MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL AVIATION UNIVERSITY
FACULTY OF AIR NAVIGATION, ELECTRONICS AND TELECOMMUNICATIONS
DEPARTMENT OF AVIONICS

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'___'___________2024

QUALIFICATION PAPER

(EXPLANATORY NOTES)

FOR THE DEGREE OF «BACHELOR»

SPECIALITY 173 'AVIONICS'

Theme: «Algorithm of Autonomous Inertial Navigation System for Aircraft»

Done by: ___________AV-410, Horbuntsov Dmytro Dmytrovych_____
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Kyiv 2024

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ
ФАКУЛЬТЕТ АЕРОНАВІГАЦІЇ, ЕЛЕКТРОНІКИ ТА ТЕЛЕКОМУНІКАЦІЙ
КАФЕДРА АВІОНІКИ

ДОПУСТИТИ ДО ЗАХИСТУ
Завідувач кафедри
_________Ю.В. Грищенко
«___» ___________2024

КВАЛІФІКАЦІЙНА РОБОТА

(ПОЯСНЮВАЛЬНА ЗАПИСКА)

ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ «БАКАЛАВР»

ЗА СПЕЦІАЛЬНІСТЮ 173 «АВІОНІКА»

Тема: «Алгоритм автономної інерціальної навігаційної системи літака»

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Київ 2024

NATIONAL AVIATION UNIVERSITY

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‘___’___________2024

TASK

for qualification paper

Oleksii Mykolayovych Starynskyi

1. Theme: ‘Reliability Culture in Aircraft Maintenance organization’, approved by order 355/ст of the Rector of the National Aviation University of 13 March 2024.

2. Duration of which is from 22 May 2024 to 30 June 2024.

3. Input data of graduation work: A safety culture is a collection of beliefs, values, and rules - formal or unspoken - about safety that are shared by all people in an organization. It effectively reflects a company's true commitment - from management to employees - to safety in its day-to-day operations and defines how safety is a priority in practice. It includes the following elements: safety management, aircraft reliability, employee responsibilities, management-employee relations, and the structure of a safety management system - or SMS.

4. Content of explanatory notes: List of conditional terms and abbreviations, Introduction, Chapter 1, Chapter 2, Chapter 3, References, Conclusions.

5. The list of mandatory graphic materials: figures, charts, and graphs.

6. Planned schedule

№	Task	Duration	Signature of supervisor
1.	Validate the rationale of the graduate work theme	22.05.2024	
2.	Carry out a literature review	24.05.2024	
3.	Develop the first chapter of the graduate work	31.05.2024	
4.	Develop the second chapter of the graduate work	07.06.2024	
5.	Develop the third chapter of the graduate work	14.06.2024	
6.	Tested for anti-plagiarism and obtained a review of the graduate work	17.06.2024	
7.	Preparation of presentation and report	22.06.2024	

8. Date of assignment: '___ ' ____________ 2024

Supervisor ______________ O.V. Kozhokhina

The task took to perform ______________ O.M. Starynskyi

ABSTRACT

Explanatory notes to graduation work 'Violations and their eliminations during aircraft maintenance' contained 69 pages, 14 figures, 2 graphs, and 22 references.

The object of research: algorithm of an autonomous inertial navigation system of an aircraft.

The subject of research: processes of measuring navigation parameters.

Purpose of the study: to develop algorithms for an inertial navigation system that provides acceptable measurement accuracy of primary navigation information.

Research methods: theory of inertial navigation systems, theory of gyroscopic devices, theory of mechanics, simulation modelling, filtering methods.

Keywords: AIRCRAFT, ALGORITHM, INERTIAL NAVIGATION SYSTEM, INERTIAL METERS, ACCELEROMETERS, FILTERING, MATHEMATICAL MODEL, SIMULATION MODELING

Contents

LIST OF ABBREVIATIONS

INS - Inertial navigation systems

Introduction

Inertial navigation systems (INS) are critical elements of modern avionics, providing accurate determination of aircraft movement and orientation parameters. These systems provide comprehensive data on heading, pitch, and roll angles, as well as acceleration, speed, and location coordinates of an object. The main advantage of INS is their complete autonomy, which allows them to function independently of external data sources, making them extremely reliable.

Inertial navigation systems (INS) are used in a variety of vehicles and machines to provide accurate determination of motion and orientation parameters.

Aviation:

Commercial airplanes: INS is used for autonomous navigation, especially when flying over oceans or in areas with limited access to ground-based navigation aids.

Military aircraft: INS provides precise navigation and orientation in combat environments where the use of external navigation signals may be difficult or impossible.

Unmanned aerial vehicles (UAVs): INS helps to control the course and stabilize the flight, especially in the absence of a GPS signal.

Marine transportation:

Military ships and submarines: INS is used for autonomous navigation underwater where GPS signals are not available.

Commercial vessels: In combination with other navigation systems, INS ensures safe movement on the high seas and coastal waters.

Ground transportation:

Military vehicles and armored vehicles: INS helps to determine location and route in environments where GPS use may be hampered by obstacles or opposition.

Autonomous vehicles: INS is used in conjunction with other sensors to provide accurate navigation and driving control in autonomous vehicles.

Spacecraft:

Satellites and spacecraft: INS is used for orientation and navigation in space where traditional navigation systems do not work.

Missiles and munitions:

Ballistic and cruise missiles: INS provides precise in-flight control and navigation, which is critical to achieving the target.

INS is a key technology in many industries where navigation accuracy and autonomy are critical.

Inertial navigation systems are divided into two main types: platform and platformless. Platform systems use a gyro-stabilized platform to keep the axes of accelerometers and gyroscopes in a given coordinate system. Despite being used successfully for many years, these systems have significant drawbacks, such as high cost and complexity of manufacturing. Platformless systems do not have a gyro-stabilized platform, which makes their design simpler and cheaper. The main advantages of platformless IMS are reduced costs and simplified design compared to platform systems. However, the disadvantage of such systems is the complexity of the algorithms required to calculate navigation parameters, but thanks to the development of information and computer technologies, these difficulties can be overcome.

INS built into aircraft provide autonomy and independence from external navigation signals, which is especially important in conditions of limited or no access to such signals. They allow the aircraft to determine its location, control the direction of movement and correct the trajectory with high accuracy, which allows it to perform various tasks with great efficiency. Increasing competition in the market forces manufacturers to introduce innovative and advanced technologies, including advanced INS, which affects the quality, price and timing of the required product.

Inertial navigation systems are characterized by numerous advantages, including autonomy in determining navigation parameters and high data output speed. They are capable of measuring a full set of navigation parameters, such as acceleration, speed, coordinates and angles of the object's position (heading, roll, pitch), as well as angular velocities of the object. Despite these advantages, like any other system, INS also has disadvantages that must be taken into account. Among them is the need to enter initial conditions, such as the initial position of the platform or object, and initial values of velocity and coordinates. It is also important to take into account the shape of the Earth and the parameters of the gravitational field at the location of the object. GNSS accumulate errors over time, so continuity of operation is required for their effective operation. Many of these drawbacks can be overcome, and the advantages of GNSS make them indispensable for many applications. Despite the challenges associated with accuracy and reliability, the advantages of these systems make them indispensable in many fields of technology and science.

Sensitive components of inertial navigation systems can include high-precision accelerometers and gyroscopes that provide a high level of measurement accuracy. To ensure reliable operation and minimize the impact of vibrations, sensitive elements can be installed in shock-absorbing devices, which ensures the stability and accuracy of the system under significant dynamic loads.

CHAPTER 1
GENERAL CHARACTERISTICS OF PLATFORM-FREE INERTIAL NAVIGATION SYSTEMS

In the field of spatial analysis and navigation, the understanding of coordinate systems is a crucial factor that affects the accuracy and efficiency of determining the position of objects. When navigating and analyzing spatial data, there is a need to use a variety of coordinate systems, as none of them is universal for all tasks. Coordinate systems are the fundamental basis for organizing spatial information, providing a standardized approach to expressing and interpreting the location of objects in a given space.

One of the main advantages of coordinate systems is their versatility, which lies in their ability to adapt to a variety of scenarios and ensure the seamless exchange of spatial data across different applications and disciplines. Whether we are plotting the path of a satellite in outer space, creating a map of a complex urban landscape, or tracking the movement of microscopic particles, the choice of the coordinate system is a critical decision that affects the accuracy and reliability of the results obtained.

Coordinate systems are not only practical tools but are also based on fundamental principles of geometry and mathematical abstraction. By defining a set of rules for assigning numerical values to points in space, they provide a basis for accurately representing spatial relationships. This mathematical framework facilitates the calculation of distances, angles, and areas and provides a universal language for spatial communication, which is critical in the modern world.

When it comes to determining the coordinates of objects, the choice of a coordinate system depends on the nature of the objects being analyzed and the context in which they exist. Cartesian coordinate systems based on orthogonal axes are often used because of their simplicity and convenience in Euclidean spaces. However, in cases where the curvature of the Earth or other celestial bodies is important, more complex systems, such as spherical or celestial coordinates, are needed to accurately describe the spatial position of objects. These systems provide the ability to accurately determine the position of objects in space, taking into account all the necessary factors.

In addition, it is important to keep in mind that understanding and correctly applying coordinate systems is critical for successful spatial analysis and navigation. This not only ensures accuracy and efficiency in various industries and applications but also allows for the creation of more complex and reliable models of spatial relationships, which is essential for the further development of science and technology. As a result, coordinate systems are a key element in the field of spatial analysis and navigation, providing accuracy, versatility and efficiency in solving a wide variety of tasks.

There are many coordinate systems developed to meet different needs and contexts in different disciplines. The geocentric inertial coordinate system OXYZ has the OX axis directed along the line of equinox to the vernal equinox, the OZ axis is directed along the axis of rotation of the Earth, and the OY axis forms a right-angled coordinate triangle with the OX and OZ axes. In the foreign literature, this coordinate system is sometimes called ECI (Earth-Centered Inertial), abbreviated by the letter i. Sometimes an inertial coordinate system is considered to be an initial coordinate system whose position relative to the i system is known.

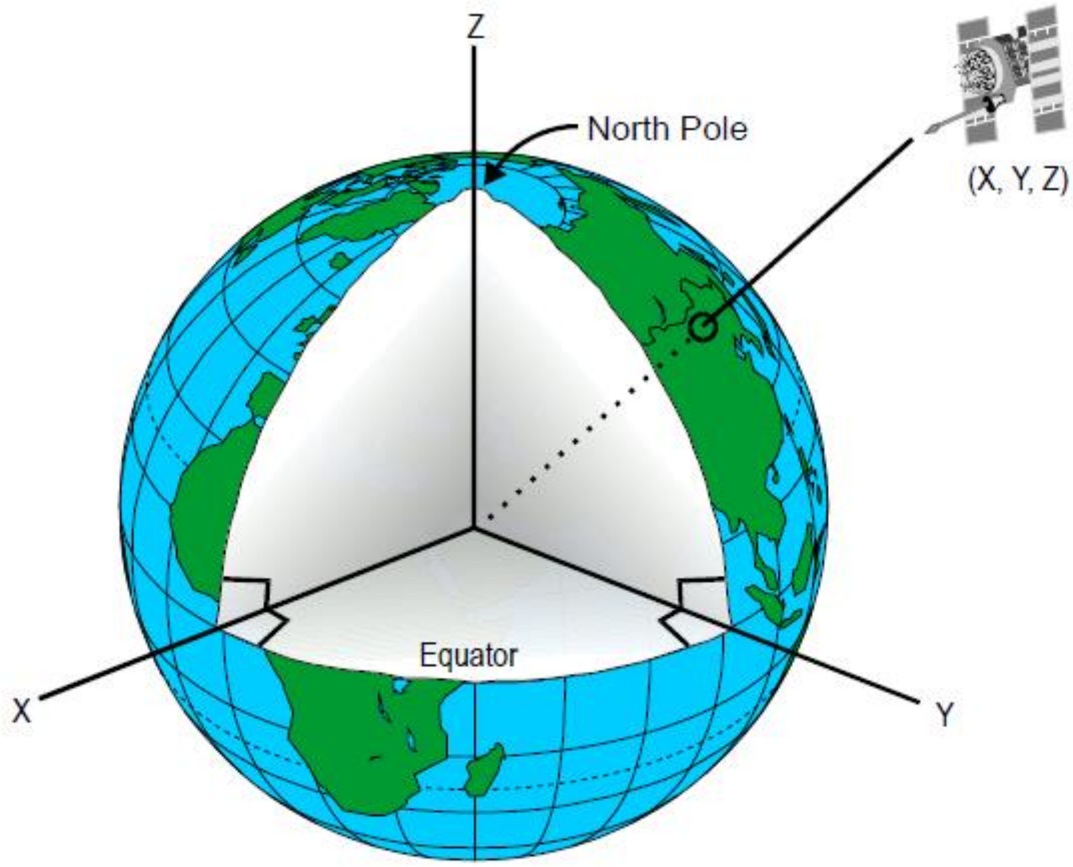


Fig 1.1 Coordinate system Earth-Centered Inertial (ECI)

The ECI (Earth-Centered Inertial) coordinate system is the fundamental basis for wide application in both satellite dynamics and celestial navigation. This inertial coordinate system plays a key role in the accurate determination and prediction of satellite orbits. Its inertial nature eliminates the need to take into account the Earth's rotation, simplifying the calculations for accurate orbit determination. In addition to predicting orbits, IMS plays an important role in controlling the orbit of satellites. Satellites often require a specific orientation for various purposes, such as communication, imaging, or scientific observation. ECI's stable, non-rotating reference frame simplifies the design and implementation of control algorithms, ensuring that satellites maintain the desired orientation.

Furthermore, in scenarios involving multiple satellites operating in close proximity or in need of coordination, the ECI serves as a common reference frame.

This facilitates seamless communication and coordination between satellites, enabling joint missions or shared tasks. The inertial stability of the ECI is especially important during orbital maneuvers, providing precise control over changes in speed and orientation without the complexities introduced by the Earth's rotation.

In the field of celestial navigation and astronomy, ECI proves invaluable for positional astronomy. Astronomers use its stability to accurately determine the location of celestial objects in the sky using precise measurements of celestial coordinates, such as direct ascension and declination, which remain constant over time. When telescopes are used in astronomical observations, ECI provides a stable reference frame for telescope pointing systems, guaranteeing accuracy without the need for constant adjustments due to Earth's rotation.

Beyond the Earth's orbit, ECI is used in interplanetary navigation, when spacecraft explore other celestial bodies. It allows for precise trajectory planning and adjustments during the journey, taking into account the gravitational effects of celestial bodies. In addition, ECI is a fundamental component in the planning and execution of celestial object-related space missions, ensuring the accuracy of mission planning and data analysis.

In both satellite dynamics and celestial navigation, the ECI consistent inertial reference frame links various aspects of space research and observation. Its use simplifies calculations, improves accuracy and facilitates coordination between spacecraft or observational instruments, making it an indispensable tool for a wide range of space research.

The ECEF (Earth-Centered Earth-Fixed) coordinate system is important for various Earth-related industries. It is used in global navigation systems, including GPS, which revolutionized global positioning and navigation. GPS satellites orbiting the Earth act as beacons, transmitting signals with important information, including ECEF coordinates and exact time. This allows the receivers to determine their location with high accuracy in real time.

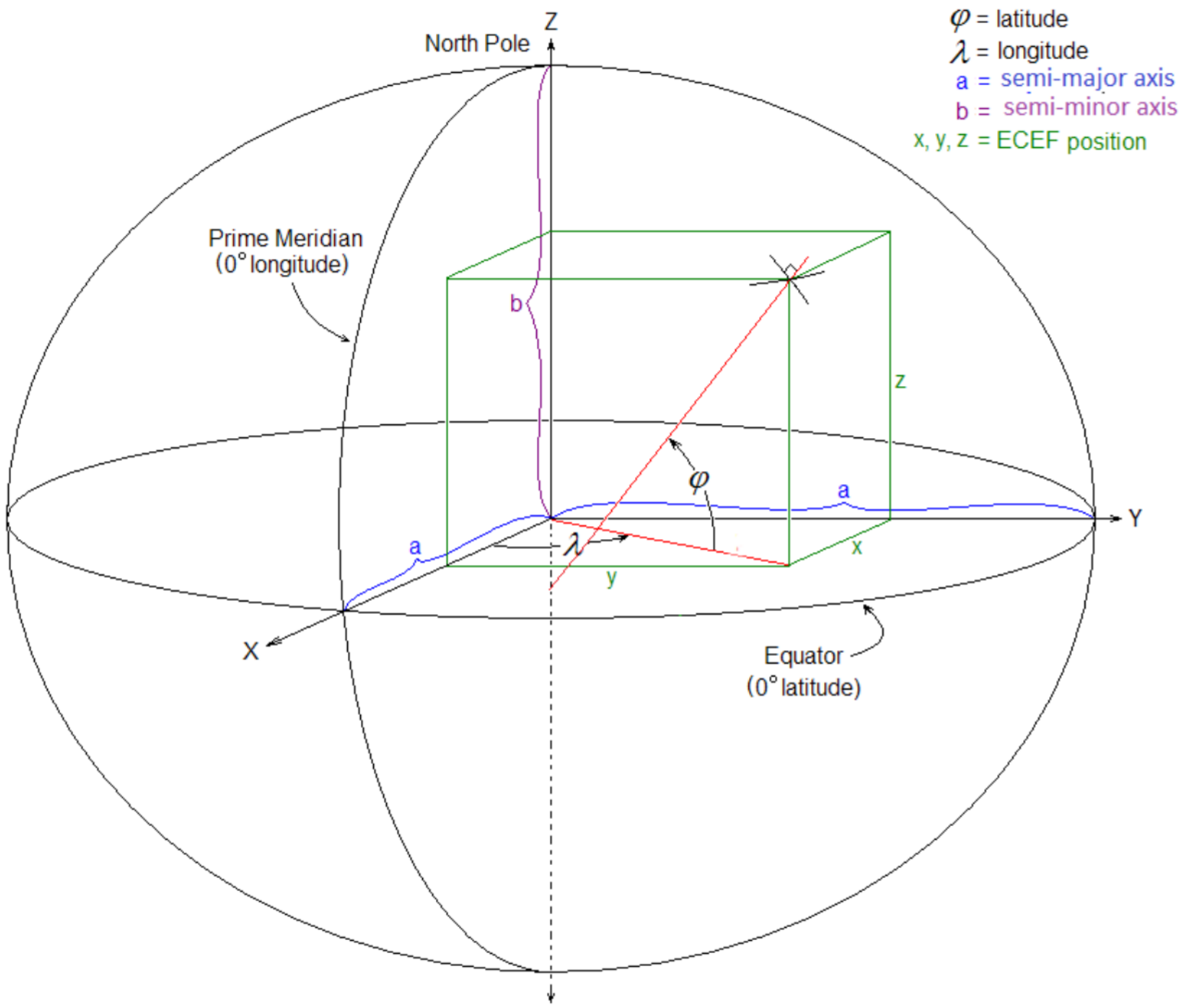


Fig 1.2 Coordinate system Earth-Centered Inertial Fixed (ECIF)

The ECEF system is also used to analyze satellite orbits, especially low-earth orbit (LEO) satellites, providing a convenient frame of reference for describing the position and velocity of satellites relative to the rotating Earth. The integration of ECEF coordinates into GPS is the basis of its operation, providing a standardized reference system for precise navigation and positioning in various applications.

In geodesy and mapping, ECEF coordinates allow you to accurately determine the position of points on the Earth's surface and simplify the calculation of distances, angles and other geometric parameters. They are an important component for accurate geographic location and analysis of geodetic data.

The ECEF coordinate system plays a key role in modern navigation and surveying technology, providing reliability, accuracy and versatility for a variety of applications and requirements.

In aviation, ECEF (Earth-Centered Earth-Fixed) coordinates are an integral part of aircraft navigation and tracking, providing a consistent frame of reference for accurate determination of aircraft position and movement. Pilots and air traffic controllers use these coordinates to provide accurate and reliable navigation during all phases of flight, including takeoff, cruise, descent, and landing.

Aviation navigation systems use ECEF coordinates to create a standardized reference system that allows accurate tracking of an aircraft's position relative to the center of the Earth. This information is critical for route planning, air traffic control and coordination of air operations. Real-time ECEF coordinates provide a consistent basis for monitoring aircraft movements, contributing to the overall safety and efficiency of air transportation.

In addition, ECEF coordinates are used in inertial navigation systems (INS) installed on board aircraft. These systems integrate data from accelerometers and gyroscopes to continuously track changes in aircraft speed and orientation. Combining inertial measurements with ECEF coordinates allows for accurate and reliable navigation, especially in conditions where GPS signals may be unavailable or limited.

The use of ECEF coordinates in aviation navigation provides a standardized and internationally recognized reference system that increases interoperability and ensures efficient data exchange between various components of the aviation infrastructure, including air traffic control, ground navigation aids and onboard avionics systems.

The use of ECEF coordinates is an essential element for accurate spatial analysis, allowing GIS users to measure distances, angles and volumes with a high degree of accuracy. These coordinates also support the integration of three-dimensional visualization methods, which allows the creation of realistic and detailed images of the Earth's surface and its features. Whether for urban planning, environmental monitoring, disaster management or any other GIS application, ECEF coordinates ensure the interoperability and accuracy of geospatial data. GIS professionals can overlay diverse data sets, perform spatial analysis, and derive meaningful conclusions, knowing that the data is tied to a globally recognized coordinate system.

The integration of ECEF coordinates into GIS applications extends the capabilities of these systems by providing a standardized and accurate basis for analyzing and presenting geospatial data, especially in scenarios where a 3D perspective is critical to informed decision-making. In addition, ECEF coordinates help create accurate maps and geospatial products based on satellite observations. Precise positioning of objects on the Earth's surface enables detailed mapping, land use classification and monitoring of changes over time, which is important for applications such as environmental impact assessment, urban planning and resource management.

ECEF coordinates are a necessary element for Earth observation satellite systems used for environmental monitoring, disaster management, agriculture and scientific research. These systems provide a consistent frame of reference for pinpointing the location of observed objects with high precision, which is critical in modern analysis and planning.

The Earth-centered, Earth-fixed (ECEF) coordinate system is non-inertial, meaning it rotates along with the Earth. It accounts for the Earth's rotation and is ideal for applications where the movement of objects on or near the Earth's surface is a primary concern. The three axes of this coordinate system are typically defined as follows: the angle of rotation of the coordinate system corresponds to the value ωt, where ω is the Earth's angular velocity, and t is time. The axis $O\eta_{\Gamma}$ is located in the plane of the Greenwich meridian.

The accompanying coordinate system, or accompanying trihedron, has its origin at a point on the Earth's surface, whose position is defined by latitude ϕ and longitude λ. If the latitude is defined as the angle between the equatorial plane and the geocentric radius (vertical), it is called geocentric and denoted by ϕ'. The axes of the accompanying geographic trihedron Oηζ (or O1O1 ENH) are directed as follows: the axis $O1\xi$O1ξ is tangent to the parallel to the east, the axis $O1\eta$O1η is directed north, and the axis $O1\zeta$O1ζ is directed vertically. This trihedron is sometimes denoted by the letter gg. Relative to it, a coordinate system free in azimuth is rotated by an angle χχ in the horizontal plane.

If the angle χχ is taken as the orthodromic course angle, then $O1\xi0\eta0\zeta0$O1ξ0 η0ζ0 is the orthodromic coordinate system. The object-associated coordinate system will be denoted as $O1xsyszs$O1xsyszs, where $O1ys$O1ys is the object's longitudinal axis, $O1xs$O1xs is the object's transverse axis (to the right side), and $O1zs$O1zs is the object's normal axis (upward). This coordinate system is sometimes denoted by the letter bb (from the word "body").

ECEF coordinates are often transformed into other coordinate systems, such as geodetic or local tangent plane coordinates, to meet specific application requirements. The transformation from ECEF to geodetic coordinates allows for the precise determination of the location of objects on the Earth's surface, considering altitude, latitude, and longitude. On the other hand, the local tangent coordinate system is useful for analyzing the movement of objects relative to the Earth's surface at a local level.

Using ECEF coordinates facilitates accurate spatial analysis, enabling GIS users to measure distances, angles, and volumes with high precision. It also supports the integration of three-dimensional visualization techniques, allowing the creation of realistic and detailed representations of the Earth's surface and its features. Whether for urban planning, environmental monitoring, disaster management, or any other GIS application, ECEF coordinates ensure interoperability and accuracy of geospatial data. GIS professionals can overlay diverse data sets, perform spatial analysis, and derive meaningful insights, knowing that the data is tied to a globally recognized coordinate system.

Integrating ECEF coordinates into GIS applications enhances these systems' capabilities, providing a standardized and accurate foundation for analyzing and presenting geospatial data, especially in scenarios where a three-dimensional perspective is crucial for informed decision-making. Additionally, ECEF coordinates help create precise maps and geospatial products based on satellite observations. Accurate positioning of objects on the Earth's surface enables detailed mapping, land-use classification, and monitoring of changes over time, which is critical for applications such as environmental impact assessment, urban planning, and resource management.

ECEF coordinates are essential for satellite Earth observation systems used for environmental monitoring, disaster management, agriculture, and scientific research. These systems provide a consistent reference frame for accurately determining the location of observed objects with high precision, which is crucial in modern analysis and planning. Integrating data from various sources and sensors in a standardized coordinate system allows seamless analysis and ensures meaningful conclusions.

ECEF coordinates also support the coordination of multiple satellites in constellations or networks. Such coordination ensures comprehensive coverage and data collection over large geographic areas, enhancing the efficiency of satellite monitoring and observation systems. This coordinate system is invaluable for applications related to objects or observations closely tied to the Earth's surface. Its non-inertial nature, accounting for the Earth's rotation, makes it suitable for navigation, mapping, and various Earth-related analyses.

The accompanying trihedron rotates in inertial space with an angular velocity whose projections can be expressed by the corresponding ratios. Projections of the angular velocity of rotation of a trihedron (SC, basis) with respect to inertial space can be written in the following form

$$\omega_\xi = \frac{-v_N}{R_2 + h}; \qquad h = h_0 + v_\zeta t$$

$$\omega_\eta = \frac{-v_E}{R_2 + h} + u cos\varphi; \tag{1.1}$$

$$\omega_\zeta = \frac{-v_E}{R_2 + h} * tg\varphi + u sin\varphi; \qquad \omega_\zeta = \omega_\eta tg\varphi$$

In addition to the previously introduced notations, $vN=v\cos\chi$vN=vcosχ represents the northern component of the object's relative velocity vector, $vE=v\sin\chi$vE=vsinχ denotes the eastern component, hh is the object's altitude, h0h0 is its initial altitude, $v\zeta$vζ is the vertical component of the velocity, and tt is time. Sometimes the angular velocity vector is expressed as the sum $\omega=u+w$0ω=u+w0, where uu is the angular velocity vector of the Earth's rotation, and w0w0 is the angular velocity vector due to the object's movement relative to the Earth (relative angular velocity). These vectors can be represented as projections onto the axes of the accompanying basis as follows:

$$\vec{u} = [0, u_{\eta}, u_{\xi}]^T; \qquad \vec{w}_{\xi}^{0} = [\omega_{\xi}^{0}, \omega_{\eta}^{0}\omega_{\zeta}^{0}]^T \tag{1.2}$$

This notation provides a clear framework for analyzing the dynamics of objects in motion relative to the Earth's surface, accounting for both the Earth's rotation and the object's movement. By decomposing the velocity and angular velocity vectors into their respective components, it becomes possible to precisely describe and predict the behaviour of objects within the Earth-centered, Earth-fixed (ECEF) coordinate system.

The absolute linear velocity is equal to the sum of the relative linear velocities of the object and the linear velocity relative to the Earth's rotation. The projection of the absolute linear velocity can be represented as follows:

$$V_N = V\cos\chi, V_E = V\sin\chi, \; V_N = v_N, V_E = v_e + (R_1 + h)u\cos\varphi \tag{1.3}$$

Hence, the form of the projection of the absolute angular velocity of the geographically accompanying trihedron can be written as follows:

$$\omega_{\xi 0} = -u\cos\chi\,\sin\chi - \frac{v}{(R+h)}$$

$$\omega_{\eta 0} = u\cos\chi; \tag{1.4}$$

$$\omega_{0\zeta} = u\cos\chi;$$

There are other ways to describe the motion of the orthodromic accompanying trihedron, particularly taking into account the ellipticity of the Earth. The direction cosine matrix between the axes of the Greenwich coordinate system $O\xi G\eta G\zeta G$OξGηGζG and the semi-free (orthodromic) azimuth coordinate system $O\xi 0\eta 0\zeta G$Oξ0η0 ζG (Fig. 1.4) is shown in Table 1.1. The table shows the direction cosines between the axes of the Greenwich and orthodromic trihedrons.

These formulas and matrices allow for precise determination and analysis of the movement of objects relative to the Earth, taking into account its rotation and shape.

Table 1.1

Direction cosines between the axes of Greenwich and orthodromic trihedra

C^{ge}	ξ_Γ	η_Γ	ζ_Γ
ξ_0	$c_{11} = -sin\varphi cos\lambda sin\varepsilon - -sin\lambda cos\varepsilon$	$c_{12} = -sin\varphi sin\lambda sin\varepsilon + sin\lambda cos\varepsilon$	$c_{13} = cos\varphi sin\varepsilon$
η_0	$c_{21} = -sin\varphi cos\lambda cos\varepsilon + +sin\lambda cos\varepsilon$	$c_{22} = -sin\varphi sin\lambda cos\varepsilon - cos\lambda sin\varepsilon$	$c_{23} = cos\varphi cos\varepsilon$
ζ_0	$c_{31} = sin\varphi cos\lambda$	$c_{32} = sin\lambda cos\varphi$	$b_{33} = sin\varphi$

The matrix is denoted as C^{ge}, where the second letter of the index denotes the initial trihedron, and the first letter of the index - the final trihedron

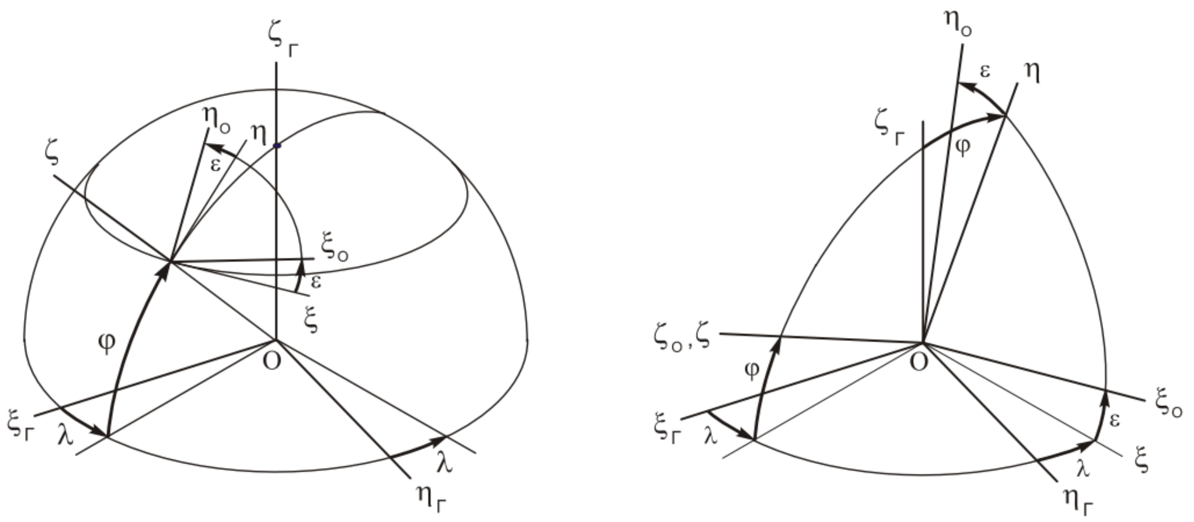


Fig. 1.2 Greenwich and orthodromic trihedra

The trihedron $O\xi 0\varepsilon_0\eta_0\zeta_0$ moves with a relative angular velocity ω^0, which is related to the path speed vv by the relation $\vec{v} = \vec{\omega}^0 * \vec{R}$, where $\vec{R}$ is the radius of curvature of the Earth ellipsoid in the plane of the trajectory. The projections of the vector $\vec{\omega}^0$ on the axes of the trihedron $0\varepsilon_0\eta_0\zeta$ can be expressed as follows:

$$\omega_{\xi 0}^{0} = -\frac{V_{\eta 0}}{R_{\eta 0}} - \frac{v_{\xi 0}}{a} e^2 b_{13} b_{23}, \quad \omega_{\eta 0}^{0} = -\frac{V_{\xi 0}}{R_{\xi 0}} - \frac{v_{\eta 0}}{a} e^2 b_{13} b_{23}, \tag{1.5}$$

Here, $R_{\eta 0}$and $R_{\xi 0}$ are the radii of curvature of the normal sections of the ellipsoid in the planes Oη0ζ and Oξ0ζ, respectively, 2e is the square of the eccentricity of the terrestrial ellipsoid. The values inverse of the radii of curvature are calculated according to the ratios:

$$\frac{1}{R_{\xi 0}} = \frac{1 - \frac{1}{2} e^2 b_{33}^2 + \frac{1}{2} e^2 b_{13}^2 - \frac{h}{a}}{a}$$

$$\frac{1}{R_{\eta 0}} = \frac{1 - \frac{1}{2} e^2 b_{33}^2 + \frac{1}{2} e^2 b_{23}^2 - \frac{h}{a}}{a} \tag{1.6}$$

Where h is the height, and a is the major semi-height of the terrestrial ellipsoid.

According to the law of universal gravitation, all bodies are attracted to each other with a force that is directly proportional to the product of the mass of these bodies and inversely proportional to the second power of the distance between them.

$$F = G\frac{Mm}{r^2}, \tag{1.7}$$

where F is the force of gravity, and M and T are the masses of two mutually attractive objects. (gravitational mass); r-distance between them; The G-coefficient of proportionality, which is called the gravitational constant and is the main physical constant of the new physical constants. on the other hand, gravity The mass m acting on a material point is determined by the formula:

$$\vec{F} = m\vec{g}, \tag{1.8}$$

where -g → is the gravitational acceleration or the acceleration of gravity. The gravitational force F r and the gravitational acceleration have the same direction. Comparing formulas (1.7) and (1.8), we find:

$$g^{`} = G\frac{M}{r^2}$$

This expression represents the simplest model of the gravitational field. The real world is more complicated. can be represented by the components of the vector g in the meridional plane. The radial component is directed towards the center of the Earth

$$g_r^{`} = -\frac{K}{r^2}\left[1 + \frac{3\mu}{2}\left(\frac{a}{r}\right)^2\left(1 - 3sin^2\varphi^{`}\right)\right] \quad (1.9)$$

Here, the general minus sign indicates that the component is directed against the direction of the radius from the center of the Earth, μ =1.09*10-10 · is the coefficient characterizing the distribution of the Earth's masses and is the semi-major axis of the ellipsoid, r is the geocentric radius, ϕ' - geocentric latitude. Transversal component, directed in the horizon plane to the equator plane (opposite to the reference latitude)

$$g_\varphi^{`} = -\frac{K}{r^2}\left[\frac{3\mu}{2}\left(\frac{a}{r}\right)^2 sin2\varphi^{`}\right] \quad (1.10)$$

The force of gravity is equal to the force of gravity and the centrifugal force (the force of inertia of centripetal acceleration due to the rotation of the Earth). Earth $\overrightarrow{W}_{цс} = \vec{u} * (\vec{u} * \vec{R})$. The acceleration of gravity is generally expressed as

$$\vec{g} = \vec{g}^1 - \vec{u} * (\vec{u} * \vec{R}) \quad (1.11)$$

The value of centripetal acceleration $\overrightarrow{W}_{цс} = u^2 Rcos\varphi$

As the height changes, the acceleration also changes $g(h) = \frac{g_\varphi^{`}}{(1+h/R)}$

The increase in the acceleration of gravity can be calculated using the formula

$\Delta g = -\frac{2g}{R}\Delta h$

The foundation of inertial navigation systems (INS) lies in the method of dead reckoning. The essence of this method is that the signals from velocity or acceleration sensors, whose sensitivity axes are maintained in a given coordinate system, are integrated. The integrals of the velocities correspond to the increments of the path, while the integrals of the accelerations correspond to the increments of the velocity.

By continuously integrating these measurements, the INS can calculate the position, velocity, and orientation of the object relative to an initial point. This process allows for precise navigation even in the absence of external signals, making INS crucial for various applications, including aviation, maritime, and space exploration.

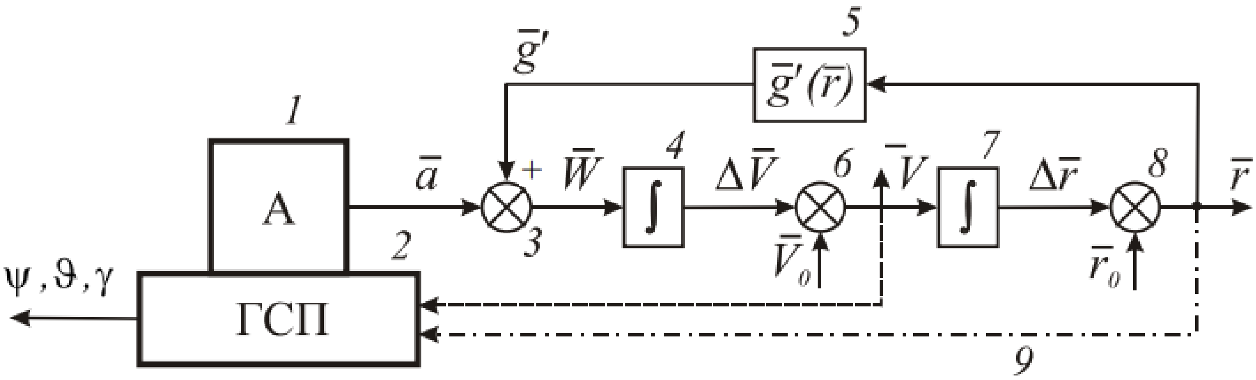


Fig. 1.3 Generalized scheme of the platform ANN

By adding the increments to the initial values of the path or velocity, the current values of the traveled path and velocity are obtained. In systems where the primary sensors are accelerometers, the accelerometer signal is integrated once to obtain the velocity, and this integral (velocity) is integrated a second time to obtain the traveled path. Knowing the directions of the path projections on the coordinate system axes, the coordinates of the moving object are determined.

An INS device that implements the method of dead reckoning can be illustrated with a generalized diagram, as shown in Fig. 1.3. This schematic represents the integration process of acceleration data to compute velocity and then position, allowing for precise tracking of the object's movement within the specified coordinate system.

1 - GSP (Gyrostabilized Platform): Maintains the sensitivity axes of the accelerometers in a given coordinate system. 2 - A (Accelerometer Unit): Block of accelerometers (three-axis accelerometer). 3, 6, 8 - Summators: Devices that sum the input signals. 4, 7 - Integrators: Devices that perform the integration of input signals. 5 - Gravity Acceleration Vector Calculator: Computes the vector of

gravitational acceleration. 9 - Feedback Loops: Connections that provide feedback control.

The following designations are introduced:

$\vec{a}$, $\vec{g}'$ r r :Vectors of fictitious and gravitational accelerations, respectively.

$\vec{W}$ $\vec{V}$: Vectors of absolute acceleration and absolute velocity, respectively.

$\Delta\vec{V}$ $\Delta\vec{r}$: Increments of absolute velocity and the radius vector of the object's position.

$r^{\rightarrow}$: Radius vector of the object's position—initial values of the vectors.

ψ,ϑ,γ: Angles of object orientation (possibly yaw, pitch, and roll).

If the system has feedback loops (9) for velocity or coordinates, it is called a closed-loop system. If there are no feedback loops, the system is referred to as an open-loop system.

This setup forms the basis of the inertial navigation system (INS) depicted in the generalized schematic (Fig. 1.3), where each component works together to calculate the current position and velocity of a moving object through integration of acceleration data.

In a Strapdown Inertial Navigation System (SINS), instead of a gyrostabilized platform, a block of gyroscopes and accelerometers is used along with a computer. The gyroscopes and accelerometers (block of inertial sensitive elements) are rigidly

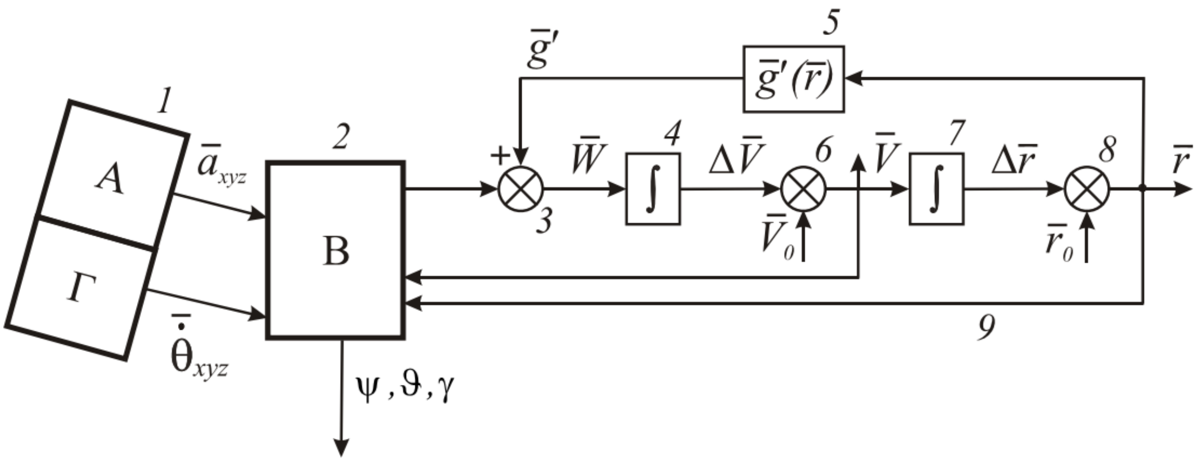


Fig 1.4 General scheme of Strapdown Inertial Navigation System (SINS)

mounted on the object's frame. In Fig. 1.4, the setup includes the following components:

1 - Block of Sensitive Elements: This block provides information about the fictitious acceleration vector $a_{\overline{\overrightarrow{xyz}}}$ in the projections on the axes of the object-related coordinate system xyz, as well as the angular velocity vector $\overline{\ddot{\theta}}_{\mathrm{xyz}}$ in the projections on the same coordinate system axes.

2 - Computer: This unit converts the projections of acceleration from the object-related coordinate system to the navigation coordinate system (for example, a geographic tracking system). To achieve this, the computer calculates the direction cosines between the axes of the respective coordinate systems based on angular velocity data (or other relevant information). Using these direction cosines, it also determines the orientation angles of the object: yaw ψ, roll ϑ, and pitch γ.

By processing the data from the inertial sensors, the computer can transform the measurements from the object-bound coordinate system to a more usable navigation frame of reference. This transformation is essential for determining the object's current position, velocity, and orientation accurately. The direction cosines and orientation angles help in aligning the inertial measurements with the global coordinate system, thus enabling precise navigation and control.

Table 2.2

Types of gyroscopic sensors used

Types of gyroscopes	Angular rate of departure	Initial parameters
Float DUS	Up to 0.01°$/hour$	Angular velocity
Laser gyroscopes	Up to 0.001°$/hour$	Angle. speed, angle
Fiber-optic gyroscopes	Up to 0.01°$/hour$	Angular velocity
Dynamically adjustable gyroscopes	Up to 10°$/hour$	Angular velocity
Micromechanical gyroscopes	Up to 10°$/hour$	Angular velocity
Solid-state wave gyroscopes	Up to 0.01°$/hour$	Angle. speed, angle
Spherical gyroscopes with electrostatic rotor suspension	Up to $10^{-5\circ}/hour$	Guide cosines
Spherical gyroscopes with magnetic rotor suspension	Up to 10^{-4} °$/hour$	Guide cosines

Spherical gyroscopes with air rotor suspension	Up to $0.01°/hour$	Guide cosines

Advantages of INS

Comprehensive Measurement of Navigation Parameters:

INS systems provide measurements of a wide range of navigation parameters, such as acceleration, speed, coordinates, object orientation angles (course, roll, pitch), angular velocity of the object, and other auxiliary parameters.

Complete Autonomy:

INS systems can operate independently of visibility of landmarks, beacons, and lights, as well as the position and movement of the object, enabling their use in any conditions.

High Speed of Calculation and Data Output:

INS systems are capable of rapid data processing and output with a frequency exceeding 100 Hz, ensuring high operational efficiency in navigation solutions.

Immunity to Relative Accelerations:

INS systems are not affected by relative accelerations, meaning there are no oscillations of the gyro-stabilized platform or its analytical counterpart in BINS during the action of relative accelerations. This minimizes errors in the output navigation data. The error oscillation frequency, which arises from various perturbing factors, generally corresponds to the frequency of Schuler pendulum oscillations.

Disadvantages of INS

Need for Initial Conditions:

To use the dead reckoning method, initial conditions such as the initial position of the platform (or object for BINS), initial speed values, and coordinates must be inputted. The shape of the Earth and the gravitational field parameters at the object's location must also be considered.

Requirement for Continuous Operation:

INS systems require continuous operation. If interrupted, initial conditions must be re-entered, which can complicate usage.

Error Accumulation Over Time:

Measurement errors in INS systems tend to accumulate over time, necessitating periodic corrections to ensure the accuracy of navigation data.

Integrating all these aspects allows INS systems to provide reliable and accurate navigation in many challenging conditions, making them an indispensable tool for aviation, maritime, and space applications.

CHAPTER 2
INERTIAL NAVIGATION SYSTEM SENSORS FOR AIRCRAFT

Inertial Navigation Systems (INS) rely on inertial sensors as critical measuring devices. These sensors include accelerometers and gyroscopes, each playing a distinct yet complementary role in tracking the movement and orientation of a moving object, such as an aircraft, spacecraft, or ground vehicle.

Gyroscopes are devices designed to measure and maintain the angular velocity or rotational speed of an object around a specific axis. They are essential components in various applications, from inertial navigation systems and aerospace technologies to consumer electronics and robotics. The operating principle of gyroscopes is based on gyroscopic stability—a fundamental concept in physics. According to this principle, a rotating object strongly resists changes in its orientation. In a gyroscope, the central component is a rapidly spinning rotor.

The key characteristic of gyroscopic stability is that a spinning rotor tends to keep its axis of rotation fixed in space. When an external force or torque attempts to change the rotor's orientation, the gyroscope exhibits a unique response known as precession. Precession is the phenomenon where the axis of a spinning object changes direction in response to an applied force. Importantly, this change in orientation occurs perpendicular to both the applied force and the initial axis of rotation. In practice, if one tries to tilt or reorient a gyroscope, the spinning rotor resists this change, demonstrating precession. The resulting motion causes the gyroscope to "tilt" in a direction perpendicular to the applied force, creating a stable and predictable response. This gyroscopic stability is utilized in various fields, from navigation systems in aircraft and spacecraft to stabilizing devices in consumer electronics and industrial equipment. The principle of gyroscopic stability underpins the reliable operation of gyroscopes, allowing them to maintain a consistent orientation in the presence of external forces. This stability characteristic makes gyroscopes invaluable for applications where precise and stable rotational information is necessary for the proper functioning of devices and systems.

Types of Gyroscopes:

Mechanical Gyroscopes: Traditional devices that use a spinning mass to measure angular velocity. They leverage the principles of gyroscopic stability, with the spinning mass providing resistance to changes in orientation. Widely used in navigation systems and aviation, mechanical gyroscopes are crucial for maintaining stability and determining orientation.

Ring Laser Gyroscopes: Operate based on the interference of laser beams circulating in a closed loop. Changes in angular velocity are detected by measuring variations in the interference pattern of the laser beams. Highly accurate and popular for their reliability in providing precise angular velocity information, ring laser gyroscopes are extensively used in aerospace and inertial navigation systems.

Fiber-Optic Gyroscopes: Use the interference of light beams passing through coiled optical fibers to measure angular velocity. Rotational changes cause variations in the interference pattern, allowing for precise measurements. Fiber-optic gyroscopes are known for their accuracy and are widely applied in navigation and other precision-based applications.

By integrating these advanced gyroscopes and accelerometers, Inertial Navigation Systems (INS) offer autonomous, high-speed data processing and accurate tracking of navigation parameters, ensuring reliable navigation even in challenging environments.

Fiber-optic gyroscopes find applications in navigation systems, robotics, and the aerospace industry, where their precision, reliability, and compactness make them well-suited for various challenging conditions. These diverse types of gyroscopes cater to different needs across various industries, each with unique advantages in terms of accuracy, size, and application. The choice of gyroscope type depends on the specific requirements of the application, ranging from traditional mechanical gyroscopes in aviation to modern ring laser gyroscopes and fiber-optic gyroscopes in advanced aerospace and navigation technologies.

Classification of Gyroscopes by Number of Axes

Gyroscopes are designed to measure angular velocity, and the number of axes around which they can perform these measurements determines their classification. Gyroscopes can be classified based on the number of axes they measure:

Single-Axis Gyroscopes: Measure rotation around a single axis. Used in specific applications where movement is mainly restricted to one plane.

Dual-Axis Gyroscopes: Capable of measuring rotation around two axes. Applied in scenarios where rotational movement occurs in two perpendicular axes, providing additional information about orientation changes.

Three-Axis Gyroscopes: Measure rotation around three axes, often referred to as three-axis gyroscopes. Widely used in various industries, including aerospace, automotive, robotics, and consumer electronics. Provide comprehensive information about orientation changes in three-dimensional space.

Classification by the number of axes highlights the ability of gyroscopes to capture rotational movement from different perspectives. The use of three-axis gyroscopes is becoming increasingly common due to their ability to provide a complete picture of an object's movement in three-dimensional space. This versatility makes three-axis gyroscopes suitable for a wide range of applications, from inertial navigation systems and aerospace technologies to consumer electronics and robotics, where precise measurement of orientation changes is crucial.

Gyroscopes play a crucial role in inertial navigation systems (INS), providing essential information about orientation changes. They work in conjunction with accelerometers to accurately determine an object's position over time, ensuring precise navigation.

Gyroscopes are extensively used in the aerospace and aviation industries. They are employed in aircraft and spacecraft for stability control, navigation, and position determination. Gyroscopic data help maintain proper orientation and stability during flight.

Gyroscopes in Consumer Electronics and Robotics

Gyroscopes are integral components of consumer electronic devices such as smartphones and gaming controllers. They enable various functions, including automatic screen rotation, gesture recognition, and motion sensing, enhancing user experience and interaction with the device.

In robotics, gyroscopes contribute significantly to stability and control. They assist in tasks such as balancing robots to prevent tipping over and ensuring precise movement. Gyroscopic data is crucial for maintaining the required orientation during the robot's operation.

Gyroscopes play a vital role in stabilization systems used in cameras, drones, and other equipment. They are employed to compensate for unwanted rotations or vibrations, ensuring that captured images or videos remain stable and blur-free.

These diverse applications demonstrate the versatility and importance of gyroscopes in various industries. From improving navigation accuracy in aerospace to enhancing user experience in consumer electronics and ensuring stability in robotics and stabilization systems, gyroscopes contribute to the functionality and performance of a wide range of technological devices and systems.

MEMS Gyroscopes

Microelectromechanical systems (MEMS) gyroscopes are compact, miniaturized versions of traditional gyroscopes. They utilize microfabrication technology to integrate mechanical and electrical components on a single chip. MEMS gyroscopes typically consist of a microscopic vibrating structure, often a tiny resonating beam or rotating mass, which responds to angular motion. Changes in angular velocity cause mechanical movement, and this movement is converted into an electrical signal for measurement.

Key advantages of MEMS gyroscopes include their small size, low cost, and low power consumption. These characteristics make them particularly suitable for integration into portable consumer electronic devices.

MEMS gyroscopes have found widespread application across various industries, primarily due to the trend towards technology miniaturization. They are commonly used in:

Phones and Tablets: MEMS gyroscopes enable features such as screen rotation, gaming, and image stabilization in portable devices.

Wearable Devices: MEMS gyroscopes are integrated into fitness trackers and smartwatches to monitor movement and provide orientation information.

Cameras and Camcorders: MEMS gyroscopes help stabilize images and videos by reducing the impact of hand movements.

Automotive Systems: These gyroscopes are used in systems such as electronic stability control (ESC) to enhance vehicle safety.

The miniature design of MEMS gyroscopes allows easy integration into small electronic devices without compromising performance. This has played a crucial role in the development of compact and multifunctional consumer electronics.

MEMS gyroscopes have been instrumental in revolutionizing consumer electronics, enabling innovative features and improving the overall user experience in various portable devices. They have made significant contributions to the advancement of technology, particularly in enhancing the functionality and user interactivity of modern electronic gadgets.

Accelerometers: Measurement and Applications

Accelerometers are devices designed to measure proper acceleration, the rate of change of velocity, or simply acceleration, along one or multiple axes. These sensors are fundamental components in various applications, including inertial navigation systems, consumer electronics, automotive safety systems, and industrial equipment.

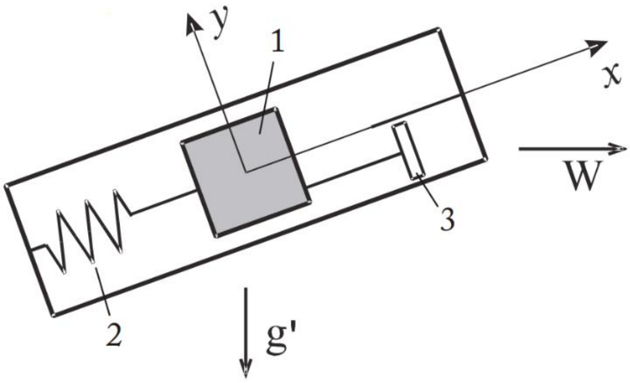


Fig. 2.1 Axial accelerometer

Working Principle

The device housing, as shown in (Figure 2.1), is mounted on a moving object and contains an inertial mass m, whose movement is restricted by a spring. When the object accelerates with an acceleration W, the inertial mass moves along the sensing axis x due to inertia until the inertial force is balanced by the spring's elastic force. In addition to the inertial force due to acceleration W, the gravitational force with the gravitational acceleration g′ also acts on the inertial mass. To reduce the transient response time, dampers are used.

The output signal (usually electrical) is proportional to the displacement of the inertial mass relative to the accelerometer housing along the X axis.

Equation of Motion

To derive the equation of motion for the inertial mass within the housing, which is attached to the xy coordinate system (connected to the object), we use the method of kinetostatics. According to d'Alembert's principle, the sum of active forces, inertial forces, and reaction forces equals zero. This can be expressed as:

$$\vec{F}_a + \vec{F}_R + \vec{F}_u = 0$$

Since the inert mass has only one degree of freedom relative to the x axis, let's make the equation of equality of force projections on this axis:

$F_{ax} = mg_x^{`} -$ gravitational forces

$F_{Rx} = -cx - f\dot{x} -$ reaction of the connection (spring and damper)

$F_{ux} = -m(W_x + \ddot{x})$ −forces of inertia

In these expressions, m is the mass, c is the linear stiffness of the spring, f is the linear damping coefficient, and the relative acceleration of the inert mass (relative to the body) Then we get:

$$-m(W_x + \ddot{x}) - cx - f\dot{x} + m\grave{g}_x = 0,$$

$$m\ddot{x} + f\dot{x} + cx = -m\left(W_x - \grave{g}_x\right) = -ma_x \qquad (2.1)$$

Here $a_x = W_x - \grave{g}_x$ - is called imaginary acceleration

In the general case, $\vec{a} = \vec{W} - \vec{g}'$. Since this feature must be taken into account during the design of the ANN algorithm, the equation is also called the basic equation of inertial navigation. Let's give examples of what accelerometer signals can be obtained as a result of its different placement. The accelerometer equation can be written from (2.1) in the form

$$\mathrm{x} = -\frac{\mathrm{m}}{\mathrm{c}}\mathrm{a_x}$$

In Fig. 2.2a. the accelerometer moves with acceleration W so that its axis of sensitivity is horizontal. In this case

$$W_x = W \; g'_x = 0 \; x = -\frac{m}{c} W \; a_x = W - 0$$

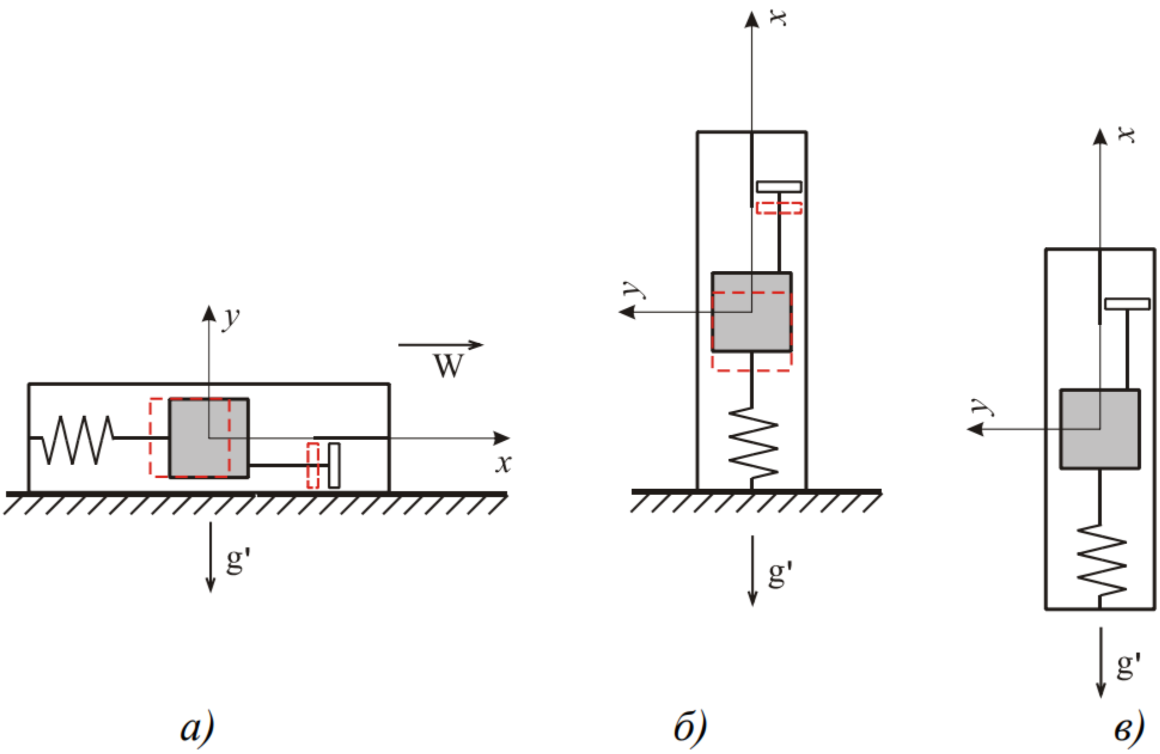


Fig. 2.2 Accelerometer behavior

In fig. 2.2b the accelerometer stands on the table so that its axis of sensitivity is vertical. There is no movement acceleration. In this case

$$W_x = 0 \; g'_x = -g'x = -\frac{m}{c} g' a_x = 0 - g'$$

and in fig. 2.2v accelerometer falls freely so that its axis of sensitivity is vertical. In this case

$$x = 0 \; a_x = -g' + g' W_x = -g' \; g'_x = -g'$$

These examples show that in order to determine the magnitude of the object's motion acceleration, it is necessary to take into account or exclude the projection of gravitational acceleration from the output signal of the accelerometer.

Types and Constructions of Accelerometers

There are numerous types of accelerometer designs, each suited for different applications and accuracy requirements. In the context of Inertial Navigation Systems (INS), pendulum accelerometers of the compensatory type are primarily used due to their high precision in measuring accelerations by minimizing the influence of gravitational forces on the sensor's readings. This is crucial in applications where distinguishing between gravitational acceleration and other accelerations is vital.

Pendulum Compensatory Accelerometers

Pendulum compensatory accelerometers are designed to provide increased measurement accuracy by compensating for gravitational forces. This is particularly important for applications where the difference between gravitational acceleration and other accelerations is critical. These accelerometers are commonly used in inertial navigation systems, where precise measurement of accelerations is essential for determining the position and orientation of an object. The pendulum mechanism helps improve the accuracy of these measurements.

The pendulum mechanism in these accelerometers also helps reduce cross-axis sensitivity. Cross-axis sensitivity is an undesired response of an accelerometer to accelerations along axes different from the intended measurement axis. Pendulum compensatory accelerometers are used in scenarios where high accuracy and reliability are paramount. These include navigation systems for aircraft, spacecraft, and other vehicles, as well as scientific instruments and industrial applications requiring precise acceleration measurements.

It is important to note that specific designs and functionalities can vary among different manufacturers and models. The use of a pendulum mechanism in accelerometers is an engineering solution to enhance the performance of these sensors in applications where gravitational effects need careful consideration and compensation.

Other Types of Accelerometers

In addition to pendulum compensatory accelerometers, several other types of accelerometers are used in various applications:

Piezoelectric Accelerometers

Piezoelectric accelerometers use piezoelectric materials, which generate an electrical charge in response to mechanical stress. These accelerometers are known for their high-frequency response and durability, making them suitable for vibration and shock measurements in industrial and automotive applications.

Piezoresistive Accelerometers

Piezoresistive accelerometers use piezoresistive materials, which change their electrical resistance under mechanical stress. These accelerometers are often used in crash testing and airbag deployment systems due to their ability to measure static and low-frequency accelerations.

Capacitive Accelerometers

Capacitive accelerometers measure changes in capacitance caused by the displacement of a proof mass under acceleration. These accelerometers are widely used in consumer electronics, automotive applications, and industrial systems due to their accuracy, stability, and low power consumption.

MEMS Accelerometers

Microelectromechanical systems (MEMS) accelerometers are compact and integrate mechanical and electrical components on a single chip using microfabrication technology. MEMS accelerometers are characterized by their small size, low cost, and low power consumption, making them ideal for portable electronic devices such as smartphones, tablets, and wearable technology.

Strain Gauge Accelerometers

Strain gauge accelerometers use strain gauges to measure deformation in response to acceleration. These accelerometers are typically used in structural health monitoring and aerospace applications, where precise and reliable measurements are critical.

Accelerometers are vital in numerous fields, offering precise data on acceleration, velocity changes, and orientation. Their design varies widely to meet the needs of different applications, from high-precision navigation systems to consumer electronics and industrial equipment. Each type of accelerometer, whether it’s pendulum compensatory, piezoelectric, piezoresistive, capacitive, MEMS, or strain gauge, provides unique benefits tailored to specific use cases.

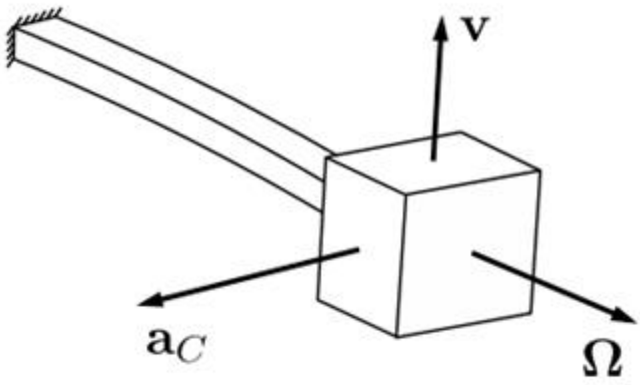
Angular velocity sensors, also known as gyroscopes, are devices designed to measure the rotational speed or angular velocity of an object around a specific axis. These sensors play a crucial role in various applications by providing information about the speed and direction of rotational movement. In the early 19th century, the French scientist G.G. de Coriolis discovered that a point moving on a rotating rigid body experiences an acceleration, now known as Coriolis acceleration. This acceleration is proportional to the velocity of the point and the rotational speed of the body:

$$a_C = 2\Omega * v$$

There are several types of angular velocity sensors, including mechanical gyroscopes, MEMS gyroscopes, fiber-optic gyroscopes, and ring laser gyroscopes.

Mechanical gyroscopes are traditional gyroscopes with a rotating mass that exhibits gyroscopic stability, resisting changes in orientation. When the orientation of the rotating mass changes, precession occurs, providing information about changes in angular velocity. They are used in various navigation systems, including aviation and marine systems, where precise measurement of angular velocity is crucial.

Microelectromechanical systems (MEMS) gyroscopes are miniature sensors that use microscopic structures, often vibrating or oscillating, to detect changes in angular velocity. MEMS angular velocity sensors typically consist of a vibrating control mass. As shown in Fig. 2.3, the bracket and control mass are excited at their resonant frequencies to induce oscillations in the vertical plane.



$$v = A\omega_n \sin(\omega_n t)$$

Fig. 2.3 Presentation of the speed angle sensor

where A is the amplitude of oscillations, and ω_n is the natural frequency of oscillations.

If the measurement axis of the angular velocity sensor is aligned along the longitudinal axis of the non-deflected bracket, then rotation about this axis will result in Coriolis acceleration in the horizontal plane. Similar to the accelerometer, the Coriolis acceleration of the control mass will cause lateral deflection of the bracket. This lateral deflection of the bracket can be registered in several ways: by means of capacitive coupling, by means of a piezoelectric charge, or by means of a change in the piezoresistance of the bracket. Whatever the conversion method, a voltage is created that is proportional to the lateral Coriolis acceleration. With the measuring axis perpendicular to the direction of oscillation, the ideal output voltage of the angular velocity sensor is proportional to the Coriolis acceleration, and is given by the expression

$$V = k_c|a_C| = 2k_c|\Omega * v|$$

Since Ω is the angular frequency of rotation around the axis of measurement of the gyroscope and v are orthogonal to each other, then

$$|\Omega * v| = \Omega|v|$$

$$V_{\text{ripo}} = 2k_C\Omega|A\omega_n \sin(\omega_n T)| = 2k_C A\omega_n \Omega = K_c\Omega$$

Where K_c is a calibration constant, and Ω represents the magnitude and direction (sign) of the angular velocity around the measurement axis. The output signal of the angular velocity sensor can be modeled as

$$\gamma_{\text{ripo}} = k_{\text{ripo}}\Omega + \beta_{\text{ripo}} + \eta'_{\text{ripo}}$$

where γ_{ripo} corresponds to the measured frequency of rotation in volts, k_{ripo} is the amplification factor that converts the angular velocity in rad/s into volts, Ω is the angular velocity in rad/s, β_{ripo} systematic error, η'_{ripo}Gaussian noise with zero mean. An approximate value for the amplification factor k_{ripo}. must be given in the list of technical characteristics of the sensor. To ensure accurate measurements, the value of this gain must be determined in the process of experimental calibration. Systematic error β_{ripo}. is highly temperature dependent and must be calibrated before each flight. For low-cost MEMS gyroscopes, the systematic error drift can be significant, and periodic systematic error zeroing should be monitored during flight. This is done assuming straight and level flight (Ω = 0) and resetting the gyro bias so that the gyro averages to zero over a period of 100 or so samples.

For simulation purposes there is an interest in simulating the calibrated gyro signals inside the autopilot. Angular velocity sensor signals are converted from analog voltages coming from the sensor to a numerical representation of angular velocities (in rad/s) inside the autopilot. It is assumed that the gyroscopes are calibrated so that 1 rad/s of angular velocity experienced by the sensor results in a numerical measurement within the autopilot of 1 rad/s (ie, the physical velocity to numerical representation within the autopilot is unity) and that systematic errors were estimated and subtracted from the measurements. It is customary to measure angular velocities around each of the body axes using three gyroscopes by aligning the axes of the gyroscope measurement along each of the i^b j^b k^baxes of the airccraft. These measurements of body angular velocity sensors p, q, and r can be modeled as

$$\gamma_{\text{ripo}\cdot,}x = p + \eta_{\text{ripo}\cdot,}x$$

$$\gamma_{\text{ripo}\cdot,}y = p + \eta_{\text{ripo}\cdot,}y$$

$$\gamma_{\text{ripo}\cdot,}z = p + \eta_{\text{ripo}\cdot,}z$$

where $\gamma_{\text{ripo}\cdot,}x$ $\gamma_{\text{ripo}\cdot,}$ and $\gamma_{\text{ripo}\cdot,}z$ are measurements of angular velocity in rad./s. The variables $\eta_{\text{ripo}\cdot,}x$, $\eta_{\text{ripo}\cdot,}y$ and $\eta_{\text{ripo}\cdot,}z$ are Gaussian processes with zero mean value and variances $\sigma^2_{\text{ripo.},x}, \sigma^2_{\text{ripo.},y}$ $\sigma^2_{\text{ripo.},z}$ respectively. MEMS gyroscopes are analog devices that the autopilot samples from. Perhaps the sample rate is set by T_s.

MEMS angle sensors are widely used in consumer electronics and portable devices. Small size, low cost, and low power consumption make MEMS gyroscopes suitable for use in smartphones, game controllers, and other electronic devices. Fiber-optic gyroscopes use the interference of light rays in coiled optical fibers to measure changes in angular velocity. The Sagnac effect, where a rotating interferometer changes the phase of light waves, is used in VOG to detect rotational motion. Commonly used in navigation systems for aircraft, ships, and other vehicles, as well as industrial applications requiring high precision. Ring laser gyroscopes detect changes in angular velocity by measuring the interference of laser beams circulating in the ring cavity. The rotation of the device affects the phase difference of the rays, providing information about the rotational movement. RLGs have high accuracy and sensitivity, which makes them suitable for use in navigation systems of aircraft, spacecraft and precision measuring instruments. Angular velocity sensors are integral components of the INS that help determine the orientation of the object and changes in its position. By accurately measuring the rate of rotation around various axes, ANNs can calculate the speed and displacement of an object, making them important for navigation in environments where external landmarks may be limited or unavailable. Gyroscopes are used in aircraft and spacecraft for stability control, navigation and position determination. Angular velocity sensors are used in aircraft and spacecraft to analyze flight dynamics. They provide real-time spin rate data to assist control systems and ensure stable flight conditions. In robotics, angular velocity sensors are used for motion control, helping robots maintain stability and precision in movement. They facilitate tasks such as robotic arm control and navigation.

Angular velocity sensors play an important role in understanding the dynamics of a moving vehicle. In cars, they contribute to stability control, especially in systems such as electronic stability control (ESC). Angular velocity sensors are used in precision machinery and manufacturing equipment to monitor and control rotational motion. They provide precise and controlled movement in various industrial processes. Virtual Reality (VR) and Augmented Reality (AR). Angular velocity sensors are used in virtual and augmented reality devices to track the movements of the user's head and provide a more immersive experience. In the marine industry, angular velocity sensors are used in the navigation systems of ships and submarines, helping to maintain course and orientation. Angular velocity sensors provide important rotational speed data and are used in a wide range of applications where accurate measurement of angular motion is essential. Their use extends to various fields, contributing to progress in navigation, control and traffic monitoring. Inertial measurement units (IMUs) often integrate both accelerometers and angular velocity sensors. By combining data from these sensors, IMUs can provide comprehensive information about the movement of an object in three-dimensional space. MEMS gyroscopes are characterized by their compact size, which makes them suitable for integration into small electronic devices such as smartphones, wearables and other portable gadgets. Their small form factor allows universal placement in limited space. The manufacturing processes involved in the manufacture of MEMS gyroscopes allow for cost-effective mass production. This availability has led to their widespread use in consumer electronics and other areas where cost considerations are critical. MEMS gyroscopes typically consume a small amount of power. This feature is beneficial for battery-powered devices as it helps extend battery life. This makes MEMS gyroscopes suitable for applications in mobile devices, IoT devices, and other battery-powered systems. The miniature design of MEMS gyroscopes allows for light weight, making them ideal for applications where weight is a critical factor. In the aerospace, automotive, and robotics industries, the lightness of MEMS gyroscopes improves overall system efficiency. MEMS gyroscopes are highly sensitive to changes in angular velocity. This sensitivity enables accurate measurements of rotational motion, enabling accurate motion tracking in a variety of applications including navigation and motion sensors. MEMS gyroscopes are often integrated with accelerometers in inertial measurement units (IMUs). Such a combination allows more complete measurement of the object's movement in three-dimensional space. By simultaneously measuring acceleration and angular velocity, IBS provide a more complete understanding of the dynamics of movement. MEMS gyroscopes are solid-state devices, meaning they have no moving parts, such as spinning disks, like traditional gyroscopes. Such a solid structure increases their durability, reliability and resistance to mechanical wear. MEMS gyroscopes find applications in a wide range of industries, including consumer electronics, automotive, aerospace and healthcare. Their versatility is due to a combination of small size, low cost and ability to provide accurate angular velocity measurements. The advantages of MEMS gyroscopes make them a popular choice for motion sensors where accurate angular velocity measurements are required and factors such as size, cost, and power consumption are critical. Angular velocity sensors are vital tools in research and development to study the dynamics of rotational motion in various fields, including physics, biomechanics, and engineering. Angular velocity sensors are fundamental components in motion reading systems, navigation devices and control systems, contributing to the development of technology in various industries.

A pressure sensor, also known as a pressure sensor or pressure transmitter, is a device that measures the force acting on a surface per unit area and converts it into an electrical signal. These sensors are widely used in various fields where pressure monitoring or control is important. Pressure sensors can work on different principles. The strain gauge pressure sensor measures the deformation of the flexible diaphragm under pressure. The membrane is usually made of a material that undergoes mechanical deformation under pressure, changing its electrical resistance. When the pressure changes, the diaphragm is deformed, causing a change in the resistance of the tensor resistor. This change in resistance is converted into an electrical signal proportional to the applied pressure. Piezoelectric pressure sensors use the piezoelectric effect, where certain materials generate an electrical charge in response to a mechanical load or pressure change. When pressure is applied to the piezoelectric material, it generates an electrical charge. This charge is then measured and converted into an electrical signal, providing a direct correlation with the applied pressure. Capacitive pressure sensors measure changes in capacitance between two plates in response to a change in pressure. When the pressure changes, the distance between the plates changes, which leads to a change in capacity. This change is detected and converted into an electrical signal that reflects the applied pressure. Resonant frequency pressure sensors monitor the change in the resonant frequency of the vibrating element under pressure. The deformation caused by the pressure changes the resonant frequency of the vibrating element (for example, a diaphragm or a tuning fork). This change is detected and converted into an electrical signal that is proportional. applied pressure. These principles enable pressure sensors to convert mechanical pressure fluctuations into measurable electrical signals, enabling accurate and reliable pressure measurements in a wide range of applications. The selection of a specific type of pressure sensor depends on factors such as the required pressure range, sensitivity, and environmental conditions for the intended application. Pressure sensors find a variety of applications in various industries due to their ability to measure and monitor pressure changes. Tire pressure monitoring systems (TPMS). Pressure sensors are used to monitor and ensure optimal tire pressure, which increases the safety and fuel efficiency of the vehicle. Engine Operation: Pressure sensors measure manifold pressure, helping to optimize fuel injection to improve engine performance.Fuel System Pressure. Control the pressure in the fuel system for efficient fuel combustion and emissions control. Pressure sensors play a critical role in monitoring and controlling pressure in industrial processes, ensuring quality and production efficiency. Used in hydraulic and pneumatic systems to maintain the exact pressure level for equipment operation. Pressure sensors are an integral part of such devices as sphygmomanometers, electronic tonometers and blood pressure monitors. Breathing systems: Used in ventilators and breathing apparatus to monitor airway pressure and provide controlled breathing. Pressure sensors regulate fluid delivery in medical infusion pumps. Pressure sensors are used in aircraft to measure altitude, which is critical to navigation and flight control systems. Used to measure flight speed, assisting in flight monitoring and control. These sensors help weather forecasting models by providing data on changes in atmospheric pressure. Fluctuations in barometric pressure are indicators of weather conditions. Pressure sensors are used to measure altitude and are integrated into meteorological applications to obtain real-time atmospheric pressure data. Wearables: Used in smart watches and fitness trackers to monitor changes in atmospheric pressure and altitude. It is used in water level measurement systems to monitor pressure fluctuations in rivers, lakes and reservoirs. Oceanography: Pressure sensors are used in oceanographic instruments to measure depth and pressure in the underwater environment. Pressure sensors are used in oil and gas wells to monitor reservoir pressure and optimize production processes. These diverse applications highlight the importance of pressure sensors for safety, efficiency and accuracy in a wide range of industries and process systems. HVAC Pressure Monitoring: Pressure sensors help regulate airflow and maintain optimal pressure levels in heating, ventilation, and air conditioning (HVAC) systems.

Accuracy and precision are critical factors when evaluating the performance of pressure sensors. Accuracy means the closeness of a measured value to its true or reference value. Accuracy is usually expressed as a percentage of the sensor's full range. The accuracy and precision of pressure sensors depend on the type of sensor and the field of application. High-precision sensors are critical in areas such as medical devices and scientific research. Different types of pressure sensors can exhibit different accuracy and reliability. For example, technologies such as strain gauges, piezoelectric and capacitive sensors have different characteristics that affect their performance. In medical applications, accuracy is critical for accurate monitoring of physiological parameters. Blood pressure monitors, infusion pumps, and ventilators require highly accurate pressure sensors to provide reliable and stable readings that are vital to patient care and treatment. Scientific experiments and research often require highly accurate measurements. Pressure sensors used in laboratories, environmental monitoring, and research instruments must provide accurate and repeatable data to ensure validity and reliability of scientific findings. In applications where accuracy is critical, even small inaccuracies can have significant consequences. For example, in medical facilities, incorrect pressure readings can affect diagnosis and treatment decisions, highlighting the need for high-precision sensors. High-precision sensors require careful calibration and regular maintenance to ensure their performance meets strict standards. Calibration against a known reference pressure is necessary to check and adjust the accuracy of the sensor. High-precision sensors often incorporate advanced manufacturing processes and materials, which can contribute to increased cost. However, the increased accuracy and reliability they offer justify their use in mission-critical applications where accuracy is paramount. In scientific research and development, the quality of the data directly affects the results. High-precision pressure sensors contribute to the accuracy of experiments, supporting accurate and reproducible results.

In conclusion, understanding the specific application requirements is critical when selecting pressure sensors. Consideration of accuracy and precision is particularly important in areas such as medical devices and scientific research, where accurate and reliable pressure measurements are integral to achieving accurate results and ensuring human well-being. Some pressure sensors are designed to operate in harsh environmental conditions, including extreme temperatures, humidity, and exposure to chemicals. This is of great importance in various fields, including industry, medicine and research. Some pressure sensors are designed to work effectively in extreme temperatures, whether very high or low. This allows their use in industrial conditions, where temperature fluctuations can be significant. For some pressure sensors, high ambient humidity is not an obstacle. This is important in environments where pressure needs to be measured in an area with high humidity, such as industrial plants or medical devices. Some sensors have chemical protection, which makes them resistant to interaction with aggressive chemicals. This can be important in industrial processes where various chemicals are present or in environments where there is a risk of contact with aggressive substances. In industries where extreme environmental conditions are the norm, such pressure sensors provide reliable and stable measurement. For example, in the field of petrochemicals, automobile manufacturing and food production. In medical devices where it is important to accurately measure pressure (for example, in cardiology or anesthesiology), pressure sensors must be resistant to treatment and sterilization conditions. Considering these environmental properties, pressure sensors can successfully function in a variety of conditions, which makes them universal and important elements in various fields of engineering and technology.

While examining various sensors such as pressure sensors, coordinate systems, gyroscopes and accelerometers, the importance of these devices in modern technology and their various applications in various industries are revealed. Gyroscopes and accelerometers are key components for measuring rotational and linear motions, respectively. They are used in modern stabilization systems, aircraft navigation, aerospace applications, smartphones, virtual reality and robotics. Gyroscopes and accelerometers are key components for measuring rotational and linear motions, respectively. They are used in modern stabilization systems, aircraft navigation, aerospace applications, smartphones, virtual reality and robotics. Considering the wide range of use of these devices, it can be noted that they not only contribute to the solution of specific technical tasks, but are also necessary for progress in many areas of life. These devices help to implement new technologies, improving the efficiency, accuracy and stability of various systems. Thanks to their application, modern technologies become more accessible and safer, opening up new opportunities for the development of science and industry.

CHAPTER 3

ALGORITHM OF FUNCTIONING OF INS FOR AIRCRAFTS

One of the most used orientation parameters used in BINS is the direction cosine. The nine cosines of the angles between the six axes of the unique coordinates determine their mutual orientation.

$$C = \begin{Vmatrix} cos\theta cos\psi & cos\gamma cos\psi sin\theta + sin\gamma sin\psi & sin\gamma cos\psi sin\theta + cos\gamma sin\psi \\ sin\theta & cos\gamma cos\theta & sin\gamma cos\theta \\ -cos\theta sin\psi & cos\gamma sin\psi sin\theta + sin\gamma cos\psi & -sin\gamma sin\psi sin\theta + cos\gamma cos\psi \end{Vmatrix}$$

The method of determining direction cosines through the angles of successive turns leads to the need to calculate the products of two or three functions of sines and cosines of the angular orientation parameters, which is a rather complicated and cumbersome process. It is quite a complicated and cumbersome process. Therefore, direction cosines are more often used as independent orientation parameters, since they can be calculated analytically if their initial values and angular velocities , $\omega_{x}, \omega_{y,} \omega_{z}$ with which the OXYZ system rotates relative to axes of the OXYZ system. rotates relative to the axes of the $OX_gY_gZ_g$ system [3.5]. If the matrix (3.1) is known on, then the yaw, pitch, and roll angles are determined through its elements.

$$\psi = arctg\frac{-c_{31}}{c_{11}}$$

$$\theta = arcsinc_{21}$$

$$\gamma = arcsin\frac{-c_{23}}{c_{22}}$$

where c_{ij} are the elements of the matrix C. Let's consider the methods of calculating the matrix of direction cosines. It is known from theoretical mechanics that the differentiation of the vector r determines the coordinates of a point in some coordinate system - OXYZ (coordinates, y, z), which gives a linear velocity

$$v = \frac{dr}{dt}\dot{r}$$

With v_x, v_y, v_zprojections on the OXYZ axis.

If this coordinate system rotates with an angular velocity relative to the angular velocity of a stationary coordinate system, that is, the absolute linear velocity of a point is determined as follows

$$v = \frac{dr}{dt} = \frac{dr}{dt} + \omega * r$$

In the right-hand side of the equality, the first term is marked with the sign ~, which represents the speed of a point in the OXYZ coordinate system, and the second takes into account the rotation factors of this system with respect to the fixed coordinate system $O\xi\eta\zeta$ in the form

$$\left.\begin{aligned} v_x &= x + \omega_y z - \omega_z y \\ v_x &= y + \omega_z z - \omega_x y \\ v_x &= z + \omega_x z - \omega_y y \end{aligned}\right\}$$

In the matrix form, the operation of differentiating the vector r by time in the coordinate system $O\xi\eta\zeta$ is determined by the expression

$$v = \frac{dr}{dt} = \begin{Vmatrix} \zeta \\ \eta \\ \zeta \end{Vmatrix}$$

where $\xi\eta\zeta$ -coordinates of the point in the $O\xi\eta\zeta$ coordinate system

The differentiation of vector r in the rotating coordinate system OXYZ is represented as follows

$$[\omega] = \begin{Vmatrix} 0 & \omega -_z & -\omega_y \\ \omega_z & 0 & -\omega_x \\ \omega_y & \omega_x & 0 \end{Vmatrix}$$

per column matrix

$$r = \begin{Vmatrix} x \\ y \\ z \end{Vmatrix}$$

Therefore, the equation can be rewritten in matrix form

$$v = \begin{Vmatrix} v_x \\ v_y \\ v_z \end{Vmatrix} = \begin{Vmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Vmatrix} + \begin{Vmatrix} 0 & \omega -_z & -\omega_y \\ \omega_z & 0 & -\omega_x \\ \omega_y & \omega_x & 0 \end{Vmatrix} * \begin{Vmatrix} x \\ y \\ z \end{Vmatrix}$$

The coordinates of a point in the moving and stationary coordinate systems are related by a matrix dependence

$$\left\|\begin{matrix} x \\ y \\ z \end{matrix}\right\| = \left\|\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\| \left\|\begin{matrix} \zeta \\ \eta \\ \zeta \end{matrix}\right\|,$$

which characterizes the transformation of coordinates, i.e. the transition from the coordinates of a point in a fixed coordinate system to the coordinates $0\xi\eta\zeta$ of the same moving coordinate system OXYZ. Matrix A with element $a_{ij}(i,j = 1,2,3)$ characterizes exactly this transition.

In the reverse transition of coordinates from points in the OXYZ system to coordinates in the $0\xi\eta\zeta$ system, the matrix $C=A^T$ transposed with respect to A is used.

The corresponding transformation of coordinates is expressed by the form dependence.

$$r = Cr'$$

To establish a relationship between the direction cosines and the angular velocities $\omega_x, \omega_y, \omega_z$ with which the moving coordinate system rotates relative to the stationary coordinate system, we differentiate with respect to the time of the expression

$$\dot{r} = C\dot{r}' + \dot{C}r'$$

Let's multiply both parts by the equality above by the matrix A and taking into account that

$$AC = E$$

where E is the unit matrix, rewrite the equation in the form

$$\dot{A} = \dot{r}' + A\dot{C}r'$$

In the equation, the left part of $A\dot{r}$ represents the absolutely linear velocity of the point v in the moving coordinate system, following from this equation the equivalents and then it follows that

$$A\dot{C} = [\omega]$$

Or

$$\dot{C} = -[\omega]A$$

The equations are well known in the theory of inertial navigation as Poisson's matrix differential equation, which relates the derivative of the matrix to the matrix itself and the angular velocity vector ω with which the OXYZ system rotates relative to the non-moving $\text{O}\xi\eta\zeta$.

Thus, if we have information about the projection of the absolute angular velocity vector on the axis of the moving coordinate system OXYZ in the form of $\omega_x, \omega_y,\ \omega_z$, then the direction cosines can be calculated in relation to the non-moving coordinate system $\text{O}\xi\eta\zeta$ by integrating the matrix Poisson's equation

$$\begin{Vmatrix} \dot{c_{11}} & \dot{c_{12}} & \dot{c_{13}} \\ \dot{c_{21}} & \dot{c_{22}} & \dot{c_{23}} \\ \dot{c_{31}} & \dot{c_{32}} & \dot{c_{33}} \end{Vmatrix} = \begin{Vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{Vmatrix} \begin{Vmatrix} 0 & \omega -_z & -\omega_y \\ \omega_z & 0 & -\omega_x \\ \omega_y & \omega_x & 0 \end{Vmatrix}$$

A matrix equation is equivalent to nine first-order differential equations

The scalar form of Poisson's equations shows that the population is divided into three separately integrated systems of three equations each.

$$\dot{c}_{11} = c_{12}\omega_z - c_{13}\omega_y, \qquad \dot{c}_{12} = c_{13}\omega_x - c_{11}\omega_z, \qquad \dot{c}_{13} = c_{11}\omega_y - c_{12}\omega_x$$

$$\dot{c}_{21} = c_{12}\omega_z - c_{13}\omega_y, \qquad \dot{c}_{12} = c_{23}\omega_x - c_{21}\omega_z, \qquad \dot{c}_{23} = c_{21}\omega_y - c_{22}\omega_x$$

$$\dot{c}_{31} = c_{32}\omega_z - c_{33}\omega_y, \qquad \dot{c}_{32} = c_{13}\omega_x - c_{11}\omega_z, \qquad \dot{c}_{13} = c_{31}\omega_y - c_{32}\omega_x$$

The scalar form of Poisson's equations shows that the totality is divided into three separate integrated parts with three parts of the equation each part. The first triad has $c_{11}\ c_{12}c_{13}$, the second - $c_{21}\ c_{22}\ c_{23}$, the third - $c_{31}\ c_{32}\ c_{33}$.

It should be noted that the vector $\omega = \|\omega_x\ \omega_y\ \omega_z\|$ and the corresponding skew-symmetric matrix $[\omega]$ in mathematics are called dual objects that are connected by Levi-Civita means.

$$[\omega_{ik}] = -\varepsilon_{ikl}\omega_l$$

If we put a skew-symmetric matrix in accordance with the angular velocity vector, the given projection into the coordinate system $\text{O}\xi\eta\zeta$.

A key parameter for the dynamic observation system described in the previous section is the observer gain L. The Kalman filter and the extended Kalman filter, which will be described in the remainder of this section, are standard methods for choosing the gain L. If the process and measurements are linear, and the process noise and measurement noise are white Gaussian noise with zero mean and known covariance matrices, then the Kalman filter gives the optimal gain, while the optimality criterion will be defined later in this chapter. There are several forms of the Kalman filter, but for the MBLA, a continuous-pass Kalman filter with discrete dimensions is used. We will assume that the (linear) dynamics of the system is described by equations.

$$x = Ax \dot{+} BU + \xi$$

$$y[n] = Cx[n] + \eta[n]$$

where y[n] = y(tn) is the nth sample of y, x[n] = x(tn) is the nth sample of x, and [n] is the measurement noise at the time tn, o is a random Gaussian noise with zero mean and covariance matrix Q, and z[n] is a random variable with zero mean and covariance matrix R. Random noise o is called process noise and represents modeling error and system disturbance. The random variable z is called measurement noise and represents sensor noise. The covariance R can be easily estimated from the sensor calibration results, but the covariance Q is generally unknown and therefore becomes a system parameter that can be tuned to improve the performance of the observer. Note that the sampling rate does not need to be fixed, the discrete-continuous Kalman filter has the form.

$$\hat{x} = \dot{A}\hat{x} + Bu$$

$$\hat{x}^{+} = \hat{x}^{-} + L(y(t_n) - C\hat{x}^{-})$$

Let's define the estimation error as $\tilde{x} = x - \hat{x}$. The covariance of the measurement error at time t is given by the expression

$$P(t) = E\{\tilde{x}(t)\tilde{x}(t)^T\}$$

Note that P(t) is a symmetric and positive semidefinite matrix, so its eigenvalues are real and nonnegative. In addition, small eigenvalues of P(t) imply a small variance, which implies a low mean estimation error. Therefore, it is necessary to choose such L(t) to minimize the eigenvalues of P(t). Let's remember that

$$tr(P) = \sum_{i=1}^{n} \lambda_i$$

where tr(P) is the trace of P, and li are the eigenvalues of P. Thus, the minimization of tr(P) leads to the minimization of the covariance of the estimation errors. The Kalman filter is obtained by finding such L that would minimize tr(P)

In the space between the dimensions Differentiating $\tilde{x}$, we get

$$\dot{\tilde{x}} = \dot{x} - \hat{x} = Ax + Bu + \xi - A\hat{x} - Bu = A\tilde{x} + \xi$$

Solving the differential equation with the initial conditions $\tilde{x}_0$, we get

$$\tilde{x}(t) = e^{At}\tilde{x}_0 + \int_0^r e^{A(t-\tau)}\,\xi(\tau)d\tau$$

One can calculate the evolution of the error covariance P as

$$\dot{P} = \frac{d}{dt}\{\tilde{x}\tilde{x}^T\} = E\{\dot{x}\tilde{x}^T + \tilde{x}\dot{\tilde{x}}^T\} = E\{A\tilde{x}\tilde{x}^T + \xi\tilde{x}^T + \tilde{x}\tilde{x}A^T + \tilde{x}\xi^T\} = AP + PA^T + E\{\xi\tilde{x}^T\} + E\{\xi\tilde{x}^T\}$$

It is also possible to calculate $E\{\xi\tilde{x}^T\}$ as

$$E\{\xi\tilde{x}^T\} = Ee^{A(t)}\tilde{x}_0\xi^T(T)\} + \int_0^t e^{A(t-\tau)}\,\xi(\tau)\xi^T(\tau)d\tau$$

$$= \int_0^t e^{A(t-\tau)}Q\delta(t-\tau)d\tau = \frac{1}{2}Q$$

where 1/2 is due to using half the area inside the delta function. And since Q is symmetric, then P evolves in the space between dimensions as

$$\dot{P} = AP + PA^T + Q$$

During the measurement we have

$$\tilde{x}^+ = x - \hat{x}^+ = x - \tilde{x}^- L(Cx + \eta - C\hat{x}^-) = \hat{x}^- - LC\tilde{x} - Ln$$

Since η and x̂⁻- are independent, then

$$E\{\tilde{x}^{-}\eta^{T}\eta^{L}\} = E\{L\eta\tilde{x}\} = 0$$

The following relations between matrices will be needed in the further derivation

$$\frac{\partial}{\partial A}tr(BAD) = B^{T}D^{T}$$

$$\frac{\partial}{\partial A}tr(ABA) = 2AB\ If\ B = B^{T}$$

Our goal is to choose such L that would minimize tr(P+). A necessary condition for this has the form

$$\frac{\partial}{\partial A}tr(P^{+}) = -P^{-} - C^{T} - P^{-} - C^{T} + 2LCP^{-}C^{T} + 2LR = 0$$

$$2L(R + CP^{-}C^{T}) = 2P^{-}C^{T}$$

$$L = P^{-}C^{T}(R + CP^{-})^{-1}$$

Substitute this into the equation and we have

$$P^{+} = P^{-} + P^{-}C^{T}(R + CP^{-}C^{T})^{-1}CP^{-} - P^{-}C^{T}(R + CP^{-}C^{T})^{-1}CP^{-} + P^{-}C^{T}(R + CP^{-}C^{T})^{-1}(CP^{-}C^{T} + R)(R + CP^{-}C^{T})^{-1}CP^{-} = P^{-} - P^{-}(R + CP^{-}C^{T}) = (I - P^{-}C^{T}(R + CP^{-}C^{T})^{-1}C)P^{-} = (I - LC)P^{-}$$

Now we can briefly describe the Kalman filter as follows. In the space between the dimensions, the equations are propagated.

$$\dot{\hat{x}} = A\hat{x} + Bu$$

$$\dot{P} = AP + PA^{T} + Q$$

where $\hat{x}$ is the state estimate, and P is the symmetric covariance matrix of the estimation errors. When a measurement is received from the ith sensor, the state estimate and error covariance are updated according to Eqs

$$L_i = P^- C^T{}_i (R_i + C_i P^- C^T \quad)^{-1}$$

$$P^+ = (I - L_i C_i) P^-$$

$$\hat{x}^+ = \hat{x}^- + L_i (y_i(t_n) - C_i \hat{x}^-)$$

where L_i is called the Kalman amplification factor of the i-th sensor. Assume that the system propagation model and the measurement model are linear. However, for many applications, which will be described later in this section, the system propagation model and the measurement model are nonlinear. In other words, the model presented in (8.19) takes the for

$$\dot{x} = f(x, u) + o$$

$$y[n] = h(x[n], y[n]) + 3[n]$$

Quaternions are mathematical entities that extend the concept of complex numbers. In the context of spatial rotation, quaternions offer an efficient and concise representation of rotation in three-dimensional space. Unlike other representations such as Euler angles, quaternions avoid problems such as gimbal locking and provide an easy way to interpolate between different rotations. In the context of spatial rotations, quaternions have advantages over other representations such as Euler angles because they avoid problems such as gimbal locking and provide a compact way to interpolate between different orientations.

The general form of the quaternion is:

$$q = a + bi + cj + dk$$

where a, b, c, d are real numbers and i,j,k are units of quaternions. In the context of quaternion rotations, unitary quaternions are often used for simplicity and efficiency. The unit quaternion representing the rotation has the form:

$$q = \cos\left(\frac{\theta}{2}\right) + \sin(\frac{\theta}{2})(xi + yj + zk)$$

the angle of rotation θ, and x,y,z are the components of the axis of rotation.

Unit quaternions: Unit quaternions are often used to ensure numerical stability and avoid problems associated with quaternion multiplication. Unit quaternions have magnitude 1 and are suitable for representing rotation. Multiplication of quaternions is defined as the multiplication of individual components by units of quaternions i,j,k. The product of two quaternions

$$p_0 + p_1 i + p_2 j + p_3 k \text{ i } q = q_0 + q_1 i + q_2 j + q_3 k$$

is given by:

$$pq = (p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3) + (p_0 q_1 + p_1 q_0 + p_2 q_3 - p_3 q_2)i + (p_0 q_2 - p_1 q_3 + p_2 q_0 + p_3 q_1)j + (p_0 q_3 - p_1 q_2 + p_2 q_1 + p_3 q_0)k$$

The rotation of the vector (x,y,z) by the quaternion q can be expressed as:

$$v' = qvq^{-1}$$

where v is the quaternion representation of a vector, and q^{-1} is the inverse of q

Quaternions provide a natural way to interpolate between two rotations. Spherical linear interpolation (slerp) smoothly transitions from one rotation to another, avoiding the singularities associated with other rotation representations. Given two unit quaternions $q_0\ and$ representing the initial and final orientations, as well as the parameter t in the range [0.1], the SLERP formula is expressed as:

$$(\,q_0, q_1, t) = \frac{\sin((1-t)\theta)}{\sin(\theta)}\ q_0 + \frac{\sin(t\theta)}{\sin(\theta)}\ q_1$$

where θ is the angle between two quaternions.

The angle θ between two quaternions q_0 and q_1 can be determined using the dot product:

$$\cos(\theta) = q_0 * q_1$$

SLERP provides a smooth and seamless transition from initial to final rotation, providing visually appealing results. Unlike some other interpolation methods, SLERP avoids gimbal lock, a situation where certain rotations become ambiguous.

The interpolation procedure $q_0 * q_1$ uses SLERP. Calculate the dot product to find cos(θ). Determine the angle θ using the inverse cosine function. Apply the SLERP formula for different values of t to obtain the intermediate quaternions representing the interpolated rotations. SLERP is commonly used in animation, computer graphics, and robotics to smoothly transition between different orientations. It is especially valuable in scenarios where smooth and visually pleasing rotations are required. The concept of quaternion interpolation extends not only to two quaternions. Squad (Spherical Quadrangle) interpolation is an extension of SLERP that allows interpolation along a more complex trajectory using a sequence of quaternions. Quaternions have several advantages. They provide a compact representation of 3D rotations using only four parameters (real and imaginary components), compared to other representations such as Euler angles, which may require three angles. Such compactness simplifies the storage, calculation and transfer of rotation information. Locking of the cardan joint is a phenomenon when the axes of rotation are aligned, which leads to the loss of one degree of freedom and, as a result, to ambiguous rotations. Quaternions inherently avoid gimbal locking, making them preferred for applications where continuous and unlimited rotation is essential. Quaternions facilitate efficient interpolation between two rotations, especially with methods such as spherical linear interpolation (SLERP). Interpolation is critical in animation, modeling, and robotics to achieve smooth and visually pleasing transitions between orientations. Unit quaternions (quaternions with magnitude 1) are often used to represent rotation. Their normalization ensures numerical stability during calculations, reducing the probability of numerical errors and providing reliable results. Unlike Euler's angles, quaternions do not suffer from the problem of repeated calculations when rotating objects multiple times. Each rotation of the quaternion is independent of previous rotations, which simplifies the application of successive rotations. Quaternions are widely used in various fields, including computer graphics, robotics, aerospace, and modeling. Their versatility makes them suitable for a wide range of applications where efficient and accurate rotation representation is critical. In computer graphics, quaternions simplify tasks such as camera orientation, object manipulation, and animation. They easily interact with graphics engines, providing an efficient way to handle rotations in three-dimensional space. Quaternions, especially with techniques such as SLERP, facilitate smooth animation transitions between different orientations. This is very important in applications where visually appealing and smooth rotations are required. Thus, quaternions offer a robust and versatile solution for rotation representations, solving common problems associated with other rotation representations. Their compactness, avoidance of gimbal blocking, and interpolation efficiency make quaternions well suited for a wide range of applications.

The proposed algorithm of BINS functioning was implemented in the MATLAB computing system. The movement of the UAV modeled using this program is presented in Fig. 3.1 - 3.4

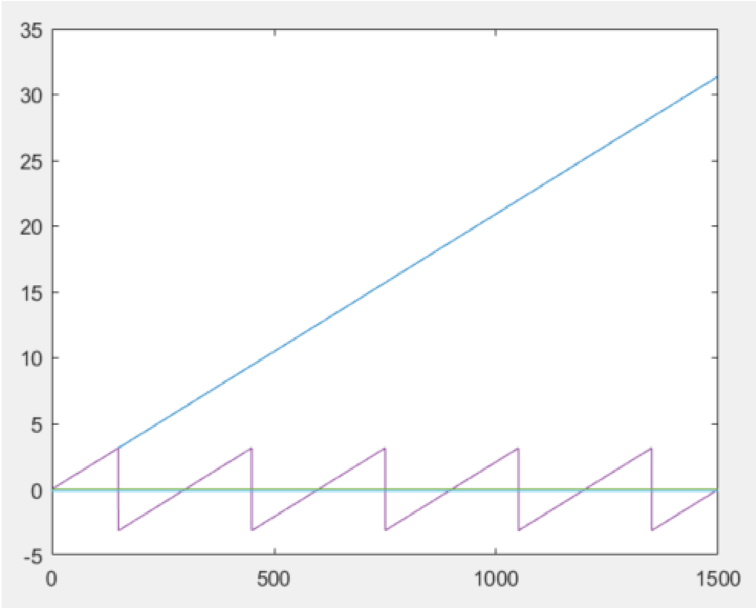


Fig. 3.1 Course of the given trajectory and estimated course (radians)

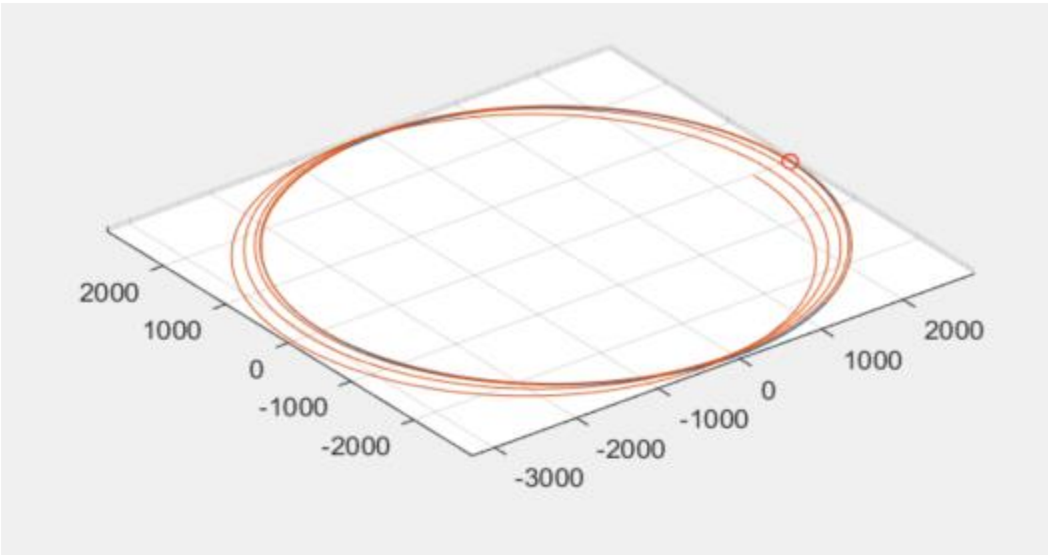


Fig. 3.2 Movement along a given trajectory with minor obstacles without correction (meters)

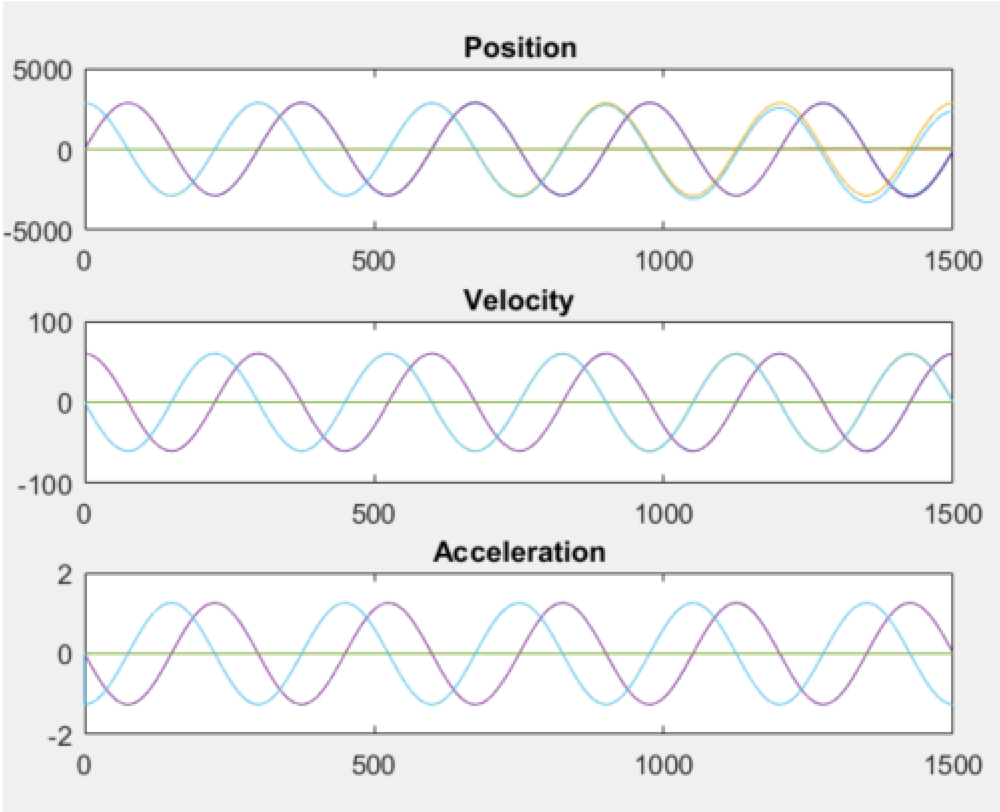


Fig. 3.3 Navigation parameters

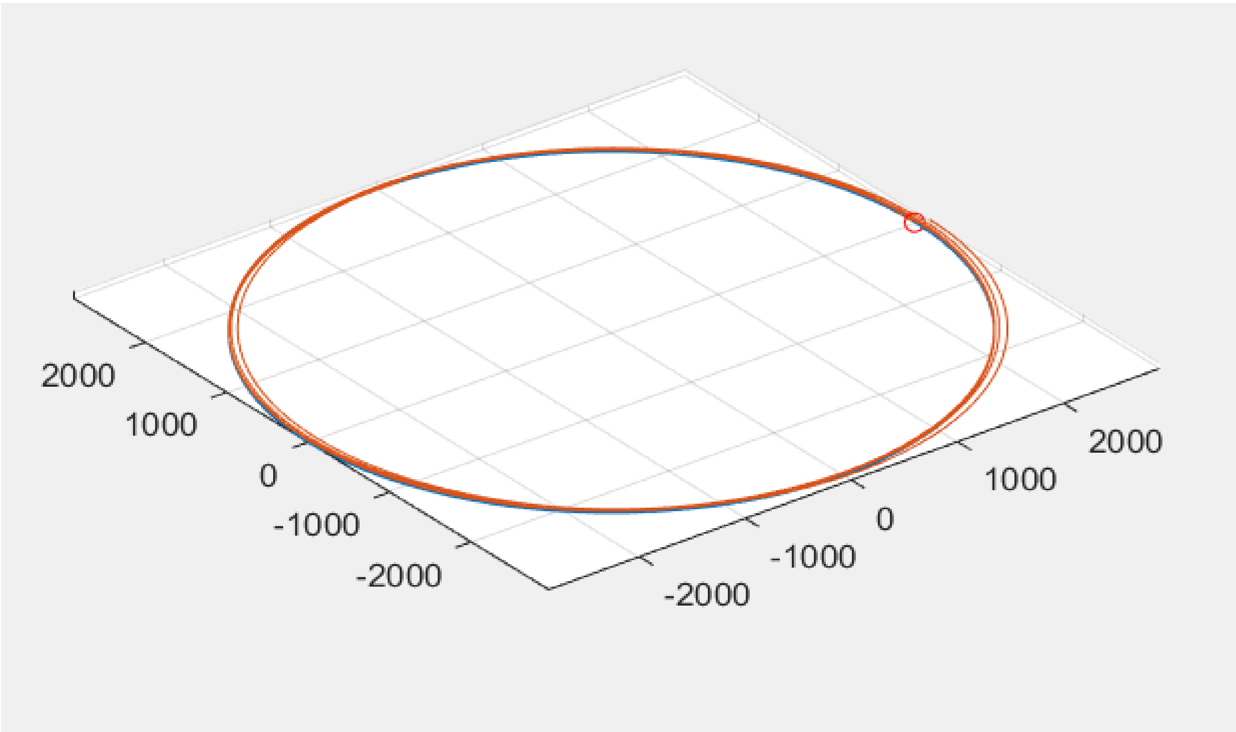


Fig. 3.4 Trajectory in the presence of obstacles with correction (meters)

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