue, surveillance, and reconnaissance.
The following are examples of things that can affect the radius range of a UAV

Battery/fuel capacity:
Fuel-Powered Aircraft: The range of an aircraft is determined by the size of its fuel tanks and the fuel efficiency of its engines. Systems for managing gasoline efficiently are essential.
Energy density and battery capacity are crucial for electric-powered aircraft. Technological developments in batteries can increase the range.

Managing Weight:
Range is impacted by the aircraft's total weight, which includes the airframe, payload, fuel, and other equipment. Because lighter aircraft use less energy, they can travel farther.

A high-end unmanned aerial vehicle (UAV) such as the MQ-9 Reaper can cover a radius of approximately 1,150 nautical miles (2,130 kilometres) based on the mission profile and payload.

**UAV-Launch and systems**

Launching and operating UAVs (Unmanned Aerial Vehicles) involves various important systems and procedures adapted to the unique type and mission of the UAV.

Various Launch methods of UAVs Include:

**Vertical Takeoff and Landing (VTOL)**

Rotors are used by UAVs with VTOL capability to enable vertical takeoff and landing, much like helicopters.
These are used in research and resucue operations due to their high manuverability. These are usually more costly than regular UAVs

Fig. 2.4. Vertical Takeoff and Landing (VTOL) UAV.

**Hand Launch**:
In this case, the operator throws the UAV into the air with their hand. This type of launch is for small fixed winged UAVs.

Fig. 2.5. Hand launch UAV

**Catapult Launch**:
In this case, the UAV is propelled into the air using a mechanical system (elastic bands, pneumatic, or hydraulic systems). This method does not require a runway, because a catapult is being used. This method is primarily for medium sized fixed winged UAVs.

Fig. 2.6. Catapult launch UAV

## Environmental conditions

UAVs (Unmanned Aerial Vehicles) operate in varied environmental situations, any of which can greatly effect their performance and mission success.

**Some environmental conditions that affect UAVs include:**

**Rain and Humidity:**

Rainfall can lead to weight gain, water intrusion into electronic components, and decreased optical sensor visibility.
Humidity can contribute to short circuits and sensor mistakes by causing condensation on electrical parts and sensors. Rainwater can build up on the UAV's surfaces, adding to its weight and reducing its ability to fly. This additional weight reduces flight endurance and may have an impact on the stability of the UAV by requiring more lift, which in turn demands more power. The aerodynamic characteristics of the UAV's wings and propellers can change in wet conditions, lowering the effectiveness of lift and propulsion. Both reduced performance and higher energy use may result from this.

**Wind**:
Wind can increase power consumption, and impact stability and control. Unpredictable movements can result from abrupt changes in wind direction and speed, which makes controlling the UAV challenging.
Flying against the wind shortens the flight duration and range since it takes more energy to sustain speed.
Winds of all directions may cause the UAV to stray from its planned flight route, necessitating frequent adjustments.

**Snow and Ice:**
Snow buildup on the UAV's wings, airframe, and propellers makes it heavier. So a heavier payload lowers lift, uses more power, and may impact the UAV's stability and balance. Apart from the build up of snow, LiDAR, cameras, and other sensors can get covered in snow. Because these sensors are covered in snow, the UAV's capacity to navigate, identify obstacles, and collect data is hampered. Snowfall can not only make it harder to see objects with optical sensors, it can alsotemper with the visual line of sight of the people operating.

**Dust and Sand**:
Because they are abrasive, sand and dust particles can damage moving components including bearings, gears, and propellers.
This results in decreased effectiveness, more upkeep, and possibly a shorter component lifespan.
Sand and dust can clog cooling systems and air intakes, causing motors and electrical components to overheat. System malfunctions, decreased performance, and possible component damage can result from overheating.
Electronic enclosures are susceptible to short circuits, corrosion, and electrical failures due to the infiltration of dust particles.
This may result in unplanned system failures, abnormal behaviour, and shutdowns of vital systems.

## Aerodynamics

The ratio of an aircraft's surface area to mass can provide an indication of how well it responds to air turbulence and, to some extent, of its relative aerodynamic efficiency.

Aerodynamic forces have the potential to upset a surface area greater in size. Its inertia, or resistance, to the applied forces, increases with mass.

Applying the scaling principles reveals that the variance in the area/mass ratio is determined by dividing the linear dimension ratio n by the package density **ρD, or n/ρD.** Stated differently, will increase as aircraft size decreases and decrease as package density increases. UAVs will usually encounter more aerodynamic disturbance in turbulent air than larger aircraft because they are frequently smaller than human aircraft; however, this can be lessened if the UAV's designer is able to achieve a higher package density. Attaining aerodynamic efficiency requires minimising the friction drag and body and wing profiles. (The aircraft's drag-to-weight ratio, which is low for efficiency and usually relates to the drag at 30 m/s airspeed, is generally used to measure this.) Again, low-packed-density aircraft usually have a higher value for this.
The smaller aircraft has more friction and profile drag of both wings and body for the same shape as a larger aircraft because it flies at a lower Reynolds number ($I_R$). The Reynolds number, a nondimensional metric that indicates the relative importance of the two force types in a fluid flow, is the ratio of the viscous and inertia forces in that flow. The inertia forces, which are larger downstream over longer surfaces at greater flow speeds, are the main forces at higher $N_R$.
The value of $N_R$ is obtained using the formula:

$$\mathbf{N_R = vl/v.}$$

Here, l is the characteristic length (like the wing chord), **v** is the flow (air) speed, and υ is the fluid's kinematic viscosity, which has a standard value of **$1.47 \times 10{-}5$ m2/s** for air.

Aerodynamic drag is greater on surfaces operating at lower $N_R$ values than it is on surfaces operating at higher values. Unmanned aerial vehicles (UAVs) usually produce more drag and less noise than human aircraft because of their smaller size and slower operating speeds.
For a range of $N_R$ values, the drag coefficient, based on surface area, of a streamlined aerofoil with a 15% thickness to chord ratio is displayed.

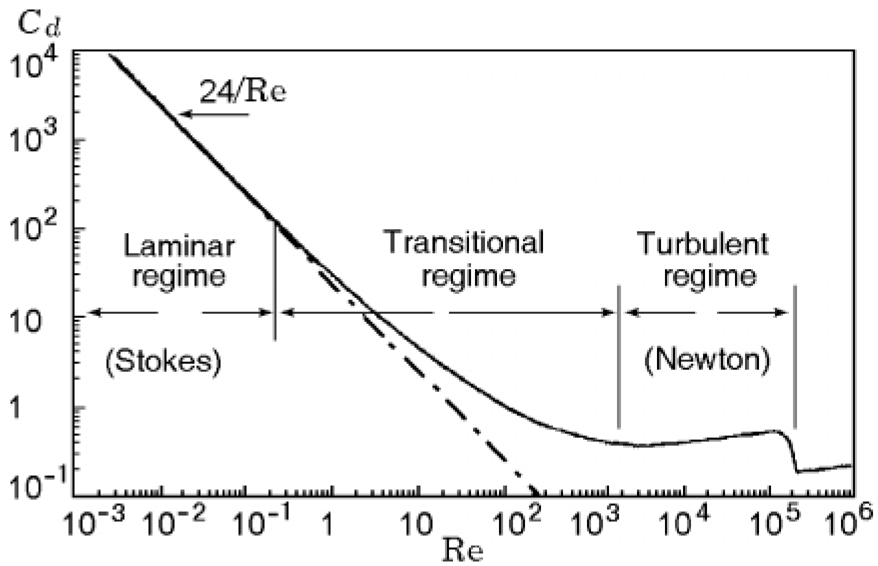


Fig. 2.7. **Standard dependence of drag coefficient on Reynolds number.**

**Reynolds Number-**

The dimensionless Reynolds number (Re) indicates the ratio of inertial forces to viscous forces for a given set of flow circumstances. Osborne Reynolds is credited with popularising the concept in 1883. Re describes a number of different flow regimes. Small UAVs operate in the range of 3×104 to 5×105 in the Re regime. The design of efficient airfoils is important at low Re. When laminar separation and turbulent flow transition to 3×104≤Re≤7×104, reasonably thick airfoils (≥6% thickness) can undergo considerable hysteresis because of the lift and drag forces generated. At low Res, where viscous forces predominate, laminar flow occurs. At high Res, when inertial forces are predominant, turbulent flow takes place.
Below are the re values for 5×104.

Laminar separation occurs in the flow, which stays unchanged and does not reattach to the airfoil surface. Re≤2×105≤7×104

The airfoil's surfaces can experience wide laminar flow, which reduces airfoil drag.

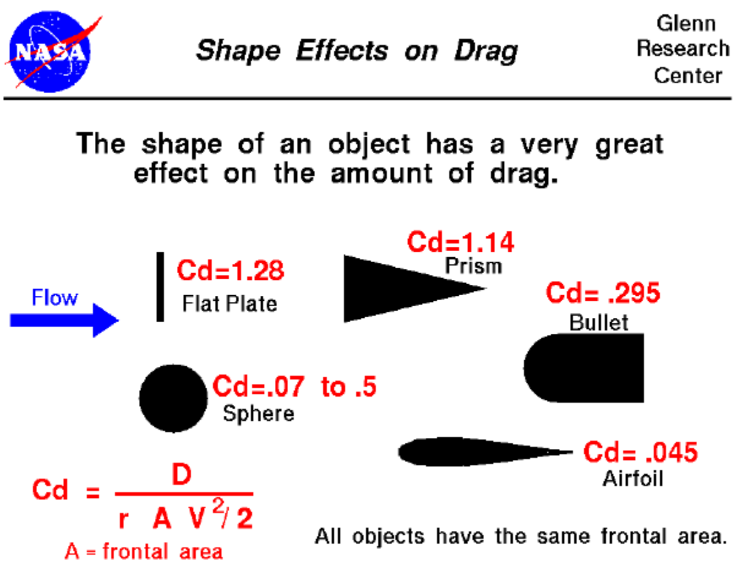



Fig. 2.8. Different shapes effect on drag.

Aerodynamicists use the drag coefficient to model different flow conditions and all of the complex interactions between drag and form inclination. By dividing the drag **(D)** by the following formula, one can find the drag coefficient **(Cd):** density **(r)** times reference area

**(A)** times half of the squared velocity (V). This slide shows the typical drag coefficient values for a variety of forms. The data displayed here were derived from experimental model installation in a wind tunnel, together with drag, tunnel velocity, and tunnel density measurements. Next, the drag equation was applied to produce the coefficient. Each object's projected frontal area served as its reference region. A sphere's coefficient of determination (Cd) ranges from.07 to.5, a bullet's i**s.295,** a typical airfoil is**.045,** a wedge-shaped prism with the wedge facing downstream is **1.14,** and a flat plate's is **1.28.**

Aircraft
Cm0 = aircraft moment coefficient at α = 0°
Cmα = aircraft moment curve slope
Vmax = aircraft maximum cruise velocity
Wing
ARw = aspect ratio of the wing planform
b = wing span
c = wing mean aerodynamic chord
Cmacw = wing moment about the aerodynamic centre
cmacw = wing airfoil moment about the aerodynamic centre
Cmcgw = wing contribution to moment about aircraft's centre of gravity
Cm0w = wing moment at α = 0°
Cmαw = wing moment curve slope
CLw = wing lift coefficient
CL0w = wing lift coefficient at α = 0°
CLαw = wing lift curve slope
cLαw = wing airfoil lift curve slope
Clαw = wing lift curve slope (per radian)
iw = wing incidence angle (mean chord to fuselage)
Sw = wing surface area
xac = distance from wing leading edge to aerodynamic centre
α = angle of attack
α0 = zero-lift angle of attack of wing airfoil
α0w = wing zero-lift angle relative to fuselage reference line
Tail
ARt = aspect ratio of the tail planform
CLαt = tail lift curve slope
cLαt = tail airfoil lift curve slope
Cmact = tail moment about the aerodynamic centre
cmact = tail airfoil moment about the aerodynamic centre
Cmαt = tail moment curve slope
Cm0t = tail moment at α = 0°
it = tail incidence angle
lt = tail moment arm
St = horizontal tail surface area
VH = horizontal tail volume ratio
α = tail angle of attack

$\varepsilon$ = downwash incidence angle
$\varepsilon 0$ = downwash angle at tail when wing is at $\alpha = 0°$
$\eta$ = ratio of tail to wing dynamic pressures (tail efficiency)
Fuselage
$Cm\alpha f$ = fuselage moment curve slope
$Cm0f$ = fuselage moment at $\alpha = 0°$
k2 − k1 = correction factor for body fineness ratio
lf = fuselage length
wf = average width of fuselage sections
x = coordinate
$\Delta x$ = length of fuselage increments
$\varepsilon u$ = local induced angle due to upwash or downwash at each fuselage segment

## 2.9. Mathematical Description of Small Unmanned Aerial Vehicle

Mathematical modelling is the process of creating mathematical formulas that describe the dynamics and control systems of a small unmanned aerial vehicle (UAV). Typically, this includes modelling the kinematics, aerodynamics, and control rules.

The mathematical modeling of a SUAV consists of some key factors which include:

Coordinate Systems

Understanding the relative orientations of different body sections is essential for studying unmanned aircraft systems. The plane's orientation with respect to the earth is the most crucial detail we must understand. We may also be curious about the orientation of a sensor, such a camera, with respect to the aircraft, or the position of an antenna with respect to a ground-based data source.

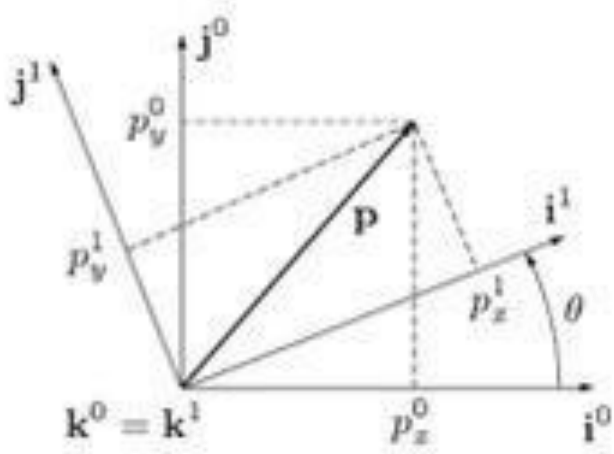



Fig 2.10. Coordinate frame rotation in 2D

**The coordinate systems includes:**

**Inertial Frame** (NED - North, East, Down): This frame is the frame that depicts the aircraft relative to the earth.

**Body Frame:** A frame that starts at the centre of gravity of the UAV and is secured to it.
Below are some of the reasons we use different coordinate frames.
-Newton's equations of motion have their roots in a fixed, inertial reference frame. Nevertheless, defining motion in a body-fixed frame is the simplest.
-Using a body-fixed reference frame makes it easier to describe the aerodynamic forces and torques operating on the aircraft body.
-Rate gyros and accelerometers are examples of on-board sensors that measure data in relation to the body frame. Alternatively, GPS determines position, ground speed, and course angle in relation to the inertial frame.
-Most mission requirements, like loiter locations and flight routes, are contained in the inertial frame. Additionally, an inertial frame with map data is offered.

**Small UAV Coordinate Frames**
A number of coorary dinate systems are of interest in order to derive and comprehend the dynamic behaviour of MAVs. The inertial frame, the vehicle frame, the vehicle-1 frame, the vehicle-2 frame, the body frame, the stability frame, and the wind frame are the coordinate frames that will be defined and discussed in this section. The remaining frames are related by rotations, and the inertial and vehicle frames are related by translations.
Roll, pitch, and yaw angles define the relative orientations of the vehicle, vehicle-1, vehicle-2, and body frames; these angles also characterise the aircraft's attitude. Euler angles are the name given to these angles in general. The angle of attack is the rotation angle that determines the relative orientation of the body, stability, and wind coordinate frames. as well as sideslip angles. The earth is assumed to be flat and non-rotating throughout the book, which is appropriate for MAVs.

Fi, the inertial frame
The defined home location serves as the origin of the inertial coordinate system, which is an earth-fixed coordinate system. The unit 208026d vectors i', i', and k' are oriented north, east, and downward, respectively, towards the earth's centre, as depicted in figure 2.3. A north-east-down (NED) reference frame is another name for this coordinate system. The terms "inertial x direction," "inertial y direction," and "inertial z direction" are frequently used to describe the directions of motion.

The vehicle frame Fu

The MAV's centre of mass is where the vehicle frame originated.

But the axes of F' line up with the inertial frame's axis.

**Kinematics**

The kinematics of the coordinate frames connect the states of a fixed-wing UAV and transform the forces and moments operating in different coordinate frames. A grasp of reference frames and their dynamics is necessary for designing guiding, navigation, and control systems. Kinematics, as it relates to UAVs (Unmanned Aerial Vehicles), is the study of motion without considering the forces that propel the motion. The position, acceleration, velocity, and orientation of the UAV are all described in a time-series format. Kinematic analysis is the foundation for path planning, navigation, and control of unmanned aerial vehicles (UAVs). It is essential to comprehend a UAV's motion across space.

Position:

**=r =v**

r here is the position vector in the inertial frame and v is the velocity vector in the inertial frame.

Orientation: Euler angles (roll $\phi$, pitch $\theta$, yaw $\psi$) describe the UAV's orientation.

$\phi = p + \tan(\theta)(q\sin(\phi) + r\cos(\phi))$

$\theta = q\cos(\phi) - r\sin(\phi)$

$\dot{\psi} = \cos(\theta) q\sin(\phi) + r\cos(\phi)/\mathrm{Cos}\ (\theta)$

where $p,q,r$ are the angular velocity components in the body frame.

**Dynamics**

Dynamics is the study and analysis of the motions and forces that regulate an unmanned aerial vehicle's behaviour in the air. It involves understanding how the UAV responds to various stimuli, such as control commands, environmental disturbances (such wind), and gravity forces, and how these responses manifest themselves in terms of the UAV's orientation and motion. Dynamics includes both rotational and translational components of motion.

**Translational Dynamics:**

Position and Velocity: The three-dimensional movement of the UAV is denoted by the terms "position" (x, y, z) and "velocity" (v).

Forces: The main forces affecting a UAV are thrust, gravity, aerodynamic lift, drag, and maybe other forces like ground effect or payload effects.

Equations of Motion: Newton's Second Law (Eq. = Eq. * F=ma) applied in three dimensions describes the acceleration of the UAV in response to net external forces.

The translational motion is described by the following equations:

□□¨=□ total

where:

*m* is the mass of the UAV.

**r**¨is the acceleration vector.

**F**total includes the sum of all external forces (thrust, gravity, aerodynamic forces).

$$m\mathbf{v}=m\mathbf{g}+\mathbf{F}T+\mathbf{F}A+\mathbf{F}D$$

*where m is the mass, g is the gravity vector, FT is the thrust force, FA is the aerodynamic force, and FD is the drag force.*

- ***Rotational Dynamics:***
- *Orientation: Describes the UAV's rotation around its centre of gravity using quaternions or Euler angles (roll, pitch, and yaw).*
- *The UAV's angular velocity and acceleration are the rates at which its orientation shifts.*
- *Moments and Torques: The UAV experiences moments or torques about its principal axes due to gyroscopic effects, aerodynamic forces, and control inputs from the propellers or control surfaces.*
- *Motion Equations: Euler's rotational motion equations, which relate the UAV's moment of inertia and applied torques to the angular acceleration.*

$$\mathbf{I}\dot{} + \omega \times (\mathbf{I}\omega) = \mathbf{M}T + \mathbf{M}A$$

*where□ is the inertia matrix, □=[□,□,□]^T is the angular velocity vector,* ***M****T is the moment due to thrust, and* ***M****A is the aerodynamic moment.*

***Aerodynamic Forces and Moments***

Aerodynamic forces and moments are essential to comprehending and controlling the flight dynamics of unmanned aerial vehicles (UAVs). These forces and moments are produced by the UAV's interaction with the air and are essential in determining the UAV's motion.

*Lift, Drag, and Side Forces:*

***Lift:- L*** *is the force perpendicular to the relative wind and the UAV's longitudinal axis. Lift is caused by the difference in pressure between the UAV's upper and lower rotor or wing surfaces as a result of its motion.*

*Formula:* $L = \frac{1}{2} \rho v^2 S C_L$

*where ρ is the air density, v is the relative airspeed, S is the wing area, and CL is the lift coefficient.*

***Drag:-*** *D is the force that runs counter to and parallel to the relative wind which is caused caused by the air resistance that the UAV encounters while flying.*

*Formula:* $D = \frac{1}{2} \rho v^2 S C_D$

*Where* $C_D$ *is the drag coefficient*

***Side Force:-*** *y is the force, usually in the horizontal plane, that is perpendicular to the drag and lift forces, because of asymmetrical circumstances like crosswinds or yawing motion.*

*Formula:* $Y = \frac{1}{2} \rho v^2 S C_Y$

*Where Y is the side force coefficient.*

*□□=[□□□□□□]=[−□□−□]*

$$\mathbf{F}_A = \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

***where FA is the aerodynamic force, and L is the lift, D is the drag, and Y is the side force.***

***Aerodynamic Moments***

$$\mathbf{M}_A = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

***where l,m,n are the roll, pitch, and yaw moments***

***Thrust and Propeller Dynamics***

- ***The thrust generated by each propeller can be modeled as***

$$T_i = k_T \omega_i^2$$

***where kT is the thrust coefficient and ωi is the rotational speed of the i-th propeller.***

***Control System***

- *PID Controllers: For controlling the UAV's position and orientation, PID (Proportional-Integral-Derivative) controllers are commonly used.*

$$u=K_p e + K_i \int e\, dt + K_d\ dt / de$$

***where e is the error between the desired and actual states.***

- *State-Space Representation:*

***x=Ax+Bu***

***y=Cx+Du***

***where x is the state vector, u is the input vector, and y is the output vector.***

***Simulation and Implementation***

- *Matlab/Simulink: Often used for simulating the UAV dynamics and control.*
- *Gazebo: For 3D simulation environments.*
- *Flight Testing: Validating the model with real-world flight data.*

***Translational Equations:***

$$m\ddot{x}=-u_1(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi)$$

$$m\ddot{y}=-u_1(\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi)$$

$$m\ddot{z}=mg-u_1\cos\phi\cos\theta$$

***Rotational Equations***

$$I_x\dot{p}=(I_y-I_z)qr+L$$

$$I_y\dot{q}=(I_z-I_x)pr+M$$

$$I_z\dot{r}=(I_x-I_y)pq+N$$

***Where L,M,N are the moments generated by the propellers and $u_1$ is the total thrust.***

The kinematic diagram of the quadcopter is shown in Fig. 2.11. In this apparatus, the front and rear motors rotate counterclockwise, while the other side motors rotate in the opposite direction, as shown in Fig. 2.11.

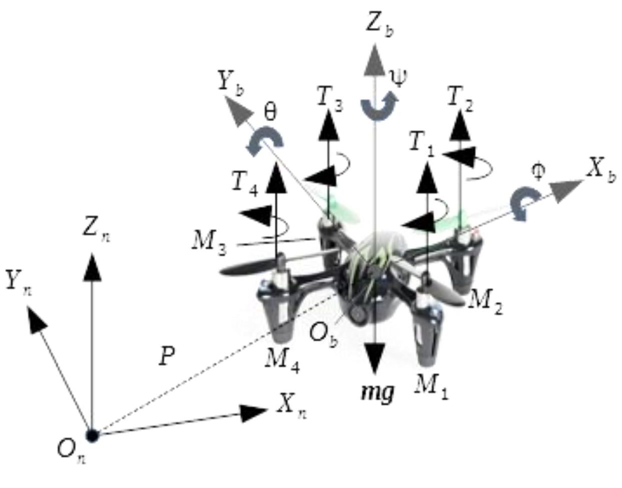



Fig. 2.11. The kinematic diagram of the quadcopter

Let's consider the principle of controlling such a device in the general case. Total power $T_z$ is the sum of capacities of every motor $T_z = \sum_{i=1}^{4} T_i$. Change of the angle of pitch $\theta$ is implemented due to increase (decrease) rotation of the rear electric motor $M_4$ with a simultaneous decrease (increase) in the speed of rotation of the front electric motor $M_2$. Change of the angle of roll occurs in a similar way with the involvement of side electric motors $M_1, M_2$.

Control of rotation around the vertical axis is carried out by increasing (decrease) the frequency of rotation of the front $M_1$ and rear $M_3$ electric motors with a simultaneous decrease (increase) of the frequency of rotation of the side electric motors $M_2, M_4$. During such maneuvers, the total thrust $T_z$ must remain stable. Changing the total thrust of all 4 engines $T_z$ provides control of the quadcopter in height.

Let $P = [p_x, p_y, p_z]$ is the radius vector of the center of gravity of the quadcopter (point $O_b$, Fig. 2.1) in the inertial (navigational) coordinate system $O_n X_n Y_n Z_n, \phi, \theta, \psi$ , are roll, pitch and yaw angles respectively, and $T_i$ is the lifting force created by the *i*-th engine $M_i$ $(i = 1, \ldots, 4)$. $O_b X_b Y_b Z_b$ is quadcopter body frame. Here and further, the original symbol denotes the transposition operation. The most generalized equation of quadcopter dynamics in terms of the Newton-Euler formalism can be formulated as follows:

$$\begin{gathered} m_Q \ddot{P} = RT, \\ J\ddot{\Theta} = C(J, \Theta, \dot{\Theta}) + Q, \end{gathered} \tag{2.1}$$

where:

$m_Q$ - total weight of the quadcopter total weight of the quadcopter;

$\mathfrak{R}$ - direct cosine matrix (DCM) from the body to the inertial coordinate system;

$T$ - vector of traction forces applied to the quadcopter, *J* is the inertia tensor of the quadcopter;

$\Theta = [\phi, \theta, \psi]$ – The vector of a position;

$C(J, \Theta, \dot{\Theta})$ – Coriolis term;

Q - vector of moments applied to the quadcopter.

System (2.1) appears with some variations in all quadcopter sources below. Decomposing the system (2.1) into a system of differential equations describing the dynamics of all components of the $P$ and $\Theta$ vectors leads to a very cumbersome system of nonlinear differential equations. Therefore, for the sake of brevity, we do not present it in this work. However, it is possible to simplify the system (2.1) and reduce it to four separate linear subsystems of differential equations, relying on some properties of the design of the quadcopter, which we will discuss next. These four systems include:

X-subsystem, which describes rotational movement relative to the X axis and translational movement along the same axis;

Y-subsystem, which describes rotational movement relative to the Y axis and translational movement along the same axis;

Z-subsystem describing translational movement along the Z axis;

$\psi$- is a subsystem describing rotational motion relative to the Z axis.

Now we can formulate the statement of the problem as follows.

Find four subsystems of differential equations in the standard Cauchy form (state space models of the four aforementioned physical subsystems) based on the known design parameters of the quadcopter. This standard form for all subsystems looks like this:

$$\frac{d\chi}{dt}(t) = A_{\chi}(t) + B_u U(t) + B_d D(t), \\ v(t) = C_{\chi}(t), \qquad (2.1)$$

where

$\chi(t)$ - state vector for each subsystem;

$U(t)$ – control input;

$D(t)$ – disturbance input (atmosphere turbulence);

$v(t)$ – observation vector;

$A$ – matrix of state augmentation;

$B_u$ – matrix of control input;

$B_d$ – matrix of disturbance input;

$C$ – observation matrix.

Linear models (2.1) for each subsystem will be used to synthesize control laws. For simulation purposes, they must be supplemented with some nonlinearities that are immanent in real systems to provide a simulation process close to real flight.

Taking into account the above statement of the problem, first of all, it is necessary to analyze the use of features of the structure of the quadcopter to simplify its mathematical model. Summarizing a significant number of works in the world literature, it is possible to formulate the following general features of the quadrotor mathematical model (2.1), which determine its decomposition.

1. The Coriolis term in the case of a quadcopter is significantly small due to the small angular velocities of the body frame, so it can be neglected.

2. The construction of the quadcopter is symmetrical with respect to all axes of the frame (see Fig. 2.3).

3. The mutual moments of inertia are very small compared to the own moments of inertia, so the inertia tensor can be represented in the form of a diagonal matrix.

All these assumptions make it possible to decompose the original system of equations into four separate systems of differential equations (2.1) describing the dynamics of the aforementioned subsystems. As follows from the previous point, the state vector of the system (2.1) will have the following form:

$$\chi = [p_x, \dot{p}_x, p_y, \dot{p}_y, p_z, \dot{p}_z, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]. \tag{2.2}$$

If the inertia of motors $M_i (i = 1, \ldots, 4)$ is neglected, then this set of components of the state vector will be complete. In some works, this assumption is accepted, especially for small indoor quadcopters. Such simplified models have proven very useful for comparing several competing flight control laws. Thus, it has been proved (based on the model with this state vector) that using an LQR control strategy based on simple state-space feedback can provide the same or even better performance of the control system than a complex nonlinear control law. Since we will return to the LQR law of flight control later, it is worth mentioning that this law is very useful not only in the case of flying a single quadcopter, but also in the case of flying as part of a wing.

It was proved that the decomposed four subsystems will be described by the following equations:

For the *X*-subsystem:

$$\frac{d^2p_x}{dt^2} = -\,g\,\frac{tantan\,_\theta}{_{cos}cos\,_\phi}, \tag{2.3}$$

$$J_x\frac{d^2\theta}{dt^2} = Q_\theta, \tag{2.4}$$

For the *Y*-subsystem:

$$\frac{d^2p_y}{dt^2} = g\ tan\ tan\,_\phi\,, \tag{2.5}$$

$$J_y\frac{d^2\phi}{dt^2} = Q_\phi, \tag{2.6}$$

For the Z-subsystem:

$$\frac{d^2p_z}{dt^2} = \frac{T_z}{m_Q}cos\ cos\,_\phi\ _{cos}\ cos\,\theta - g\ , \tag{2.7}$$

For the ψ-subsystem:

$$J_z\frac{d^2\psi}{dt^2} = Q_\psi, \tag{2.8}$$

In equations (2.2) - (2.8): $g$ – gravity acceleration 9.81 м/с; $T_z$ - total traction force of 4 screws:$Y_z = \sum_{i=1}^{4} \quad T_i$ . Variables $Q_\theta, Q_\phi, Q_\psi$ - control moments rotating the quadcopter relative to the corresponding axes (components of the vector Q in (2.1)). We can easily linearize these equations for small Euler angles $\varphi, \theta, \psi$ ($_{tan}\,tan\,_\phi \approx \phi$,$cos\,cos\,_\phi \approx 1$) and transform them into systems (2.1) for each subsystem. Note that equations (2.5) - (2.8) describe the movement of the quadcopter as a holonomic system. Therefore, the angle of inclination can be fixed at an arbitrary value, which does not affect other values.

Regarding the controlling influences, which are the traction forces of each propeller $T_i$ and their reactive moments $Q_i$ , it can be stated that they are proportional to the square of their rotation speed: $T_i = K_i\omega_i^2$ and $Q_i = K_Q\omega_i^2$, where $K_T, K_Q$ are force coefficients and moment respectively. In most of the world's publications, the torque coefficients $K_Q$ and thrust $K_T$ are evaluated using test benches that include small real-world models of quadcopters, data acquisition and processing systems. Most of these small real quadcopters are designed for indoor use. In the case of heavy outdoor quadcopters, such experiments require equipment that costs much more and takes more time. That is why the ultimate goal of this paper is to avoid such experiments in order to create an approximate but plausible model of quadcopter dynamics. The state vector model describes the dynamics of the quadcopter, neglecting the inertia of the electric motors. This assumption is accepted in many publications. However, it is not acceptable for heavy street quadcopters.

The electric motors of quadcopters are controlled by the standard "Electronic speed control" unit. At the output of EPC there is a current of electric motors. Therefore, we will include the dynamics of electric motors in the quadcopter model, which can be approximated by the following transfer function:

$$W_{em}(s) = \frac{K_{em}}{\tau_{em}s+1}, \tag{2.9}$$

where $K_{em}$ and $\tau_{em}$ are the static amplification factor (between the motor current $I(t)$ at the input and the propeller rotation frequency $\Omega(t)$ at the output) and the electromechanical time constant, respectively.

The actual control input for all 4 subsystems of the quadcopter is the increase in the rotation speed of the motors, taking into account the rotation speed $\Omega_0$ in the hover mode. This corresponds to the equality of the total traction force: $T_z = m_Q g$ . For example, for the $Y$ axis (see Fig. 2.1) we have: $\Omega_2 = \Omega_0 + \Delta\Omega_2$, $\Omega_4 = \Omega_0 - \Delta\Omega_2$. Therefore, to account for the inertia of the motors, we must include these $\Delta\Omega_i$ variables in the state vector. Note that in real conditions this variable is immeasurable. This circumstance must be taken into account in the procedure of synthesis of the control law.

Finally, it should be noted that the drag force is also neglected. However, in the case of a heavy quadcopter, it must be taken into account and included in the mathematical model.

### 2.12. Simplified Mathematical Model of Quadcopter

The simplified quadcopter model is based on the following assumptions:

1. Small values of the Euler angles in controlled flight allow the basic equations to be linearized.

2. The design of the quadcopter is symmetrical with respect to the vertical z axis, therefore, four motion control systems can be considered independently, especially for small values of the state variables:

2.1. Rotation relative to the $Y$ axis by roll angle and linear movement along the $X$ axis.

2.2. Rotation about the $Y$-axis by the pitch angle and linear movement along the $X$-axis.

2.3. Rotation relative to the $Z$ axis by yaw angle.

2.4. Linear movement along the $Z$ axis to adjust height.

3. Coriolis accelerations and gyro effects can be neglected for engineering design purposes.

Since we can reduce the quadcopter dynamics to the four motions just listed, we can design a flight control system for each partial motion. As mentioned above, here we consider the flight of the quadcopter in the horizontal plane. Therefore, we will design a control system for partial movements of the quadcopter, which include rotation relative to the -axis by an angle and linear movement along the axis.

The linearized UAV model can be presented in the following standard form:

$$\{\frac{d\chi}{dt}(t) = A_{\chi}(t) + B_u u(t) + B_d(t)d(t);\ v(t) = C_{\chi}(t), \quad (2.13)$$

where the state vector χ and matrices $A, B_u, B_d$ look like:

$$\chi = [\chi_1, \chi_2, \chi_3, \chi_4, \chi_5]^T = [y, dy/dt, \varphi, d\varphi/dt, \Delta\Omega]^T, \quad (2.14)$$

where $y$ is the position of the quadcopter along the $Y$ axis, $dy/dt$ is its linear speed, $\varphi, d\varphi/dt$ is the roll angle and roll speed, respectively, $\Delta\Omega$ is the increase in engine rotation speed, $d(t)$ is the external disturbance (instantaneous value of the speed of the turbulent wind), $u$ is the input control signal.

It is also necessary to include in the model a term that determines the inertia of the engine. It can be approximated by a 2nd-order or 1st-order transfer function. Considering the small inductance of the rotor circuit, the simplest model of the 1st-order electric motor can be used:

$$W_{em}(s) = \frac{K_{\Sigma}}{\tau_{em}s+1}, \quad (2.15)$$

where $K_{\Sigma}$ - static gain,

$\tau_{em}$ – a time constant of the electric motor.

The input of this unit is the current gain (control input $u$) produced by the electronic speed controller (ESC), and the output is the gain of the angular velocity of the electric motor $\Delta\Omega$, which produces the control moment applied to the $X$-axis of the quadcopter according to the 2nd equation in system (2.10).

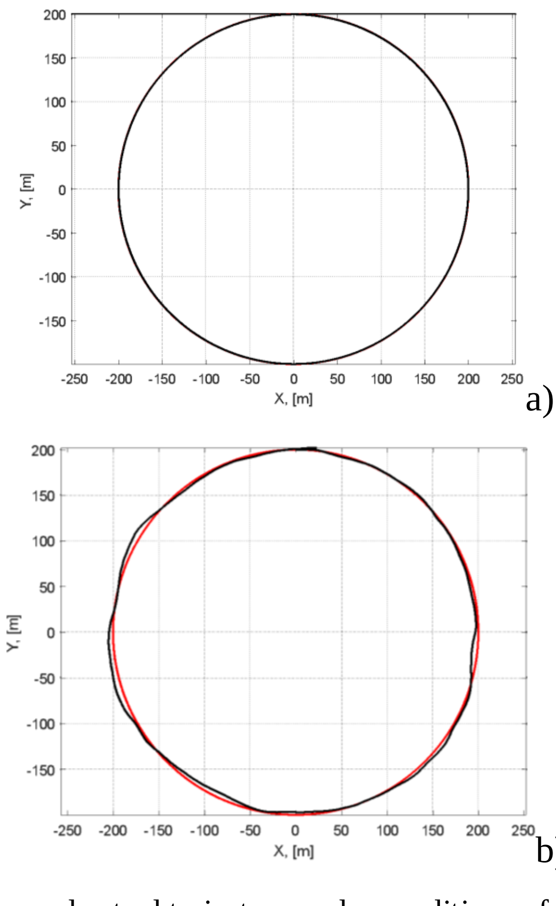
Based on all these assumptions and the following numerical values of the matrices $A, B_u, B_d$ and $C$ included in (2.10), the following numerical values were calculated:

$$A = [0\ 1\ 0\ 0\ 0\ 0\ -0.28\ 9.81\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 13.51\ 0\ 0\ 0\ 0\ -8.33\ ],$$

$$B_u = [0\ 0\ 0\ 0\ 10.42\ ], B_d = [0\ 1\ 0.0285\ 0\ 0\ ], A = [1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ ]. \tag{2.16}$$

# CHAPTER 3
# MODELING RESULTS OF SMALL UNMANNED AERIAL VEHICLE

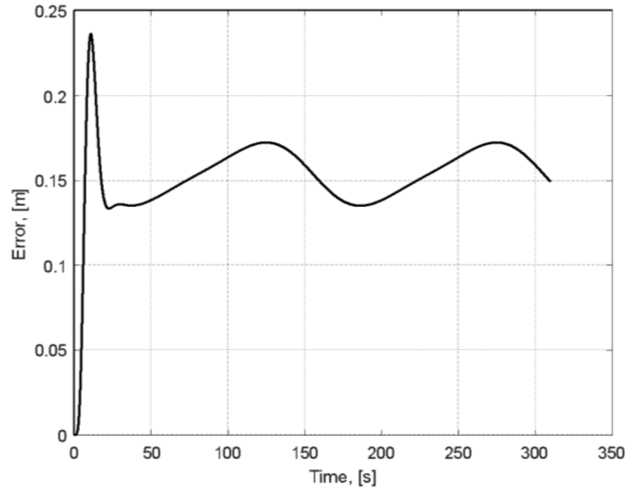
In Fig. 3.1, various aspects of modeling UAV movement in the horizontal plane in conditions of calm and turbulent atmosphere are shown. To simulate the turbulent atmosphere, the Dryden model was used for the case of flight at low altitude with moderate turbulence. As can be seen from Figures 3.1 – 3.6, the proposed control system provides quite good results for both cases: calm and turbulent atmosphere. Even with the influence of wind on the UAV, the error of the horizontal trajectory did not exceed 7 meters, and the speed was about 5 m/s, which are typical values for a simulated quadcopter. A positive point is also that the roll and pitch angles did not exceed 10 degrees in both cases, which ensured smooth control.



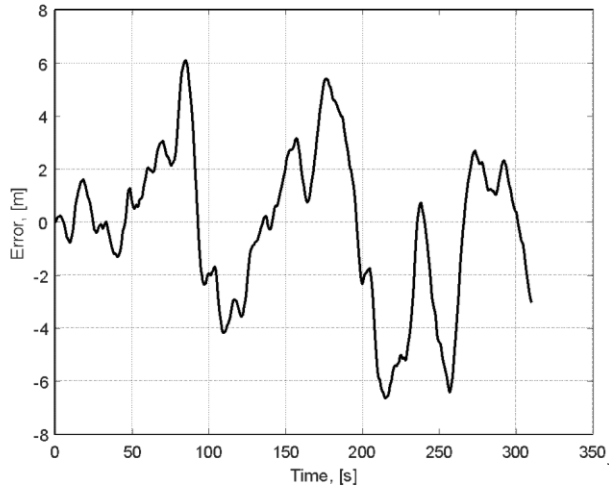
a)

b)

Fig. 1.3 Reference trajectory and actual trajectory under conditions of UAV movement in: a) calm and b) turbulent atmosphere

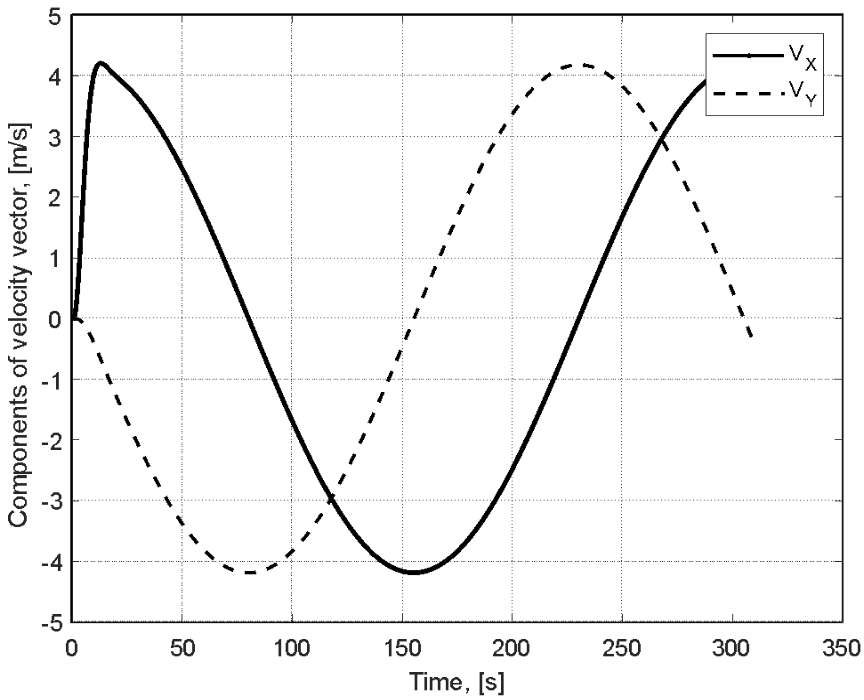



a)

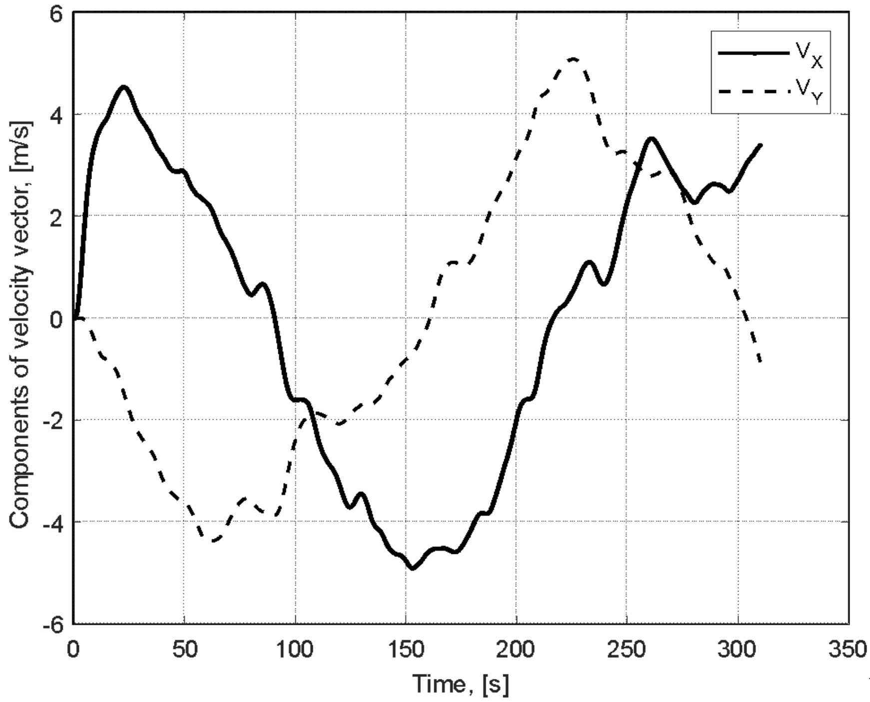



b)

Fig. 3.2. Error of the horizontal trajectory in the conditions of UAV movement in: a) calm and b) turbulent atmosphere

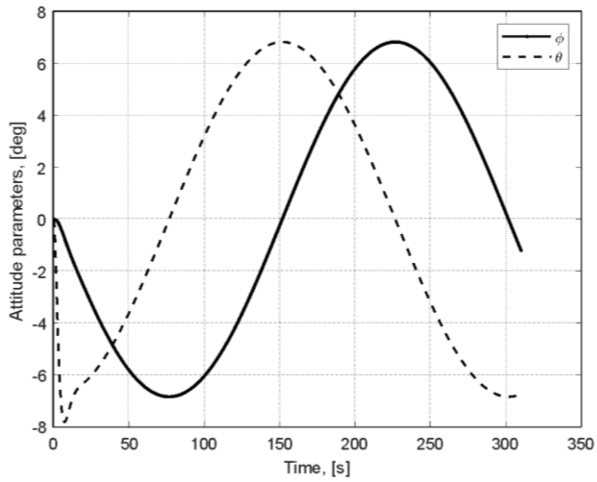



a)

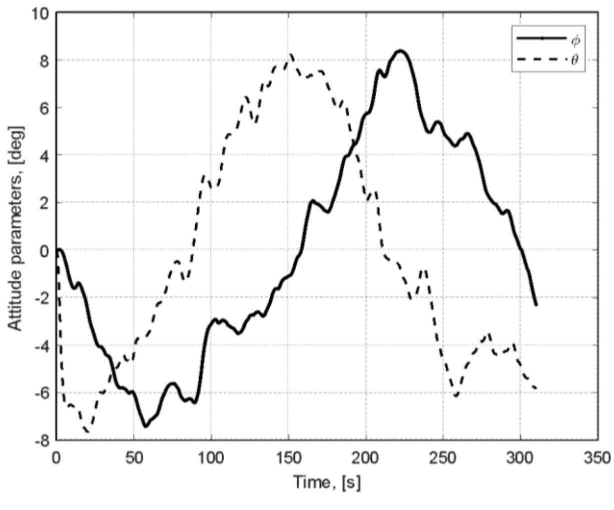



b)

Fig. 3.3 Speeds in the conditions of UAV movement in: a) calm and b) turbulent atmosphere




a)




b)

Fig. 3.4. Angle of roll and pitch in conditions of UAV movement in: a) calm and b) turbulent atmosphere

## CONCLUSIONS

The paper developed a mathematical model of quadcopter dynamics based on the physical laws of motion and UAV parameters. This made it possible to synthesize an effective control system without complex experiments.

The method of determining the parameters of the mathematical model based on the known characteristics of the quadcopter is proposed. This simplifies the process of modeling and designing the control system.

A robust control system with static output feedback based on the method of linear matrix inequalities has been synthesized. The synthesized system provides resistance to external disturbances.

A successful simulation of the movement of the quadcopter along a circular trajectory in conditions of a calm and turbulent atmosphere was carried out. The results meet the specified requirements.

The developed quadcopter control system is efficient, reliable and meets modern environmental standards. It can be used as a basis for further research and improvements.

So, the set tasks were successfully solved, and the developed quadrocopter control system demonstrates high efficiency and can be practically applied. Further research can be aimed at expanding the functionality of the system and adapting it to specific tasks.

## REFERENCES


1. G.M. Hoffman, S.L. Waslander, and C.J. Tomlin, "Quadrotor Helicopter Trajectory Tracking Control," Proc. of the AIAA Guidance, Navigation and Control Conference and Exhibition, 18-21 August 2008, Hawaii, Honolulu, pp. 1–14.
2. H. Bouadi, F. Mora-Camino, D. Choukroun, "Space-Index Control for Aircraft Vertical Guidance with Time Constraint", Journal for Guidance, Control and Dynamics, vol. 37, no.4, 2014, https://doi.org/10.2514/1.62118.
3. T.S. Tsay, "Guidance and Control Laws for Quadrotor UAV," WSEAS Trans. on Systems and Control," vol. 9, 2014, pp. 606–613.
4. S. Bouabdallah and R. Siegwart, "PID vs LQ Control Techniques Applied to an Indoor Microquadrotor," Proc. of 2004 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Sept. 28 – Oct. 2, Sendai, Japan, 2004, pp. 2451–2456.
5. S. Bouabdallah and R. Siegwart, "Full Control of a Quadrocopter," Proc. of 2007 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Oct. 29 – Nov. 2, San Diego, CA, USA, 2007, pp. 153–158.
6. R.W. Beard and T.W. McLain, "Small Unmanned Aircraft. Theory and Practice," Princeton University Press, 2012.
7. Anastasios M. Lekkas, Andreas R. Dahl, Morten Breivik, Thor I. Fossen/ Continuous-Curvature Path Generation Using Fermat's Spiral // Modeling, Identification and Control. — 2013. — Vol. 34, № 4. — P. 183-198.
8. Castillo, P., Dzul, A., & Lozano, R. (2004). Real-time stabilization and tracking of a four-rotor mini rotorcraft. *IEEE Transactions on Control Systems Technology*. https://doi.org/10.1109/TCST.2004.825052
9. A. Wahyudie, T.B. Susilo, H. Noura. (2013) Robust PID Controller for Quad-rotors. Journ. of Unmanned System Technology, Vol.1, №1,-2013.-pp.14-19.
10. V.B. Larin and A.A. Tunik. "On Problem of Synthesis of Control System for Quadrocopter," International Applied Mechanics, vol. 53, no. 3, 2017, pp. 342–348.
11. A. Wahyudie, T.B. Susilo, and H. Noura, "Robust PID controller for quad-rotors," Journ. of Unmanned System Technology, vol. 1, pp. 14–19, 2013.
12. V.L. Timchenko and D.O. Lebedev, "Optimization of Stabilization Processes of Quadrocopter for Maritime Traffic's Monitoring," Electronics and Control Systems, 3(53), pp. 32–39, 2017.

13. A.A. Tunik, S.I. Ilnytska, O.A. Sushchenko, Synthesis of Quadrotor Robust Guidance and Control System via Parametrization of all Stabilizing H-infinity State Feedback Gains, Electronics and Control Systems. – 2019. – № 4. – P. 33–41.
14. V.B. Larin, A.A. Tunik. Synthesis of the Quad-Rotor Control Algorithms in the Basic Flight Modes. TWMS Journal of Pure and Applied Mathematics"- Vol.9 No.2, 2018.- pp.147-158.
15. A. Tayebi and S. McGilvray, "Attitude Stabilization of a VTOL Quadrotor Aircraft," IEEE Trans. on Control Syst. Technology, vol. 14, no. 3, 2006, pp. 562–571.
16. I.C. Dikmen, A. Arısoy, H. Temeltas. (2009) Attitude Control of a Quadrotor. 2009 4th International Conference on Recent Advances in Space Technologies. DOI: 10.1109/RAST.2009.5158286
17. H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati and S. Smailzadeh, "Attitude Control of a Quadrotor with Optimized PID Controller," *Intelligent Control and Automation*, Vol. 4 No. 3, 2013, pp. 335-342. doi: 10.4236/ica.2013.43039 .
18. H.R. Jafari, M. Zareh, J. Roshanian, A. Nikkhah. An Optimal Guidance Law Applied to Quadrotor Using LQR Method. Transactions of the Japan Society for Aeronautical and Space Sciences, 53(179), May 2010:pp.32-39.
19. Castillo P., Lozano R., Dzul A. Stabilization of a Mini Rotorcraft with Four Rotors. IEEE Control Systems Magazine. December, 2005. –pp. 45-55.
20. A. Tunik, O. Sushchenko, S. Ilnytska, Algorithm of Processing Navigation Information in Systems of Quadrotor Motion Control, International Journal of Image, Graphics and Signal Processing, 2023, vol. 15(1), pp. 1–11
21. R. Beard. Quadrotor Dynamics and Control Rev 0.1, 2008. Available at: https://scholarsarchive.byu.edu/facpub/13252
22. R.C. Leishman, J.C. Macdonald, R.W. Beard, T.W. McLain. Quadrotors and Accelerometers. IEEE Control Systems Magazine, February 2014, pp.28-41. Application of CAD Systems in Microelectronics, pages 10–14, 2021.
23. M.Sc. Kambushev M. PhD.1 , M.Sc. Biliderov S. PhD.1, Development and study of a mathematical model used in the automatic control systems of small- size UAVS Year XXIV, Volume 1, P.P. 39-42 (2016

24. Gu, Y.; Zhang, G.; Bi, Y.; Meng, W.; Ma, X.; Ni, W. Pitch Mathematical Modeling and Dynamic Analysis of a HALE UAV with Moving Mass Control Technology. *Aerospace* **2023**, *10*, 918. https://doi.org/10.3390/aerospace10110918
25. Tom Benson, Shape effects on drag.
26. Bautista-Medina, J.A.; Lozano, R.; Osorio-Cordero, A. Modeling and Control of a Single Rotor Composed of Two Fixed Wing Airplanes. *Drones* **2021**, *5*, 92. https://doi.org/10.3390/drones5030092
27. Bianchi, D.; Di Gennaro, S.; Di Ferdinando, M.; Acosta Lùa, C. Robust Control of UAV with Disturbances and Uncertainty Estimation. *Machines* **2023**, *11*, 352. https://doi.org/10.3390/machines11030352